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Form-finding approach for flexibly-formed concrete elements

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b

d

P

p	is the hydrostatic pressure
T	is the tensile force in fabric
R	is the radius of curvature of a fabric-formed profile
θ	is the angle between two successive straight lines of a fabric-formed profile
Δl	is the length of the straight line step of a fabric-formed profile
ρ	is the density of wet concrete
g	is the gravitational acceleration
z	is the hydrostatic height
c	is constant defining a fabric-formed profile
x	is the horizontal ordinate
y	is the vertical ordinate

is the breadth of a fabric-formed cross section

is the depth of a fabric-formed cross section

is the depth of a fabric-formed cross section

Abstract:

There is no reason why concrete elements should be prismatic. Concrete is mouldable and can be cast in efficient forms, which follow the stresses varying along the length of a concrete element. One option to achieve this is to use fabric as flexible formwork. Fabric deforms under the hydrostatic pressure exerted by wet concrete during construction creating the shape of hardened concrete. The final shape needs to be known in advance in order to be able to perform stress analysis and design of structural elements. This paper presents a form-finding approach capable of predicting the shape in cross section of flexibly-formed concrete elements. The approach provides acceptable in practice results when compared with an analytical solution, particularly when applied to complex shapes. An experimental investigation has been undertaken to compare predicted and as-built shapes. The influence of construction tolerances, practical aspects and limitations of the approach are also discussed.

1. Introduction

Casting concrete in flexible textile sheets, known as fabric formwork, is a novel and yet well-investigated construction method, which offers unique opportunities in terms of freedom of form and material expression. One of the main advantages of fabric formwork is the possibility to shape concrete elements appropriately in order to use the concrete material more efficiently.

The method has been successfully applied to create a wide range of concrete forms varying from sculptural works to structurally efficient elements (Chandler and Pedreschi, 2007). A detailed historical overview of the development of fabric formwork methods can be found in Veenendaal et al. (2011a). Numerous realistic scale examples of beams, slabs, columns, walls and shells have been built and described in detail by one of the pioneers in fabric formwork construction Professor Mark West (West, 2016), while the Japanese architect Kenzo Unno is well known for his 'zero-waste' formwork system utilising fabric formwork for construction of aesthetical concrete walls (Umi Architectural Atelier). Fabric formwork systems for foundations and circular columns are commercially available in Canada and the USA via company Fab-Form Industries (Veenendaal et al., 2011a). Hybrid cable net/fabric systems have also been investigated for the opportunity to build long-span shell structures (Veenendaal and Block, 2014).

A large range of synthetic textiles, including polyester, polyamide, polypropylene and polyethylene fibres, have been found suitable for fabric formwork construction. They are typically applied as reusable flat sheets but may be cut in patterns and sewn. Being considerably lighter than traditional timber or steel formwork systems, fabric formwork can facilitates transportation and lifting processes. The porosity of fabric materials has proven to be another advantage of the construction method due to the loss of excessive surface water during construction providing enhanced durability of concrete (Orr, 2013; Lee, 2012; Ioannou et al., 2016).

Despite all of the potential advantages, the design of flexibly-formed structural elements presents a major challenge to the industrial application of fabric formwork. The flexible textile sheets deform under the hydrostatic pressure of wet concrete and require additional effort for predicting the hardened concrete shapes. An optimisation process must also be integrated to ensure that the final form will be structurally efficient.

The shape assumed by fabric during construction may be related to that of pre-tensioned membranes structures. However, Veenedaal et al. (2011b) defined a number of differences, such as the influence of 'pinch' points and the allowance for wrinkling not typical for minimal surface form-finding of pretensioned membranes, which require further development to adapt available methods. Veenendaal (2008) combined evolutionary structural optimisation algorithms with dynamic relaxation methods and structural analysis of fabric-formed beams in ANSYS. Modelling the wrinkling behaviour of fabric was achieved by Veenendaal and Block (2012) through the development of a design tool, based on a combination of the natural force density method and an elastic stiffness matrix method, which was written in Python and implemented as a toolbar in Rhino. Van Mele and Block (2010) proposed an approach for the design of thin flexibly-formed anticlastic concrete shells, based on the force density method finding an equilibrium surface closest to a given target under specified loads, and calculating the required prestress in fabric formwork. The form-finding methodology developed by Tysmans et al. (2011) involved dynamic relaxation with kinetic damping and structural analysis using the finite element software Abaqus for the design of anticlastic shell shapes made of a fire-safe textile reinforced cement composite. Bak et al. (2012) applied a detecting collision approach to 'drape' fabric over an optimised concrete shape determined using topology optimisation, based on the BESO algorithm. Meshless techniques, which model the physical structure of concrete by randomly

positioned particles connected by bars, have also been considered as a potentially powerful tool for numerical modelling of fabric-formed concrete shapes (Williams, 2012).

All described methods and techniques are particularly effective for studying the three-dimensional nature of fabric-formed concrete shapes and are imperative for the design of three-dimensional structures such as shells. The presented work offers a simplified form-finding approach for linear structural elements, typically designed by cross-sectional analysis, such as the fabric-formed beam shown in *Figure 1*. Avoiding the need for use of advanced software and knowledge may present a significant advantage for these type of elements and make fabric formwork construction more accessible to engineers and architects.



Figure 1 Flexibly-formed beam cast in hung from fixed edge supports

2. Research significance

The proposed form-finding approach herein uses a numerical procedure in order to provide fast and reliable results, which are compatible with conventional cross-sectional analysis of reinforced concrete elements such as beams or columns, and have been verified through experiments.

The approach is used to find the concrete profile of a cross section for specified construction parameters and can be implemented as a short algorithm within the capacity calculation for any cross section along the length of a flexibly-formed element. This gives the opportunity to perform a relatively simple structural optimisation process by dividing an element into a sufficient number of sections. The final output of the process are the construction parameters required to achieve the desired shape.

Therefore, the major significance of this research is that it allows the design of flexibly-formed concrete elements for material efficiency using conventional design methods. Although it is recognised that a two-dimensional form-finding method cannot provide a completely accurate representation of the three-dimensional form, three-dimensional visualisations may also be obtained by creating surfaces defined by every two adjacent cross sections in order to assess the aesthetics and elegance of the hardened concrete forms.

3. Previous studies related to the proposed approach

Empirical studies on flexibly-formed beams cast in fabrics hung between two fixed edges involving an experimental investigation were carried out at the University of Bath, and aimed to establish a relationship between the cross-sectional breadth, depth and perimeter (Bailiss, 2006; Garbett, 2010). For this type of construction the breadth is fixed by the distance between the edges, while the depth is dependent on the perimeter of the hung fabric at each cross section of the beam. Knowing the relationship amongst these three parameters allowed prediction and control of the depth of cross sections. However, the actual concrete profiles could not be determined. Furthermore, it was not possible to take into account the specific material properties of the concrete and fabric to find the tensile force in the fabric, which may well be an important factor in the choice of an appropriate fabric material.

A more analytical approach was required to fully describe the shape of a flexibly-formed section. losilevski (2010) derived a closed form analytical solution for finding the shape of an open two-dimensional soft container filled with liquid. Due to the fluidity of wet concrete, the problem is very similar to form-finding a flexibly-formed cross section. However, the solution comprised incomplete elliptical integrals, which would be impractical for design purposes. Also it considered only a single case of external constraints where the fabric is held at its ends and allowed to deflect along its full perimeter. As shown later in this paper, this would not always result in the most efficient structural shapes.

Foster and Ibell (2016) proposed a numerical solution, implemented in an EXCEL spreadsheet, which produced results closely matching the outputs provided by Iosilevski (2010). The numerical procedure was also capable of solving part-full or over-full cases. Although not entirely useful on its own, this feature allowed for further development in order to solve more complex shapes. A detailed description of the procedure is presented in the next section.

4. Theoretical approach

4.1. Basis of the approach

The present work is based on the numerical solution proposed by Foster and Ibell (2016) and follows a simple procedure where the perimeter of a flexibly-formed plane vertical section is represented by a large number of straight lines. In a freely hung fabric, all resulting sections will be symmetrical and, therefore, only half of the section is considered (refer to *Figure 2*).

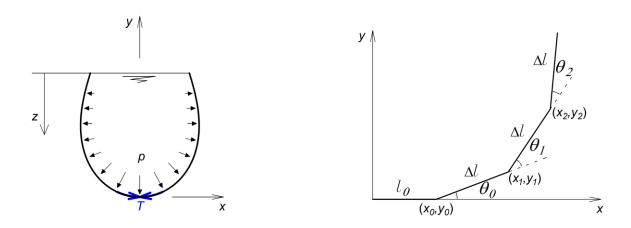


Figure 2 Flexibly-formed section perimeter represented by straight lines

The fabric is assumed to be inextensible and is subjected to a hydrostatic pressure, p. The tensile force in the fabric, T, can then be expressed as a function of the hydrostatic pressure, p, and the radius of curvature, R:

$$T = pR$$
 Equation 1

If θ_i is the angle between two successive straight lines and Δl_i is the length of the second straight line, the expression can be written at any point as:

$$p_i R_i = rac{
ho g z_i \Delta l_i}{ heta_i}$$
 , Equation 2

where the hydrostatic pressure is expressed as a function of the density of wet concrete, ρ , the gravitational acceleration, g, and the hydrostatic height, z_i at the point.

In his formulation of the closed form analytical solution, losilevski (2010) proved that the tensile force in the fabric, T, is constant along the full length of the curved perimeter. Therefore, for equal lengths Δl , all constant values can be substituted with a single constant c:

$$\frac{\rho g \Delta l}{T} = c$$
 Equation 3

Then the angle θ_i can be expressed as a function of only z_i :

$$\theta_i = cz_i$$
 Equation 4

The initial angle θ_I is calculated assuming an initial straight line, so the coordinates of each straight line can be found from trigonometry:

$$[x_{i+1}, y_{i+1}] = [x_i + \Delta l \cos \sum_{j=0}^{i} (\theta_j), y_i + \Delta l \sin \sum_{j=0}^{i} (\theta_j)]$$
 Equation 5

The procedure, as proposed by Foster and Ibell (2016), uses a fixed number of straight lines or a fixed length of a test curve, which is adjusted to provide the profile of a section with predefined breadth and depth. As the coordinates of the top corner of the flexibly-formed section are known, the solution relies on finding a point along the test curve closest to the top corner. This is achieved by applying a standard goal seek algorithm, which minimises the least square of the coordinate differences through iteration of the constant c.

The procedure allows for form-finding shapes of sections in a part-full or over-full fabric. In the first case the maximum hydrostatic height is less than the depth of the hanging fabric, which practically represents any stage during concreting. The over-full scenario occurs when the breadth of a section is fixed over a part of the depth at the top i.e. the maximum hydrostatic height is greater than the depth of fabric allowed to deform freely. The hydrostatic height is defined independently of the freely hung fabric depth during the procedure and, therefore, a full range of theoretical shapes can be obtained. In the extreme case when the fixed top breadth approaches zero and the maximum hydrostatic height tends to infinity, the unrestrained fabric will be subjected to a uniform normal pressure and deform in a circular shape.

4.2. Main observations from the existing procedure

There are two main observations that can be made from testing the existing procedure. Firstly, the success of the solution depends highly on the initial choice of parameters, including the length of the

test curve and the constant c. The length of the test curve is equal to the number of lines multiplied by the length of each line and should be greater than half of the perimeter, which is unknown in advance. Figure 3a compares different initial guesses of c for cross sections where the maximum hydrostatic height z is equal to the depth of the cross section d. As can be seen, beyond a critical value of c the curve goes into a loop, making the goal seek algorithm slow and difficult to converge to a solution. Therefore, a good initial guess of c is very important. Figure 3b demonstrates that the shape is also dependent on the maximum hydrostatic height c, in the case of part-full (c) or over-full scenarios (c), and the test curve could go into a loop for the same value of c when c varies.

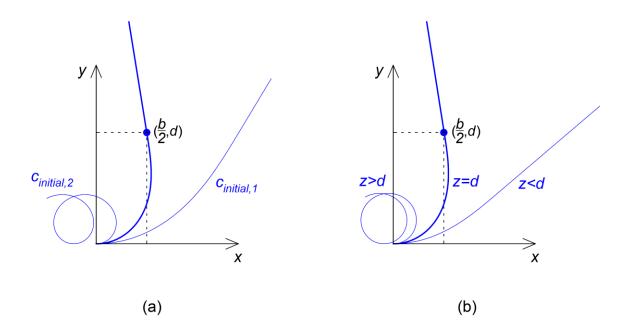


Figure 3 Sensitivity analysis of test curve shape: (a) c varies, z=d (b) z varies for same values of c and d

The second observation is illustrated in *Figure 4* and concerns the material efficiency of the predicted cross-sectional shapes. The profiles of sections with different breadth-to-depth (b/d) ratios for z=d, plotted in *Figure 4a*, demonstrate that for lower ratios the bulging of the cross section becomes more pronounced. *Figure 4b* indicates that the area of the bulb can even become larger than that of a rectangular section with the same breadth and depth. However, the additional area is located near the centroid of the section and, therefore, would not contribute to the bending capacity of an element, leading to a less efficient design in the case of flexural elements. It is possible to reduce the redundant area through pinching of fabric by means of formwork ties or external web formers (Garbett et al., 2010).

The percentage values presented in *Figure 4b* are the potential material savings that could be achieved in comparison with a rectangular section, based purely on the reduced area of cross sections formed in restrained fabric. The current procedure is limited to form-finding of the lowest bulb of the restrained cross sections in *Figure 4b*. However, as can be seen there is a considerable advantage in applying various restraints to control the shape of fabric during construction, which highlights the need for developing capability to predict such shapes.

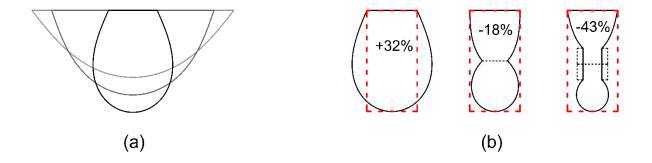


Figure 4: Concrete profiles: (a) single-bulb section for different b/d ratios (b) sections formed in restrained fabric compared with a rectangular section

4.3. New development

The approach proposed by Foster and Ibell (2016) has been further developed based on the following objectives:

- To improve the algorithm by solving current convergence problems related to the initial guess of parameters
- To allow for more boundary conditions in order to increase structural efficiency

A control statement "while $(y_i \le d)$ and $(y_i > y_{i+1})$ " for the vertical ordinate, y, has been introduced to eliminate the possibility for looping of the test curve for the single bulb solution, as indicated in *Figure 5b*. This has also allowed speeding up of the algorithm by minimising the error only for ordinate x_n . The initial angle is more accurately determined from symmetry for sections cast in a freely hung fabric, as illustrated in *Figure 5c*. The proposed changes have been implemented in a short program written in MATLAB, which allows for a fast simultaneous solution of multiple sections. Furthermore, the final curves can easily be reproduced from the obtained constant c and there is no need to store the coordinate data for each section.

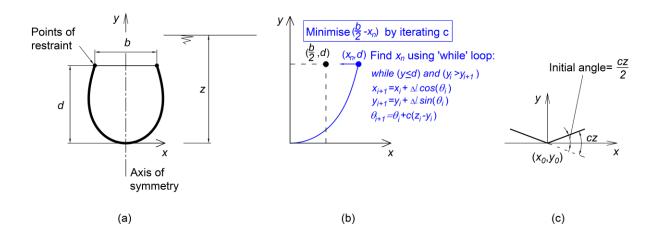


Figure 5 Single-bulb solution: (a) final cross section (b) form-finding algorithm (c) initial angle

In the experimental part of this research, internal restraint methods have been investigated to fully utilise the benefit of freely hung fabric formwork. As shown in *Figure 6*, discrete or continuous dents can be achieved by tying the sides of the fabric using plastic wire or standard formwork ties in plastic tubes. Continuous web formers may also be attached without the need for external supports. More intricate patterns can be sought for architectural expression.



Figure 6 Internal restraints in fabric formwork using discrete ties or continuous web formers

The major challenge in the form-finding of sections created by restraining fabric formwork during construction is to determine the initial angle of upper bulbs. *Figure* 7 illustrates a two-bulb section where the length and position of the horizontal tie is known. The initial angle of the lower bulb curve is expressed by the relationship in *Figure* 5c, while the initial angle of the upper bulb depends on the final

angle of the lower curve. Therefore, the solution for any section with multiple bulbs needs to start with form-finding of the lowest bulb.

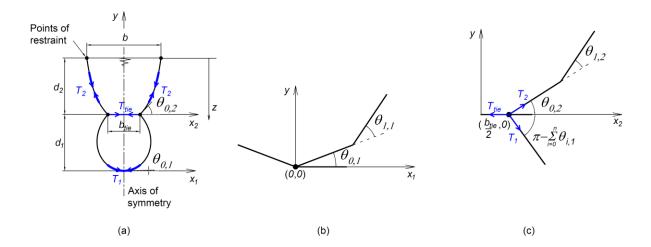


Figure 7 Two-bulb solution: (a) final cross section (b) lower bulb (c) upper bulb

The initial angle of the upper bulb, $\theta_{0,2}$, can be found from equilibrium of vertical forces at the tie point:

$$T_1 sin \left(\pi - \sum_{i=0}^n \theta_{i,1} \right) = rac{
ho g \Delta l}{c_2} sin \left(heta_{0,2}
ight)$$
 , Equation 6

where c_2 is the curve constant for the upper bulb:

$$c_2 = \frac{\rho g \Delta l}{T_2}$$
 Equation 7

The shape of the upper bulb is then form-found by iteration of c_2 . More bulbs can be resolved in a similar manner with each successive bulb depending on the lower one. The algorithm is applicable to sections with web formers, although the final section has also to include the fully restrained web area in this case. Restraints in the vertical direction may also be analysed using equilibrium of forces at tie points. Such restraints may be imposed by flexible reinforcement attached to fabric during construction in order to use the selfweigth of wet concrete to form curved reinforcement profiles following the profile of hardened concrete in the longitudinal direction (Kostova, 2016).

It should be noted that the algorithm is still based on the assumption of inextensible fabric. However, the tensile force in each bulb can be calculated and the extension of fabric determined for known material properties. If not negligible for practical applications, the extension may be subtracted from the

curve perimeter to compensate for the stretching of fabric. This approach may not be suitable for highly extensible fabrics. It is not expected that such fabric materials will be used for construction of structural elements although they may be explored for architectural applications.

5. Experimental investigation and results

The proposed form-finding approach has been verified through physical experiments as part of a comprehensive study on the structural behaviour of flexibly-formed beam elements. Different construction techniques, concrete types and reinforcement materials have been tested to ascertain the predictability and repeatability of results. A major objective of the study has been to further develop the construction method itself, explore the physical possibilities and feed back into the design. The beams have been optimised for the bending moments and shear force effects under specified test loads, based on physical boundaries determined by the experimental construction. A detailed description of all experiments is given elsewhere (Kostova, 2016). The photographs in Figure 1 and Figure 6 show examples of the test specimens and construction. The density of wet plain concrete used to predict the final concrete shapes was taken as 25 kN/m³ in accordance with BS EN 1991-1-1 (2002). Theoretically, the approach allows also for use of lightweight concrete by changing the density of concrete in Equation 3. Design patterns of the predicted shapes were produced for each beam and printed on the fabric, which was then stapled to the edge of the formwork supporting frames along the pattern lines to minimise the chance for errors during construction (see Figure 1). The survey methods for obtaining as-built geometries included laser scanning (Smith, 2012), use of profile gauges and physically sawcutting the beams after being tested to failure.

The chosen fabric was a commercially available polypropylene geotextile, specifically used for casting concrete in marine environments, which had an optimal pore size of 0.25mm, capable of retaining the cement particles while allowing the surface water to flow through it, creating a dense fabric-like texture at the surface of the concrete. The fabric was easily stripped from the hardened concrete without the need for using chemical formwork release agents. The extension of fabric calculated from the manufacturer's strain-stress curve was found to be practically negligible, which was further confirmed by the experiments. Three main findings of the experimental study related to the form-finding approach are discussed in detail below.

5.1. Effect of construction tolerances

For a single-bulb section, the two construction parameters are the opening between top restraints, defining the breadth, and the length of the hung fabric perimeter. Both of them can vary along the length of the element (see *Figure 1*). When a mid-restraint is introduced as illustrated in Figure 7, the length of the tie and its position are also required for construction. The tolerance on any of these dimensions would affect the final depths and hardened cross-sectional shapes along a concrete element, as described earlier.

The results of the as-built survey showed a good accuracy of the theoretical predictions, typically within less than 5 mm deviation from the design concrete profiles in any direction. More significant deviations were recorded for sections where the top breadth of a bulb also deviated considerably from its design value. Re-running the form-finding algorithm with the as-built dimension of the top breadth produced well matching curved profiles in all cases, which indicated the importance of defining an appropriate construction tolerance for the top breadth parameter.

In order to achieve an adequate control of the depth and during construction, it is necessary to understand the effect of variation of the top breadth dimension on the final depth of a fabric-formed cross section. The curves plotted in *Figure 8* represent the relationship between the perimeter and depth of bulb for different breadth values, taken in a practical range of 250 mm to 1000 mm.

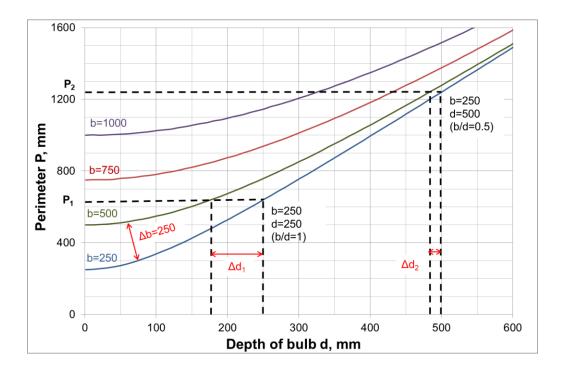


Figure 8 Influence of top breadth increase on depth of a fabric-formed section

Consider two fabric-formed cross sections with the same breadth, for example 250 mm, and depths of 250 mm and 500 mm, respectively. The perimeters of each section, P_I and P_2 , can be obtained from the relevant 'breadth' curve and are assumed to remain constant during construction. If for illustrative purposes the breadth is constructed with an exaggerated tolerance of +250 mm, the depth of the cross sections can be obtained from the 500 mm 'breadth' curve. The differences between the original and constructed depths for both sections, labelled d_I and d_2 , are highlighted in *Figure 8*. As can be seen, the change in the depth does not vary linearly with the change in the top breadth and depends on the breadth-to-depth ratio of a cross section. A further observation indicates that for sections with sufficiently low breadth-to-depth ratios, even a significant change in the top breadth may have a negligible effect on the final depth, while for large breadth-to-depth ratios the final depth would be highly dependent on the constructed breadth. This conclusion agrees with the experimental results. The plots also suggest that it is possible to control the depth by adjusting the fabric perimeter during construction, if required.

Based on the presented analysis, defining absolute permissible construction tolerances may not be feasible for flexibly-formed concrete elements. However, the form-finding algorithm can be used to back-calculate individual maximum and minimum dimensions of the top breath, and any other construction parameter, which correspond to a specified tolerance on the design depth. Producing a full range of depth-perimeter curves may also be used as a practical tool for checking individual section tolerances and determining required perimeter adjustments.

5.2. Effect of fixed edge thickness

For the construction of all test beams, fabrics were hung from the top of horizontal plywood sheets (refer to *Figure 1*) and, therefore, the plywood thickness formed a part of the hydrostatic height. This was considered to be an over-full scenario (i.e. z>d), following the original procedure proposed by Foster and Ibell (2016). However, a deviation from the design concrete profiles was identified for a number of shallow-depth sections, where the top of the bulb curve coincided with the top of the plywood. As illustrated in *Figure 9*, the actual level of side restraint for the bulb depends on the top angle of the curve, which is not known in advance. When the angle with the horizontal is greater than 90°, the fabric pulls away from the edge and the over-full scenario condition is no longer valid. The form-finding algorithm was updated to include a check of the angle and predict the shape based on the correct hydrostatic depth scenario, i.e. z>d or z=d.

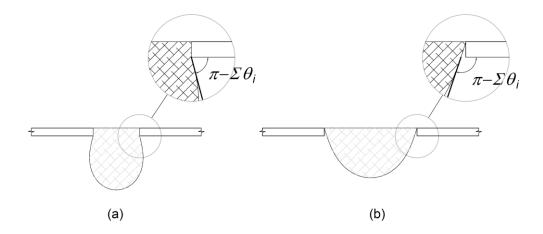


Figure 9 Effect of fixed edge thickness on concrete shape: (a) z>d (b) z=d

5.3. Effect of fabric wrinkling

The test beam specimens were cast in flat unstitched fabrics due to the high reusability of the material, which is considered to be another advantage of fabric formwork construction. It is recognised that the presented cross-sectional form-finding approach cannot be used to prevent wrinkling of flat fabrics but this is not normally an objective for flexibly-formed concrete. On the contrary, controlled wrinkling is often seen as a desired architectural feature. Nevertheless, avoiding excessive wrinkles is still necessary wherever it can affect the structural performance or durability of an element.

Overall, no significant problems were identified during the experimental investigation of structurally optimised beams with both the depth and breadth allowed to vary along the length. The example of excessive wrinkling, presented in *Figure 10*, occurred in a beam specimen purposefully designed with a sudden change in the size and profile of adjacent sections in order to demonstrate the limitation of the current approach. The wrinkle resulted in a total loss of concrete cover locally. Therefore, it was concluded that the proposed approach can be used for casting concrete in flat fabrics, only when a smooth transition between varying cross sections is ensured along the length of an element, which as demonstrated by the experiments would be appropriate for most practical applications

A more detailed analysis of the three-dimensional surfaces formed by connecting all cross-sectional profiles in a beam element may be performed in readily available software for creating flat patterns. Sewn fabrics may ultimately be utilised to achieve greater freedom of form without wrinkling effects.

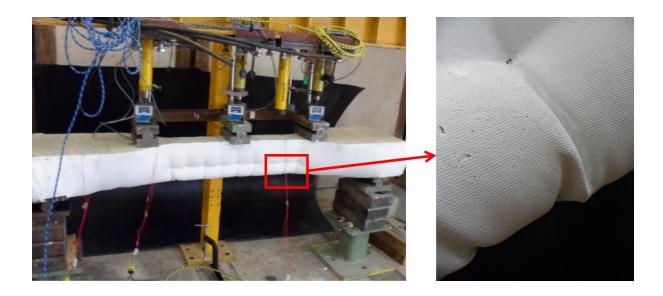


Figure 10 Undesirable fabric wrinkling in tensile reinforcement cover zone

6. Conclusions and future work

The form-finding approach presented in this paper provides a practical solution for determining the shape of flexibly-formed concrete elements, suitable for cross-sectional analysis, such as beams and columns. It has been demonstrated that every curved shape is defined by a single constant parameter, which is sufficient to reproduce the full curve coordinates once the form-finding has been performed. The output data then can be used for design and structural optimisation of the elements in order to fully utilise the flexibility of forms cast in fabrics.

The form-finding results have been verified through an experimental study, which has helped to draw important conclusions on the effect of construction tolerances and how they can be considered in the design and construction of flexibly-formed concrete elements. The form-finding algorithm has also been improved to address unforeseen buildability aspects. Practical guidance has been provided on how to avoid the limitations of using a two-dimensional approach for three-dimensional form-finding.

The structural behaviour of flexibly-formed elements has not been discussed in this paper. However, achieving the correct design depth and position of reinforcement at critical cross sections during construction has proven to be a major factor for ensuring adequate structural performance. Therefore, developing appropriate reinforcement techniques is a key priority for further studies on flexibly-formed concrete. The possible use of fabric formwork as external reinforcement has remained a subject of

future interest, mainly due to the risks associated with exposed reinforcement, but could evolve into an exciting area of research on sustainable 'zero-waste' participating formwork systems.

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