The U-shape of income inequality over the 20th century: The role of education*

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Abstract

We propose an overlapping generations model with three social classes to investigate the effects of higher education on the evolution of inequality. Initially, no social class invests in higher education and inequality is driven by wealth accumulation/bequests. Once the rich surpass a certain income threshold, they invest in higher education and their children's incomes start to grow faster. Over time, the middle class and potentially the poor follow suit. Overall, this framework provides a candidate explanation for i) the U-shaped evolution of income inequality, ii) the fall and rise of inheritance flows, and iii) differential investments in higher education.

Keywords: Evolution of inequality, bequests, differential education, human capital accumulation, growth regime switch.

JEL Classifications: I23, I24, I25, O11, O41.

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I Introduction

The most salient features of the evolution of income inequality and inheritance flows in rich countries are i) comparatively high income inequality and high inheritance flows as a share of total income at the beginning of the 20th Century; ii) a substantial drop of both series after World War II; iii) a rather constant level of both series throughout the 1950s, 1960s, and 1970s; and vi) strongly rising income inequality and inheritance flows thereafter (Atkinson et al., 2011; Piketty and Saez, 2003; Piketty, 2014; Piketty and Zucman, 2015; Alvaredo et al., 2017a). We show this pattern in Figure 1 for the evolution of the top 10% income share (Alvaredo et al., 2017b) and in Figure 2 for the flow of inheritances in terms of GDP (Alvaredo et al., 2017a) for the case of the United States.[1]

Many explanations for the U-shaped evolution of income inequality and inheritances have been proposed: most important are the disruptions of World War II, which had two main effects. First, they reduced wealth because assets were destroyed. Since assets were disproportionately owned by the richer parts of the population, this dampened wealth inequality and inequality in terms of asset incomes. Second, the high costs of the war led to comparatively high taxes on wealth and to financial repression (i.e., a situation in which the rate of return on investing at home is low but capital controls and generally low capital mobility ensure that investors cannot easily move abroad). Substantial inheritance taxes and high marginal income tax rates in the period 1950-1970 therefore exacerbated the drop in inheritance flows and income inequality after World War II. As far as the increase in inheritance flows and income inequality from the late 1980s onwards is concerned, potential explanations include i) decreases in marginal income tax rates and inheritance taxes (particularly in the United States and the United Kingdom), ii) skill-biased technological change, which disproportionately benefited the well-educated, iii) globalization, which put additional pressure on low incomes because low-skilled labor-intensive production has often been outsourced to low-wage countries, and iv) that wealthier households invest a greater fraction of their wealth in assets with

[1]The U-shaped evolution of the top 10% income share and of inheritances is also documented for most other rich countries in Piketty (2014), Piketty and Zucman (2015), and Alvaredo et al. (2017a).

a higher rate of return (see Acemoglu, 2002; Goldin and Katz, 2009; Acemoglu and Autor, 2012; Elsby et al., 2013; Piketty, 2014; Coibion et al., 2017; Kasa and Lei, 2018, for different arguments and explanations regarding the reductions in inequality after World War II and the increase in inequality since the 1980s).

Figure 1: Share of total income of the 10% with the highest incomes in the United States (excluding capital gains).

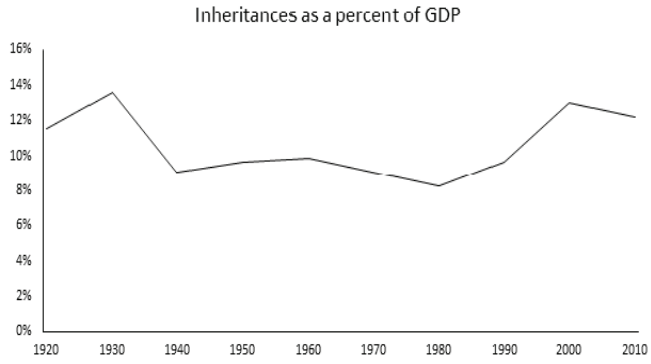

Figure 2: Inheritances as a percent of GDP in the United States under the assumption that gifts evolve as in Germany (Alvaredo et al., 2017a).

We propose a complementary mechanism to explain the joint U-shaped evolution of income inequality and inheritance flows throughout the 20th Century. The central driving

force in this mechanism is the increase in costly higher education after World War II, when the most wealthy groups were the first to invest massively in the college education of their children. This slows down intergenerational wealth accumulation among the rich because household's resources are re-allocated away from bequests (wealth accumulation) toward higher education (human capital accumulation). The fact that a certain amount of time elapses between children's education and their labor market entry implies that inheritance flows and income inequality decrease in the short run. However, the increase in rich children's skills raises their wage income once they enter the labor market and leads to a steeper age-income profile later on in their lives. Thus, after about one generation, income inequality rises again. While income inequality is mainly driven by differential wealth accumulation before the rich start to invest massively in their children's college education, income inequality afterwards originates in differences in wage incomes due to differences in educational attainment. This pattern is consistent with the data on income inequality and inheritances and with the established view on the driving factors of the evolution of income inequality at different points in time (Atkinson et al., 2011; Meschi and Scervini, 2014; Piketty, 2014).

The increase in the importance of higher education is clearly visible in the data and the timing coincides with the timing that our model implies: While in 1940 only 4.6% of the population above the age of 25 had a college degree, 32% did so in the year 2015 (United States Census Bureau, 2015). As far as the joint evolution of income and higher educational attainment between the different income groups is concerned, the predicted pattern of our model is consistent with the data reported by The Pell Institute (2015) as summarized in Table 1. They show that in 1970, 40% of the dependent members of families in the top income quartile had a bachelor's degree by the age of 24. This number almost doubled to 77% in 2013. In the second highest income quartile, 15% of dependent family members had a bachelor's degree in 1970, which more than doubled to 34% in 2013. In the lowest income quartile, however, only 6% of the dependent family members had a bachelor's degree in 1970 and the number barely rose over time to 9% in 2013. The timing of the rising importance of higher education is also reflected in Figure 3, which displays the

increase of the population share holding a bachelor's degree from 1920 to 2010 according to the National Center for Education Statistics (2019). Finally, empirical evidence supports a positive and independent impact of family income on children's educational attainment with the effect becoming stronger over the past decades (for the US, see Clark-Kaufman et al., 2003; Belley and Lochner, 2007; Lochner and Monge-Naranko, 2011).[2]

Table 1: Bachelor's degree attainment by age 24 depending on the family's income quartile (Source: The Pell Institute, 2015)

	1970	2013
Top	40%	77%
Third	15%	34%
Second	11%	17%
Bottom	6%	9%

Note: The data refer to unmarried young adults until 1986 and to dependent young adults from 1987 onwards.

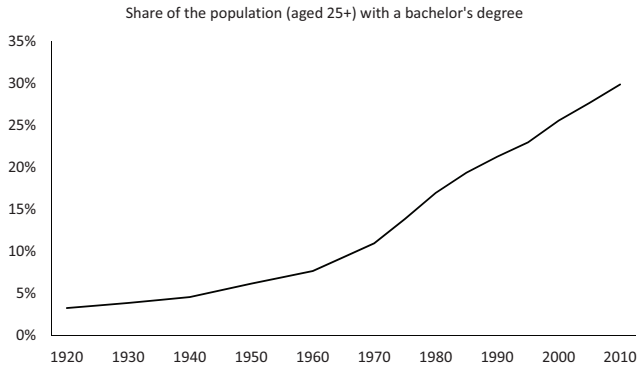

Figure 3: Share of the population above the age of 25 with a bachelor's degree (Source: National Center for Education Statistics, 2019).

We conceptualize our mechanism by assuming the presence of three social classes, the

[2]Blanden et al. (2007) account for over 80% of the rise in the intergenerational correlation coefficient by the stronger relationship between family income and education. Galindo-Rueda and Vignoles (2005) demonstrate that the role of cognitive test scores in determining educational attainment has declined between the 1958 and 1970 cohorts. Furthermore, Acemoglu and Pischke (2001) suggest large effects of family income on enrollment with a 10 percent increase in family income being associated with a 1.4 percentage point increase in the probability of children attending a four-year college. Finally, given the observed increases in education inequality, our model predicts that lifetime income inequality should have been on the rise over the recent decades, which is consistent with recent empirical evidence for Germany and the United States (Bönke et al., 2015; Guvenen et al., 2017).

rich, the middle class, and the poor.[3] A setting with fewer social classes might capture some aspects of our mechanism, i.e., that investments in higher education imply declining bequests and inequality in the short run but increasing inequality later due to accelerated human capital investments of the rich. Such a framework, however, does not capture the U-shaped evolution of wealth inequality. Our mechanism hinges on the fact that population groups differ in their levels of human capital investments because teachers are paid according to their level of human capital. This means that it is comparatively cheaper to educate a child for parents with higher human capital and comparatively more expensive for parents with less human capital. This mechanism is important for the emergence of a U-shaped evolution of wealth inequality because the switch to the high-skill regime amplifies the differences between educational investments and bequests for rich and poor households — to the extent that the poor may not invest in higher education at all. Nevertheless, the remainder of the mechanism survives also with two population groups.

Initially, income inequality is driven by the accumulation of wealth because the rich are able to save more than the other social classes. During this stage of economic development, which we call the *neoclassical regime*, investments in college education are not yet widespread. However, at some point in time, an income threshold is surpassed, above which the rich start to invest in college education of their children. In the aftermath of the regime switch to the *high-skill regime*, income inequality and inheritance flows decline because the rich reduce their savings and therefore the intergenerational accumulation of wealth. Afterwards, income inequality rises again because the better educated children of the rich earn higher labor incomes, are thus able to bequeath more wealth to their own children, and are also able to invest more in their children's education than the less well educated children of the middle class and the poor. Subsequently, incomes of the

[3]While a comprehensive treatment of inequality dynamics should take social mobility into account, such a model could not be solved analytically. Since social mobility is rather low in the United States (Corak, 2013; Chetty et al., 2014) and the education level of parents crucially impacts on the education level of their children (Black et al., 2005; Plug and Wim, 2003), the small sacrifice in realism by abstracting from social mobility is a small price to pay for the benefit of being able to trace the dynamics of income inequality, education, and inheritances over generations analytically. For a model that focuses on social mobility in the context of public versus private schooling see Davies et al. (2005).

middle class and potentially of the poor surpass the thresholds above which investments in their children's higher education becomes a utility-maximizing strategy. Crucial for the emergence of declining income inequality is that all social classes switch to the high-skill regime. As we show, this is by no means guaranteed because the poor face the risk of being disconnected from the growth process. In this case, targeted policy interventions such as stipends and tuition fee waivers for the poor would be an effective instrument to reduce inequality.

Related to our research and to the recent increase in inequality, Böhm et al. (2015) develop a directed technical change model to analyze the extent to which low-skilled workers benefit from trickle-down effects of public education policies that are targeted toward high-skilled workers. They show that such policies raise inequality in the short run, whereas they are beneficial for low-skilled workers in the long run. While Böhm et al. (2015) consider an exogenous distribution of skilled and unskilled households, the switch from the neoclassical regime to the high-skill regime and, thus, the distribution of skills is endogenous in our setting. Finally, Böhm et al. (2015) focus on the phase of increasing inequality, whereas our paper addresses the whole U-shaped pattern in the joint evolution of income inequality and inheritances. For the sake of clarity, however, we abstract from skill-biased technical change because it would merely reinforce the increase in income inequality after the regime switch and therefore lead to a more pronounced U-shaped pattern.[4]

Our paper is also related to Galor and Zeira (1993) who show that initial wealth disparities matter for the long-run distribution of incomes if capital markets are imperfect and education investments are indivisible and to the subsequent literature of Galor and Moav (2004), Galor and Moav (2006), and Galor et al. (2009). In contrast to these papers, we are concerned with the connection between *higher* education in terms of a college degree—which gained importance in the second half of the 20th Century—and

[4]Introducing skill-biased technical change would complicate our model considerably. For the conceptual problems arising from a directed technical change setting with endogenous human capital formation, see Schaefer (2014). Thus, our mechanism should be seen as complementary to the explanations advanced in the literature on skill-biased technical change by Goldin and Katz (2009), Acemoglu and Autor (2012), and Böhm et al. (2015).

the patterns of income inequality after World War II. We therefore explain the phase of increasing inequality throughout the period 1980-2010 and show that, under certain circumstances, inequality might decline again in the future.

The paper is organized as follows: Section II contains the model description, Section III the analytical analysis of the dynamics, and Section IV an illustration of the model solution by means of a numerical example. In Section V, we conclude and describe potential policy measures to reduce income inequality and its negative economic effects.

II The Model

Firms

Consider an economy populated by individuals whose life cycle consists of three distinct phases: childhood, adulthood, and retirement. Time is discrete, indexed by t, and ranges from 0 to ∞. We assume that one time period lasts for approximately 30 years such that short-run changes unfold over a 30 year period, medium-run changes unfold within 90 years (three generations), and long-run changes capture effects that last beyond this time range. A continuum of small firms $i \in (0,1)$ produce aggregate output Y_t with the representative firm having access to a constant returns to scale production function of the Cobb-Douglas type

$$Y_t = AK_t^{\alpha} H_{Y,t}^{1-\alpha}, \tag{1}$$

where K_t is aggregate physical capital, $H_{Y,t}$ is human capital employed in production, $A > 0$ refers to the total factor productivity (TFP), and $\alpha \in (0,1)$ denotes the elasticity of output with respect to physical capital. Perfect competition implies that the wage rate per unit of human capital, w_t, and the capital rental rate, R_t, are given by

$$w_t = (1-\alpha) A k_t^{\alpha}, \qquad \text{and} \qquad R_t = \alpha A k_t^{\alpha-1}, \tag{2}$$

where $k_t = K_t / H_{Y,t}$ denotes physical capital per unit of human capital. We assume that

physical capital depreciates entirely over the course of one generation, i.e., the interest rate is given by $r_t = R_t - 1$. This is a reasonable assumption given that a generation covers a time span of 30 years. Moreover, the interest rate is determined at the integrated world capital market such that $R_t = R$ is constant and the equilibrium physical capital stock per unit of human capital pins down to

$$k_t \equiv k = \left(\frac{\alpha A}{R}\right)^{\frac{1}{1-\alpha}}. \tag{3}$$

This in turn implies that the wage rate per unit of human capital is constant and given by

$$w_t \equiv w = (1-\alpha)A\left(\frac{\alpha A}{R}\right)^{\frac{\alpha}{1-\alpha}}. \tag{4}$$

Households

Each member of generation t belongs to one of the three social classes that are indexed by $j = r, m, p$: the rich are referred to by r, the middle class by m, and the poor by p. Adults are endowed with one unit of time, $h_{j,t}$ units of human capital, and they get a bequest $b_{j,t}$ from their parents. When we refer to $b_{j,t}$, we use the terms assets and wealth interchangeably. Adults work, consume the amount $c_{j,t}$, give birth to $n > 0$ children, and potentially provide each child with $e_{j,t} \geq 0$ units of higher education. Moreover, adults save the amount $s_{j,t}$ to cover their own consumption needs when old, $c_{j,t+1}$, and to bequeath the amount $nb_{j,t+1}$ to their offspring.

We describe the lifetime utility of agent j, who was born in $t-1$, by the utility function

$$u_{j,t} = \log(c_{j,t} - \bar{c}) + \gamma \log(h_{j,t+1}) + \beta\left[\log(c_{j,t+1}) + \theta \log(b_{j,t+1})\right], \tag{5}$$

where $\bar{c} > 0$ is the subsistence level of consumption, $\beta \in (0,1)$ represents the discount factor, $h_{j,t+1}$ refers to the level of human capital per child, $\gamma > 0$ denotes the utility weight that parents attach to the human capital level of their children, and $\theta \in (0,1)$ represents the utility weight of the bequests to each child. The fact that bequests and the human

capital level of children appear in the utility function is motivated by the "warm glow" motive of giving (Andreoni, 1989). This is a short-cut formulation that leads to the same tradeoffs between own consumption and future consumption of the children as in case of a dynastic utility function. The reason for the similarity is that the consumption level of the children is determined by their income, which is in turn determined by their education and their inheritance. As usual, the logarithmic specification allows obtaining analytical results. Note that our utility function is less restrictive than the standard specification because, due to the presence of subsistence consumption, $\bar{c} > 0$, it allows the saving rate to depend positively on the income level. This is consistent with the available empirical evidence (see Steger, 2000; Dynan et al., 2004; Strulik, 2010, for growth models with subsistence consumption and for empirical evidence on the relation between income and saving).

We treat population growth as exogenous because we do not analyze the quality-quantity trade-off, the demographic transition, and the take-off to long-run economic growth.[5] Instead, we assume that the economy already escaped the Malthusian stagnation in the past and is now industrialized. This means that the transition from the Malthusian regime to a Solovian regime has already occurred (Hansen and Prescott, 2002; Galor and Moav, 2004). What we are concerned with is the next transition from a physical capital based economy to an economy in which economic growth is primarily driven by human capital accumulation. We are thus interested in the transition from neoclassical to endogenous growth (Lucas, 1988; Romer, 1990).

Higher education is financed by parents and provided by the education sector that employs lecturers who are members of the middle class. The alternative assumption that lecturers belong to the rich is difficult to motivate from an empirical point of view. Furthermore, it has the drawback that the regime switch becomes more difficult for the middle class such that they could be disconnected from the growth process. Since we observe a takeoff of education for the middle class, our assumption implies a better fit between

[5]For an appropriate treatment of the historical take-off toward sustained economic growth see the Unified Growth Theory, in particular, Galor and Weil (2000), Galor (2011), Hansen and Prescott (2002), Doepke (2004), and Strulik et al. (2013). See Córdoba et al. (2016) for a recent paper that focuses on the contribution of endogenous fertility to the evolution of inequality in the long run.

the theoretical implications and the observed educational trajectories. Altogether, human capital evolves according to

$$h^{nc}_{j,t+1} = \bar{e}_j, \qquad \text{if} \quad e_{j,t} = 0, \tag{6}$$

$$h^{h}_{j,t+1} = (\bar{e}_j + e_{j,t})\kappa h^{l}_{m,t} \qquad \text{if} \quad e_{j,t} > 0, \tag{7}$$

where κ denotes the productivity of the education sector and $\bar{e}_j$ represents the baseline level of education that each child of each group j obtains by observing her parents and peers (see, for example, Strulik et al., 2013) and/or because it is provided costlessly by the community in the form of public schooling.[6] If $e_{j,t} > 0$, the corresponding social class is in the high-skill regime as indicated by the superscript $l = h$ and otherwise it is in the neoclassical regime as indicated by the superscript $l = nc$. We treat $\bar{e}_j$ as exogenous for brevity. Public schooling could, however, be endogenized via income taxes such that the tax-financed level of $\bar{e}_j$ would depend on the structural parameters of our model (see de la Croix and Doepke, 2004, for more details).[7]

To reduce notational clutter, we omit the superscript whenever this is possible. It is straightforward to assume that $\bar{e}_r \geq \bar{e}_m \geq \bar{e}_p$, i.e., that the rich acquire no less baseline education than the middle class, which in turn acquire no less baseline education than the poor. While the strict inequality follows in a straightforward manner from the fact that the peer group that children observe tends to consist of members of the same social class, it is important to note that our parameter restriction allows for the equality of baseline

[6]Note that our formulation is a special case of de la Croix and Doepke (2003, 2004) and Glomm and Ravikumar (1992): $h_{j,t+1} = (\bar{e}_j + e_{j,t})^{\eta} h^{\nu}_{j,t} \bar{h}^{1-\nu}_{j,t}$, where $\bar{h}_{j,t}$ denotes average human capital, ν the intergenerational transmission of human capital, and η the impact of education on human capital. From a conceptual point of view, the presence of average human capital can also be interpreted as a spillover effect which is sizable according to recent findings (see Choi, 2011). We set $\eta = 1$ and $\nu = 0$ for convenience and without changing our results. Moreover, we allow the productivity of the education sector, κ, to differ from 1.

[7]Note, at this stage, that the outcome of the parental optimization problem does not depend on whether parents derive utility directly from investing in education or from the human capital level that the child attains. The reason is that the human capital level of the child, $h_{j,t+1}$, is the product of the parental choice of education and the productivity of the higher education sector. Since the latter is taken as given from the perspective of optimizing parents, the result of the optimization problem is the same, irrespective of whether the education choice of parents or the human capital level of their children enters the utility function. Analogously, for the optimal bequest it is irrelevant whether bequests enter the utility function or the capital stock of children. This is because the interest rate is exogenous in the optimization problem of parents.

education levels among the different social classes, i.e., $\bar{e}_r = \bar{e}_m = \bar{e}_p$. Our main qualitative result regarding the U-shaped evolution of income inequality due to a differential regime switch of the different social classes holds irrespective of whether or not the assumption is fulfilled with strict equalities or strict inequalities.

The budget constraints of adults and retirees are given by

$$I^l_{j,t} = \begin{cases} c^h_{j,t} + s^h_{j,t} + wh^l_{m,t} ne^h_{j,t}, & \text{if } e_{j,t} > 0, \\ c^{nc}_{j,t} + s^{nc}_{j,t}, & \text{if } e_{j,t} = 0, \end{cases} \tag{8}$$

$$s^l_{j,t} = \frac{c^l_{j,t+1} + nb^l_{j,t+1}}{R}, \tag{9}$$

with income being denoted by $I^l_{j,t} = wh^l_{j,t}(1 - zn) + Rb^l_{j,t}$, where $z \in (0, 1)$ is the time share necessary to raise one child to adulthood. Note that the term $Rb^l_{j,t}$ refers to the bequests (plus interest payments) that a member of the cohort born in $t-1$ gets from her parents. As an example, if the rich are in the high-skill regime, whereas the middle class is still in the neoclassical regime, the budget constraint reads $I^h_{r,t} = c^h_{r,t} + s^h_{r,t} + wh^{nc}_{m,t} ne^h_{r,t}$. The following lemma summarizes households' optimal decisions.[8]

Lemma 1.

(i) If $l = nc$, agents maximize (5) subject to (8) and (9), such that

$$c^{nc}_{j,t} = \frac{I^{nc}_{j,t} + [(1+\theta)\beta]\bar{c}}{1 + (1+\theta)\beta}, \tag{10}$$

$$c^{nc}_{j,t+1} = \frac{\beta}{1 + (1+\theta)\beta}[I^{nc}_{j,t} - \bar{c}]R, \tag{11}$$

$$b^{nc}_{j,t+1} = \frac{\theta\beta}{n[1 + (1+\theta)\beta]}[I^{nc}_{j,t} - \bar{c}]R, \tag{12}$$

with $e_{j,t} = 0$ implying that $h^{nc}_{j,t+1}$ is constant according to (6).

[8] See Diamond (1965) for the analysis of capital accumulation in a neoclassical setting without human capital accumulation.

(ii) If $l = h$, agents maximize (5) subject to (7), (8), and (9), such that

$$c^h_{j,t} = \frac{I^h_{j,t} + [\gamma + (1+\theta)\beta]\bar{c} + wh^h_{m,t}n\bar{e}_j}{1+\gamma+(1+\theta)\beta}, \tag{13}$$

$$c^h_{j,t+1} = \frac{\beta}{1+\gamma+(1+\theta)\beta}[I^h_{j,t} - \bar{c} + wh^h_{m,t}n\bar{e}_j]R, \tag{14}$$

$$b^h_{j,t+1} = \frac{\theta\beta}{n[1+\gamma+(1+\theta)\beta]}[I^h_{j,t} - \bar{c} + wh^h_{m,t}n\bar{e}_j]R, \tag{15}$$

$$e^h_{j,t} = \frac{\gamma}{wh^h_{m,t}n[1+\gamma+(1+\theta)\beta]}(I^h_{j,t} - \bar{c}) - \frac{[1+(1+\theta)\beta]\bar{e}_j}{1+\gamma+(1+\theta)\beta}, \tag{16}$$

with $h^h_{j,t+1}$ evolving according to (7).

Proof. See Online Appendix A.1. □

We observe from Lemma 1 that, *ceteris paribus*, consumption and bequests increase with income ($I_{j,t}$); bequests decrease with population growth (n); and second period consumption and bequests increase with the discount factor (β) and with the interest rate (R), whereas they decrease with the subsistence consumption level ($\bar{c}$). The existence of $\bar{c} > 0$ implies a hierarchy of needs for households: expenditure shares of first period consumption are declining with income, whereas expenditure shares of second period consumption and of bequests are increasing with income. Hence, richer households save more and bequeath more wealth to their children, which is a well-known fact and which is the driver of income inequality in the neoclassical regime.

In the high-skill regime, educational investments are positive and decrease with the preference for bequests (θ), population growth (n), the discount factor (β), the subsistence consumption level ($\bar{c}$), and the baseline education level ($\bar{e}$), whereas they increase with the preference for education (γ). Moreover, in light of (16), we observe that the regime switch from the neoclassical to the high-skill regime occurs if and only if the level of income, $I^l_{j,t}$, is sufficiently high. This is expressed formally in the following proposition.

Proposition 1.

A member of social class $j = r, m, p$ invests in eduction, i.e., $e_{j,t} > 0$, if her income

exceeds the critical threshold $I'_{j,t}$ *defined as*

$$I'_{j,t} = \frac{wh^l_{m,t}\bar{e}_j}{\gamma}[1 + (1+\theta)\beta] + \bar{c}, \qquad l = nc, h. \tag{17}$$

Hence, $e_{j,t} = 0$ *if* $I^{nc}_{j,t} \leq I'_{j,t}$.

Proof. See Online Appendix A.2. □

Regarding the critical level of income that induces the regime switch to the high-skill regime, several remarks are in order. First, the threshold level of income necessary to induce the regime switch depends positively on the level of subsistence consumption, $\bar{c}$, on the sum of the weights of first and second period consumption, $1 + \beta$, on the weight of bequests, $\theta\beta$, on the level of baseline education, $\bar{e}_j$, and on education cost, $wh^l_{m,t}n$. By contrast, a higher preference for education, γ, reduces the critical income level.

Second, in light of Proposition 1, the high-skill regime applies to households of class j if their income level is sufficiently high such that investments in education above the basic level (that can be acquired costlessly) deliver a higher amount of additional utility than using the same amount of income for consumption or bequests. In this case we have $I_{j,t} > I'_{j,t}$, such that $e_{j,t} > 0$. The optimal solutions are then described by item (ii) of Lemma 1. If the income level of households in social class j falls short of the critical threshold $I'_{j,t}$, these households find it optimal not to invest in education of their children because the children acquire a certain amount of human capital costlessly and the additional investments in human capital deliver less additional utility than if the same amount of income was instead spent on consumption or was bequeathed. These households find themselves in the neoclassical regime, i.e., $l = nc$ and $e_{j,t} = 0$, such that their optimal decisions are described by item (i) of Lemma 1.

Third, as long as the middle class does not switch to the high-skill regime, $I'_{j,t}$ is constant. If the middle class starts to invest in higher education of their children, $I'_{j,t}$ increases at the same rate as $h_{m,t}$. If, furthermore, the middle class switches before the poor and after the rich, the described mechanism becomes crucial for the poor because they are at risk of $I'_{p,t}$ growing faster than their incomes such that a regime switch would never

occur for the poor without policy interventions in terms of publicly financed universities, college education subsidies such as stipends, or tuition fee waivers (Rauh, 2017). Note that an alternative modeling of the neoclassical regime would not affect our results qualitatively. If we allowed for perpetual growth in the neoclassical regime by assuming an AK-production technology (see, for example, Mierau and Turnovsky, 2014), the critical threshold level of income ($I'_{j,t}$) would rise with the productivity growth rate of the middle class. If all classes benefit equally from productivity gains, both the critical threshold and incomes would grow at the same constant rate. The omission of other engines of growth is therefore a convenient shortcut. Similarly, in a closed economy, capital accumulation increases wages and, thus, incomes, while $I'_{j,t}$ increases by the same amount. Hence, the small open economy assumption is innocuous with respect to the regime switch, unless incomes of the poor grew faster than wages in the education sector, which, however, does not seem to be an empirically relevant case.

Fourth, in case of $\bar{e}_r = \bar{e}_m = \bar{e}_p$, it follows that all social classes exhibit the same threshold of income for the regime switch but that, nevertheless, social classes switch at different dates depending on the distribution of wealth. Thus, even without any differences in the levels of basic education among the social classes, there is a risk that the poor will be disconnected from the growth process with their incomes lagging behind their critical income level $I'_{p,t}$. In case of $\bar{e}_r > \bar{e}_m > \bar{e}_p$, it follows that $I'_{r,t} > I'_{m,t} > I'_{p,t}$. However, a social group with a higher baseline level of education reaches its higher threshold income level earlier if its accumulation rate of wealth is sufficiently high.[9] This implies that differences in basic education or in innate abilities with an initial advantage for the rich would only reinforce our mechanism.

Equilibrium Implications

[9]The critical level of wealth compatible with $I'_{j,t}$ reads $b'_{j,t} = (1/R)[wh_{m,t}\bar{e}_j[1 + (1 + \theta)\beta)]/\gamma + \bar{c} - w(1 - zn)\bar{e}_j]$. The distance of social group j to $b'_{j,t}$ shrinks at rate $b'_{j,t} - b_{j,t+1}$. An increase in $\bar{e}_j$ reduces the latter if $\partial(b'_{j,t} - b_{j,t+1})/\partial\bar{e}_j < 0$, which is the case if $h_{m,t} < \gamma(1 - zn)\left[1/[1 + (1 + \theta)\beta] + \theta\beta R^2/\{n[1 + (1 + \theta)\beta]^2\}\right]$.

Taking into account the time required for child care ($z \cdot n$), each social class supplies

$$L_{j,t} = (1 - zn)N_{j,t} \tag{18}$$

units of raw labor, where $N_{j,t}$ is the number of individuals who belong to social class j. The aggregate supply of human capital is the sum over all social classes of the product of class-specific labor supply ($L_{j,t}$) with average class-specific human capital ($h_{j,t}$):

$$H_t = h_{r,t}L_{r,t} + h_{m,t}L_{m,t} + h_{p,t}L_{p,t}. \tag{19}$$

Taking into account that human capital used for production ($H_{Y,t}$) is equal to aggregate human capital net of the human capital that is employed in the education sector, which is recruited from the middle class, we have

$$H_{Y,t} = \begin{cases} H_t - \left(\sum_j e_{j,t}N_{j,t}\right) h_{m,t}L_{m,t}, & \text{if } l = h \quad \text{for at least one } j, \\ H_t, & \text{if } l = n \quad \text{for all } j. \end{cases} \tag{20}$$

Before we discuss the dynamics, we define the market equilibrium as follows.

Definition 1. *The market equilibrium of the economy is defined by the solution of the system of equations (10)-(12) for $l = nc$ and by the solution of the system of equations (13)-(16) for $l = h$, where $I^{nc}_{j,t}$ and $I^{h}_{j,t}$ are given by (9), the wage rate per unit of human capital and the capital rental rate are determined by (4), the capital stock per unit of human capital is given by (3), and the market for human capital is cleared according to (20). Since the markets for physical capital and human capital clear according to this definition, Walras' law implies that also the goods market is cleared.*

III Dynamics

Neoclassical Regime

In light of Proposition 1, the neoclassical regime is characterized by $I_{j,t} \leq I'_{j,t}$, such that $e_{j,t} = 0$. Consequently, the level of human capital is constant and equal to the level of

baseline education: $h^{nc}_{j,t+1} = \bar{e}_j$. Thus, income of a member of social class j is given by

$$I^{nc}_{j,t} = w\bar{e}_j(1 - zn) + Rb^{nc}_{j,t}. \tag{21}$$

According to (12) and (21), the accumulation of wealth within social class j is governed by

$$b^{nc}_{j,t+1} = \frac{\theta\beta}{n[1 + (1 + \theta)\beta]}[w\bar{e}_j(1 - zn) + Rb^{nc}_{j,t} - \bar{c}]R. \tag{22}$$

The unique and stable steady state, denoted by an asterisk in the subscript, is

$$b^{nc}_{j,*} = \frac{\theta\beta R}{n[1 + (1 + \theta)\beta] - \theta\beta R^2}[(1 - zn)w\bar{e}_j - \bar{c}], \tag{23}$$

given that $\theta\beta R^2 < n[1 + (1 + \theta)\beta]$, which implies that the slope of the $b^{nc}_{j,t+1}$-locus is smaller than 1. Overall, labor income must exceed the level of subsistence consumption, i.e., $(1 - zn)w\bar{e}_j > \bar{c}$, otherwise the steady state is economically meaningless. Moreover, global stability of $b^{nc}_{j,*}$ requires that $\theta\beta R^2 < n[1 + (1 + \theta)\beta]$.[10] In addition, note that the location of the b^{nc}_j-locus as defined by (22) depends positively on $\bar{e}_j$. If $\bar{e}_r > \bar{e}_m > \bar{e}_p$, it follows that $b^{nc}_{r,*} > b^{nc}_{m,*} > b^{nc}_{p,*}$.

Given that labor incomes are stationary in the neoclassical regime, the accumulation of wealth via bequests is the only source of income growth. Income, $I^{nc}_{j,*} = w\bar{e}_j(1-zn)+Rb^{nc}_{j,*}$, is thus constant when bequests approach their steady-state level, $b^{nc}_{j,*}$. This implies that a regime switch from the neoclassical regime to the high-skill regime can only occur if the threshold levels of income that are compatible with the regime switch are below their steady-state levels. In mathematical terms, a necessary condition for the regime switch is $I^{nc}_{j,*} > I'_{j,t}$, where $e_{j,t} > 0$ if and only if the income of a member of social class j in period t ($I^{nc}_{j,t}$) exceeds the threshold level $I'_{j,t}$, which requires that $I^{nc}_{j,*} > I'_{j,t}$. Note that this would be the case even in the presence of productivity growth fueled by other sources than human capital accumulation (e.g., by technological progress) because disposable incomes

[10] For $(1 - zn)w\bar{e}_j < \bar{c}$, a steady state exists only if $\theta\beta R^2 > n[1 + (1 + \theta)\beta]$, which implies, in turn, that $b^{nc}_{j,*}$ is globally unstable.

and the threshold level $I'_{j,t}$ were growing at the same rate. The following proposition specifies this aspect in more detail. A graphical illustration is presented in Figure 4.

Proposition 2. *The regime switch requires* $I_{j,*} = (1-zn)w\bar{e}_j + Rb_{j,*} > I'_{j,t}$*, which implies in light of (21) and (23) that* $b^{nc}_{j,*} > \frac{wh_{m,t}n\bar{e}_j}{\gamma R}[1+(1+\theta)\beta] + \frac{\bar{c}}{R} - \frac{(1-zn)w\bar{e}_j}{R}$ *and*

$$\bar{e}_j > \hat{e}^h \equiv \frac{\gamma\bar{c}}{\{\gamma(1-zn)w - wh^h_{m,t}[n(1+(1+\theta)\beta) - \theta\beta R^2]\}} \tag{24}$$

if $l = h$*, and*

$$\bar{e}_j > \hat{e}^{nc} \equiv \frac{\gamma\bar{c}}{\{\gamma(1-zn)w - w\bar{e}_m[n(1+(1+\theta)\beta) - \theta\beta R^2]\}} \tag{25}$$

if $l = nc$*.*

Proof. See Online Appendix A.3. □

A transition into the high-skill regime requires that the long-run value of bequests in social class j as given by $b^{nc}_{j,*}$ is greater than the threshold level of wealth $b'_{j,t}$ that is associated with the threshold level of income $I'_{j,t}$.[11] This is only possible if $\bar{e}_j > \hat{e}$. In light of our discussion around (23) we know that a feasible and globally stable steady state, $b^{nc}_{j,*} > 0$, requires that $n[1+(1+\theta)\beta] - \theta\beta R^2 > 0$. As $\gamma\bar{c} > 0$ and $\bar{e}_j > 0$, (24) and (25) hold only if $\gamma(1-zn)w > \{n[1+(1+\theta)\beta] - \theta\beta R^2\}$. If the middle class is in the neoclassical regime, the poor need a minimum $\bar{e}_p > \hat{e}^{nc}$, otherwise their steady-state income falls short of their threshold income. If the middle class already experienced a regime switch, costs of higher education are increasing such that the threshold level of income necessary for the regime switch is also increasing. As long as the baseline level of education of the poor fulfills (24), the regime switch is still possible for them.

If $\bar{e}_j \leq \hat{e}^l$, accumulated assets of social class j sustained by the neoclassical regime fall short of $I'_{j,t}$ such that a switch to the high-skill regime cannot occur for social class j. If $\hat{e}^{nc} > \bar{e}_r$, no social class would ever invest in higher education of their children. In this case

[11] This is a direct implication of Proposition 1 and the requirement that $I'_{j,t} < I^{nc}_{j,*}$, or – equivalent to the latter – that $b'_{j,t} < b^{nc}_{j,*}$, where $b'_{j,t} = (1/R)\{wh_{m,t}\bar{e}_j[1+(1+\theta)\beta)]/\gamma + \bar{c} - w(1-zn)\bar{e}_j\}$. Otherwise, the critical threshold level would be greater than the long-run value and could not be reached if $b^{nc}_{j,0} < b^{nc}_{j,*}$.

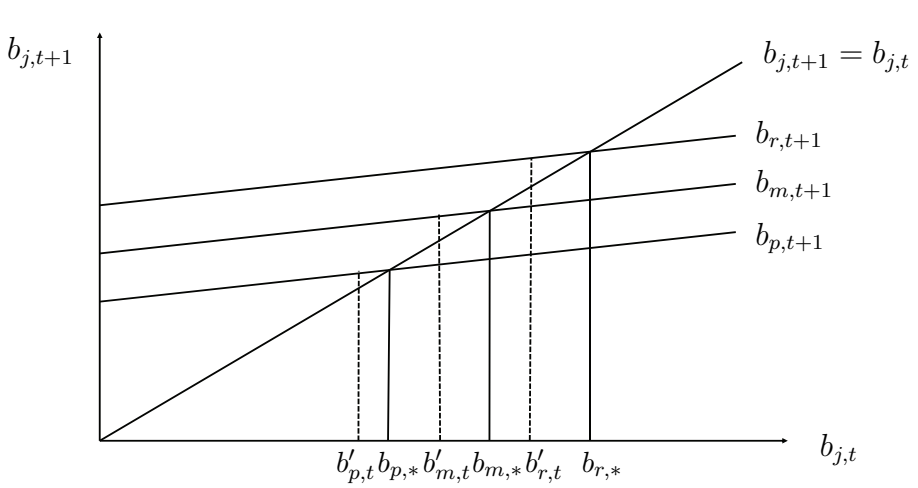

Figure 4: Neoclassical regime, with $\bar{e}_r > \bar{e}_m > \bar{e}_p$.

the economy is trapped in the neoclassical regime and growth ceases at a certain point. This could be a relevant description of the "middle income trap", i.e., that countries, which escaped the phase of stagnation at the subsistence level successfully, cannot manage to switch to an innovation-based high-skill economy (cf. Eichengreen et al., 2012, 2013, who observe that middle income traps are less likely to be an obstacle for countries with a well educated population). By contrast, a switch of all social classes to the high-skill regime is guaranteed for $\bar{e}_p > \hat{e}^l$.

High-Skill Regime

In the high-skill regime, income of social class j exceeds $I'_{j,t}$, such that the parents belonging to class j invest in higher education of their children. Their optimal decisions are represented by item (ii) of Lemma 1 and the evolution of human capital in social class j is then governed by (7). We summarize the dynamic behavior in the high-skill regime in the next proposition.

Proposition 3.

(i) The switch to the high-skill regime induces growing wage incomes, $wh^h_{j,t}$, and increasing levels of bequests, such that total incomes, $I^h_{j,t}$, grow as well. After the regime switch, the ratio between bequests and human capital is constant and the same for all social classes

$$\frac{b^h_{j,t+1}}{h_{j,t+1}} = \frac{w\theta\beta R}{\gamma\kappa} = const. \tag{26}$$

impying that incomes grow at the same rate.

(ii) Let $x^h_{j,t}$ denote the ratio of human capital between social class j and the middle class, i.e., $x^h_{j,t} = h^h_{j,t}/h^h_{m,t}$, such that

$$x^h_{j,t+1} = \frac{\left[(1-zn) + \frac{\theta\beta R^2}{\gamma}\right] x^h_{j,t} - \frac{\bar{c}}{w h^h_{m,t}} + \bar{e}_j n}{(1-zn) + \frac{\theta\beta R^2}{\gamma} - \frac{\bar{c}}{w h^h_{m,t}} + \bar{e}_m n}. \tag{27}$$

The stationary solution is then given by

$$x^h_{j,*} = \lim_{t\to\infty} x_{j,t+1} = \bar{e}_j/\bar{e}_m, \tag{28}$$

with $x^h_{j,*} \gtreqless 1$, if $\bar{e}_j \gtreqless \bar{e}_m$.

It follows that the long-run ratio of bequests between social class j and the middle class converges to $\lim_{t\to\infty} \frac{b^h_{j,t+1}}{b^h_{m,t+1}} = \frac{b^h_{j,*}}{b^h_{m,*}}$, where $\frac{b^h_{j,*}}{b^h_{m,*}} \gtreqless 1$ for $\bar{e}_j \gtreqless \bar{e}_m$.[12]

(iii) The growth factor of human capital converges to

$$\lim_{t\to\infty} \frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa(1-zn+\bar{e}_m n) + \theta\beta R^2}{n[1+\gamma+\beta(1+\theta)]}. \tag{29}$$

Proof. See Online Appendix A.4. □

In light of item (i) of Proposition 3, the ratio of bequests to human capital, $b^h_{j,t+1}/h_{j,t+1}$, declines with the weight of children's education in the parental utility function, γ, and the productivity of the education sector, κ. By contrast, the ratio of bequests to human capital increases with those factors that are responsible for an increase in bequests, i.e., wages, w, which also increase education costs, the weight of bequests in the parental utility function, $\theta\beta$, and the capital rental rate, R.

Income inequality is determined by the initial distribution of wealth, the differential evolution of bequests, and the differential investment in human capital between the social classes. In item (ii), we capture the evolution of inequality in terms of education by the

[12]Note that $\frac{b^h_{j,*}}{b^h_{m,*}} = \frac{w\frac{\bar{e}_j}{\bar{e}_m}(1-zn) + \frac{w\theta\beta R^2}{\gamma\kappa} - \bar{c} + wn\bar{e}_j}{w(1-zn) + \frac{w\theta\beta R^2}{\gamma\kappa} - \bar{c} + wn\bar{e}_m}$.

dynamics of the ratio between human capital of social class j and the level of human capital of the middle class, i.e., $x_{j,t} = h^h_{j,t}/h^h_{m,t}$. As regards the evolution of relative human capital, two cases have to be distinguished. (1) If social classes do not differ with respect to their baseline levels of education such that $\bar{e}_r = \bar{e}_m = \bar{e}_p$, it follows from (27) that the influence of subsistence consumption on the evolution of $x_{j,t}$ approaches zero in the long run ($\lim_{t\to\infty} \bar{c}/wh^h_{j,t} = 0$). Thus, $x_{j,t}$ converges to 1 as t approaches infinity. Item (i) implies that all social classes bequeath the same amount of assets to their children, such that incomes are equal among the social classes in the long run. Thus, there is only scope for long-run income inequality in this case if not all social classes switch to the high-skill regime. Transitory income inequality is determined by the initial distribution of wealth for a given constellation of $I'_{j,t} < I^{nc}_{j,*}$ determining the timing of the regime switch in social class j. It is important to note in this context that the transition phase of these types of models typically lasts for a very long time (several generations) such that income inequality could be observed for centuries if such a model represented the underlying data generating process. (2) If, in turn, $\bar{e}_j \gtrless \bar{e}_m$, it follows that x_j approaches $\bar{e}_j/\bar{e}_m \gtrless 1$ if $\bar{e}_j \gtrless \bar{e}_m$. Thus, differences in the level of baseline education translate into differences in relative human capital endowments, differences in the levels of bequests, and differences in the levels of income. A constant $b/h-$ ratio implies then that households with lower human capital endowments also exhibit lower bequests, even in the long run.

The growth rate of human capital [item (iii)] depends positively on the $b/h-$ratio, positively on time devoted to work, and positively on $\gamma/\{n[1+\gamma+(1+\theta)\beta]\}$, which drives the expenditure share of higher education. Moreover, the growth rate of human capital is positively affected by the baseline level of education, $\bar{e}_j$, and the productivity of the education sector, κ. During the transition, the growth rate of human capital is adversely affected by subsistence needs, $\bar{c}$, and by $x^h_{j,t}$. The latter reflects a neoclassical convergence mechanism. In the long run, due to increasing wage incomes, the impact of subsistence needs on the evolution of human capital approaches zero. Altogether, $x_{j,t}$ converges to $\bar{e}_j/\bar{e}_m$, such that the growth factor of human capital converges to expression (29). Since differences in the baseline level of education drive long-run differences in

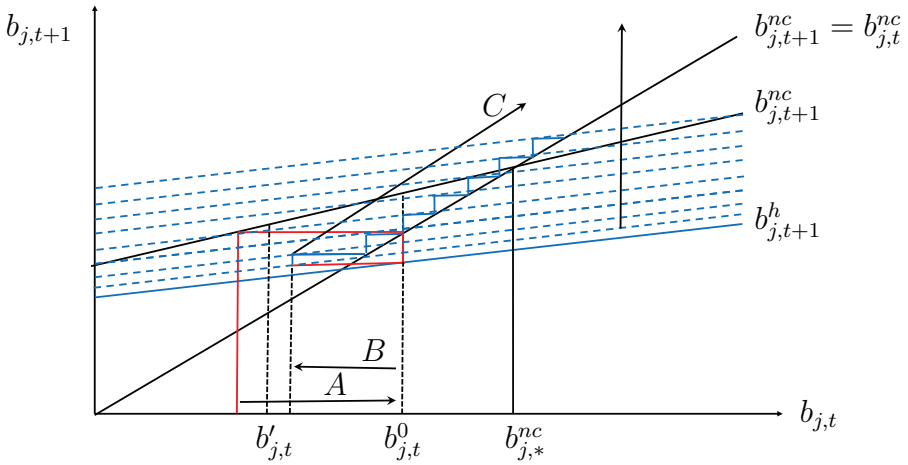

Figure 5: Regime switch to the high-skill regime

human capital, it follows that differences in bequests across social classes are also driven by differences in $\bar{e}_j$ [last part of item (ii)].

Income Inequality: Basic Mechanism

Crucial for our theory is the emergence of first declining and then increasing income inequality after the regime switch from the neoclassical to the high-skill regime. The main argument is summarized in the following proposition. The possible decline in income inequality in a later phase of the high-skill regime will be discussed further below.

Proposition 4. *A social class that reaches the critical income level $I'_{j,t}$ reduces the accumulation of wealth.*

Proof. At the moment of the regime switch, a member of social class j is equipped with a wealth level of at least $b'_{j,t}$, which assures the necessary level of income that generates the regime switch ($I'_{j,t}$, as defined by Proposition 1). In the neoclassical regime, according to Lemma 1, an income level of $I^0_{j,t}$ induces a level of bequests of

$$b^{nc}_{j,t+1} = \frac{\theta\beta R}{n[1+(1+\theta)\beta]}[I^0_{j,t} - \bar{c}]. \tag{30}$$

In the high-skill regime, the level of bequests is given by

$$b^{h}_{j,t+1} = \frac{\theta\beta R}{n[1+\gamma+(1+\theta)\beta]}[I^0_{j,t} - \bar{c} + w_t \bar{h}^h_t n \bar{e}_j]. \tag{31}$$

From the last two expressions, we obtain $b^{nc}_{j,t+1} > b^{h}_{j,t+1}$ if

$$I^{0}_{j,t} > [1 + (1 + \theta)\beta]\frac{wh^{h}_{m,t}n\bar{e}_{j}}{\gamma} + \bar{c}, \tag{32}$$

which, in light of Proposition 1, implies that $I^{0}_{j,t} > I'_{j,t}$. □

According to Proposition 4, a household switching to the high-skill regime starts to invest in higher education of the children but at the expense of bequests per child. To put it differently, households shift resources from the accumulation of wealth to the accumulation of human capital. The rich are the first social class that experiences the regime switch and reduces bequests, followed by the middle class and possibly also the poor. Given that the expenditure share of bequests is, due to the existence of subsistence needs, increasing with income, the decline in bequeathed assets in the highest income percentiles happens at a time when there are increasing levels of bequests in the other social classes, such that income inequality falls. As labor incomes of the children of the rich are increasing because of human capital accumulation (which, in turn, stimulates increasing levels of bequests and education), income inequality starts to rise again at a later stage. The reason for inequality to fall in the short- to medium run is that the greater human capital level of the children leads to higher wages only later on in their lives, whereas the effect of lower bequests on inequality occurs immediately. Altogether, income inequality and the flow of bequests follow the U-shaped pattern as observed in the data described in the introduction.

A graphical illustration of our arguments is presented in Figure 5. The evolution of bequests follows the arrows A, B, and C. In period t, the members of social class j exhibit a level of assets indicated by $b^{0}_{j,t}$, which exceeds $b'_{j,t}$ associated with the threshold income $I'_{j,t}$. Thus, social class j switches to the high-skill regime and leaves the $b^{nc}_{j,t+1}$-locus. From Proposition 4 it follows that the $b^{h}_{j,t+1}$-locus compatible with $b^{0}_{j,t}$ must be located below the $b^{nc}_{j,t+1}$-locus, such that the level of bequests shrinks (B). Since the evolution of social class j is now described by Proposition 3, item (i), the $b^{h}_{j,t+1}$-locus moves upwards such that bequests increase again (C). Obviously, the speed of the upward shift depends (ceteris

paribus) on the quality of the education sector reflected by the education productivity parameter κ. Thus, κ also affects the evolution of the flow of bequests and the evolution of income inequality.

Regarding the evolution of income inequality after the regime switch, it is important to note that the discussed fall and rise of inequality is a transitory phenomenon. Relative human capital stocks will converge to their initial values and income inequality will decline toward its long-run value. Nevertheless, and this will be clarified further below, the emergence of declining income inequality and its potential amount depend crucially on whether or not all social groups experience a switch to the high-skill regime. In light of Lemma 1, expenditures for higher education are zero in the neoclassical regime, such that we obtain from Proposition 3, item (ii), initial relative human capital endowments in the neoclassical regime as

$$x_{r,t}^{nc} = \frac{\bar{e}_r}{\bar{e}_m} > 1, \qquad x_{m,t}^{nc} = 1, \qquad x_{p,t}^{nc} = \frac{\bar{e}_p}{\bar{e}_m} < 1, \tag{33}$$

which are equal to the corresponding long-run values in the high-skill regime, $x_{j,*}^{h}$ [see (28)]. Due to a greater level of wealth and a faster speed of convergence, the rich surpass their critical threshold income first, followed by the middle class and possibly also by the poor. Higher education of the rich can be expressed as

$$e_{r,t} = \frac{\gamma}{n[1+\gamma(1+\theta)\beta]}\left[(1-zn)+\frac{R^2\theta\beta}{\gamma}\right]x_{r,t}^{h} - \frac{\bar{c}}{wh_{m,t}} - \frac{[1+(1+\theta)\beta]\bar{e}_r}{1+\gamma(1+\theta)\beta}. \tag{34}$$

With $h_{m,t}$ being constant, $x_{r,t}^{h}$ increases after the regime switch of the rich above $x_{r,t}^{nc}$ and induces increasing expenditures on eduction, which amplifies the income gap to the other social classes later on. After the middle class experiences a regime switch, the resulting increase in $h_{m,t}$ affects the relative endowments of the other social groups because the price of education changes, while $x_{m,t}$ remains at 1. If the poor are still not investing in higher education, their relative human capital stock shrinks and falls short of $x_{p,t}^{nc}$. Moreover, their threshold income ($I'_{p,t}$) is now increasing with the growth rate of human capital in the middle class. If, under these circumstances, $I'_{p,t}$ is increasing above $I_{p,*}^{nc}$, the

poor will never switch (or they may just switch temporarily) to the high-skill regime.

Initially, $e_{r,t}$ increases because of the increase in $x^h_{r,t}$, indicating that education is comparatively cheap for the rich. When the middle class starts to invest in education, there is a dampening effect on $x_{r,t}$ because of the increase in education costs due to the increase in $h_{m,t}$, but a second reinforcing effect on education setting in through the diminishing role of subsistence needs, $\bar{c}/(wh_{m,t})$. The latter, however, is only a transitory effect that becomes smaller and smaller as the middle class accumulates human capital. Thus, the rich reduce the growth rate of expenditures on eduction below the level of the middle class such that $x^h_{r,t}$ converges from above to its long-run value $x^h_{r,*} = x^{nc}_{r,t}$. A symmetric argument holds for the poor. The regime switch of the middle class adversely affects the poor's relative human capital stock, $x_{p,t}$. Thus, their expenditures fall short of the level achieved in the middle class and $x^h_{p,t}$ shrinks below $x^{nc}_{p,t}$. However, the shrinking importance of subsistence needs dampens the decline over time, which implies that the growth in expenditures on education of the middle class ceases to the extent that $x^h_{p,t}$ adjusts from below to its long-run value $x^h_{p,*} = x^{nc}_{p,t}$. The convergence of relative human capital stocks to their initial values is precisely the mechanism responsible for the decline in income inequality. But again, it is important to stress the feasibility of the regime switch for the poor. Since the poor may start to invest in higher education after the regime switch of the middle class has occurred, their threshold income may grow faster than their actual incomes. Thus, the poor might never (or only temporarily) switch to the high-skill regime. In this case, their relative human capital shrinks toward zero and the initial distribution of wealth will affect long-run income inequality. In this context, the overall amount of income inequality is a poor predictor for the feasibility of the regime switch for all social classes. What matters is the income gap between the poor and the middle class. A comparatively large distance between the poor and the middle class induces an early switch of the middle class to the high-skill regime, which is responsible for a comparatively fast growing threshold income of the poor.

Altogether, we can summarize our analytical results on the evolution of income inequality over time as follows: (1) At early stages of economic development, the accumu-

lation of wealth and bequests is the engine of income growth. Subsistence consumption constraints imply that saving rates increase with income such that the richer parts of the population accumulate (and bequeath) wealth at a higher rate than the other social classes. This leads to rising income inequality. (2) If the economy manages to avoid the middle income trap, the rich start to invest in higher education of their children, which is costly and crowds out bequests. At that stage, wealth accumulation of the rich is reduced and income inequality starts to shrink. This phase lasts for one generation, i.e., for around 30 years. (3) Over time, the better educated children of the rich start to earn higher wage incomes. The engine of growth becomes human capital accumulation and income inequality starts to rise again. During that phase, the driving force behind the evolution of income inequality is the skill differential between those with higher education and those without. (4) Eventually, all social classes might be able to invest in higher education and the growth rates of human capital converge. As a consequence, the level of income inequality decreases.

The described development of the model economy is consistent with the data on wealth concentration, income inequality, the flow of bequests, and the fraction of individuals that receive higher education depending on parental income as described in Figures 1 and 2, and the literature cited in the introduction. Furthermore, the evolution of income inequality that our model predicts is consistent with the notion that the main driver of income inequality before World War I was the differential accumulation of wealth, whereas, in modern times, the wage gap between highly educated individuals and poorly educated ones is another crucial driver of income inequality (Atkinson et al., 2011; Meschi and Scervini, 2014; Piketty, 2014).

IV Numerical Experiments

In this section, we conduct numerical experiments to illustrate the theoretical results from the previous sections and to get an impression of the transitional dynamics that our framework implies. In so doing, we analyze the effects of different amounts of initial wealth inequality, i.e., $b_{j,0}$, on the evolution of inequality over time. We shall empha-

size that, despite implementing reasonable parameter values, our goal is to characterize transitional dynamics qualitatively rather than quantitatively. On the one hand, for a quantitative analysis, our two-period overlapping-generations structure is too stylized. On the other hand, the simplicity allows us to gain a solid intuition for the underlying economic mechanisms.

We choose the parameters of the model such that the balanced growth path fits to empirical observations of rich economies considering that, in our model, one period has a length of thirty years. We fix the capital income share in the production of output, α, at 0.3. The real interest rate is set to 4% per year, i.e., $R = 1.04^{30}$. The literature on business cycles suggests a discount factor of future consumption of around 0.99 per quarter, such that $\beta = 0.99^{120}$. The projections of the United Nations suggest a stationary world population in the long run such that we set $n = 1$. As far as the child rearing time is concerned, we fix the time share necessary to raise one child to adulthood, z, at 0.075. This implies an opportunity cost of around 15% of parents' time endowment per child.[13] The remaining values are calibrated in an iterative way assuring long-run human capital growth (per year) between one and two percent and that expenditures on education are not exceeding 6%. Moreover, the initial value of the Gini coefficient of wealth should be above 80 matching the value of wealth inequality at the beginning of the 19th century in the United States.[14] To match this level of inequality, we need at least four income classes. Note that the qualitative results are not dependent on the existence of a fourth or more income classes. We split the middle class into upper and lower middle class denoted by um and lm, respectively. Education is provided by the lower middle class (lm). Moreover, we adjust population shares such that they correspond to a skilled-unskilled population ratio of about 1.5 (see Hornstein et al., 2005). This implies for our framework that the poor do not invest in higher education. Population shares of population group $j = p, lm, um, r$ correspond to $\{0.4, 0.4, 0.1, 0.1\}$. The initial values of wealth are $b_{p,0} = 0$, $b_{lm,0} = 900$,

[13] de la Croix and Doepke (2003) report that the opportunity cost of a child is equivalent to about 15 percent of the parents' time endowment. Assuming that children depend on child care for 15 years and that the adult period lasts for 30 years, the overall time cost should be 50 percent of the time cost per year with the child present.

[14] For historic values in the United States at the end of the 19th and the beginning of the 20th century, see, for example, Steckel and Moehling (2001) and for the last six decades, see Kuhn et al. (2018).

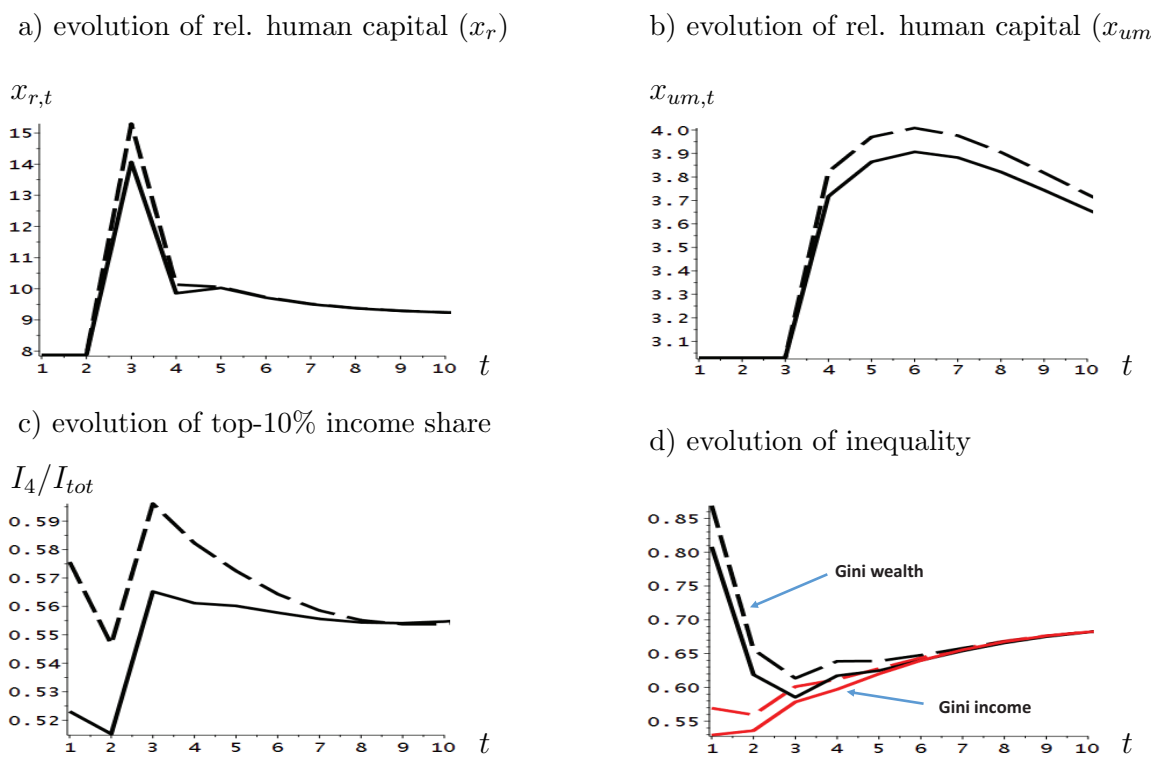

Figure 6: Evolution of relative human capital levels for the richest and the second richest population group ($j = r, um$), the incomes share of the richest, and overall inequality.

$b_{um,0} = 1000$, and $b_{r,0} = 25000$. The remaining parameters are $A = 4400$, $\kappa = 4.5$, $\theta = 0.4$, $\gamma = 0.3$, $\bar{c} = 6700$, $\bar{e}_p = 0.25$, $\bar{e}_{lm} = 0.33$, $\bar{e}_{um} = 1$, and $\bar{e}_r = 2.6$.[15]

In Figure 6, we depict the evolution of relative human capital for the rich and the upper middle class,[16] the income share of the top-10 percent and the evolution of the Gini coefficients of wealth and income. Solid lines represent low initial inequality of wealth and dashed lines high initial inequality of wealth.[17] As population groups are characterized by different levels of baseline education ($\bar{e}_r > \bar{e}_{um} > \bar{e}_{lm} > \bar{e}_p$), agents transit along different trajectories toward different steady states in the neoclassical regime (see Proposition 2 and Figure 4) in the sense that $b^{nc}_{r,*} > b^{nc}_{um,*} > b^{nc}_{lm,*} > b^{nc}_{p,*}$. Moreover, Proposition 1 implies that these agents exhibit different threshold incomes: $\hat{I}_r > \hat{I}_{um} > \hat{I}_{lm} > \hat{I}_p$. Due to a greater

[15] de la Croix and Doepke (2003) set γ to 0.271 to achieve $n = 1$ along the balanced growth path. In our setting the number of children is exogenous, such that there are no fertility differentials affecting inequality. Thus, our γ is slightly higher.

[16] We omit population groups p and lm because the former's relative human capital is converging to zero and the latter's is equal to one. The evolution of bequests can be found in Online Appendix A.5.

[17] The initial values of wealth in the high inequality scenario are $b_{p,0} = 0$, $b_{lm,0} = 400$, $b_{um,0} = 1000$, and $b_{r,0} = 40000$.

level of assets, the rich surpass their threshold level of income first and start to invest in higher education of their children. This is reflected by the increase in relative human capital of the rich in period 2. In the previous section, we argued that the regime switch to the high-skill regime induces a reduction in bequests (see Proposition 4 and Figure 5); hence wealth inequality declines as does the income share of the rich. If this effect is strong enough, income inequality will also shrink. Nevertheless, increasing investments in education increase labor incomes of the rich in subsequent periods such that the level of bequests increases again. Therefore, the income share of the rich, inequality in wealth, and incomes rise again. In subsequent periods, less wealthy population groups surpass their threshold levels of income ($\hat{I}_j$) and switch to the high-skill regime. Thus, the income share of the rich increases again because the other population groups have to reduce their bequests. This dampens the decline in wealth inequality such that income inequality rises. As next generations are equipped with higher levels of human capital, income growth accelerates and the income share of the top 10 percent shrinks again. In light of Proposition 3, item (ii), relative human capital stocks converge to $\bar{e}_j/\bar{e}_{lm}$, such that the subsequent increase in income inequality and wealth is driven by different investment levels in higher education at the steady state determined by different levels in baseline education. Regarding the evolution of the income share of the rich, the observed increase in recent years is just a transitory phenomenon and reflects—according to our mechanism in isolation—a comparatively early switch of richer population groups to the high-skill regime. The magnitude of the subsequent decline is determined by the long-run levels of relative human capital stocks. At the same time it is important to note that the decline in the income share of the rich is not necessarily associated with a decline in overall inequality. Depending on the differences in relative human capital, income inequality and wealth inequality increase because relative human capital determines households' capacity to accumulate wealth and to invest in higher education. After the population group lm has switched to the high-skill regime, the education sector is characterized by growing levels of human capital. Thus, from the perspective of the poor, this mechanism

accelerates the increase in income and wealth inequality.[18]

Our theoretical mechanism and its numerical illustrations are consistent with the contemporaneous evolution of inequality and the income share of the rich. Along these lines, Kuhn and Rios-Rull (2016) document a strong increase in wealth inequality, a modest increase in earnings inequality, but only a weak increase in income inequality. In subsequent periods, our model neglects further shocks and wealth heterogeneity in terms of different types of assets, such that income and wealth inequality are highly correlated.[19]

The consequences of higher initial wealth inequality are depicted by dashed lines in Figure 6. In this scenario, the rich are—compared with the benchmark scenario—equipped with more wealth and the lower middle class with less wealth. Hence, education is comparatively cheaper for the upper middle class and the rich, such that their relative human capital endowments exceed the levels of the reference case. Higher initial wealth inequality implies a smaller drop in the income share of the rich after they have switched to the high-skill regime. This translates into a smaller drop in the Gini coefficient of wealth. Since the subsequent generation is equipped with higher levels of human capital, the income share of the rich peaks at a higher level. Thus, income inequality increases also at a higher level towards its steady state.

[18]Because of increasing costs of education after the regime switch of population group lm, this effect would be mitigated but not disappear if the poorest population group also invested in higher education. Regarding the regime switch of the poor, a sufficiently high endowment of wealth alone does not generate a regime switch. What matters is a sufficiently high endowment of baseline education $\bar{e}_p$ and wealth. Based on the calibration of our reference scenario we find that a level of $\bar{e}_p = 0.32$ and a minimum level of initial wealth of $b_{p,0} = 650$ are just sufficient to generate a regime switch for the poor. For $\bar{e}_p = 0.31$, the poor start to invest in higher education but switch back to the neoclassical regime after one period. Further reductions in $\bar{e}_p$ would require an initial level of wealth above the one of the middle class.

[19]Kuhn et al. (2018) argue that wealth heterogeneity and a differential evolution of asset prices are responsible for the wedge between the evolution in wealth and income inequality. This feature is beyond the research question of this paper but certainly a limitation of models abstracting from different types of assets. Moreover, as we abstract from other channels that increased inequality over the same period such as the evolution of housing prices and technological change, wealth inequality is under-shooting observed levels of inequality. It is also worth noting that in our overlapping generations framework, initial conditions have a long-lasting effect on education during the transition because one period lasts for about 30 years. However, the steady state characteristics remain unaffected by changes in the initial conditions.

V Conclusions

We set up a novel overlapping generations model with three social classes: the rich, the middle class, and the poor. After a certain threshold level of income is surpassed, richer households start to invest in higher education of their children. This, however, reduces their savings and therefore their bequests, which reduces income inequality for at least one generation. The corresponding earlier onset of higher education and therefore faster human capital accumulation of the children of the rich leads to an increase of their incomes. Subsequently, also the middle class and possibly the poor start to invest in higher education. Since different social groups accumulate human capital at different rates, income inequality starts to rise again. This pattern is consistent with the data on bequests, income inequality, and education depending on parental income as described in the introduction. Furthermore, the pattern is also consistent with the view that the main driver of income inequality has been the differential accumulation of wealth in the past but that nowadays a large part of income inequality can be explained by differences in wage incomes based on differences in educational attainment (Atkinson et al., 2011; Meschi and Scervini, 2014; Piketty, 2014). Finally, the implications of our model with respect to rising lifetime income inequality is consistent with the empirical evidence for Germany and the United States over the past decades (Bönke et al., 2015; Guvenen et al., 2017). Our proposed mechanism should be understood as a complementary force to the ones that have already been analyzed in the literature (e.g., the disruptions of World War II, changes in taxation, globalization, and skill-biased technological change) and it gives rise to an additional explanation for the observed U-shaped evolution of income inequality and inheritance flows.

Crucial for the emergence of declining income inequality is that all social classes switch to the high-skill regime. However, the poor face a risk of being disconnected from the growth process. Regarding the feasibility of the regime switch for the poor, the initial distribution of wealth and, thus, initial income inequality is not a good predictor. What matters is the distance between the poor and the middle class. A central policy implication of our framework to reduce income inequality is to invest in public universities or in

education subsidies for the poor, e.g., in the form of stipends and tuition fee waivers (Rauh, 2017).

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Online Appendix

The U-shape of income inequality over the 20th century: The role of education

by Klaus Prettner and Andreas Schaefer

A.1 Lemma 1

(1) Item (i):

If $l = nc$, a member of social class j maximizes (5) subject to (8) and (9). The associated first-order conditions read

$$\frac{1}{c_{j,t}^{nc} - \bar{c}} = \lambda, \tag{A.1}$$

$$\frac{\beta}{c_{j,t+1}^{nc}} = \frac{\lambda}{R}, \tag{A.2}$$

$$\frac{\theta\beta}{b_{j,t+1}^{nc}} = \frac{\lambda n}{R}. \tag{A.3}$$

Combining (A.1) with (A.2) and (A.1) with (A.3) yields

$$c_{j,t+1}^{nc} = \beta R(c_{j,t}^{nc} - \bar{c}), \tag{A.4}$$

$$b_{j,t+1}^{nc} = \frac{\theta}{n} c_{j,t+1}^{nc}. \tag{A.5}$$

Combining the last two expressions with the budget constraint gives item (i).

(2) Item (ii):

If $l = h$, agents maximize (5) subject to (8), (9), and (6). The associated first-order conditions are given by (A.1)-(A.3) for $l = h$ and

$$\frac{\gamma}{e_{j,t}^{h} + \bar{e}_j} = w h_{m,t}^{h} n \lambda. \tag{A.6}$$

From the last expression and (A.1), we obtain

$$e^h_{j,t} = \frac{\gamma}{n}(c^h_{j,t} - \bar{c}) - \bar{e}_j. \tag{A.7}$$

Combining the last expression with (A.4) and (A.5) verifies item (ii).

A.2 Proposition 1

Noting (16), we obtain $e^l_{j,t} \leq 0$, if

$$\begin{aligned} \frac{\gamma I_{j,t}}{w h^h_{m,t} n} &\leq [1 + (1+\theta)\beta]\bar{e}_j + \frac{\bar{c}}{w h^h_{m,t} n}, & \text{(A.8)} \\ \Rightarrow I'_{j,t} &= \frac{w h^h_{m,t} n \bar{e}_j [1 + (1+\theta)\beta]}{\gamma} + \bar{c}. & \text{(A.9)} \end{aligned}$$

A.3 Proposition 2

The regime switch requires $I_{j,*} = (1 - zn)w\bar{e}_j + Rb_{j,*} > I'_{j,t}$, which implies in light of Proposition 1 that

$$b^{nc}_{j,*} > \frac{w h^h_{m,t} n \bar{e}_j}{\gamma R}[1 + (1+\theta)\beta] + \frac{\bar{c}}{R} - \frac{(1 - zn)w\bar{e}_j}{R}. \tag{A.10}$$

Substituting now for $b^{nc}_{j,*}$ by using (23) yields

$$\begin{aligned} & \frac{\theta\beta R^2}{n[1 + (1+\theta)\beta] - \theta\beta R^2}[(1 - zn)w\bar{e}_j - \bar{c}] \\ > \; & \frac{w h^h_{m,t} n \bar{e}_j}{\gamma}[1 + (1+\theta)\beta] + \bar{c} - (1 - zn)w\bar{e}_j. \end{aligned} \tag{A.11}$$

From the last expression, we obtain a minimum level of $\hat{e}^l$ that assures a regime switch in the future

$$\hat{e}^l = \frac{\gamma\bar{c}}{\{\gamma(1 - zn)w - w h^l_{m,t}[n(1 + (1+\theta)\beta) - \theta\beta R^2]\}}, \tag{A.12}$$

such that

$$\hat{e}^h = \frac{\gamma \bar{c}}{\{\gamma(1-zn)w - wh^h_{m,t}[n(1+(1+\theta)\beta) - \theta\beta R^2]\}}, \tag{A.13}$$

and

$$\hat{e}^{nc} = \frac{\gamma \bar{c}}{\{\gamma(1-zn)w - w\bar{e}_m[n(1+(1+\theta)\beta) - \theta\beta R^2]\}}. \tag{A.14}$$

Obviously, $\hat{e}^{nc}$ is constant and a regime switch is possible if $\bar{e}_j > \hat{e}^h$.

If $l = h$, it follows that $\hat{e}^h$ is growing with $h^h_{m,t}$, thus moving the critical level of income to the right, such that a regime switch of classes poorer than the middle class becomes infeasible if the threshold level has become greater than their steady-state level of bequests. Note also that $\hat{e}^{nc}$ and $\hat{e}^h$ exhibit a vertical asymptote at

$$\tilde{h}_{m,t} = \frac{(1-zn)\gamma}{n[1+(1+\theta)\beta] - \theta\beta R^2}. \tag{A.15}$$

A.4 Proposition 3

(i) The ratio between bequests and human capital is constant along the BGP.

Note that

$$\bar{e}_j + e_{j,t} = \frac{\gamma(I^h_{j,t} - \bar{c} + wh^h_{m,t}\bar{e}_j n)}{wh^h_{m,t} n[1+\gamma+(1+\theta)\beta]}. \tag{A.16}$$

Thus

$$\frac{h^h_{j,t+1}}{h^h_{j,t}} = (\bar{e}_j + e_{j,t})\kappa \frac{h^h_{m,t}}{h_{j,t}} = \frac{\gamma\kappa(I^h_{j,t} - \bar{c} + wh^h_{m,t}\bar{e}_j n)}{wh^h_{j,t} n[1+\gamma+(1+\theta)\beta]}. \tag{A.17}$$

Note further that

$$\frac{b^h_{j,t+1}}{h^h_{j,t}} = \frac{\theta\beta R}{n[1+\gamma+(1+\theta)\beta]}\left[\frac{I^h_{j,t} - \bar{c} + wh^h_{m,t} n\bar{e}_j}{h^h_{j,t}}\right] \tag{A.18}$$

$$\Rightarrow \frac{b^h_{j,t+1}}{h^h_{j,t+1}} = \frac{h^h_{j,t}}{h^h_{j,t+1}} \frac{\theta\beta R}{n[1+\gamma+(1+\theta)\beta]}\left[\frac{I^h_{j,t} - \bar{c} + wh^h_{m,t} n\bar{e}_j}{h^h_{j,t}}\right]. \tag{A.19}$$

Combining the last expression with (A.17) yields

$$\frac{b^h_{j,t+1}}{h^h_{j,t}} = \frac{w\theta\beta R}{\gamma\kappa} = const. \tag{A.20}$$

(ii) The evolution of relative inequality. Noting that $x^h_{j,t} = h^h_{j,t}/h^h_{m,t}$, we obtain

$$x^h_{j,t+1} = \frac{\bar{e}_j + e_{j,t}}{\bar{e}_m + e_{m,t}} = \frac{\frac{I^h_{j,t}-\bar{c}}{wh^h_{m,t}} + \bar{e}_j n}{\frac{I^h_{m,t}-\bar{c}}{wh^h_{m,t}} + \bar{e}_m n} \tag{A.21}$$

$$\Rightarrow x^h_{j,t+1} = \frac{\left[(1-zn) + \frac{Rb^h_{j,t}}{wh^h_{j,t}}\right] x^h_{j,t} - \frac{\bar{c}}{wh^h_{m,t}} + \bar{e}_j n}{(1-zn) + \frac{Rb^h_{m,t}}{wh^h_{m,t}} - \frac{\bar{c}}{wh^h_{m,t}} + \bar{e}_m n}. \tag{A.22}$$

Taking into account (A.20), we obtain

$$x^h_{j,t+1} = \frac{\left[(1-zn) + \frac{\theta\beta R^2}{\gamma}\right] x^h_{j,t} - \frac{\bar{c}}{wh^h_{m,t}} + \bar{e}_j n}{(1-zn) + \frac{\theta\beta R^2}{\gamma} - \frac{\bar{c}}{wh^h_{m,t}} + \bar{e}_m n}. \tag{A.23}$$

As $\lim_{t\to\infty} \frac{\bar{c}}{wh^h_{m,t}} = 0$, we obtain from the last expression that

$$x_{j,*} = x_{j,t+1} = x_{j,t} = 1 \tag{A.24}$$

if $\bar{e}_j = \bar{e}_m$ and

$$x_{j,*} = x_{j,t+1} = x_{j,t} = \frac{\bar{e}_j}{\bar{e}_m} \gtrless 1 \tag{A.25}$$

if $\bar{e}_j \gtrless \bar{e}_m$.

(iii) The growth factor of human capital.

From (A.17) and the definition of households' incomes, we obtain

$$\frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa}{n[1+\gamma+(1+\theta)\beta]}\left[(1-zn) + \frac{Rb^h_{j,t}}{wh^h_{j,t}} - \frac{\bar{c}}{wh^h_{j,t}} + \frac{\bar{e}_j n}{x_{j,t}}\right]. \tag{A.26}$$

Combining the last expression with (A.20), we obtain

$$\frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa}{n[1+\gamma+(1+\theta)\beta]}\left[(1-zn)+\frac{\theta\beta R^2}{\gamma}-\frac{\bar{c}}{wh^h_{j,t}}+\frac{\bar{e}_j n}{x_{j,t}}\right], \quad \text{(A.27)}$$

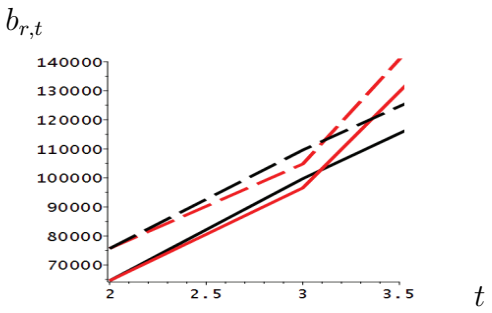
such that, in light of (ii), the growth factor of human capital reads

$$\lim_{t\to\infty}\frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa(1-zn+\bar{e}_m n)+\theta\beta R^2}{n[1+\gamma+(1+\theta)\beta]}. \quad \text{(A.28)}$$

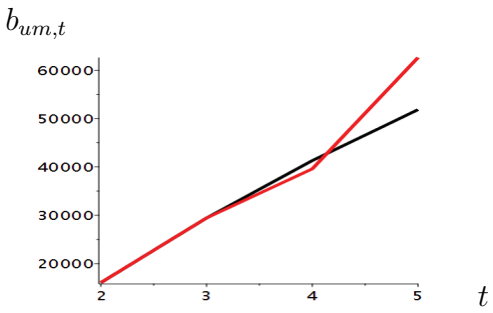
A.5 Evolution of bequests

In Figure A.1 we depict the evolution of bequests and illustrate the argument of Proposition 4. The switch to the high-skill regime reduces bequests because some of household income is diverted to higher education investments for children. This raises rich children's incomes in later periods such that bequests increase again in the future. Dashed lines reflect the scenario of higher inequality. The curves for the lower middle class do not change because their initial level of wealth remains unchanged in this experiment. The poor do not invest in higher education for their offspring at all.

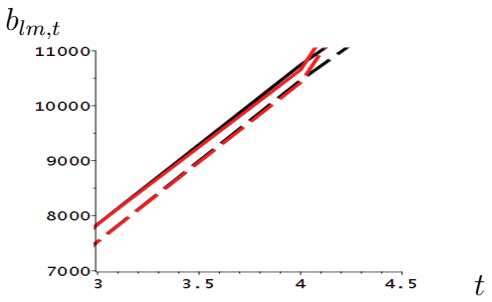
a) rich



b) upper middle class



c) lower middle class



d) poor

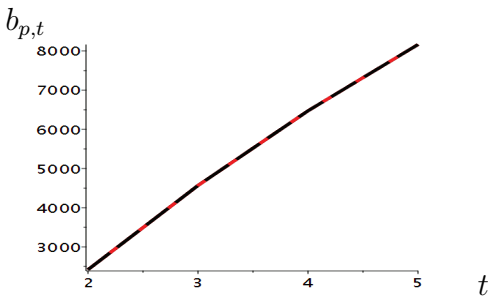


Figure A.1: Evolution of bequests.