Towards a more comprehensive framework for Central Bank communication*

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Abstract

There has been a profound change in the communication strategy of central banks since the Great Recession. Besides offering guidance on the path of the policy rate, central banks have also started releasing news on uncertainty about the economic outlook to the public. Has this enhanced communication strategy, which we define as comprehensive, proven effective in anchoring inflation expectations? To address this question, we let a central bank communicate noisy information on the multiple shocks that hit the economy via its comprehensive strategy. At the same time, agents update their beliefs in a Bayesian way and infer the shocks for which the central bank has been informed about. We show that a comprehensive central bank communication strategy renders certain inflation paths incompatible with equilibrium conditions, facilitating, therefore, the anchoring of inflation expectations.

Keywords: Central Bank communication, monetary policy, inflation, news on uncertainty.

JEL Classification: E31, E52, E58.

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1 Introduction

Over the last two decades, the role of central bank communication has taken centre stage in policy debates. The communication strategies of major central banks have radically shifted towards greater transparency, and wider and deeper engagement with the public. Especially during and after the Great Recession, communication has become a policy tool in its own right. Understanding how communication shapes market outcomes is therefore of high importance.

There is a recent trend among central banks towards greater transparency (Table 1). Specifically, in August 2013, the Bank of England introduced explicit state-contingent forward guidance, and since March 2015 it releases the minutes of its policy meetings and the Inflation Report[1] at the same time as its policy decisions;[2] since 2012, the Federal Reserve adopted an explicit inflation target of 2%, began publishing its members' individual projections for the federal funds target rate (dot plot), introduced a mix of state-contingent and date-based forward guidance, and since 2017 it start publishing fan charts to their economic forecasts and the risk assessment of the FOMC members around projections.[3] In 2001, the Bank of Japan introduced state-contingent guidance, in 2013 it adopted an explicit inflation target of 2%, and in 2015 it stepped up its communication by releasing each Policy Board member's forecast and risk assessment of the economic outlook;[4] in 2013, the European Central Bank offered forward guidance on the future path of its key interest rates, and in 2014 it start publishing more detailed background for its forecasts.[5] The upshot is that announcements about the future path of the policy rate have been enhanced

[1]The Inflation Report contains the projections for GDP and inflation of the MPC committee; and presents these projections in the form of fan charts (with information on the variance and the skew).

[2]See Warsh (2014), Bank of England (2014) and Haldane (2017) for details on the changes of the Bank's communication framework.

[3]In fact, Mester (2016) recommended that the FOMC should publish distributional information and risk assessments of its members; and Chair Yellen used distributional information in her speech at Jackson Hall in 2016 (Yellen, 2016). For a graphical illustration of risk and uncertainty assessments, see, for example, the Summary of Economic Projections that accompany the minutes of the December 2019 FOMC meeting.

[4]See Bank of Japan (2015) for details on how each member's risk assessment is released.

[5]Since December 2013, the ECB staff macroeconomic projections are complemented with fan charts on inflation and GDP forecasts; see European Central Bank (2016) for the timeline of these changes.

with (simultaneous) announcements containing news on uncertainty about the economic outlook. We define the combination of announcements about the future path of the policy rate and information on uncertainty about the economic outlook as comprehensive communication.

In light of these profound changes, we offer a novel characterisation in favour of greater transparency. In our set up, conventional monetary policy targets expected inflation (in deviation from its target) with a Taylor-type rule. Comprehensive communication takes the form of announcements about the central bank's forecasts of expected inflation (information on levels) and the variance of inflation (news on uncertainty). This strategy is closer to the Bank of England's framework, where the members of the Monetary Policy Committee agree on a single fan chart that represents their best collective judgement about the current and future economic conditions. Therefore we abstract from dispersion of beliefs among members of the committee, as it is reflected in the announcements of other central banks. The contribution of this paper is to draw attention to the effectiveness of comprehensive communication in anchoring inflation to its target.

We consider a stochastic, infinite-horizon environment where the central bank receives noisy signals about the state of the economy, and in the absence of any information transmission, there exist multiple inflation paths that support the (unique) real allocation. The multiplicity does not derive from the indeterminacy of a steady state, in which case it is well-known that active policy rules restore determinacy. We take our cue from Nakajima and Polemarchakis (2005), who showed that in finite or infinite horizon stochastic monetary economies, interest rate or money supply policies do not suffice to determine the "distribution" of inflation across date-events.[6] The role of communication is to anchor beliefs to the inflation path closest to, or at the policy target. Information exchange takes place between two rational agents: the central bank and a representative agent. The central bank announces its posterior beliefs to the agent, updating takes place and information sets are equalised. Announcements convey information on the fundamentals, which allow agents to infer that certain inflation paths are not feasible—violate equilibrium conditions—and the dimension of the indeterminacy is reduced.

[6]See Castillo-Martinez and Reis (2019) for a relevant discussion of this type of indeterminacy.

Comprehensive communication is necessary when the state of the economy consists of multiple shocks—here, demand and supply shocks. Announcements containing forecasts of expected inflation lead to an identification problem: they map to a noisy signal at the central bank's information set which might convey information on demand or supply or both shocks. Without exact identification, announcements are not informative and anchoring of inflation fails. Therefore, additional information on higher moments is needed. Our comprehensive framework is related to the work of Hansen et al. (2019), who showed that news on uncertainty, extracted from the Bank of England's Inflation Report, have big effects along the yield curve. In the present setting, we characterise how news on uncertainty can anchor the stochastic path of inflation.

We focus on linear rational expectations equilibria, where inflation consists of two components: the first depends on past and contemporaneous shocks, while the second depends on expectations of future shocks. Multiplicity arises because dependence on some of the shocks is not restricted by equilibrium conditions without information away from priors. The variance of inflation is a linear function of its two components. We define as "fundamental uncertainty" the variance of the first component, which is determined unambiguously with the arrival of information and places a lower bound on the inflation variance; and we define as "reducible uncertainty" the variance of the second component, which depends arbitrarily on expectations of future shocks, and goes to zero with improvements of information over longer horizons. Taking advantage of linearity, information (only) on "fundamental uncertainty" allow agents to solve the identification problem; and forecasts of expected inflation allow agents to back out the signals for which the central bank has been informed. Hence, updating of beliefs about shocks add additional equilibrium restrictions and anchoring takes place.[7]

At each date, the central bank announces its forecasts about fundamental uncertainty over the entire horizon. Identification works through a static and a dynamic channel. The static channel works as follows. The agent compares each element of the announced vector of forecasts with announcements that correspond to different partitions of the central bank's information set, in order to infer if the central bank has received information about

[7] These notions of "uncertainty" are adapted from the work of Hansen et al. (2019).

one or both shocks or any information at all. Subsequently, static identification proceeds through elimination. For example, if each element is consistent with information on both shocks, then it should not be equal to zero (no signals whatsoever) or be consistent with information on only one of the shocks.

Next, in our dynamic framework, the agent needs to identify also the information horizon, as of the current date, for which the central bank has received noisy information, in order to update beliefs about the inflation path. To explain the dynamic channel, we use the following example. Suppose the central bank receives noisy information τ periods ahead every time. Then, announcements up to and including τ periods ahead consist of prior variances plus the non-zero change in variance attributed to the arrival of information in the current period, while announcements from $\tau + 1$ onwards are equal to prior variances only, reflecting the fact that additional information will arrive in the future, but it is not available yet. Dynamic identification proceeds by comparing the non-zero change in variance between announcements up to τ periods ahead and announcements from $\tau+1$ onwards, as of the current date.[8]

Related Literature

Our work is related to the literature on communication and transparency. The seminal work of Morris and Shin (2002) showed that public information can reduce welfare when private information is precise. Angeletos and Pavan (2007) clarified that this arises because of misaligned incentives between individuals and the social planner in the use of information.[9] Gaballo (2016) studied the link between central bank communication and rational inattention, and showed that information can be welfare reducing by increasing the volatility of prices. Bassetto (2019) considered a cheap talk game between agents and the central bank, and showed that communication has social value when the central bank has private information. Here, we abstract from strategic interactions, incomplete infor-

[8]Dynamic identification is related to the methodology of identification through heteroskedasticity (Rigobon, 2003).

[9]Other contributions include Angeletos et al. (2013), Hellwig (2005), Myatt and Wallace (2014), Morris and Shin (2005).

mation and costs of processing information. Our framework follows the methodology of Geanakoplos and Polemarchakis (1982), where rational agents announce posteriors and agreement is reached. To the best of our knowledge, we are the first to study the link between communication and multiplicity of equilibria; and characterise how greater transparency reduces multiplicity.

The literature on multiplicity in rational expectation monetary models is extensive.[10] As we already discussed, we take our cue from Nakajima and Polemarchakis (2005), who index indeterminacy with the distribution of the price of state-contingent nominal bonds. In that context, Adão et al. (2014) and Magill and Quinzii (2014) showed that fixing the term structure of interest rates determines the path of inflation; and Dreze and Polemarchakis (2001) argued that fixing the interest rate or the money supply, and the price of Arrow securities, determines the distribution of inflation.

The notion of comprehensive communication is related to the so-called "Delphic" forward guidance, which Campbell et al. (2012) defined as the announcements of forecast of macroeconomic outcomes and likely monetary policy actions. Our results highlight the importance of "Delphic" guidance in anchoring the path of inflation. Finally, the communication channel we propose is important if one takes into account the increasing evidence that uncertainty has macroeconomic effects (Bloom, 2009; Baker et al., 2016; Fernandez-Villaverde et al., 2011).

The paper is organised as follows. Section 2 presents the basic model, where fluctuations are driven only by TFP; section 3 characterises communication in the basic model; section 4 incorporates demand shocks and characterises comprehensive communication; section 5 discusses extensions; section 6 concludes; and, Appendix A includes derivations.

[10]Other notable contributions include Woodford (1994), Sargent and Wallace (1975), McCallum (1981), Clarida et al. (2000), Cochrane (2011), Ascari et al. (2019), Castillo-Martinez and Reis (2019).

2 The Model

Log-linear environment

Consider a frictionless, cashless, representative agent economy. Time is discrete and extends to infinity. The equilibrium conditions in log-linear form are given by

$$y_t = a_t, \tag{1}$$

$$i_t - E_t[\pi_{t+1}] = E_t[y_{t+1}] - y_t, \tag{2}$$

where y denotes output, a productivity, i the nominal interest rate, π denotes inflation (lowercase variables refer to logs).[11] (1)-(2), respectively, denote market clearing and the Euler equation (stochastic Fisher equation).

The central bank (CB) sets the nominal interest rate according to an interest-rate rule that is known to agents. Let the CB target expected inflation with a zero-inflation target:

$$i_t = \phi E_t[\pi_{t+1}] + v_t, \tag{3}$$

where $\phi \neq 1$, $\phi \geq 0$ and v denotes the monetary policy shock. Eqs. (2)-(3) imply the Euler equation can be rewritten as

$$E_t[\pi_{t+1}] = \frac{1}{\phi - 1}\left(E_t[a_{t+1}] - a_t - v_t\right). \tag{E}$$

Shocks processes and announcements

We assume that log-productivity, a_t, follows a white noise process:

$$a_t = \epsilon_t, \tag{4}$$

where ϵ is i.i.d. with $\epsilon \sim N(0, \sigma_\epsilon^2)$. Also, v is i.i.d. with $v \sim N(0, \sigma_v^2)$. The CB receives information on productivity with noisy signals

$$s_{t+\tau} = a_{t+\tau} + e_{t+\tau}, \tag{5}$$

where $s_{t+\tau}$ refers to a noisy signal about productivity in period $t+\tau$, with $\tau \geq 1$, and e is an i.i.d. *noise* shock with $e \sim N(0, \sigma_e^2)$ and σ_e finite. The two shocks ϵ and e are mutually

[11] Appendix A.1 offers details.

independent, and so are they with the policy shock v. We assume that all agents have common priors on productivity and policy shocks, and the representative agent knows the distribution of noise.

All agents in the economy are Bayesian and, hence, in the presence of a noisy signal about the productivity innovation in $t + \tau$, their expectations of it are

$$E_t\left[a_{t+\tau}\right] = \mu\, s_{t+\tau}, \tag{6}$$

with $\mu = \left(1 + \sigma_e^2/\sigma_\epsilon^2\right)^{-1}$.

We focus on indirect exchange of information, where the CB communications information on productivity innovations through announcements about its systematic component of monetary policy; moreover, the CB makes announcements about its non-systematic component. Let $\bar{\pi}_{t+\tau,t}$ denote the CB's forecast as of t about expected inflation (systematic component) at $t+\tau$, and $v_{t+\tau,t}$ be its announcement as of t about policy shocks (non-systematic component) at $t+\tau$. The CB is transparent with respect to information transmission, so that announcements contain all that there is to know about its information set, without hiding or containing arbitrary information.

The information sets of the representative agent (RA) and the CB in period t are given by

$$\begin{aligned} \Omega_t^{RA} &= \left\{(\ldots,\, a_{t-1}\,,\, a_t),\, (\ldots,\, v_{t+\tau,t}), (\ldots,\, \bar{\pi}_{t+\tau+1,t}, \ldots)\right\}, \quad \tau \geq 1 \\ \Omega_t^{CB} &= \left\{(\ldots,\, a_{t-1}\,,\, a_t),\, (\ldots,\, v_{t+\tau,t}), (\ldots,\, s_{t+\tau,t})\right\}, \quad \tau \geq 1. \end{aligned} \tag{7}$$

The mathematical expectation of a variable X as of date t is taken with respect to information contained in Ω_t, that is, $E_t\left[X\right] = E\left[X|\Omega_t\right]$. In equilibrium, following CB announcements, the information sets of both agents are identical and posterior beliefs coincide.

Linear Rational Expectations Equilibria

We focus on linear rational expectations equilibria (LREE). To this end, we consider conjectures of inflation of the following form:

$$\pi_{t+1} = \sum_{j=0}^{\infty} \theta_j E_{t+1}\left[a_{t+j}\right] + \sum_{j=0}^{\infty} \varpi_j E_{t+1}\left[v_{t+j}\right], \tag{C1}$$

where θ's and ϖ's are time-invariant coefficients that correspond to the productivity and monetary policy shocks respectively. More specifically, the coefficients are determined, if possible, by matching expected inflation generated by conjecture (C1) with the Euler equation (E).

(C1) requires inflation to depend on past and contemporaneous shocks, but also on expectations on future shocks. However, expectations are deviations from the prior mean which, in that case, is zero. Communication perturbs expectations away from its prior mean and, hence, they matters for inflation determination.

Some remarks are in order. First, the representation in (C1) is not unique. Specifically, in Section 5 we consider conjectures that correspond to alternative shocks processes and an interest rate rule that targets current inflation, and show that all our results remain intact. Second, in order to isolate and highlight the role of information transmission in anchoring the distribution of inflation, we restrict the analysis to conjectures that depend only on fundamental and monetary policy shocks, for which the central bank either receives noisy information or can manipulate with its own actions, and show that communication pins down the stochastic path of inflation.[12]

Benchmark of "no communication"

To motivate the importance of information transmission and illustrate our main point, we analyse first the benchmark case of "no communication". The agent's current information set includes current and past realisations of productivity and policy shocks, and agents beliefs coincide with the priors.

Consider the following result.

Proposition 1. *Under the benchmark of "no communication", the stochastic path of inflation is indeterminate, with indeterminacy indexed by the pair* (θ_1, ϖ_1) *.*

[12]We do not index the inflation conjecture with arbitrary white noise processes ("sunspots"). In that case, it is well-known that determinacy obtains by combining (1), (2) with a current inflation targeting rule that satisfies the Taylor principle, and also imposing a terminal condition on the inflation path. See Castillo-Martinez and Reis (2019) for an explanation of the so-called elusive terminal condition; and Cochrane (2011) for a critique of this argument.

The proof is as follows. Take expectations of (C1) as of date t and match coefficients with the Euler equation (E). It follows that

$$\theta_0 = \varpi_0 = -\frac{1}{\phi - 1} \tag{8}$$

and coefficients (θ_j, ϖ_j) for all $j \geq 1$ are not determined given that expectations of shocks are consistent with the prior mean and, hence, these coefficients vanish when we match expectations of (C1) with (E).

The stochastic path of inflation reduces to

$$\pi_{t+1} = -\frac{1}{\phi - 1}\left(a_t + v_t\right) + \theta_1\, a_{t+1} + \varpi_1 v_{t+1}, \tag{9}$$

where the pair (θ_1, ϖ_1) takes arbitrary values. Hence, the distribution of inflation across date-events is indeterminate.

Communication policies

The objective of communication is to anchor inflation expectations and determine the stochastic path of inflation. Since productivity is the only driver in the basic model—apart from monetary policy shocks—, we provide the following definition of "simple communication", as follows:

Definition 1. *(Simple Communication)*

1. *(Commitment) Binding announcements about deviations from systematic policy up to τ periods ahead, $(v_{t+1,t}, ..., v_{t+\tau,t})$, as of date t;*

2. *(Forecasts) Non-binding announcements about the systematic policy component for the entire horizon, $(\bar{\pi}_{t+1,t}, ..., \bar{\pi}_{t+\tau,t}, ...)$, as of date t.*

Taken together, commitments about deviations from inflation targeting and forecasts about expected inflation amount to announcements about the path of the expected nominal interest rate, $(i_t, E_t\left(i_{t+1}\right), E_t\left(i_{t+2}\right), ...)$. Following Campbell et al. (2012), the distinction between commitment and forecasts in our set-up map to their distinction between Odyssean and Delphic forward guidance respectively.

Information on productivity innovations is transmitted via the interest rate rule and the stochastic Fisher equation. In particular, following an announcements from the central bank, the agents update beliefs about productivity through (E), and backs out the exact value of noisy signals. Inference in our set up requires Delphic guidance and cannot be made possible with Odyssean commitments.[13]

The previous discussion leads to the following result.

Proposition 2. *Under simple communication, the agent backs out the noisy signals for which the central bank has been informed.*

Suppose the central bank receives noisy information about productivity innovations τ periods ahead. Consider the following announcements:

$$(v_{t+1,t}, ..., v_{t+\tau,t}) \, ,$$

and

$$(\bar{\pi}_{t+1,t}, ..., \bar{\pi}_{t+\tau+1,t}, 0, 0, ...) \, ,$$

with Delphic announcements up to, and including $\tau + 1$ different from zero, and with zeros indicating that information beyond this point coincides with priors. Optimality conditions (E), as of date t, reduce to

$$\bar{\pi}_{t+1,t} = \frac{1}{\phi - 1} [E_t[a_{t+1}] - a_t - v_t], \tag{10}$$

$$\vdots$$

$$\bar{\pi}_{t+\tau+1,t} = \frac{1}{\phi - 1} \left[E_t[a_{t+\tau+1} - a_{t+\tau}] - v_{t+\tau,t}\right], \tag{11}$$

$$\bar{\pi}_{t+\tau+2,t} = \bar{\pi}_{t+\tau+3,t} = \ldots = 0. \tag{12}$$

Combining announcements with (10) and (6), agents update beliefs on productivity innovations at $t + 1$ and back out the signal s_{t+1}. Subsequently, iterating forward up to τ

[13] However, see Nakamura and Steinsson (2018), who characterised the effect of communication on long-run expectations in a stylised New Keynesian model, where monetary policy shocks transmit information on fundamentals.

periods ahead, and combining it with announcements and (6), they back out all remaining signals $(s_{t+2}, ..., s_{t+\tau})$. Moreover, it follows from (11) that expected inflation $\tau + 1$ periods ahead is consistent with $E_t[a_{t+\tau+1}] = 0$, and announcements $\bar{\pi}_{t+\tau+1}$ have to be nonzero and consistent with signal $s_{t+\tau}$ and Odyssean commitment $v_{t+\tau,t}$. Finally, it follows from (12) and the commitment horizon that beliefs on productivity innovations beyond τ periods ahead coincide with priors. It is important to note that this argument does not require deviations from systematic policy—however, at the end of section 3, we argue that in the presence of the zero lower bound, these deviations are important.

3 Simple communication

Suppose the central bank possesses noisy information about productivity τ periods ahead; and, at the beginning of each period, it receives additional noisy signals so that the information horizon is kept constant to τ periods ahead. Announcements consists of commitments about non-systematic deviations up to τ periods ahead, and (nonzero) forecasts up to $\tau + 1$ periods ahead. The central bank refines its own forecasts each period, keeping its non-zero forecasts horizon to $\tau + 1$ periods, and makes additional commitments, keeping its commitment horizon to τ periods. We speak of asymptotic communication, when $\tau \to \infty$ and the central bank wants to transmit information about the entire path of innovations; in that case, it refines its forecasts every period.

Let us start with announcements involving commitments to deviate from systematic policy in the future.

Proposition 3. *Under announcements $(v_{t+1,t}, ..., v_{t+\tau,t})$, the stochastic path of inflation is indeterminate, with indeterminacy indexed by the pair $(\theta_1, \varpi_{\tau+1})$. As $\tau \to \infty$, indeterminacy is indexed by θ_1.*

The argument is as follows. Suppose the central bank, as of date t, commits to a sequence of policy shocks $(v_{t+1,t}, ..., v_{t+\tau,t})$. Combining these announcements with (E) and with expectation, as of date t, of (C1), and matching coefficients, it follows that

$$\theta_0 = \varpi_0 = -\frac{1}{\phi - 1} \text{ and } \varpi_1 = \cdots = \varpi_\tau = 0;$$

and state contingent inflation is equal to

$$\pi_{t+1} \ = -\frac{1}{\phi-1}\,(\,a_t+v_t) \ + \ \theta_1\,a_{t+1} \ + \varpi_{\tau+1}v_{t+\tau+1}.$$

Announcements perturb expectations of agents about future policy shocks away from priors and add additional equilibrium restrictions: $\varpi_1 = \cdots = \varpi_\tau = 0$. At $t+1$, the CB makes one additional commitment to keep the information horizon constant. However, as of date t, $E_t\left[v_{t+\tau+1}\right] = 0$, since there is no announcement at that point, and equilibrium does not restrict $\varpi_{\tau+1}$. Asymptotic communication renders future expectations of policy shocks irrelevant, but does not anchor inflation expectations, which are indexed by θ_1.

Consider the following result.

Proposition 4. *Under announcements* $(v_{t+1,t}, ..., v_{t+\tau,t})$ *and* $(\bar{\pi}_{t+1,t}, ..., \bar{\pi}_{t+\tau+1,t}, 0, 0, ...)$*, with* $\bar{\pi} \neq 0$*, the stochastic path of inflation is indexed by the pair* $(\theta_{\tau+1}, \varpi_{\tau+1})$*. As* $\tau \to \infty$*, the stochastic path of inflation is determined.*

As before, announcements perturb expectations away from priors and add equilibrium restrictions:

$$\theta_0 \ = \ \varpi_0 \ = \ -\frac{1}{\phi-1}, \ \ \theta_1 \ = \ \frac{1}{\phi-1}, \ \ \theta_2 = \cdots = \theta_\tau = 0, \ \ \varpi_1 = \cdots = \varpi_\tau = 0.$$

The equilibrium restrictions $\theta_1 = 1/(\phi-1)$, $\theta_2 = \cdots = \theta_\tau = 0$ are derived from updating beliefs about productivity, and the matching of coefficients. The stochastic path of inflation is equal to

$$\pi_{t+1} \ = -\frac{1}{\phi-1}\,(\,a_t+v_t) \ + \frac{1}{\phi-1}a_{t+1} \ + \theta_{\tau+1}\,E_{t+1}[a_{t+\tau+1}] \ + \varpi_{\tau+1}v_{t+\tau+1},$$

with $E_{t+1}[a_{t+\tau+1}] \neq 0$, since forecasts about expected inflation are updated at each date, so that the information horizon is kept constant at τ periods ahead. Hence, state contingent inflation depends on the arbitrary pair $(\theta_{\tau+1}, \varpi_{\tau+1})$.

As $\tau \to \infty$, equilibrium restrictions modify to

$$\theta_0 \ = \ \varpi_0 \ = \ -\frac{1}{\phi-1}, \ \ \theta_1 \ = \ \frac{1}{\phi-1}, \ \ \theta_2 = \cdots = \theta_\tau = \cdots = 0, \ \ \varpi_1 = \cdots = \varpi_\tau = \cdots = 0,$$

and inflation reduces to

$$\pi_{t+1} \ = -\frac{1}{\phi-1}\,(\,a_t+v_t) \ + \frac{1}{\phi-1}a_{t+1}. \tag{13}$$

Asymptotic communication renders all future expectations of productivity innovations and policy shocks irrelevant, so that it pins down the stochastic path of inflation.

Remark 1. *In section 5, we show that the results apply intact to interest rate rules that target current inflation and to alternative shock processes.*

Myopia

The determination of the stochastic path of inflation requires the communication horizon to match the planning horizon of agents. This requirement is restrictive when agents are infinitely lived. A more appropriate framework requires equilibria that feature "myopia". Consider the following subclass of (C1):

$$\pi_{t+1} = \theta_0 a_t + \sum_{j=1}^{\infty} \tilde{\theta}_j E_{t+1}\left[a_{t+j}\right] + \varpi_0 v_t + \sum_{j=1}^{\infty} \tilde{\omega}_j E_{t+1}\left[v_{t+j}\right], \tag{C2}$$

with $\tilde{\theta}_j = \delta^{j-1}\theta_j$, $\tilde{\omega}_j = \delta^{j-1}\varpi_j$, for $j \geq 1$, and $|\delta| < 1$, so that coefficients of expectation terms converge to zero. All the previous results apply intact to (C2.) However, with myopic beliefs, communication finite periods ahead is justifiable on the grounds that the influence of arbitrary future expectations on state contingent inflation is negligible.

Zero lower bound

The analysis so far has ignored the zero lower bound; and in that case, keeping inflation close to its target requires only announcements which contain forecasts about expected inflation (deviations from systematic policy are not relevant, so we can as well set them to zero, $v_t = 0$ for all paths).

To incorporate the zero lower bound, we modify (3) as follows:

$$i_t = \max\left\{\phi E_t[\pi_{t+1}] + v_t, 0\right\}. \tag{14}$$

If the lower bound binds, then the ability of the central bank to communicate its forecasts via its interest rate rule, and anchor the path of inflation, is limited. Here, we characterise how deviations from systematic policy can keep the policy rate away form the lower bound, in paths that it would have been binding under pure expected inflation targeting ($v_t = 0$). The central bank's commitment policy with respect to these deviations is path-dependent.

Without loss of generality, let us assume that the central bank receives noisy information for the entire future, as of date zero ($\tau \to \infty$). Stochastic paths where the zero bound binds under $v_t = 0$, require

$$\frac{\phi}{\phi - 1}\Big(E_t\left[a_{t+1}\right] - a_t\Big) < 0, \tag{15}$$

which is derived by substituting (E) into (14), and setting $v_t = 0$. Alternatively, credible promises to deviate from pure expected inflation targeting succeed in keeping the policy rate away from the zero bound if and only if

$$\frac{1}{\phi - 1}\Big\{\phi(E_t\left[a_{t+1}\right] - a_t) - v_t\Big\} > 0, \tag{16}$$

which is derived similarly to (15).

Starting from date zero, and considering all possible stochastic paths in the future, the optimal commitment policy is as follows. For those paths that satisfy (15), the central bank makes credible promises, as of date zero, to deviate from expected inflation targeting according to (16); while for those paths that do not satisfy (15), it commits to zero deviations from expected targeting. The commitment policy is clearly path-dependent. This argument resonates with Eggertsson and Woodford (2003)—albeit in an environment without sticky prices—who argued that in the presence of the zero lower bound, the optimal commitment policy is history-dependent; and also with Krugman (1998), who argued that exiting the liquidity trap requires the central bank to promise to be irresponsible in the future.

4 Comprehensive communication

We extend the set-up to incorporate demand shocks as well. As before, we continue assuming that only the central bank receives noisy information about shocks. To simplify the argument, we abstract from monetary policy shocks, so that announcements about the expected path of inflation are equivalent to announcements about the expected path of interest rates.

Consider a white noise preference (demand) shock η, with $\eta \sim N(0, \sigma_\eta^2)$. The central bank receives either a noisy signal on a or η, or two independent noisy signals on a and η, which, in turn, are combined to a composite noisy signal on $a - \eta$. The information set of the central bank modifies to

$$\Omega_t^{CB} = \{(\ldots, a_{t-1}, a_t), (\ldots, \eta_{t-1}, \eta_t), (\ldots, s_{t+\tau,t})\}, \quad \tau \geq 1, \tag{17}$$

where $s_{t+\tau,t}$ is a noisy signal as of date t about shocks at $t+\tau$.[14]

The Euler equation and (C1), respectively, modify to

$$E_t[\pi_{t+1}] = \frac{1}{\phi - 1}[E_t[a_{t+1} - \eta_{t+1}] - (a_t - \eta_t)] \tag{E2}$$

and

$$\pi_{t+1} = \sum_{j=0}^{\infty} \theta_j E_{t+1}[a_{t+j}] + \sum_{j=0}^{\infty} \kappa_j E_{t+1}[\eta_{t+j}]. \tag{C4}$$

The necessity of a more comprehensive framework for central bank communication arises because announcements about the expected path of the nominal interest rate lead to an identification problem. To see this, consider announcements $(\bar{\pi}_{t+1,t}, ..., \bar{\pi}_{t+\tau+1,t}, 0, 0, ...)$, with $\bar{\pi} \neq 0$. Then, (E2) reduce to

$$\bar{\pi}_{t+1,t} = \frac{1}{\phi - 1}[E_t[a_{t+1} - \eta_{t+1}] - (a_t - \eta_t)], \tag{18}$$

$$\vdots$$

$$\bar{\pi}_{t+\tau+1,t} = \frac{1}{\phi - 1} E_t[(a_{t+\tau+1} - \eta_{t+\tau+1}) - (a_{t+\tau} - \eta_{t+\tau})], \tag{19}$$

$$\bar{\pi}_{t+\tau+2,t} = \bar{\pi}_{t+\tau+3,t} = \ldots = 0. \tag{20}$$

[14]We continue assuming that each shock is i.i.d. and normally distributed with mean zero and finite variance; all shocks are independent with each other; and, as before, the representative agent has common priors with the central bank and knows the distribution of noise.

Since $\bar{\pi} \neq 0$, announcements unambiguously reveal to the agent that the central bank has received noisy information different from priors. However, and contrary to the previous section, the agent can not back out the value of the noisy signal by combining the announcement with (18)-(20). The reason is that the announcement do not map uniquely to the "type" of information that is available in the central bank's information set. For example, announcements might be consistent with noisy information on a or on η or on both shocks for some periods, and only on one of the shocks for the rest of the communication horizon, and so on. Thus, (18)-(20) allow for multiple possibilities, and exact identification fails. However, exact identification is required to pin down the pairs (θ_j, κ_j), for $j \geq 0$, in (C4) and determine the path of inflation. (Imposing the additional equilibrium restriction $\theta_j = -\kappa_j$, implies that inflation depends arbitrarily on $a - \eta$, rather than on each shock separately. Thus, exact identification is not required and following (18)-(20), anchoring proceeds as in Section 3. However, this is an unappealing solution because anchoring of beliefs over one shock, implies anchoring of beliefs over all other shocks; or equivalently, the fact that the central bank has limited information about the fundamentals of the economy is not relevant for monetary policy design.)

To overcome this identification problem, we propose the following characterisation.

Proposition 5. *(Comprehensive communication) Effective communication requires to supplement forecasts about the expected path of inflation with forecasts about the variance of inflation (news on uncertainty).*

Suppose the central bank receives noisy (independent) signals about both shocks τ periods ahead; and, at the beginning of each period, it receives additional noisy signals so that the information horizon is kept constant to τ periods ahead. Announcing forecasts about the expected path of inflation (interest rates), $(\bar{\pi}_{t+1,t}, ..., \bar{\pi}_{t+\tau+1,t}, 0, 0, ...)$, with $\bar{\pi} \neq 0$, lead to an identification problem. The objective is to supplement these announcements with forecasts about the variance of inflation. To that end, consider the following thought experiment. Suppose the central bank, as of date t, were to communicate information *directly*, by announcing noisy composite signals on $a - \eta$—no identification problem with direct communication—and rational agents were to update beliefs about shocks accordingly. Then, following the argument in Section 3, the path of inflation would have been

$$\pi_{t+1}=-\frac{1}{\phi-1}\left(a_t-\eta_t\right)+\frac{1}{\phi-1}\left(a_{t+1}-\eta_{t+1}\right)+\theta_{\tau+1}E_{t+1}\left[a_{t+\tau+1}\right]+\kappa_{\tau+1}E_{t+1}\left[\eta_{t+\tau+1}\right],$$

with $\theta_0=-\kappa_0=-1/\left(\phi-1\right)$, $\theta_1=-\kappa_1=1/\left(\phi-1\right)$, and the pair $\left(\theta_{\tau+1},\kappa_{\tau+1}\right)$ is arbitrary since expectations are perturbed away from priors because of the updating of beliefs. The conditional variance, as of date t, yields

$$Var_t\left[\pi_{t+1}\right]=\underbrace{\left(1/\left(\phi-1\right)\right)^2 Var_t\left(a_{t+1}-\eta_{t+1}\right)}_{\text{“fundamental uncertainty”}}+\underbrace{Var_t\left(\theta_{\tau+1}E_{t+1}\left[a_{t+\tau+1}\right]+\kappa_{\tau+1}E_{t+1}\left[\eta_{t+\tau+1}\right]\right)}_{\text{“reducible uncertainty”}}. \tag{21}$$

We decompose the conditional variance into two components: “fundamental uncertainty” (FU, henceforth) and “reducible uncertainty” (RU, henceforth). The former is determined unambiguously with the arrival of noisy information at date t about shocks at $t+1$, and imposes a lower bound on inflation volatility; while the latter is a function of the (non-random) arbitrary pair $\left(\theta_{\tau+1},\kappa_{\tau+1}\right)$ and reduces to zero as $\tau\to\infty$.[15]

Continuing with our thought experiment, the path of inflation at $t+j$, $j\geq 2$, would have been

$$\pi_{t+j}=-\tfrac{1}{\phi-1}\left(a_{t+j-1}-\eta_{t+j-1}\right)+\tfrac{1}{\phi-1}\left(a_{t+j}-\eta_{t+j}\right)+\theta_{\tau+1}E_{t+j}\left[a_{t+\tau+j}\right]+\kappa_{\tau+1}E_{t+j}\left[\eta_{t+\tau+j}\right],$$

with expectations determined by composite signals received at $t+j$ about $t+\tau+j$. The expression for FU at $t+j$, for $2\leq j\leq\tau$, as of date t, is equal to

$$\text{FU}_{t+j,t}=\left(\frac{1}{\phi-1}\right)^2\left[Var_t\left(a_{t+j-1}-\eta_{t+j-1}\right)+Var_t\left(a_{t+j}-\eta_{t+j}\right)\right]; \tag{22}$$

at $t+j$, for $j=\tau+1$, as of date t, is equal to

$$\text{FU}_{t+j,t}=\left(\frac{1}{\phi-1}\right)^2\left[\underbrace{Var_t\left(a_{t+\tau}-\eta_{t+\tau}\right)}_{t+j-1}+\underbrace{\left(\sigma_\epsilon^2+\sigma_\eta^2\right)}_{t+j}\right]; \tag{23}$$

[15] The notation $Var\left(X\right)$ denotes the posterior variance of random variable X, conditional on a noisy signal. FU is the variance of $a_{t+1}-\eta_{t+1}$, conditional on a composite noisy signal received at date t; and is equal to the inverse of the sum of the signal's and prior's precision. RU is the variance of the sum of the expected value of each shock at $t+2$, conditional on a composite signal received at $t+1$. Updating requires that each term is computed as in (6). Hence, RU is equal to the variance of the composite signal as of date t. The covariance between FU and RU is zero because of independence.

and at $t+j$, for $j > \tau+1$, as of date t, is equal to

$$\mathrm{FU}_{t+j,t} = \left(\frac{1}{\phi - 1}\right)^2 \Big[\underbrace{\left(\sigma_\epsilon^2 + \sigma_\eta^2\right)}_{t+j-1} + \underbrace{\left(\sigma_\epsilon^2 + \sigma_\eta^2\right)}_{t+j}\Big]. \tag{24}$$

Taken together, $\{\mathrm{FU}_{t+j,t}\}_{j=2}^{\infty}$ is decomposed into a term attributed to shocks at $t+j-1$, and into another term attributed to shocks at $t+j$. Note that the $t+j$ term of (23) and the entire path of (24) are determined by prior variances. This is because the information horizon for which the central bank receives noisy signals is always τ periods ahead as of the current date t. This concludes our thought experiment.

Consider the following set of announcements:

$$\left(\bar{\pi}_{t+1,t}, .., \bar{\pi}_{t+\tau+1,t}, 0, 0, ..\right), \ \left(\mathrm{FU}_{t+1,t}, \mathrm{FU}_{t+2,t}, .., \mathrm{FU}_{t+j,t}, ..\right). \tag{25}$$

Static identification proceeds through elimination of alternatives. Upon observing $\mathrm{FU}_{t+1,t} = (1/(\phi-1))^2\, Var_t\left(a_{t+1} - \eta_{t+1}\right)$, the agent infers that the central bank has received noisy signals, otherwise $\mathrm{FU}_{t+1,t} = 0$; and this information maps to both shocks, otherwise $\mathrm{FU}_{t+1,t} = (1/(\phi-1))^2\, Var_t\left(a_{t+1}\right)$ or $\mathrm{FU}_{t+1,t} = (1/(\phi-1))^2\, Var_t\left(\eta_{t+1}\right)$. Subsequently, elimination proceeds recursively. In particular, let us start with $\mathrm{FU}_{t+j,t}$, $j=2$, given by expression (22). Inference on the first term has been established upon observing $\mathrm{FU}_{t+1,t}$. Next, the agent juxtaposes the second term with the following alternatives: zero, which denotes no arrival of noisy signals whatsoever; $(1/(\phi-1)^2)\,\sigma_\epsilon^2$ or $(1/(\phi-1)^2)\,\sigma_\eta^2$, which denotes the arrival of noisy signals at $t+1$ about either productivity or demand shocks next period; and $(1/(\phi-1)^2)\,Var_t\left(a_{t+2}\right)$ or $(1/(\phi-1)^2)\,Var_t\left(\eta_{t+2}\right)$, which denotes the arrival of noisy signals at date t about either productivity or demand shocks at $t+2$. Hence, the agent infers that the central bank has or will receive information about both shocks. The argument with respect to $\mathrm{FU}_{t+j,t}$, $j \geq 3$, is similar.

Dynamic identification proceeds by comparing the change in variances between announcements. To see this, let us express (22)-(24) as follows:

$$\mathrm{FU}_{t+2,t} = \mathrm{FU}_{t+1,t} + (1/(\phi-1))^2 \Big[\underbrace{\left(\sigma_\epsilon^2 + \sigma_\eta^2\right) + \Delta V_{t+2,t}}_{t+2}\Big], \tag{26}$$

$$\mathrm{FU}_{t+j,t} = (1/(\phi-1))^2 \Big[\underbrace{\left(\sigma_\epsilon^2 + \sigma_\eta^2\right) + \Delta V_{t+j-1,t}}_{t+j-1} + \underbrace{\left(\sigma_\epsilon^2 + \sigma_\eta^2\right) + \Delta V_{t+j,t}}_{t+j}\Big], \ 3 \leq j \leq \tau, \tag{27}$$

$$\text{FU}_{t+\tau+1,t} = (1/(\phi-1))^2 \left[\underbrace{\left(\sigma_\epsilon^2+\sigma_\eta^2\right) + \Delta V_{t+\tau,t}}_{t+j-1} + \underbrace{\left(\sigma_\epsilon^2+\sigma_\eta^2\right)}_{t+j} \right],\ j=\tau+1, \tag{28}$$

$$\text{FU}_{t+j,t} = (1/(\phi-1))^2 \left[\underbrace{\left(\sigma_\epsilon^2+\sigma_\eta^2\right)}_{t+j-1} + \underbrace{\left(\sigma_\epsilon^2+\sigma_\eta^2\right)}_{t+j} \right],\ j>\tau+1, \tag{29}$$

which are obtained by adding and subtracting $\left(\sigma_\epsilon^2+\sigma_\eta^2\right)$ in (22)-(24), that is, $\Delta V_{t+j,t} \equiv Var_t\left(a_{t+j}-\eta_{t+j}\right) - \left(\sigma_\epsilon^2+\sigma_\eta^2\right)$, for $2 \le j \le \tau$. The term $\Delta V_{t+j,t}$ denotes the change in variance at $t+j$ as of date t, relative to the prior distribution. The identification argument relies on the condition $\Delta V_{t+j,t} \neq 0$ (heteroskedasticity), so that announcements up to and including τ periods ahead are away from priors, while announcements from $\tau+1$ onwards are consistent with prior variances. This is evident by observing (26)-(29). The upshot is that heteroskedasticity between announcements allow the central bank to communicate its information horizon effectively.

Next, the agent backs out the value of the signals from $(\bar{\pi}_{t+1,t},..,\bar{\pi}_{t+\tau+1,t},0,0,..)$, following the argument of Proposition 2, update beliefs about shocks accordingly, and the argument of Section 3 applies. Finally, note that the linearity structure of the equilibrium separates "reducible" from "fundamental" uncertainty. Hence, it allows us to construct announcements (25) based only on the latter type of uncertainty.

5 Extensions

Current inflation targeting

The analysis extends to interest rate rules that target current inflation:

$$i_t = \phi\pi_t + v_t,\ \phi>0, \tag{30}$$

with a zero inflation target. Combining (30) with (2), yields

$$E_t\left[\pi_{t+1}\right] = \phi\,\pi_t - \left(E_t\left[a_{t+1}\right] - a_t - v_t\right), \tag{E3}$$

and taking into account (E3), the inflation conjecture (C1) modifies as follows:

$$\pi_{t+1} = \phi\pi_t + \sum_{j=0}^{\infty}\theta_j E_{t+1}\left[a_{t+j}\right] + \sum_{j=0}^{\infty}\varpi_j E_{t+1}\left[v_{t+j}\right]. \tag{C5}$$

Under the benchmark of "no communication", the path of inflation is equal to

$$\pi_{t+1} = \phi\pi_t + a_t + v_t + \theta_1 a_{t+1} + \varpi_1 v_{t+1},$$

with $\theta_0 = \varpi_o = 1$ by matching the expectation, as of date t, of (C5) with (E3). As before, the stochastic path of inflation depends on the arbitrary pair (θ_1, ϖ_1).

The result of Proposition 4 apply, and the stochastic path of inflation reduces to

$$\pi_{t+1} = \phi\pi_t + a_t + v_t - a_{t+1} + \theta_{\tau+1} E_{t+1}[a_{t+\tau+1}] + \varpi_{\tau+1} v_{t+\tau+1},$$

with $\theta_0 = \varpi_o = 1$, $\theta_1 = -1$; and as $\tau \to \infty$, the path of inflation reduces to

$$\pi_{t+1} = \phi\pi_t + a_t + v_t - a_{t+1}. \tag{31}$$

The path of inflation is unique up to the initial price level. Moreover, if $\phi > 1$, inflation grows unboundedly large absent a terminal condition that requires inflation to converge to its target. However, explosive solutions are well-defined equilibria in our set up since the transversality condition rules out real but not nominal explosions (see Cochrane, 2011, for a detailed explanation of this argument).

Shock processess

The analysis extends to non-zero mean processes, $a = \epsilon \sim N(\mu, \sigma_\epsilon^2)$, where $\mu \neq 0$ and finite. Conjecture (C1) modifies as follows:

$$\pi_{t+1} = \sum_{j=0}^{\infty} \theta_j \left(E_{t+1}\left[\epsilon_{t+j}\right] - \mu\right) + \sum_{j=0}^{\infty} \varpi_j E_{t+1}\left[v_{t+j}\right]. \tag{C6}$$

Setting $\tilde{\epsilon} = \epsilon - \mu$, (C6) reduces to

$$\pi_{t+1} = \sum_{j=0}^{\infty} \theta_j E_{t+1}\left[\tilde{\epsilon}_{t+j}\right] + \sum_{j=0}^{\infty} \varpi_j E_{t+1}\left[v_{t+j}\right], \tag{C7}$$

and by adding and subtracting μ to (E), the analysis applies intact to (C7).

Alternatively, suppose productivity innovations follows an AR(1) process

$$a_t = \gamma a_{t-1} + \epsilon_t\,,\ 0 < \gamma < 1, \tag{32}$$

where ϵ is i.i.d. with $\epsilon \sim N(0, \sigma_\epsilon^2)$. Conjecture (C1) modifies as follows:

$$\pi_{t+1} \;=\; \theta_0 a_t + \sum_{j=1}^{\infty} \theta_j \left(E_{t+1}\left[a_{t+j}\right] - \gamma E_{t+1}\left[a_{t+j-1}\right]\right) + \sum_{j=0}^{\infty} \varpi_j E_{t+1}\left[v_{t+j}\right], \tag{C8}$$

and substituting (32) into (C8), yields

$$\pi_{t+1} \;=\; \theta_0 a_t + \sum_{j=1}^{\infty} \theta_j E_{t+1}\left[\epsilon_{t+j}\right] + \sum_{j=0}^{\infty} \varpi_j E_{t+1}\left[v_{t+j}\right]. \tag{C9}$$

Under no communication, and matching expectations of (E) and (C9), yields

$$\theta_0 = -\frac{1-\gamma}{\phi - 1}, \;\; \varpi_0 = -\frac{1}{\phi - 1}, \tag{33}$$

and state contingent inflation is indexed by (θ_1, ϖ_1). The derivation of (33) requires the addition and subtraction of γa_t to (E), and then do the matching of expectations.

6 Conclusion

We have developed a theoretical framework to study the role of comprehensive communication in anchoring inflation expectations. Communication of news on uncertainty about the future economic outlook (high dimensional signals) provide an effective tool for central baks to steer market expectations to their targeted path. This channel is particularly important during times of crisis and market distress, where uncertainty about the future outlook increases and traditional communication tools are not effective.

Turning to future work, it would be interesting to investigate the robustness of our results in a set-up where the stochastic path of inflation affects the total allocation of resources and the welfare of agents in the economy. In that case, information has a dual role. On the one hand, it allows agents to resolve the identification problem, as discussed in this paper, and on the other hand it has social value by affecting the allocation of resources and welfare of agents.

A Appendix

A.1 The economy

The frictionless economy in Section 2 corresponds to agents having preferences given by

$$E_{-1} \sum_{t=0}^{\infty} e^{\eta_t} \beta^t \left(\log\left(C_t\right) - \frac{1}{1+\zeta} N_t^{1+\zeta} \right), \tag{34}$$

where C_t denotes consumption in period t, N_t denotes employment in t, $\beta \in (0,1)$ denotes the agents' discount factor, and $\zeta > 0$ denotes the inverse "Frisch" labor elasticity and η is an i.i.d. preference shock with $\eta \sim N(0, \sigma_\eta^2)$.

Agents face a sequence of budget constraints given by

$$P_t\, C_t + Q_t\, B_{t+1} = B_t + W_t\, N_t + \Psi_t\,, \tag{35}$$

where P_t denotes the consumption-good price in period t, B_{t+1} denotes holdings of a nominal riskless bond (in zero net supply) purchased in t and maturing in $t+1$, Q_t denotes the nominal bond price, W_t denotes the nominal wage, and Ψ_t denotes the firm's profits.

A perfectly competitive firm operates a linear technology given by

$$Y_t = A_t\, N_t\,, \tag{36}$$

where Y_t denotes production and A_t denotes productivity. Profits are given by

$$\Psi_t = P_t\, Y_t - W_t\, N_t\,. \tag{37}$$

A monetary authority sets the inverse nominal bond price (nominal interest rate) to target expected inflation. In a rational expectations equilibrium under an expected inflation targeting rule, agents solve their problems, which we specify below, and markets clear at the stated prices, that is $Y_t = C_t$, $N_t^d = N_t^s$, and $B_{t+1} = 0$ for all t with $B_0 = 0$.

Given $B_0 = 0$ agents decide how much to consume, save in the nominal bond, and work to maximize their expected utility (34) subject to the sequence of budget constraints (35) and a standard no-Ponzi-scheme constraint. Their optimality conditions are

$$N_t^{\zeta} = \frac{W_t}{P_t\, C_t} \tag{38}$$

$$Q_t \;=\; \beta\, E_t\left[e^{\eta_{t+1}-\eta_t}\frac{1}{\Pi_{t+1}}\frac{C_t}{C_{t+1}}\right]. \tag{39}$$

Equation (38) is the usual intratemporal labor supply condition and (39) is the Euler equation.

Firms maximise profits and they accommodate any labor supplied at a wage given by

$$W_t \;=\; P_t\, A_t\,. \tag{40}$$

We restrict attention to linear rational expectations equilibria and, to this end, we consider an interest-rate rule given by

$$i_t \;=\; -\,\log\beta \;+\; \phi\, E_t\left[\pi_{t+1}-\overline{\pi}\right]+v_t, \tag{41}$$

where we define $i_t \;\equiv\; -\,\log Q_t$ and let π_t denote log-inflation, defined as $\pi_t \;\equiv\; p_t \;-\; p_{t-1}$, $\overline{\pi}$ the inflation target and v are i.i.d. policy shocks with $v \sim N(0, \sigma_v^2)$.

The optimality conditions (38)-(40), technology (36) and market clearing, in log-linear form are

$$\zeta n_t \;=\; w_t \;-\; p_t \;-\; c_t, \tag{42}$$

$$c_t \;=\; const \;-\; i_t \;+\; E_t\left[c_{t+1} \;+\; \pi_{t+1}\right]+\eta_t - E_t\left[\eta_{t+1}\right], \tag{43}$$

$$w_t \;=\; a_t \;+\; p_t, \tag{44}$$

$$y_t \;=\; n_t + a_t, \tag{45}$$

$$y_t \;=\; c_t. \tag{46}$$

Expressions (42)-(46) lead to eqs. (1) and (2) in the main text—abstracting from preference shocks that will be introduced in section 4.

A.2 The equilibria without approximations

In the previous section we considered a log-linear approximation of the Euler equation. This is with no loss of generality because higher-order (here, only second-order) terms appear as constants, which we have suppressed. To "unsuppress" the constant terms, we will take the same steps as in the Appendix of Rousakis (2013). Without loss of generaliy, and to simplify the argument, we abstract from preference and policy shocks.

Define $x_t \equiv \log(X_t)$. We can rewrite (39) as:

$$e^{\log Q_t} = e^{\log \beta} E_t \left[e^{-\pi_{t+1} + a_t - a_{t+1}} \right], \tag{47}$$

where we have used the equilibrium condition $y_t = c_t = a_t$. Using (41) (remember $i_t \equiv -\log Q_t$) and assuming a white noise productivity process, $a_t = \epsilon_t$, (47) simplifies to

$$e^{-\phi E_t [\pi_{t+1} - \overline{\pi}]} = E_t \left[e^{-\pi_{t+1} + \epsilon_t - \epsilon_{t+1}} \right]. \tag{48}$$

We allow noisy information about productivity up to τ periods ahead, where $\tau \geq 0$, given by (5). Next, conjecture (C1), allowing a constant term, abstracting from policy shocks and taking into account information up to τ periods ahead, reduces to

$$\pi_{t+1} = \theta_{-1} + \sum_{j=0}^{\tau+1} \theta_j E_{t+1} [\epsilon_{t+j}]. \tag{AC}$$

Using (AC), the LHS in (48) becomes:

$$e^{-\phi E_t [\pi_{t+1} - \overline{\pi}]} = e^{-\phi \left\{ \theta_{-1} + \sum_{j=0}^{\tau} \theta_j E_t [\epsilon_{t+j}] - \overline{\pi} \right\}}, \tag{49}$$

where we have used the fact that $E_t [\epsilon_{t+\tau+1}] = 0$ because, by assumption, agents have information up to τ periods ahead.

The RHS in (48) becomes:

$$E_t \left[e^{-\pi_{t+1} + \epsilon_t - \epsilon_{t+1}} \right] = e^{\epsilon_t} E_t \left[e^{-(\pi_{t+1} + \epsilon_{t+1})} \right] = e^{\epsilon_t - E_t[\pi_{t+1} + \epsilon_{t+1}] + \frac{1}{2} Var_t [\pi_{t+1} + \epsilon_{t+1}]}, \tag{50}$$

where the last equality follows from the fact that $\pi_{t+1} + \epsilon_{t+1}$ is normally distributed (hence $e^{\pi_{t+1} + \epsilon_{t+1}}$ is log-normally distributed), while $Var_t [\cdot]$ denotes variance of a variable conditional on Ω_t. Using (AC), eq. (50) becomes:

$$E_t \left[e^{-\pi_{t+1} + \epsilon_t - \epsilon_{t+1}} \right] = e^{\epsilon_t - \{\theta_{-1} + E_t [\epsilon_{t+1}] + \sum_{j=0}^{\tau} \theta_j E_t [\epsilon_{t+j}]\} + \frac{1}{2} \{[(1+\theta_1)^2 + \sum_{j=2}^{\tau} \theta_j^2] (\sigma_e^{-2} + \sigma_\epsilon^{-2})^{-1} + \theta_{\tau+1}^2 \sigma_\epsilon^2\}}, \tag{51}$$

where we use the fact that

$$\begin{aligned}
\epsilon_{t+1} &\sim N\,(\mu\, s_{t+1}, (\sigma_e^{-2} \,+\, \sigma_\epsilon^{-2})^{-1}),\\
\epsilon_{t+2} &\sim N\,(\mu\, s_{t+2}, (\sigma_e^{-2} \,+\, \sigma_\epsilon^{-2})^{-1}),\\
&\vdots\\
\epsilon_{t+\tau} &\sim N\,(\mu\, s_{t+\tau}, (\sigma_e^{-2} \,+\, \sigma_\epsilon^{-2})^{-1}),\\
\epsilon_{t+\tau+1} &\sim N\,(0, \sigma_\epsilon^2)
\end{aligned}$$

and $\mu \,=\, (\sigma_e)^{-2}/\left((\sigma_e)^{-2} + (\sigma_\epsilon)^{-2}\right)$.

Matching coefficients in (49) and (51) yields:

$$-\,\phi\,(\theta_{-1} \,-\, \overline{\pi}) \,=\, -\,\theta_{-1} \,+\, \frac{1}{2}\{[(1+\theta_1)^2 + \sum_{j=2}^{\tau}\theta_j^2]\,(\sigma_e^{-2} \,+\, \sigma_\epsilon^{-2})^{-1} \,+\, \theta_{\tau+1}^2\,\sigma_\epsilon^2\} \tag{52}$$

$$-\,\phi\,\theta_0 \,=\, 1 \,-\, \theta_0 \tag{53}$$

$$-\,\phi\,\theta_1 \,=\, -\,(1 \,+\, \theta_1) \tag{54}$$

$$\theta_2 \,=\, \ldots \,=\, \theta_\tau \,=\, 0\,. \tag{55}$$

We can rearrange (52-(55) as:

$$\theta_{-1} \,=\, -\,\frac{1}{2\,(\phi-1)}\left\{\left(\frac{\phi}{\phi-1}\right)^2(\sigma_e^{-2} \,+\, \sigma_\epsilon^{-2})^{-1} \,+\, \theta_{\tau+1}^2\,\sigma_\epsilon^2\right\} \,+\, \frac{\phi}{\phi-1}\,\overline{\pi} \tag{56}$$

$$\theta_0 \,=\, -\,\frac{1}{\phi-1} \tag{57}$$

$$\theta_1 \,=\, \frac{1}{\phi-1} \tag{58}$$

$$\theta_2 \,=\, \ldots \,=\, \theta_\tau \,=\, 0\,. \tag{59}$$

Observe that the solution is the same as the one in the main text and, crucially, $\theta_{t+\tau+1}$ is indeterminate. The only difference compared to the main text is the presence of the now unsuppressed constant term, θ_{-1}, which captures the slope of the Euler equation. Observe that the constant term depends on $\theta_{t+\tau+1}$, hence the constant term is indeterminate too. One may, therefore, be tempted to argue that, unlike the point made in Nakajima and Polemarchakis (2005), it is also mean inflation that remains indeterminate; this is not correct. What we have shown is that the mean of log-inflation is indeterminate, which should come as no surprise given that the mean of a log-normally distributed variable depends on its variance.

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Tables

Table 1: Timeline of selected communication and transparency innovations for major central banks

	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Bank of England	Inflation report launched. Quarterly forecasts.	Publish minutes of Governor-Chancellor policy meetings.			Operational independence. Minutes published. Press conferences.	More timely minutes.	Publish analysis of forecast accuracy.					
Bank of Japan						Regular monetary policy meetings. Announce policy. Publish minutes. Publish transcripts.	Qualitative forward guidance.	Publish forecasts.	State-contingent forward guidance.		Publish more timely policy rationale.	Publish policy model.
European Central Bank						Established with formal inflation objective and operational independence.		Publish (staff) semi-annual forecasts.	Publish policy model.			Publish quarterly forecasts.
US Federal Reserve	Publish minutes. Publish past transcripts.	Announce rate changes. Explain rationale.	Publish future transcripts.	Quantitative announcement and policy rationale.			Rationale even when no policy change.			Immediately announce individual voting record.	Qualitative forward guidance.	More timely minutes.

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Bank of England									State-contingent forward guidance.	Publish more detail about forecast. Independent evaluation office.	Publish transcripts. Publish minutes on day of decision.		
Bank of Japan				Policy rationale even when policy unchanged. More timely minutes.	Publish quarterly forecasts.				Formal inflation objective.		Publish individual forecasts and risk assessments.	Publish more detailed backgrounds for forecasts.	
European Central Bank									Qualitative forward guidance.	Publish more detail about forecast.	Publish minutes.		
US Federal Reserve			Publish quarterly forecasts.				Press conferences. Time-based forward guidance.	Formal inflation objective. Individual interest rate projections. State-contingent forward guidance.					Publish fan charts.

Note: Reprinted from Haldane (2017).