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Spin and Valley Polarization Dependence of Resistivity in Two Dimensions

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We demonstrate that spin polarization and valley polarization have quantitatively similar effects on the resistivity of a two-dimensional electron gas in a silicon-on-insulator quantum well. In-so-doing, we also examine the dependence on disorder, leading to a coarse but global phenomenology of how the resistivity depends on its key parameters: spin- and valley-polarization, density, disorder and temperature.

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In addition to the spin degree of freedom which is widely known to be pivotal in determining the resistivity of two dimensional electronic systems (2DES) [1–4], charge carriers in a number of important and topical materials such as silicon, diamond, graphene, MoS₂ and aluminium arsenide possess a valley degree of freedom due to band degeneracy. Science and technology exploiting this degree of freedom, now often referred to as valleytronics, is a subject of surging interest [5]. However, unlike spin which usually couples strongly to an external magnetic field, techniques for controlling valley splitting and valley polarization are more material specific, raising challenges for exploring its impact on resistivity.

Experiments probing the physics of valley polarization in steady-state transport have been largely limited to the forerunning work on AIAs [6], in which valley polarization is achieved by applying symmetry breaking strain. These have revealed remarkable valley dependent phenomena such as in the behaviour of fractional quantum Hall states [7] and impressive similarities between spin and valley, for example, in the behaviour of valley susceptibility [8] and the qualitative role played by valleys in the apparent metal-insulator transition [9]. However, these experiments are complicated by the in-plane anisotropy of effective mass, and the extent to which the results are general and applicable to silicon, the archetypical semiconductor in which most studies of the metal insulator transition have been undertaken [1–3] and which continues to reveal new phenomena [10], remains to be fully established.

In (001) silicon quantum wells, the two valleys are isotropic, and similar experiments would enable direct comparisons between the roles of valley and spin in transport. By using a relatively recently discovered technique of exploiting a particular Si-SiO₂ interface [11], we have been able to demonstrate that valley-polarization also enhances resistivity in (001) silicon [12]. However, the out-of-plane electric field required to change valley po-

larization in these experiments also leads to a change in the bare disorder potential through the interface roughness, which also strongly affects resistivity, preventing direct comparisons from being made.

Here, we compare the magnitude of resistivity with and without spin and valley polarization under equivalent conditions of out-of-plane electric field and disorder. This is achieved by using both of the Si-SiO₂ interfaces of silicon-on-insulator devices, one of which is a standard interface with negligible valley splitting while the other is an interface with giant valley splitting [13]. Magnetic field up to 28 T is used to spin polarize electrons over a large density range. In combination, this enables us to complete a coarse but global description of the resistivity, demonstrating how electron density, spin-valley polarization and disorder determine the resistivity in two dimensions. The data demonstrate that the temperature dependence can be changed in sign by each of these parameters and that magneto-resistivity and valley-resistivity are quantitatively, strikingly similar.

Our samples consist of a nominally 10 nm thick layer of (001) silicon-on-insulator (SOI) patterned into Hall bars whose Ohmic contacts are degenerately doped with ion-implanted phosphorus. The active silicon layer is bound between a top-oxide layer formed by standard thermal oxidation and a lower buried oxide (BOX) layer formed by ion implantation of oxygen followed by high temperature annealing (“SIMOX” process) [Fig. 1(a)]. The latter is known to give rise to a giant valley splitting as previously described in detail [11]. The electron density n is controlled by front and back gates with capacitances $C_F = 463 \mu\text{Fm}^{-2}$ and $C_B = 91.2 \mu\text{Fm}^{-2}$ respectively and is given by $n = n_B + n_F$; n_F and n_B being electron densities contributed by the two respective gates. Valley splitting is controlled by tuning an effective out-of-plane electric field we quantify by a parameter $\delta n = n_B - n_F$ which can be tuned independently of the total density n . The size of the valley splitting is approximated by a

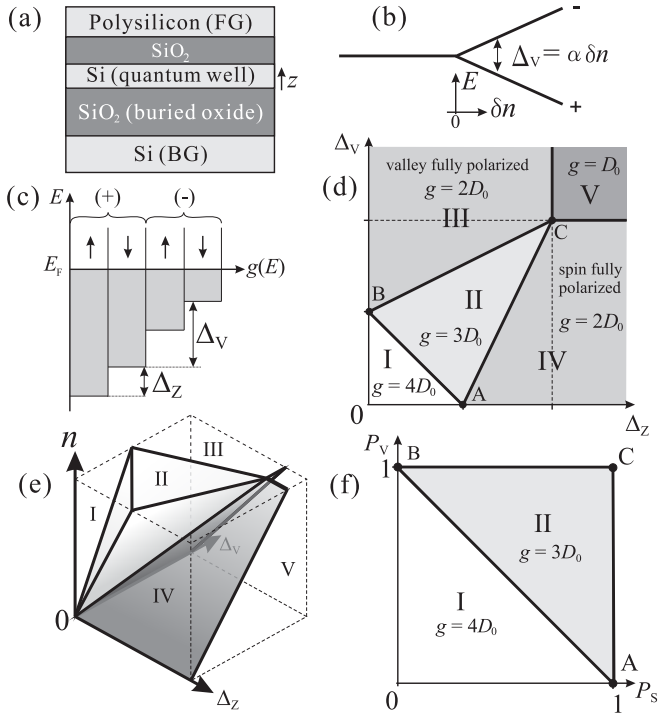


FIG. 1: (a) Schematic diagram of the sample. (b) Dependence of valley splitting Δ_V on δn . (c) Spin and valley split subbands. (d) Single particle phase diagram at constant density. (e) A 3d visualization of (d) with n as a further parameter. (f) A phase diagram in terms of polarization.

phenomenological formula: $\Delta_V = \alpha \delta n$ when $\delta n > 0 \text{ m}^{-2}$ where the valley factor α for the Si-buried oxide interface in this study has a value of about $0.49 \text{ meV}/10^{15} \text{ m}^{-2}$. The valley splitting remains small when δn is negative [Fig. 1(b)]. Standard transport measurements were performed in a helium-4 variable temperature insert in a resistive magnet with field up to 28 T. The spin splitting is controlled by applying an in-plane magnetic field B_{\parallel} parallel to the current. Parallelism between the 2DES and the magnetic field is ensured by using a rotation stage and eliminating the Hall resistance.

In a single particle picture in which each spin and valley split subband has density of states $g = D_0 = m^*/2\pi\hbar^2$ [Fig. 1(c)] where m^* is the in-plane effective mass, a simple phase diagram can be constructed where the boundaries represent the onset of occupation of a spin-valley-split subband at a constant sheet density n [Fig. 1(d)]. The boundaries are symmetric about the line $\Delta_V = \Delta_Z$ and each boundary scales with n linearly, collapsing to the origin at $n = 0$ on the (Δ_Z, Δ_V) plane [Fig. 1(e)]. The diagram can be simplified by defining spin and valley polarization: $P_S = (n_{\uparrow} - n_{\downarrow})/(n_{\uparrow} + n_{\downarrow})$ and $P_V = (n_{+} - n_{-})/(n_{+} + n_{-})$ where n_{\uparrow} and n_{\downarrow} are densities of electrons with up and down spin respectively while n_{+} and n_{-} are densities of electrons in two respective valleys [Fig. 1(f)].

Experimentally, the trajectory along the Δ_Z axis ($\Delta_V \sim 0$ [Fig. 1(d)]) is achieved by keeping $\delta n \leq 0 \text{ m}^{-2}$ while sweeping magnetic field. Thick black lines in Figs. 2(a to f) show resistivity ρ_{xx} at $\delta n = 0 \text{ m}^{-2}$ displaying the usual increase in resistivity with magnetic field as spins polarize, followed by saturation at full polarization [2, 14–16]. With increased density n , the field at which saturation occurs (B_P) increases as the Fermi energy increases [18]. On the other hand, the trajectory along $\Delta_Z \sim 0$ [Fig. 1(d)], along the Δ_V axis, is achieved by sweeping δn (for $\delta n > 0 \text{ m}^{-2}$) and the evolution of resistivity in these samples has already been described in detail [12]. Resistivity increases with δn in a similar manner as with spin polarization, followed by a shoulder feature rather than saturation, and continues to increase. In Fig. 2(d), ρ_{xx} is plotted at equal intervals of δn and at zero magnetic field, there is a slight bunching of the traces near point B corresponding to the onset of full valley polarization. With increasing $|\delta n|$, there is an increase in the disorder potential due to interface roughness, as the out-of-plane wavefunction is pushed closer to the Si-SiO₂ interface and this in turn strongly enhances the resistivity, and leads to an absence of saturation. This coupling of δn to disorder had previously prevented direct comparisons between valley resistance and magnetoresistance.

In order to assess the disorder potential in the absence of valley effects, we have recently performed experiments in which both hole and electron mobility were measured in a sample whose Si-SiO₂ interfaces were prepared by identical methods as those used in the present study [13]. Holes, which do not possess the valley degree of freedom show mobility which is almost symmetric in δn , in particular at large $|\delta n|$ where interface roughness is the dominant scattering mechanism. This indicates that the scale of interface roughness at the two interfaces are very alike and that the out-of-plane electric field dependence of the disorder potential are also very similar. We can therefore make a comparison of the resistivity with positive and negative δn , that is, with or without valley polarization, knowing that the bare disorder potential is similar for the same $|\delta n|$.

For negative δn at the lowest density ($n = 2.5 \times 10^{15} \text{ m}^{-2}$ [Fig. 2(a)]), the resistivity increases with $|\delta n|$ and the entire magnetoresistance trace shifts upwards on the logarithmic plot. The increase in zero field resistivity indicates an increasingly large disorder potential, but the relative similarity of the curves shows that the factor by which the resistivity is enhanced by spin polarization is relatively constant and that there is not much qualitative change in the nature of the disorder [20, 21]. At higher density [Fig. 2(c) $n = 3.5 \times 10^{15} \text{ m}^{-2}$ and (e) $n = 4.5 \times 10^{15} \text{ m}^{-2}$], the effects of negative δn are less dramatic [22]. This is presumably due to the ability of the electrons to screen the background disorder potential.

We now compare resistivity under valley polarization and spin degeneracy against spin polarization and valley

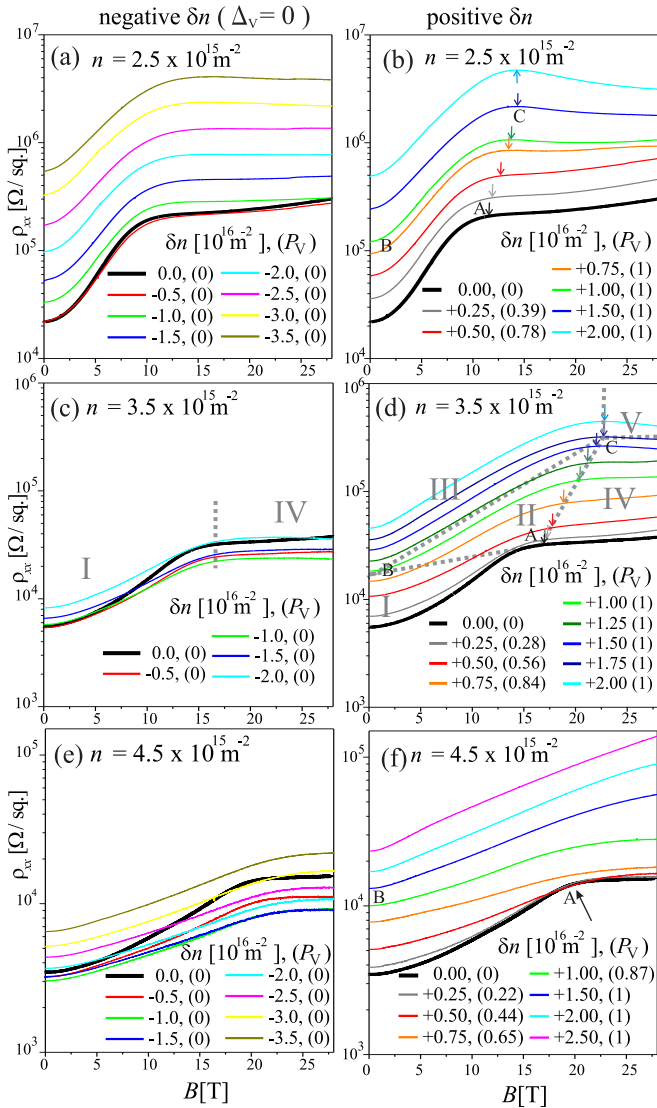


FIG. 2: *Color online.* Magnetotransport at $T = 1.4$ K as a function of δn . The left column (a), (c) and (e) shows data at negative δn ($\Delta_V = 0$, $P_V = 0$) while the right column (b), (d) and (f) shows data at positive δn . Each row corresponds to different density where (a) and (b): $n = 2.5 \times 10^{15} \text{ m}^{-2}$, (c) and (d): $n = 3.5 \times 10^{15} \text{ m}^{-2}$ and (e) and (f): $n = 4.5 \times 10^{15} \text{ m}^{-2}$. For each density (row), identical colors are used for identical values of $|\delta n|$. Small arrows are guides to the eye, marking the feature described in the body text. P_V in parentheses [11] show values expected at zero magnetic field. Roman numerals in (c) and (d) show corresponding regions in fig. 1(d) where the dotted line boundaries are straight line guides to the eye joining points A, B and C.

degeneracy. These conditions correspond to $(P_S, P_V) = (0, 1)$ $((1, 0))$, which corresponds to point B (A) in figure 1(f) but a locus of points along the Δ_V (Δ_Z) axis in region III (IV) in figure 1(d). Data is shown for two different densities and $|\delta n|$ in figure 3. It can clearly be seen that there is considerable agreement between the data sets, providing strong evidence that valley and spin

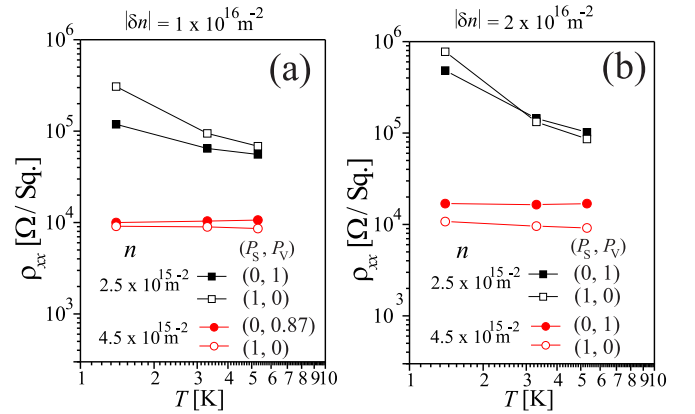


FIG. 3: *Color online.* Comparison of the resistivity between spin-degenerate-valley-polarized and spin-polarized-valley-degenerate electrons at equal out-of-plane bias. (a) $|\delta n| = 1 \times 10^{16} \text{ m}^{-2}$ and (b) $|\delta n| = 2 \times 10^{16} \text{ m}^{-2}$. Spin polarized data are taken at $B = 28$ T.

polarization play quantitatively equivalent roles in determining the value of the resistivity. The data do also show disagreement, especially at low density, in that data-sets at $(P_S, P_V) = (0, 1)$ and $(1, 0)$ differ in temperature dependence, despite their similarity in the actual values of ρ_{xx} . For example, the low density data in Fig. 3(b) shows a cross-over and indeed, the discrepancy can be expected to grow at lower temperature. There will inevitably be differences in disorder at the two interfaces even if the hole mobilities are quantitatively very close [13], and there is no direct correspondence between the value of magnetic field used (28 T) and the ‘valley field’ to expect exact agreements. However, we can also expect more physically fundamental differences such as differences between intervalley scattering and spin-flip scattering underlying the differences observed and makes an interesting prospect for future investigation.

We now address how the magnetoresistance changes with valley polarization at positive δn . Data are shown in Figs. 2(b, d and f). At a density of $n = 2.5 \times 10^{15} \text{ m}^{-2}$, valley polarization occurs at around $\delta n = 0.64 \times 10^{16} \text{ m}^{-2}$ at zero magnetic field [24]. In contrast to data at negative δn , positive δn clearly increases the field B_P at which the shoulder is observed. However, the single particle picture predicts a doubling of this field [Fig. 1(d)] but this is clearly not what is observed, and only increases by a couple of Tesla, qualitatively consistent with previous measurements in AIsAs [25, 26]. The shift in B_P is clearer at higher density. At $n = 3.5 \times 10^{15} \text{ m}^{-2}$ [Fig. 2(d)], saturation occurs at around $B_P = 15.7$ T and this is seen to increase with increasing δn before saturating at around 22 T. Saturation of B_P with δn indicates the attainment of both spin and valley polarization [region V in Fig. 1(d)] and demonstrates that we can access all regions of the spin-valley phase diagram [Fig. 1(f)] [27, 28]. At a higher density of $n = 4.5 \times 10^{15}$ [Fig. 2(f)], B_P in-

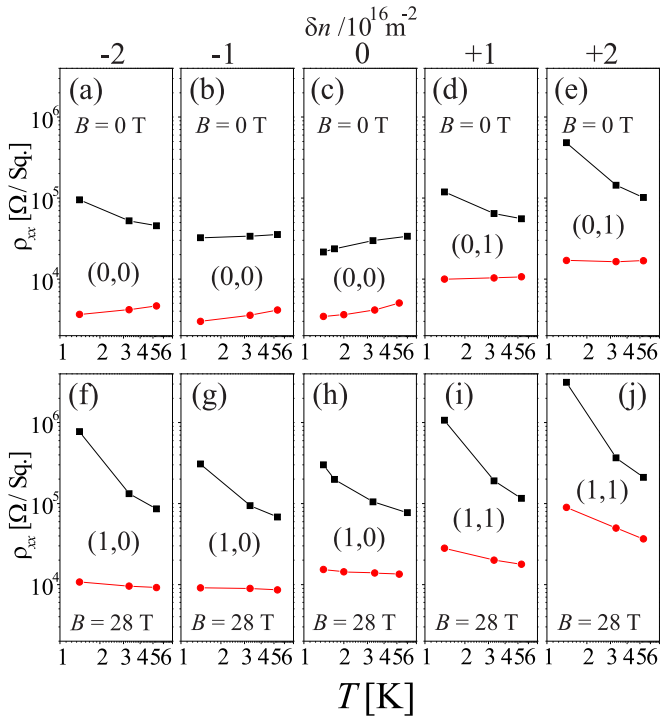


FIG. 4: *Color online.* Temperature dependence of resistivity (squares: $n = 2.5 \times 10^{15} \text{ m}^{-2}$, circles: $n = 4.5 \times 10^{15} \text{ m}^{-2}$). Data at $B = 0 \text{ T}$ are shown in the top row (a to e) while the lower row (f to j) shows data at $B = 28 \text{ T}$. Each column corresponds to a value of δn . Polarization (P_s, P_v) for $n = 2.5 \times 10^{15} \text{ m}^{-2}$ is indicated in parentheses for each graph. Polarization at $n = 4.5 \times 10^{15} \text{ m}^{-2}$ is the same except in (d) where $P_v = 0.87$ and (i) and (j) where the degree of spin-valley polarization is substantial but not full.

creases from 20.8 T to beyond the field range of these experiments.

We now describe the global phenomenology of how the temperature dependence of resistivity changes with the key parameters [Figs. 4 (a) to (j)]. At zero magnetic field ($B = 0 \text{ T}$) and symmetry ($\delta n = 0$) [Fig. 4(c)], both sets of data ($n = 2.5 \times 10^{15} \text{ m}^{-2}$ and $n = 4.5 \times 10^{15} \text{ m}^{-2}$) are on the metallic side of the so-called metal insulator transition (MIT) [1–3] and the resistivity decreases with decreasing temperature. With negative δn , disorder is increased and the critical density n_c of the MIT increases. The 2DEG becomes insulating for $n = 2.5 \times 10^{15} \text{ m}^{-2}$ representing a disorder driven transition, while data at $n = 4.5 \times 10^{15} \text{ m}^{-2}$ still display metallic behavior at $\delta n = -2.0 \times 10^{16} \text{ m}^{-2}$ [Fig. 4(a)] and the absolute values are not very different to those at symmetry.

Under a strong magnetic field of 28 T, the 2DEG becomes insulating for both values of density, for all values of δn . This is a magnetic field induced transition to insulating behavior driven by spin polarization of the electrons. At low density, if the 2DEG shows insulating behavior [eg. Fig. 4(a)], the insulating temperature dependence becomes even stronger [Fig. 4 (f)] under spin

polarization. In contrast, at higher density, the insulating temperature dependence is relatively mild. Similarly, with valley polarization at zero magnetic field [Figs. 4(d) and e)], low density data show a striking transition to insulating behaviour while at high density ($n = 4.5 \times 10^{15} \text{ m}^{-2}$) the system is very mildly metallic at $\delta n = 1 \times 10^{16} \text{ m}^{-2}$ [Fig. 4(d)] but almost temperature independent by $\delta n = 2 \times 10^{16} \text{ m}^{-2}$ [Fig. 4(e)].

Finally, at large positive δn and high magnetic field, both spin and valley are fully polarized [Figs. 4(i and j)] for the lower density. Here, we find very strong insulating behavior which can be expected from the large values of resistivity, enhanced by both spin and valley polarization.

To summarize, we have presented a coarse but global phenomenology of how the resistivity depends on its key parameters: spin- and valley-polarization, density and disorder. Similarities between the phenomenology when spin and valley polarization are exchanged provides strong evidence for quantitative equivalence in their roles played in the underlying physics.

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