# Electronic supplementary material for 'Predicting resilience profiles of the run-up to regime shifts in nearly-1D systems'

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## Appendix B - Determining the minimum reachable population density during chaotic cycles in 1D discrete-time maps

Consider a discrete time map described by

$$x_{t+1} = f\left(x_t\right), \tag{1}$$

where $f$ is a unimodal function, i.e. it has a single peak at $\hat{x} \in [0, x_{\max}]$ such that

$$\begin{array}{ll} f'\left(x\right) > 0, & \text{if } x \in [0, \hat{x}) \\ f'\left(x\right) < 0, & \text{if } x \in (\hat{x}, x_{\max}]\,. \end{array} \tag{2}$$

An example of such a map is given by the shifted Ricker equation

$$x_{t+1} = \left(x_t - a\right) \exp\left(r\left(1 - \frac{x_t - a}{K}\right)\right) + b, \tag{3}$$

whose graph is shown in Figure S.1.

In this system, there are two equilibria, $x_1^*$ and $x_2^*$. The system has a single attractor which either is or surrounds the uppermost equilbrium $x_2^*$. If $x_2^*$ is unstable this attractor will either be a periodic or chaotic orbit around it.

A lower bound on the minimum value it is possible to reach in this attractor will be given by

$$a = f\left(\max\left\{f\left(x\right)\left|x \in [0, x_{\max}]\right.\right\}\right) = f \circ f\left(\hat{x}\right) \tag{4}$$

because of the unimodal shape of the function: to reach lower values would necessitate starting at a higher value the timestep before, which is impossible.

Initial conditions $x_0 < x_1^*$ or large enough that $f\left(x_0\right) < x_1^*$ will converge to negative values, corresponding to extinction of the population. $x_1^*$ therefore serves as a critical threshold below which the system cannot drop without transitioning to a different dynamic regime. If the computed lower bound on the

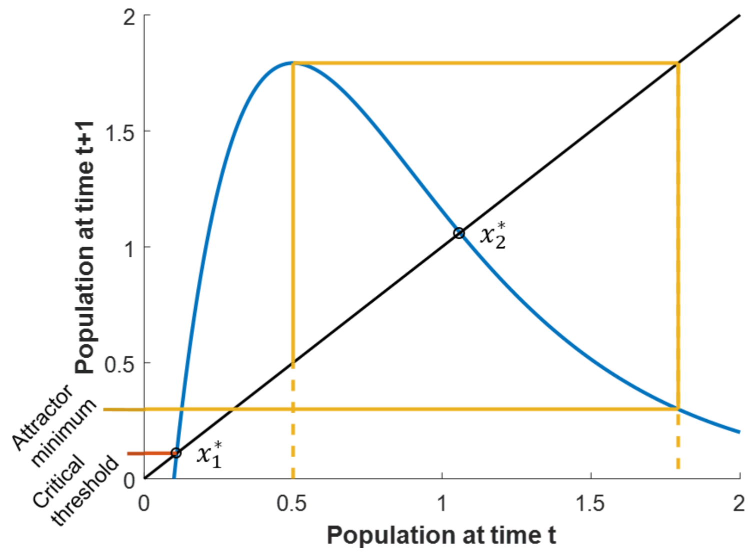


Figure S.1: Example of a shifted Ricker map (blue curve), showing how the critical threshold (orange line) and the attractor minimum (yellow lines) are found.

attractor minimum is above this threshold, persistence of the system is guaranteed in the absence of external perturbations or noise. If this attractor minimum is below the critical threshold, it indicates the possibility that a transition in the system via a boundary crisis has taken place. Whether or not the system will persist at this point will depend on the extent to which the attractor fills out its possible range of values, between $f(\hat{x})$ and $f \circ f(\hat{x})$.

# Appendix C - Testing on a time series without a transition for false positives

In order to determine under which conditions the approach is likely to produce false positives, here we apply it to a time series for the lake phosphorous model in which there is no ramping in the inflow of phosphorous from surrounding ground water is constant, so that a regime shift does not take place (Fig. S.2A).

Figure S.2B shows the time at which a given calibration window–characterised by start time and length–predicts a regime shift. White regions correspond to calibration windows that correctly predict that there is no transition coming. Coloured regions correspond to calibration windows that yield a false positive, with the colour scheme such that in dark regions the transition is predicted to be very soon–and therefore representing a more severe false positive–while lighter grey regions represent milder false positives where the transition is predicted to be a long way in the future.

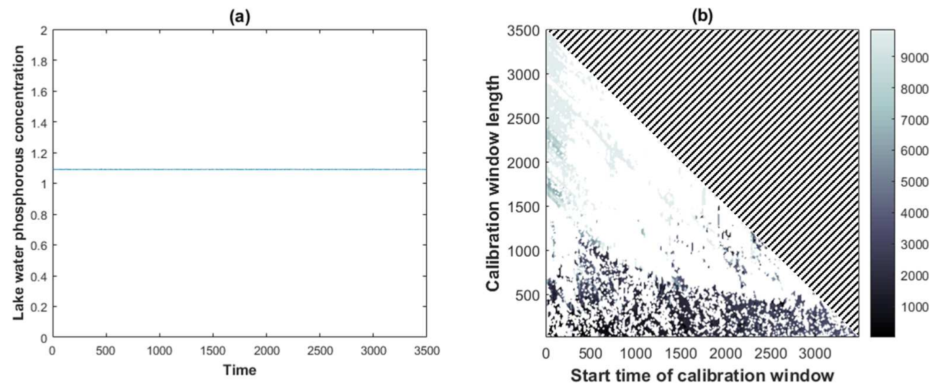


Figure S.2: A, Time series for the lake phosphorous model with phosphorous inflow fixed at $U = 800$, such that no regime shift takes place. All other parameters are kept the same as in Fig 1. B. Predicted times of regime shifts for different calibration windows of the lake phosphorous time series without a regime shift. Here the legend corresponds to the predicted time of the regime shift. All transitions predicted to take place at 10000 time units or more, or not at all, are represented by the same light grey colour.

Overall, severe false positives take place almost exclusively when there is limited calibration data. Calibration windows longer than 1000 rarely report false positives, and when they do the predicted time of the regime shift is usually at least 10000 time units in the future. The requirements of the calibration data therefore seem to be similar with respect to false positives as they are to the accuracy of predicting regime shifts (c.f. Fig 7).