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# Contingent wage subsidy

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## Abstract

This paper proposes a policy aimed at tackling unemployment that arises from macroeconomic coordination failure. The policy offers firms wage subsidies payable only if the total number of new hires made across the economy is below a prespecified threshold. Subsidies provide incentives for firms to create jobs but the policy's goal is to generate a sufficiently large amount of employment spillovers to set off hiring complementarities taking employment beyond the threshold. Thus, subsidies are not distributed but the policy achieves a Pareto improvement. The market structure is important for policy design. Aggregative game techniques prove useful for the oligopsonistic case.

## 1 | INTRODUCTION

New hiring by one firm is a reason for new hiring by other firms because of employment externalities related to additional aggregate demand, new trading opportunities, or production synergies. Without a coordinated action, however, the virtuous hiring cycle may not start, stranding the economy in a low-employment, low-spending equilibrium as in the aftermath of the 2007–2009 financial crisis (OECD, 2016). The traditional approach to this problem emphasizes a “big push,” when one large player like the government spends enough to convince others to spend. In this paper, we show how a “zero push” can achieve the same results.

With the economy in a low-employment equilibrium, we propose a policy that offers firms wage subsidies for new hires payable only if the total number of new hires made in the economy *does not* exceed a prespecified threshold. An example would be a promise to cover all new labor costs contingent on that less than, say, 100,000 new jobs are created in total. From a firm's perspective two outcomes can occur from this policy. One outcome is when the number

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of new jobs is less than the threshold, in which case the firm has its additional labor costs covered while keeping all the additional revenue. The second outcome is when the threshold is met and no subsidies are paid. The firm then benefits from employment spillovers generated by a substantial increase in total employment which makes hiring profitable even without any subsidies. With hiring profitable in both scenarios and, thus, all firms hiring, the threshold for new hires is reached, bringing the economy to high-employment equilibrium without any subsidies paid.<sup>1</sup>

For more intuition, it is useful to illustrate the contingent wage subsidy with a hiring game between two firms as shown in Figure 1. Each firm has a hiring decision to make by choosing either *status quo* or *hire*. Because of externalities hiring is profitable only if both firms hire; otherwise it is not (see panel (a)). The resultant game is a coordination game with two symmetric equilibria of  $\{status\ quo, status\ quo\}$  and  $\{hire, hire\}$ . Now consider a policy that pays a subsidy to a hiring firm contingent on the other firm choosing *status quo* as shown in panel (b). In the modified game hiring becomes a dominant strategy resulting in the unique hiring equilibrium. The contingent subsidy policy effectively transforms the coordination game into a “prisoner’s dilemma” game with the desirable equilibrium outcome but no subsidies disbursed.

In the main text, we use the labor market model with private information and adverse selection developed by Weiss (1980). In this model, there are multiple Pareto-ranked equilibria due to the employment externalities which arise from adverse selection because the expected productivity of a new hire is larger when more people are employed.<sup>2</sup> In particular, in a low-wage and low-employment equilibrium the wage is low because firms hold low expectations about workers’ productivity which results in only less able workers accepting employment. A wage rise and subsequent increase in total employment create a positive externality as firms favorably update their beliefs about new hires’ productivity. If the total amount of externality is sufficiently large, it can start the virtuous process of hiring complementarities bringing the economy to higher employment equilibrium. We assume that without coordinated effort this process cannot be started or that no individual firm is big enough to kickstart it unilaterally.

The purpose of contingent wage subsidies is to generate a sufficiently large aggregate externality that would set off hiring complementarities. We demonstrate for competitive and oligopsonistic labor markets that it is possible to design a contingent wage subsidy policy such that it achieves high-employment equilibrium without distributing any subsidies. However, we note that the market structure is a very important factor for policy design. Not all policies that are effective for the competitive market would also be effective for the oligopsonistic market. In particular, the insurance policy that only compensates for losses would turn ineffective.

The analysis of the oligopsonistic case is important in light of mounting evidence for increasing employer market power in labor markets (Naidu, Posner, & Weyl, 2018). We undertake this analysis using the aggregative game techniques developed by Martimort and Stole (2012). We consider the labor market where firms are not wage-takers and hire labor to maximize their profits which also depend on total employment as it determines the wage rate in the economy.

<sup>1</sup>The proposed policy is an application of the “dominant assurance contract” of Tabarrok (1998), which is an extension of the “assurance contract” of Dybvig and Spatt (1983). See Related Literature for further discussion.

<sup>2</sup>Our modeling choice is motivated by the tractability and compactness of the model in Weiss (1980) that requires only few preliminary digressions. Generally, however, the modeling approach is immaterial for our results as long as it generates multiple equilibria and strategic complementarities in employment. For instance, one alternative would be to use the human capital model with random matching of Acemoglu (1996), which also features pecuniary employment externalities and multiple Pareto-ranked equilibria. Another alternative would be to use technological employment externalities as in, for example, Romer (1986) or Lucas (1988).

**FIGURE 1** Two-firm hiring with employment externalities. (a) No subsidy; (b) Contingent subsidy

		(a) No subsidy		(b) Contingent subsidy	
		Status quo	Hire	Status quo	Hire
Status quo	Status quo	0, 0	0, -1	0, 0	0, 0.5
	Hire	-1, 0	2, 2	0.5, 0	2, 2

But as it is only total employment that matters for profits, not its distribution among firms, the resultant oligopsony is an aggregative game. Furthermore, as our interest lies with equilibrium aggregates, the *aggregate concurrence principle* developed by Martimort and Stole (2012) is applicable to determine the equilibrium aggregates before and after the introduction of contingent wage subsidy policy. Specifically, we apply this principle to demonstrate that our constructed policy eliminates the low-employment equilibrium leaving only the high-employment equilibrium whereas no new equilibria are created.

The remainder of this paper is organized as follows. Following a literature review in Section 2, we study the competitive labor market in Section 3 where we also introduce the contingent wage subsidy policy. In Section 4 we study the case of oligopsonistic labor market. Section 5 concludes the study. All omitted proofs are in the Appendix.

## 2 | RELATED LITERATURE

It is a long established view in economics that macroeconomic coordination failures can be a cause of unemployment. Such failures can arise due to aggregate demand externalities (Hart, 1982; Heller, 1986; Shleifer, 1986); production input externalities (Matsuyama, 1991; Weitzman, 1982), trading and firm entry externalities (Chatterjee, Cooper, & Ravikumar, 1993; Diamond, 1982), and information externalities (Lockwood, 1991). See Cooper and John (1988) for a general unifying framework of strategic complementarities in employment; for recent examples, see Chamley (2014), Michailat and Saez (2015), Kaplan and Menzio (2016), and Guimaraes and Machado (2018).

The contingent wage subsidy is an example of the assurance contract proposed by Dybvig and Spatt (1983). In collective action problems participation externalities are public goods and, therefore, decentralized outcomes can be suboptimal. To encourage participation Dybvig and Spatt (1983) propose an insurance scheme dubbed as the “assurance contract” that compensates participants for their losses in the event of low participation. The best known example is the deposit insurance scheme, which eliminates the bank run equilibrium at no cost (Diamond and Dybvig (1983). Kiyotaki (1988) proposes a similar insurance policy to eliminate the pessimistic capital investment equilibrium. Another example is crowdfunding’s “all-or-nothing” fundraising method which refunds contributions if the threshold for contributions is not reached.

The compensation of losses may not be sufficient to eliminate low-participation equilibria. Even with insurance against losses agents may abstain from participating if they think others will abstain. Tabarrok (1998) suggests an extension to the assurance contract which not only compensates for losses but also offers a bonus to participating agents in the event of low participation. Then, participation becomes a dominant strategy which ensures high-participation equilibrium and no costs incurred (see Zubrickas, 2014 for an application to crowdfunding). Cason and Zubrickas (2017, 2019) experimentally show that such an extension offers a substantial economic improvement to a simple assurance contract. Finally, the present paper is the first study of assurance contracts in an environment beyond price-taking firms.

### 3 | COMPETITIVE LABOR MARKET

#### 3.1 | Setup

There are a unit continuum of workers and a set  $\mathcal{N}$  of firms, indexed by  $i \in \mathcal{N}$ , that can hire workers.<sup>3</sup> Let the number of firms, denoted by  $N \equiv |\mathcal{N}|$ , be larger than 1. Firms produce the same output using an identical constant returns to scale technology, where workers' labor is the only input. The price of a unit of output is normalized to 1. Firms are risk neutral profit maximizers.

Workers differ in the amount of output they produce if hired by a firm. A worker's productivity, denoted by  $\theta$ , can take values from an interval  $[\underline{\theta}, \bar{\theta}]$ ,  $0 < \underline{\theta} < \bar{\theta} < \infty$ , and levels of productivity are distributed in the population according to distribution function  $F(\theta)$  with the density function  $f(\theta) > 0$ . A worker's productivity  $\theta$  is private information and his reservation utility is given by a nondecreasing continuous "home production" function  $r(\theta)$ . A worker accepts employment if and only if his wage is at least  $r(\theta)$ .

As firms cannot distinguish among workers by their productivity, we assume that workers are all employed for the same wage. Then, the aggregate supply of labor  $S(w)$  for a given wage  $w$  is determined by

$$S(w) = \begin{cases} 0 & \text{for } w < r(\underline{\theta}), \\ F(r^{-1}(w)) & \text{for } r(\underline{\theta}) \leq w \leq r(\bar{\theta}), \\ 1 & \text{for } w > r(\bar{\theta}), \end{cases} \quad (1)$$

where  $r^{-1}(w) = \sup\{\theta: r(\theta) = w\}$ . The expected productivity of a worker employed at a wage  $w \geq r(\underline{\theta})$  is given by

$$\mu(w) = \begin{cases} \underline{\theta} & \text{for } w = r(\underline{\theta}), \\ \frac{\int_{\underline{\theta}}^{r^{-1}(w)} \theta dF(\theta)}{S(w)} & \text{for } r(\underline{\theta}) < w \leq r(\bar{\theta}), \\ \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta)}{S(w)} & \text{for } w > r(\bar{\theta}). \end{cases} \quad (2)$$

On the demand side for labor, each firm can hire at most  $\bar{e}$  workers, where  $\bar{e} \in (0, 1)$  and  $N\bar{e} > 1$ . For a given wage  $w$  a firm hires workers up to its capacity if and only if their expected individual productivity  $\mu(w)$  is at least  $w$ . Let  $e_i \in [0, \bar{e}]$  denote the population share of workers hired by firm  $i$ ,  $i \in \mathcal{N}$ , and let  $E = \sum_i e_i$  denote the total population share of workers in employment, also referred to as total employment.

By competitive labor market we understand a labor market with a large number of firms and free entry, where each firm takes total employment  $E$ , wage  $w$ , and expected productivity  $\mu$  as fixed. In particular, by free entry we understand that in the set  $\mathcal{N}$  there is always a firm with the zero level of own employment that is ready to hire labor if it becomes profitable.

<sup>3</sup>In this section, we draw on the textbook exposition of the labor market model of Weiss (1980) presented in Mas-Colell, Whinston, and Green (1995, Ch. 13).

### 3.2 | Equilibrium employment

First, we define the market clearing wage and firms' expected profits. We observe that the conditions of  $N\bar{e} > 1$  and of hiring at  $\mu(w) \geq w$  imply that there is no rationing in the labor market. For our analysis, it is convenient to define the market clearing wage as a function of total employment  $E$ , which is done by equating it with the supply of labor:  $S(w) = E$ . Thus, we obtain the clearing wage function

$$w(E) = r(F^{-1}(E)). \quad (3)$$

To avoid multiple clearing wages at full employment, we assume

**Assumption 1.**  $\mu(w(1)) \leq w(1)$ .

In words, at full employment we require the expected productivity be less than the lowest wage necessary for full employment. In addition to the purpose of inverting the market clearance condition, this assumption ensures that firms' demand for labor is always met.

Firm  $i$ 's expected profits as a function of own and total levels of employment are given by

$$\pi(e_i, E) = e_i(\mu(w(E)) - w(E)). \quad (4)$$

From (4) we observe that the expected marginal net value of labor,  $\mu(w(E)) - w(E)$ , depends on total employment in the economy. The expected productivity depends on the population of workers employed, which also determines the market clearing wage. Thus, a firm's profits depend not only on own workers but also on workers hired by other firms through their effect on the expected marginal net value of labor.<sup>4</sup>

In our analysis, we predict the outcomes of the labor market to be in equilibrium. As the focus is on aggregate levels of employment, we define equilibrium employment as

**Definition 1.** A total level of employment  $E$  is equilibrium if no firm can unilaterally increase its profits by changing its level of employment.

Assumption 1 implies that our definition of equilibrium employment in the competitive labor market can be expressed as the condition of zero expected profits

$$\mu(w(E)) = w(E), \quad (5)$$

which, after transformation, can be rewritten as

$$\frac{\int_{\theta}^{F^{-1}(E)} \theta dF(\theta)}{r(F^{-1}(E))} \equiv \eta(E) = E. \quad (6)$$

<sup>4</sup>If we extend the model with the market for the consumption good, the employment externality discussed in the text can also be related to aggregate demand externality. The clearance of the labor market implies the clearance of the market for the consumption good. Then, total employment determines aggregate demand for the consumption good which, in turn, affects firm profits. But this extension does not affect our analysis presented below and, therefore, is not modeled.

The employment equilibria are determined by the fixed points of the function  $\eta$ . Graphically, the fixed points are found at the intersection of the graph of  $\eta$  and the 45-degree line (see Figure 2). We also observe that our assumptions on reservation utility  $r$  and distribution function  $F$  imply that the function  $\eta$  is continuous and nondecreasing with its range and domain of  $[0, 1]$ .

In what follows, we restrict our attention to scenarios with a multiple but finite number of nonzero equilibrium levels of employment that can be Pareto-ranked. (In a competitive market an equilibrium with a lower level of employment is Pareto-dominated by an equilibrium with a higher level of employment.) We also require employment be a locally stable equilibrium which we define as

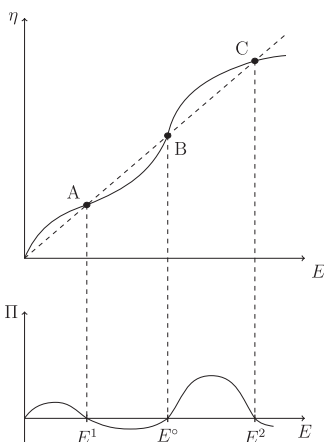
**Definition 2.** Equilibrium  $E < 1$  is locally stable if there is  $\varepsilon$  such that  $\mu(w(E')) > w(E')$  for  $E' \in (E - \varepsilon, E)$  and  $\mu(w(E')) < w(E')$  for  $E' \in (E, E + \varepsilon)$ . If  $E = 1$  is equilibrium, then it is locally stable if there is  $\varepsilon$  such that  $\mu(w(E')) > w(E')$  for  $E' \in (E - \varepsilon, E)$ .

In words, profits are negative to the right and positive to the left of a locally stable equilibrium. Observing that for employment levels  $E$  at which the value  $\eta(E)$  is below the 45-degree line profits are negative and otherwise are positive, we have that an equilibrium is locally stable if its corresponding fixed point is crossed from above the 45-degree line. The implication is that a locally stable employment equilibrium is robust to relatively small perturbations in employment.

For the competitive market environment, we impose

**Assumption 2.** There are two locally stable equilibria,  $\{E^1, E^2\}$ , where  $0 < E^1 < E^2$ .

The restriction to two locally stable equilibria is made for the convenience of exposition and without any loss of generality as the analysis presented is applicable to any (finite) number of locally stable equilibria. As it is relevant for subsequent analysis, we note that Assumption 2 implies that there must be at least one other equilibrium between the two locally stable equilibria due to the continuity of  $\eta$ . It is implicitly assumed, and as it follows from our modeling of competitive labor market, no firm can unilaterally shift the economy from  $E^1$  to any



**FIGURE 2** Equilibrium employment in a competitive labor market

other employment equilibrium. We also note that due to Assumption 1 there cannot be other equilibrium to the right from the highest locally stable equilibrium. As an illustration, Figure 2 presents a commonly encountered example (see, e.g., Cooper & John, 1988) with three nonzero equilibria  $\{E^1, E^\circ, E^2\}$ ,  $0 < E^1 < E^\circ < E^2$ , determined by fixed points A, B, and C, respectively, where  $E^1$  and  $E^2$  are locally stable equilibria.

### 3.3 | Contingent wage subsidy

Consider the labor market shown in Figure 2 and let the initial state of the economy be at point A with employment equilibrium  $E^1$ . Suppose that the government plans a policy aimed at bringing the economy to point C with employment equilibrium  $E^2$ . First of all, in relationship to the local stability of  $E^1$ , we observe that if a policy takes total employment between  $E^1$  and  $E^\circ$ , the effect of this policy will be under the pressure of reversal by firms because of negative (regular) profits in this region. For a policy to be successful, the leap in new jobs created needs to be sufficiently large so that firms find themselves in the region of positive profits or where the expected marginal value of labor exceeds its costs, that is between  $E^\circ$  and  $E^2$ . Once in this region, firms' profit maximization together with free entry would ultimately shift employment to  $E^2$ .

Next we propose a policy which solely aims at generating the leap in employment described above and, by design, is not expected to bear any costs for the government. The policy offers firms contingent wage subsidies for new hires payable only if the total number of new hires counted across all firms does not exceed a prespecified threshold. Sufficiently generous subsidies provide incentives for firms to hire until the threshold is reached. But if we set the threshold so that upon reaching it the resultant total employment falls within the region of positive (regular) profits (i.e., in the interval  $(E^\circ, E^2)$  in Figure 2), then market forces (profit maximization and/or free entry) will take employment further toward the sought-after outcome. As the threshold for new hires is exceeded in equilibrium, no subsidies need to be paid.<sup>5</sup>

Formally, our proposed policy takes the form of contingent wage subsidy, denoted by  $S(h, H, T)$ , which is the amount paid to a firm with  $h$  new hires when the total number  $H = \sum_i h_i$  of new hires, counted across all the firms, does not exceed the threshold for new hires set at  $T$ . Specifically, we say

**Definition 3.** A function  $S: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$  is a contingent wage subsidy if  $S(h, H, T) \geq 0$  for  $H \leq T$  and  $S(h, H, T) = 0$  for  $H > T$ .

For the ease of notation, we let  $\Xi = \{S(h, H, T), (e_i^*)\}$  denote a policy profile that, besides the policy, also includes the prepolicy vector of firm employment levels,  $(e_i^*) \equiv (e_1^*, \dots, e_N^*)$ . A firm's after-policy payoff, denoted by  $\pi^\Xi$ , is the sum of (regular) profits  $\pi$  and the amount of contingent subsidy  $S$

$$\pi^\Xi(e, E) = \pi(e, E) + S(e - e^*, E - E^*, T), \quad (7)$$

<sup>5</sup>It is assumed throughout that in the nonequilibrium event when the threshold for new hires is not reached, the government can resort to debt-based methods to fund subsidies. From a practical perspective, we also note that the contingent wage subsidy policy can also be designed in the form of loans or loan guarantees issued to cover firms' additional labor expenses. Then, the condition is that loans need to be discharged only if the threshold for total new jobs is exceeded; otherwise, loans are forgiven.



where  $e$  and  $E$  are the after-policy levels of the firm's own employment and of total employment, respectively, and  $e^*$  and  $E^* = \sum_i e_i^*$  are the corresponding prepolicy levels of employment.

Next, we show that there is a contingent wage subsidy that brings total employment to its highest equilibrium without any expense on subsidies borne.

**Proposition 1.** *Consider a competitive labor market. Suppose that Assumptions 1 and 2 hold. Let the level of employment be at the lowest locally stable equilibrium  $E^1$ . There is a contingent wage subsidy  $S(h, H, T)$  that strictly implements the highest locally stable equilibrium  $E^2$  without any wage subsidies paid in equilibrium.*

In words, provided that there is a region of positive profits at employment levels up to the targeted outcome, a sufficiently generous subsidy scheme needs to be designed to bring the economy to this region from which market competition takes employment further. Regarding how generous the scheme needs to be, there is a lower bound on it as the scheme needs to ensure positive profits  $\pi^\Xi$  for all levels of total new hires up to the threshold but there is no upper bound on its generosity. The upper bound appears, however, in the case of oligopsonistic labor market.

#### 4 | OLIGOPSONISTIC LABOR MARKET

We relax the assumptions of free entry and a large number of firms and allow an individual employment strategy to have an effect on expected productivity and wage. We still require that  $N > 1$ . As profits  $\pi$  in (4) depend on own strategy  $e$  and aggregate strategy  $E$  only, we can model an oligopsonistic labor market as the aggregative game  $\Gamma = \{\mathcal{N}, [0, \bar{e}], \pi\}$ . As in the case of competitive labor market, we are interested in equilibrium aggregates  $E$  only. For this purpose, we express total profits  $\Pi$  in the economy as a function of total employment

$$\Pi(E) = E(\mu(w(E)) - w(E)). \quad (8)$$

In our analysis of the game  $\Gamma$ , we draw on the aggregate concurrence principle introduced by Martimort and Stole (2012), which is based on the observation that in equilibrium all players must agree on the aggregate. According to their Definition 2, our aggregative game  $\Gamma$  is quasi-linear because the payoff function  $\pi$  in (4) is a bilinear form.<sup>6</sup> By Lemma 2 of Martimort and Stole (2012), if  $E^*$  is an equilibrium aggregate and  $(e_i^*)$  is a vector of equilibrium employment levels then  $E^*$  must be a maximizer of the aggregate objective function  $\Lambda(E, E^*)$  defined as

$$\Lambda(E, E^*) = \frac{1}{N} \sum_{i \in \mathcal{N}} \pi \left( E - \sum_{j \neq i} e_j^*, E \right) \quad (9)$$

<sup>6</sup>A bilinear form on a vector space  $\mathcal{V}$  is a map  $\mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}$  that is linear in each (functional) argument separately. In the game  $\Gamma$ , the profit function, expressed as  $\pi = e\beta(E)$ , is linear in  $e$  and  $\beta(E)$ .

or, using that the game  $\Gamma$  is of the quasi-linear form,

$$\Lambda(E, E^*) = \pi\left(E - \frac{N-1}{N}E^*, E\right). \quad (10)$$

Thus, for any equilibrium  $E^*$  it has to hold that

$$E^* = \arg \max_{E \in [E^* - \bar{e}, E^* + \bar{e}]} \Lambda(E, E^*). \quad (11)$$

We observe that since firms' strategy sets are restricted to  $[0, \bar{e}]$ , the equilibrium aggregates are the local maximizers of the aggregate objective function  $\Lambda(E, E^*)$ . Throughout, we assume differentiability and that every equilibrium aggregate is an interior maximizer found from the first-order condition  $\Lambda_1(E^*, E^*) = 0$ .<sup>7</sup>

## 4.1 | Total profits and equilibrium aggregate

For subsequent analysis, it is useful to relate the equilibrium condition  $\Lambda_1(E^*, E^*) = 0$  to the total profits  $\Pi$ . In the following lemma, we establish that there cannot be an equilibrium in the region of increasing total payoffs as otherwise firms would be willing to hire more labor.

**Lemma 1.** *Let  $E^*$  be an equilibrium aggregate of the game  $\Gamma$  determined by  $\Lambda_1(E^*, E^*) = 0$  and let  $\Pi(E^*) > 0$ . Then, we have  $\Pi'(E^*) < 0$ .*

As before, for the convenience of exposition we assume

**Assumption 3.** There are two equilibrium aggregates  $E^{m1}$  and  $E^{m2}$ , with  $E^{m1} < E^{m2}$  and  $0 < \Pi(E^{m1}) < \Pi(E^{m2})$ .

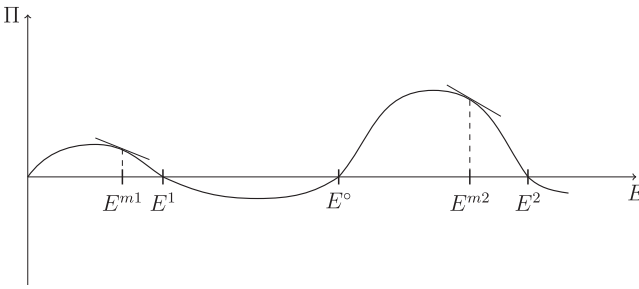
The condition  $\Pi(E^{m1}) < \Pi(E^{m2})$  is imposed to have that the higher employment aggregate  $E^{m2}$  is socially superior to the lower equilibrium aggregate  $E^{m1}$ . Furthermore, by the existence of two equilibria it is implicitly assumed that the difference between the equilibrium aggregates is sufficiently large so that no firm can unilaterally move the economy from one equilibrium to the other.

In the oligopsonistic labor market with Assumption 3 satisfied, the negative derivatives  $\Pi'(E^{m1}) < 0$  and  $\Pi'(E^{m2}) < 0$ , stemming from Lemma 1, imply together with  $0 < \Pi(E^{m1}) < \Pi(E^{m2})$  that the equilibrium aggregates  $E^{m1}$  and  $E^{m2}$  must lie to the right of local maximizers of the total profit function  $\Pi$ . These observations are illustrated in Figure 3, where we use the same specification as in the case of the competitive labor market.

## 4.2 | Contingent wage subsidy

Before we present our main result for the case of oligopsonistic labor market, we introduce some more notation and definitions. First, as contingent wage subsidy policy changes the payoff

<sup>7</sup>Here, and in what follows, a function's subscript  $j$  denotes its partial derivative with respect to the  $j$ th argument.



**FIGURE 3** Equilibrium employment in an oligopsonistic labor market

structure of the labor market game  $\Gamma$ , we denote the after-policy game with policy profile  $\Xi = (S(h, H, T), (e_i^*))$  by  $\Gamma^\Xi = \{\mathcal{N}, [0, \bar{e}], \pi^\Xi\}$ , where the payoff function  $\pi^\Xi$  is given by (7). Noting that policy  $S(h, H, T)$  does not change the aggregative structure of the game, we define the aggregate objective function of the game  $\Gamma^\Xi$  as

$$\Lambda^S(E, E^\#) = \frac{1}{N} \sum_{i \in \mathcal{N}} \pi^\Xi \left( E - \sum_{j \neq i} e_j^\#, E \right), \tag{12}$$

where  $(e_i^\#)$  is an equilibrium of  $\Gamma^\Xi$  and  $E^\#$  is the equilibrium aggregate.

We also define

**Definition 4.** Contingent wage policy  $S(h, H, T)$  is quasi-linear if  $\Gamma^\Xi$  is a quasi-linear aggregative game.

Put differently, policy  $S(h, H, T)$  is quasi-linear if the after-policy payoff function  $\pi^\Xi$  defined in (7) is a bilinear form or, more simply,  $S(h, H, T)$  is a bilinear form for a fixed threshold  $T$ . This implies that the marginal return of labor, including the subsidy per worker, depends on total employment only or that aggregate payoff  $\Pi^\Xi(E) \equiv \sum_i \pi^\Xi(e_i, E)$  is a function of aggregate employment  $E$  only.

Note that Lemma 1 continues to hold with quasi-linear and differential policy  $S(h, H, T)$ .

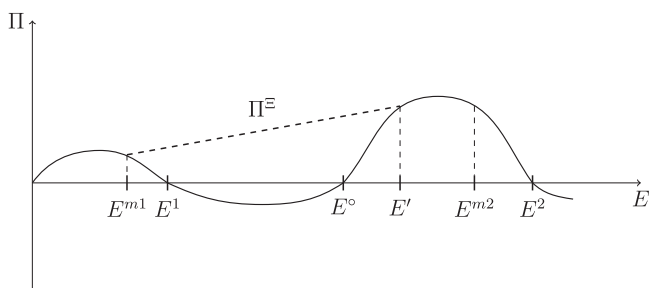
**Lemma 1'.** Let  $E^\#$  be an equilibrium aggregate of the game  $\Gamma^\Xi$  determined by  $\Lambda_1^\Xi(E^\#, E^\#) = 0$  and let  $\Pi^\Xi(E^\#) > 0$ . Then, we have  $\partial \Pi^\Xi(E^\#) / \partial E < 0$ .

The next proposition shows that contingent wage subsidy policy can also be effective in oligopsony as it can steer the economy to its highest employment equilibrium without any expense borne.

**Proposition 2.** Consider an oligopsonistic labor market and suppose that Assumption 3 holds. Let the level of employment be at the lowest equilibrium aggregate  $E^{m1}$ . There is a contingent wage subsidy  $S(h, H, T)$  that strictly implements the highest equilibrium aggregate  $E^{m2}$  without any wage subsidies paid in equilibrium.

Figure 4 illustrates Proposition 2. The economy is initially at the employment level  $E^{m1}$ . After the policy  $S(h, H, T)$ , described in the proposition, is introduced, the total payoff becomes increasing in the region where the policy is in effect (see the dashed segment over the interval

**FIGURE 4** Contingent wage subsidy



$[E^{m1}, E']$ ). However, in this region no new equilibrium aggregate arises as we are always bound to find a firm willing to hire more labor. Outside the policy's region, firms earn positive (regular) profits, the competition for which ultimately takes the total employment to its highest equilibrium.

Importantly, unlike in the case of competitive labor market, here subsidies should not be too generous so that in the policy's working region total payoffs do not exceed total payoffs outside this region as it may create new equilibria that lead to the payout of subsidies. At the same time, we also note that the insurance policy of compensating for losses only is not effective in oligopsony. While ensuring positive profits is sufficient to eliminate bad equilibria in competitive markets, it is not sufficient in noncompetitive markets. The mere elimination of the negative-profit region  $[E^1, E^o]$  in Figure 4 does not eliminate the low-employment equilibrium  $E^{m1}$ .

Kiyotaki (1988) studies an economy with a large number of firms, where “pessimistic” and “optimistic” investment equilibria can arise. He proposes, without formalization, an insurance program under which if one firm invests in a large amount of capital corresponding to an optimistic equilibrium but a pessimistic equilibrium is nevertheless realized, the government will transfer a certain amount to that firm. Then, firms will invest optimistically if the sum of this amount and the profit from optimistic investment is larger than the profit from pessimistic investment under the pessimistic equilibrium. Our analysis suggests, however, that the conclusion of Kiyotaki (1988) that the optimistic equilibrium will be realized without any subsidies paid depends on the assumption of price-taking firms. For this conclusion to hold with oligopolistic firms, the amount of subsidy would have to be set to depend on aggregate investment to ensure monotonically increasing total profits.

Lastly, there are several theoretical observations worth noting about the contingent wage subsidy. Our results depend only on the aggregate of individual actions which means that firms are not required to act symmetrically. Neither do our results depend on firms being homogeneous provided the quasi-linear aggregative structure of the underlying competition game is preserved with heterogeneous firms. Furthermore, an important implication of Lemma 1' is that the contingent wage subsidy is coalition-proof. By this lemma, the aggregate equilibrium outcomes lie to the right of the corresponding local maxima of the total profit function. Since coalitions do not change the total profit function of the underlying game and the contingent wage subsidy policy is effective only to the left from the global maximum, the change in the number of firms brought by a coalition cannot lead to the disbursement of contingent subsidies in equilibrium.

## 5 | CONCLUDING REMARKS

In this paper, we propose a new policy aimed at shifting the economy from low- to high-employment equilibrium at no cost for the government. The policy offers firms contingent wage subsidies for new

hires payable only if the prespecified threshold for total new hires is not reached. The role of contingent subsidies is to generate a sufficiently large externality that would start the mechanism of strategic employment complementarities taking employment further and beyond the threshold. The second contribution of the paper is theoretical as we demonstrate how assurance contracts need to be designed and applied in noncompetitive markets. In particular, we show that simple assurance contracts which only compensate for losses may not be effective in noncompetitive markets.

The contingent wage subsidy policy can be related to policies on wage regulation. Similarly to larger payroll expenses serving the purpose of the “big push” (Murphy, Shleifer, & Vishny, 1989), an increase in the mandated minimum wage can generate an aggregate demand externality justifying the increase; for empirical evidence, see Magruder (2013) and for recent discussions on minimum wages, see Economides and Moutos (2016) and Bennett and Chioveanu (2017). Unlike wage regulation, the contingent wage subsidy is not a coercive measure mandating firms to take required actions, which can be bankrupting for some firms, but rather a device to help firms coordinate on the socially desirable outcome.

In contrast to typical fiscal policy, the contingent wage subsidy does not distort the economy towards the areas where the government spends. Under the proposed policy market prices determine resource allocation where all expansions must pass a market test. In general, the labor market application studied in this paper is only one of possible applications of contingent subsidies. Such subsidies could be applied to any “big push” economic problem the successful resolution of which requires the accumulation of critical mass.

The main limitation of our model is that the conditional wage subsidy policy is derived under the assumption that the cause of low employment is known to be a macroeconomic coordination failure (e.g., in the aftermath of a pandemic). Generally, a reduction in employment could also be caused by a real economic shock such as an adverse productivity shock. Future research could examine the contingent wage subsidy policy under a more general setup. For instance, one direction could be to assume two aggregate states, high and low, about which the government does not have perfect information. Aggregate productivity in the low state is low enough so that only the bad equilibrium exists. In the high state aggregate productivity is such that both the good and the bad equilibria exist. Making an analogy to the bank runs model (e.g., Diamond & Dybvig, 1983), we can have two types of crisis: (a) fundamental crisis when the economy is in the low state; (b) expectation driven crisis when the economy is in the high state but in the bad equilibrium. The contingent wage subsidy is effective and implies no disbursements only if the crisis is expectation driven; otherwise subsidies would have to be disbursed. An interesting research problem is to find the optimal form of contingent wage subsidy policy when the type of crisis is uncertain and whether this policy would still be preferable to typical “big push” fiscal policy.

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## DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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## APPENDIX A: PROOFS

*Proof of Proposition 1.* Due to the continuity of  $\eta$  there must be at least one equilibrium between  $E^1$  and  $E^2$  and denote the closest equilibrium to  $E^2$  by  $E^\circ$ . Consider a policy profile  $\Xi = \{S(h, H, T), (e_i^1)\}$ , where  $(e_i^1)$  is the prepolicy equilibrium and the policy  $S(h, H, T)$  has  $T \in (E^\circ - E^1, E^2 - E^1)$  and the amount of subsidy such that a firm's resultant payoff satisfies (a)  $\pi^\Xi(e, E) > 0$  for any  $E \leq E^1 + T$  and  $e > e^1$ , where  $e^1$  is the firm's prepolicy level of employment, and (b)  $\pi^\Xi(e, E)$  is increasing in  $e$  for  $E < E^1 + T$ . Such a policy exists as there is no upper bound on subsidies. Clearly,  $E^1$  is not a locally stable equilibrium anymore but neither is any  $E < E^1 + T$  because firms can increase their payoff by increasing their employment levels. The total employment  $E = E^1 + T$  is not an equilibrium either because (regular) profits  $\pi$  are strictly positive in the neighborhood of  $E^1 + T$  which implies the possibility of profitable entries into the market. Ultimately, strictly positive regular profits  $\pi(e, E)$  for  $E \in [E^1 + T, E^2)$  result in firms' hiring until total employment reaches the locally stable equilibrium  $E^2$ . This also establishes that  $E^2$  is the unique equilibrium.  $\square$

*Proof of Lemma 1.* The equilibrium condition  $\Lambda_1(E^*, E^*) = 0$  implies from (10) that

$$\pi_1\left(\frac{E^*}{N}, E^*\right) + \pi_2\left(\frac{E^*}{N}, E^*\right) = 0. \quad (\text{A1})$$

The condition of positive profits,  $\Pi(E^*) > 0$ , implies that in equilibrium the expected marginal net return of labor, given by the partial derivative  $\pi_1$ , must be positive, that is  $\pi_1\left(\frac{E^*}{N}, E^*\right) > 0$ . Then, it follows from (A1) that  $\pi_2\left(\frac{E^*}{N}, E^*\right) < 0$ .

As the profit function  $\pi$  is a bilinear form, we have

$$\pi\left(E - \frac{N-1}{N}E^*, E\right) = \pi(E, E) - \frac{N-1}{N}\pi(E^*, E) \quad (\text{A2})$$

which implies, after noting that  $\Pi(E) = \pi(E, E)$ ,

$$\Lambda(E, E^*) = \Pi(E) - \frac{N - 1}{N} \pi(E^*, E). \tag{A3}$$

Differentiating both sides with respect to  $E$  and using the equilibrium condition  $\Lambda_1(E^*, E^*) = 0$ , we find

$$\Pi'(E^*) = \frac{N - 1}{N} \pi_2(E^*, E^*) < 0$$

as required. □

*Proof of Lemma 1'.* It is analogous to the proof of Lemma 1. □

*Proof of Proposition 2.* Let  $E' \in (E^{m1}, E^{m2})$  be the employment level closest to  $E^{m1}$  such that  $\Pi(E') = \Pi(E^{m2})$ . Such level  $E'$  must exist because of  $0 < \Pi(E^{m1}) < \Pi(E^{m2})$  by Assumption 3 and  $\Pi'(E^{m1}) < 0$  and  $\Pi'(E^{m2}) < 0$  from Lemma 1. Consider policy profile  $\Xi = \{S(h, H, T), (e_i^{m1})\}$ , where  $(e_i^{m1})$  is the prepolicy equilibrium and the policy  $S(h, H, T)$  has the following properties: (i) it is quasi-linear, (ii) the threshold is set at  $T = E' - E_1^m$ , (iii) the resultant total payoff function  $\Pi^\Xi(E)$  is twice differentiable and strictly increasing in  $E \in [E_1^m, E']$ , and (iv)  $S(h, T, T) = 0$  or  $\Pi^\Xi(E') = \Pi(E')$ . See Figure 4 for an illustration of such a policy. Next, we show that in the after-policy game  $\Gamma^\Xi$  there is no employment level  $E \in [E^{m1}, E']$  that is an equilibrium aggregate. Since  $\Gamma^\Xi$  is quasi-linear due to property (i), each equilibrium aggregate  $E^\#$  must satisfy  $E^\# = \arg \max_E \Lambda^\Xi(E, E^\#)$ . The previous equilibrium aggregate  $E^{m1}$  is no longer equilibrium because  $\Lambda^\Xi(E, E^{m1})$  increases in  $E$  to the right from  $E = E^{m1}$  due to property (iii). Because of properties (ii) and (iii), Lemma 1' implies that there is no equilibrium aggregate in the interval  $(E^{m1}, E')$ . If the employment level  $E'$  were an equilibrium aggregate, then it would also be an equilibrium aggregate in the prepolicy game  $\Gamma$  because of  $\Lambda^\Xi(E', E') = \Lambda(E', E')$  due to property (iv) but this contradicts Assumption 3. We obtain an analogous contradiction for any other  $E \in \overline{[E_1^m, E']} \cap E^{m2}$ . Thus,  $E^{m2}$  is the only equilibrium aggregate. □