Vying for Support: Lobbying a Legislator with Uncertain Preferences*

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Abstract

We consider a model where two opposing lobbyists bid for the support of a legislator with an integrity threshold and an uncertain bias towards either lobbyist. Our results show that at low levels of uncertainty, lobbyists bid aggressively. The potential bias of the legislator towards a lobbyist's policy is not high enough to risk losing the legislator's support, and each lobbyist bids high amounts to stay competitive. Conversely, at high levels of uncertainty, lobbyists take into account the possibility that the legislator has a strong preference towards a lobbyist's policy position and bid lower. This is in line with empirical evidence that lobbying is often done towards legislators who already agree with them. Our results take this a step further and indicate that as long as there is a possibility that a given legislator has strong preference for the lobbyist's position, it would be more cost efficient to vye for them. Under moderate levels of uncertainty, asymmetric equilibria are observed and lobbyists bid just enough to ensure that the legislator chooses a policy to support.

Keywords: lobbying; uncertainty; integrity threshold; legislatures

JEL Classification: D72, D80

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1 Introduction

Lobbying is ubiquitous in most legislative systems. It is also one of the more controversial feaures of democratic governance. The persistent negative reputation of lobbying can be attributed to its close association with corruption. In an effort to shift this negative image, there has been a marked increase in national and supranational lobbying regulations from five in 2000 to eighteen in 2020 (McKay and Wozniak, 2020). This also means that a significant amount of lobbying done in the rest of the world is still largely unregulated and with little to no information on the process, its inputs, and the corresponding outcomes.

Uncertainty on preferences can provide politicians with more options when interacting with lobbyists. Politicians are less likely to seek rent where there is increased scrutiny. For example, the sectors with the highest levels of lobbying spending in the United States in the past five years do not include hot button issues such as abortion and gun laws (Center for Responsive Politics, 2017). Schneider (2012), in his study on the role of the agenda-setter and lobbying, found that for issues with low salience, committee chairs have more incentive to propose more extreme policies and reap monetary rewards. Low salience sectors, including finance and health care, have the highest levels of lobbyist spending for 2018 (Center for Responsive Politics, 2018). Business interests, trade associations, and professional groups have been shown to employ more lobbyists per issue and spend more (Baumgartner and Leech, 2001; de Figueiredo and Richter, 2013; McKay, 2012), accounting for 84% and 86% of total lobbying expenditures at the U.S. Federal and state level (de Figueiredo, 2004). The views of legislators are often undisclosed in these sectors, and this uncertainty in preferences provides politicians with opportunities for gain at the expense of the collective good. Increased information on lobbying activities to the public, and the reflection of public sentiment by non-profit associations, may influence legislator preferences, and consequently the outcomes of the lobbying process to improve public welfare.

The paper studies the effect of this opacity and uncertainty in lobbying behaviour. Uncertainty, specifically in legislator preferences, is key to understanding unregulated lobbying environments as the uncertainty in a legislator's preference provides lobbyists with a unique opportunity to obtain this once "unaccounted" support. The model captures this by looking at the degree of uncertainty on a legislator's preference and her known level of integrity. We approach opacity in the lobbying process through a simultaneous lobbying structure. Where lobbyist-legislator interactions are kept private, the opportunity for lobbyists to counteroffer may not exist. The simultaneous lobbying approach takes this into account and retains focus on how interactions center on the uncertainty of legislator preferences.

Our results show that the degree of uncertainty about the legislator preference directly affects the bidding strategy of lobbyists. At low levels of uncertainty, lobbyists bid aggressively. The potential bias of the legislator towards a lobbyist's policy is not high enough to risk losing the legislator's support, and each lobbyist bids high amounts to stay competitive. Conversely, at high levels of uncertainty, lobbyists take into account the possibility that the legislator has a strong preference towards a lobbyist's policy position and bid lower. This is in line with empirical evidence that lobbying is often done towards legislators who already agree with them (Hojnacki and Kimball, 1998; Hall and Miler, 2008; de Figueiredo and Richter, 2013). Our results take this a step further and indicate that as long as there is a possibility that a given

legislator has strong preference for the lobbyist's position, it would be more cost efficient to vye for them. Under moderate levels of uncertainty, asymmetric equilibria are observed and lobbyists bid just enough to ensure that the legislator chooses a policy to support.

Empirical evidence suggests that the effects of cash-for-favour lobbying activities on policy are marginal (Grossman and Helpman, 1994; Ansolabehere et al., 2003; de Figueiredo and Richter, 2013). Despite this, public awareness on lobbying is centred largely on the perception of transactional lobbyist-legislator interactions. The impact of lobbying activities from non-profit associations on legislation is significant enough for the US congress to attempt to legislate restrictions on their participation in lobbying in the mid-1990s (Balassiano and Chandler, 2010). An article from the New York Times in 2013 reported that the influence of Wall Street in Washington has grown substantially (Lipton and Protess, 2013). In one of the bills passed by the House Financial Services committee in May 2013 exempting a large portion of financial trades in new regulation, the recommendations of Citigroup were reflected in seventy of the eighty five lines of the bill (Lipton and Protess, 2013).This alludes to a prevalence of shadow lobbying - one where lobbying proceeds without any regulatory oversight, within one of the strictest lobbying environments in the world.

Even in the UK, which has been deemed as one of the most transparent governments in the world, shadow lobbying can also be observed. According to Transparency International Report - Lifting the Lid on Lobbying *" The vast majority of lobbying in the UK occurs behind closed doors and is not disclosed ... great deal of policy-making and lobbying takes place elsewhere, as our interviews with both policy-makers and lobbyists confirmed."*(David-Barrett, 2015, p.15). In the same 2015 report, Transparency International UK outlined lobbying and public sector scandals and its most recent occurrence. Scandals where 'money is exchanged for access to politicians and party policy committees', 'local councillors acting as paid lobbyists', and 'former government and military officers selling access and influence for money' have been recorded up to a year before the review was written. The proliferation of cash-for-favour exchange scandals in the UK government indicate that this practice is still commonplace despite its illegality.

One can understand shadow lobbying better through the study of legislator-lobbyist interactions beyond initial introductions. In his continuum access model, Wright (1996) called this interaction the'messaging' stage. Heberlig (2005) explored Wright's continuum model further and found that instead of a straight path regarding legislator interaction from introduction ('positioning') to exchange ('messaging'), the lobbyists appeared to gather information on legislators first, with varied target legislators at the 'positioning' and 'messaging' stages. We take this into account by incorporating uncertainty on the legislator preferences in our model - focus on the transition from the 'positioning' to the 'messaging' stage - where the lobbyist secures the support of the legislator. We move away from the sequential lobbying structure introduced by Groseclose and Snyder (1996), instead opting for a simultaneous sealed bid lobbying structure to better recreate shadow lobbying. When lobbyists and legislators meet behind closed doors, the opportunity for competing lobbyists to counteroffer may not exist. Che and Gale (1998) looked at lobbying under capped expenditures through an all pay auction structure. We also move away from using an all pay auction as it considers all the legwork done prior to the discussions to gain access to the legislator. We argue that any interactions prior to the actual exchange, such as the dinners and perks

received, although helpful in securing access do not always come into play when the decision making is done. The legislator needs to consider whether the "loot is commensurate to the punishment" and can only feasibly do so using a forward looking approach (*i.e.* if I do this for you, how much will I get in return?). In our model, the legislator accepts the first offer that meets her expectations. The decision to support the lobbyist has to be made quickly as the risk of discovery through holding repeated negotiations will be high and the consequences substantial.

Preferences of legislators are often private and unknown to the lobbyists (Heberlig, 2005). We explore this uncertainty through studying lobbyist behaviour given uncertain legislator preferences similar to Buzard and Saiegh (2016) and Dekel et al. (2006).There has been a significant amount of work on the effects of lobbying on policy outcomes, either through information or transactional exchanges (Austen-Smith, 1993; Groseclose and Snyder, 1996; de Figueiredo, 2002; Hall and Deardorff, 2006), although less have explored uncertainty in lobbying. Austen-Smith and Wright (1992) looked at lobbying as information transmission at the agenda-setting and voting stages, and found that when there is occasional uncertainty on how informed the lobbyist is, more information transmission can occur. Buzard and Saiegh (2016) looked at sequential vote buying models, and specifically, at the allocation of bribes amongst three legislators. Dekel et al. (2006) looked at vote buying and explored as an extension the presence of uncertainty in legislatures. They found that with a large enough body of legislators, one can predict who the winning lobbyist is. Tyutin and Zaporozhets (2017) focus on the uncertainty on legislator types in a legislature and the interaction of the legislature and a single lobbyist. Uncertainty is rarely explored on its own, we address this by looking at lobbyist interactions over one non-strategic legislator. We remove the dimension of budget allocation and instead assume that the lobbyists are willing to pay at most the value of the legislator support. To the best of our knowledge, we are the first to study how uncertainty on the position of a single legislator can affect the behaviour of competing lobbyists.

Our model also features a measure of the legislator's level of integrity. The politician does not hold an open sale for her support given restrictions on vote-buying. Instead, the politician chooses the lobbyist that promises to contribute enough, and if there is more than one lobbyist that offers enough, she goes for the lobbyist offering the most value to her. We can think about this from the perspective of a reputation conscious politician. A reputation conscious politician may not want to be associated with more than one lobbying group for an issue, and would like to be viewed as consistent and honest by her constituents. The more controversial an issue is, the more likely it is for the politician's threshold to be higher. Lobby's revolving-door phenomenon, whereby former stasaffers turned lobbyists use their connections to incumbents to push for legislation (Blanes i Vidal et al., 2012; LaPira and Thomas, 2014; Lazarus et al., 2016), explains why the integrity of the legislator is known to lobbyists. As politicians move from public service to the private sector, they take their expertise on both the workings of the political system and the proclivities of their former colleagues with them. Politicians would know who among their colleagues can be wooed for support, and who are strong in their convictions from previous interactions. Moving through the revolving door as lobbyists, they provide lobbies with information on the level of integrity of each legislator. This does not mean that the specific preferences of each legislator are known to the lobbyists. As the preferences vary across issues, the reliability of the information on preferences would be less than that of the integrity of the legislator, making a range of possible legislator

preference values more likely than a clear cut one.

The rest of the paper is organized as follows. Section 2 introduces the model. Sections 3 and 4 show the expected utility and the computation of best responses. The equilibrium of the game is characterized in full in Section 5. The results are discussed in Section 6, and the paper is concluded in Section 7. The proofs are shown in full in the appendices.

2 The Model

Consider a legislator and two lobbyist. The legislator has a policy bias b distributed uniformly $b \sim \mathcal{U}(-d, d)$, with $d > 0$, and an integrity threshold $t > 0$. A positive b indicates a legislator bias towards policy two, while a negative b indicates a preference for policy one. At $b = 0$ the legislator is unbiased. The legislator's utility is given by,

$$U_L = \begin{cases} p_1 - t - b & \text{if Lobbyist 1 wins,} \\ p_2 - t + b & \text{if Lobbyist 2 wins,} \\ 0 & \text{otherwise,} \end{cases}$$

where p_i is the bid submitted by lobbyist i.[1]

The lobbyists support opposing policy positions and try to sway the legislator. The game can be viewed through the lens of a policy dimension, with each lobbied policy on the opposite end of a one-dimensional policy space. The preference of each lobbyist is fully aligned with their policy of choice, and lies on the extremes of the policy space. The lobbyist preferences are fully revealed from the onset with each lobbyist supporting only their policy of choice.

The lobbyists know the distribution of the legislator's bias and the integrity threshold. Both bid simultaneously $p_i \geq 0$ to win the legislator's support. Bids are only considered by the legislator when they are above the bias-adjusted threshold (i.e. $t \pm b$). The bid that provides the legislator with the highest utility to the legislator wins. Only the winning bid is collected. Lobbyist i gains $w \in \mathbb{R}_+$ upon winning.

The utilities of each lobbyist i are given below:

$$U_1 = \begin{cases} w - p_1 & \text{if } p_1 > t + b \text{ and } p_1 > p_2 + 2b, \\ 0 & \text{otherwise.} \end{cases}$$

$$U_2 = \begin{cases} w - p_2 & \text{if } p_2 > t - b \text{ and } p_2 > p_1 - 2b, \\ 0 & \text{otherwise.} \end{cases}$$

[1] Note that we assume that in the absence of lobbying the legislator finds it optimal to not implement a new policy and this gives the legislator an outside option of 0 utility. When either policy is approved, we assume the legislator suffers a loss of t, the integrity threshold.

The game is summarised in the symmetric simultaneous-move game (N, P, b, U), defined formally as follows:

1. Three players ($N = 3$), two lobbyists and one legislator;
2. Lobbyist's bids belong to $p_i \in P = [0, w]$;
3. b is the legislator's realised type, $b \sim \mathcal{U}(-d, d)$;
4. U_i, for $i \in \{1, 2\}$, is the lobbyist's utility, and U_L the legislator's utility.

Note that the legislator is a non-strategic player. The legislator's utility function can be viewed as the rules of the game. The winner is determined not by any strategic decision making on the legislator's side. As we are looking at shadow lobbying, it stands to reason that the legislator will accept the first offer that meets her expectations, as the risk of discovery through holding repeated negotiations will be high and the consequences substantial.

A lobbyist only wins if the bid is considered sufficient and provides the most payoff to the legislator. The probabilities of winning for each lobbyist are obtained in full in the appendix and are given as follows:

$$P(1 \text{ wins}) = \begin{cases} \dfrac{p_1 - t + d}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{p_1 - p_2 + 2d}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{1}$$

$$P(2 \text{ wins}) = \begin{cases} \dfrac{d - t + p_2}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{2d - p_1 + p_2}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{2}$$

3 Expected Utilities

The expected utility of lobbyist i is given by the probability of winning, expressions (1) and (2), and the utility of the lobbyist, U_i. As with the probabilities, the computation of the expected utility depends on the relationship of the average bid to the integrity threshold of the legislator. We compute the expected utility of each lobbyists when the average bid is below the threshold, $(p_1 + p_2)/2 \leq t$, and when it is above the threshold, $(p_1 + p_2)/2 \geq t$. The expected utility is continuous at the point where the average bid is equal to the threshold. At each scenario, we compute the optimal bids, with $\underline{p_i}$ and $\overline{p}_i$ denoting the optimal bids when the average bid is below and above the threshold, respectively.

Given the symmetric environment, it is sufficient to compute the expected utility of one lobbyist, say lobbyist one. The expected utility of lobbyist 1 is

$$EU_1(p_1, p_2) = \begin{cases} \dfrac{p_1 - t + d}{2d}(w - p_1) & \text{if } \frac{p_1 + p_2}{2} \leq t, \quad (3) \\ \dfrac{p_1 - p_2 + 2d}{4d}(w - p_1) & \text{if } \frac{p_1 + p_2}{2} \geq t. \quad (4) \end{cases}$$

Scenario 1: Average bid below the threshold ($\frac{p_1+p_2}{2} \leq t$)

Given an opposing bid p_2, we solve for the maximum of the expected utility (3). The optimal bid satisfies $p_1 = (w + t - d)/2$. Note that the function (3) is concave, with second derivative, $-1/d$, that is always negative. The lobbyist's bids must be feasible, that is, $p_1 \in [t - d, 2t - p_2]$, at the given p_2, in order to keep the average bid below the threshold, $(p_1 + p_2)/2 \leq t$, and exceed the effective lower bound, $t - d$. Note that the feasible bids of lobbyist one belong to a well-defined interval as long as $p_2 < t + d$.[2]

The identification of $\underline{p_1}$ is shown graphically in figure 1. The expected utilities when the average bid is below the threshold are outlined in 5.

$$EU_1\left(\underline{p_1}, p_2\right) = \begin{cases} \dfrac{(d - t + w)^2}{8d} & \text{if } p_2 \leq \dfrac{3t - w + d}{2}, \\ \dfrac{(d - p_2 + t)(p_2 - 2t + w)}{2d} & \text{otherwise.} \end{cases} \tag{5}$$

The lower bound, $t-d$, is the minimum possible bid for the lobbyist. One can look at the lower bound as the lowest possible bias adjusted threshold—the legislator's integrity threshold, t, adjusted to account for the maximum possible bias the legislator can have for the lobbyist, $-d$. As the legislator only considers bids that are at least the bias adjusted threshold, bidding below the lower bound renders lobbyist's bid irrelevant. We assume that the winning valuation, w, always exceeds the minimum possible bid for the lobbyists.

Scenario 2: Average bid above the threshold ($\frac{p_1+p_2}{2} \geq t$)

We follow the same process in the identification of expected utilities for scenario 2. Given an opposing bid p_2, we solve for the maximum of the expected utility (4). The optimal bid satisfies $p_1 = (w + p_2 - 2d)/2$. The function (4) is always concave with a negative second derivative, $-1/2d$. For the average bid to be greater than or equal to the threshold, for any given p_2, lobbyist one has to bid at least $2t - p_2$, and the feasible set of bids reduce to $p_1 \in [max\{p_2 - 2d, 2t - p_2\}, w]$. For the average bid to be greater than or equal to the threshold, for any given p_2, lobbyist one has to bid at least $2t - p_2$, and the feasible set of bids reduce to $p_1 \in [max\{p_2 - 2d, 2t - p_2\}, w]$.

The identification of $\overline{p}_1$ is shown graphically in figures 2. the expected utility of lobbyist one is equal to

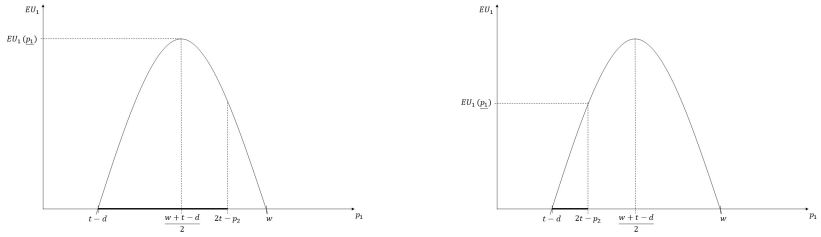
$$EU_1\left(\overline{p}_1, p_2\right) = \begin{cases} \dfrac{(2d - p_2 + w)^2}{16d} & \text{if } p_2 \geq \dfrac{4t + 2d - w}{3}, \\ \dfrac{(d - p_2 + t)(p_2 - 2t + w)}{2d} & \text{otherwise.} \end{cases} \tag{6}$$

Under this scenario, the lobbyist must bid at least $p_2 - 2d$ for the expected utility to be positive. When the average bid is above or equal to the threshold, it is certain that one of the lobbyists wins the support of the legislator. The lobbyist shifts from making sure that the minimum eligibility condition, the bias adjusted threshold, is met to ensure that his bid is competitive enough to remain in the game. If both

[2]When $p_2 \geq t + d$, lobbyist one does not choose to keep the average below the threshold. All feasible bids will neither reach the threshold nor beat the opposing bid. Lobbyist two wins for sure if lobbyist's one bidding strategy keeps the average bid below the threshold.

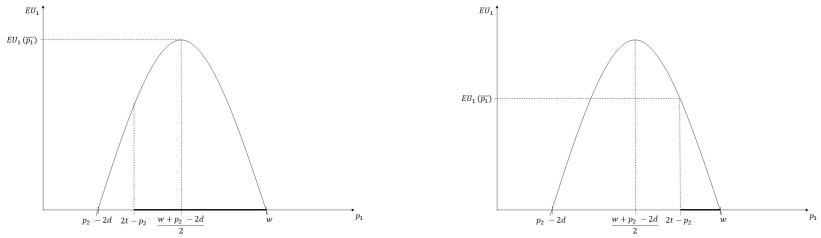
bids exceed their respective bias adjusted thresholds, lobbyist one only wins once his bid, adjusted with the bias, exceeds that of his opponent (*i.e.* $p_1 - b > p_2 + b$). Recall that a negative bias is advantageous for lobbyist 1, obtaining the maximum possible advantage when $b = -d$. For lobbyist one to have an eligible bid when the average bid is above or equal to the threshold, the bid must at least be greater than $p_2 - 2d$. The bid is capped by the winning valuation w.

Note that the optimal bids alternate between $\underline{p_1}$ and $\overline{p}_1$, depending on where the lobbyist's two bid p_2 is, in relation to thresholds $(3t - w + d)/2$ and $(4t + 2d - w)/3$. As the minimum possible bid cannot exceed the winning valuation, $t - d \leq w$, it follows that $(3t - w + d)/2 \leq (4t + 2d - w)/3$. These thresholds are referred to as the lower bound and the upper bound.



(a) $\frac{w+t-d}{2} \leq 2t - p_2$ (b) $\frac{w+t-d}{2} \geq 2t - p_2$

Figure 1: Identification of $\underline{p_1}$



(a) $\frac{w+p_2-2d}{2} \geq 2t - p_2$ (b) $\frac{w+p_2-2d}{2} < 2t - p_2$

Figure 2: Identification of $\overline{p_1}$

4 Best Responses

Each lobbyist compares the expected utility from bidding $\underline{p_i}$ and $\overline{p_i}$, given the opponent's bid, and chooses the bid that provides him with the highest utility. The best response of lobbyist i is equal to

$$BR_i(p_{-i}) = \begin{cases} \dfrac{w+t-d}{2} & \text{if } p_{-i} \leq \dfrac{3t-w+d}{2}, \\ 2t - p_{-i} & \text{if } \dfrac{3t-w+d}{2} < p_{-i} < \dfrac{2d+4t-w}{3}, \\ \dfrac{w-2d+p_{-i}}{2} & \text{otherwise.} \end{cases} \tag{7}$$

We determine the best responses of lobbyist 1 from the sets of possible bids of lobbysit 2 identified in C.

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

If lobbyist two bids at or below $\frac{3t-w+d}{2}$, lobbyist one can choose to bid either $\underline{p_1} = (w+t-d)/2$ and keep the average bid below the threshold, or bid $\overline{p_1} = 2t - p_2$ and push the average bid to the threshold.

Whenever $p_2 \leq (3t-w+d)/2$, a quick comparison of expected utilities show that $EU_1\left(\underline{p_1}, p_2\right) \geq EU_1\left(\overline{p}_1, p_2\right)$. This means that the utility of lobbyist 1 is higher if he bids to keep the average below the legislator's integrity threshold and the best response of lobbyist one is $BR_1\left(p_2\right) = (w+t-d)/2$.

When the opponent's bid is low enough, the lobbyist chooses to bid conservatively. It may appear counterintuitive at first as the lobbyist can potentially secure the legislator's support immediately. By increasing his bid, he increases the chances of winning the support of legislator, but this is at the expense of lowering his take-home win. As his opponent is not bidding aggressively, the lobbyist may end up paying more if he decides to bid just enough to ensure that the game ends, $\underline{p_1} = 2t - p_2$. The lobbyist can afford to bid conservatively as the winning probability of the opponent is not high enough to warrant a price war. This is evident from the fact that his best response stays constant regardless of the opponent's bid.

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t+2d-w}{3}$

For a given p_2 between the lower and upper bound, the best response of lobbyist's one is $BR_1\left(p_2\right) = 2t - p_2$. As before, each lobbyist is faced with the trade-off between a higher chance of winning the support of the legislator and a lower take-home win. When the opposing bid is not low enough to justify bidding conservatively, but also not high enough to justify bidding aggressively, his best response is to bid enough so that the game ends with a winning bid.

Case 3: $p_2 \geq \frac{4t+2d-w}{3}$

When lobbyist 2 bids at or above $\frac{4t+2d-w}{3}$, lobbyist one can choose to bid either $\underline{p_1} = 2t - p_2$ to keep the average bid at the threshold or bid $\overline{p}_1 = (w + p_2 - 2d)/2$ to push the bid above the threshold.

Whenever $p_2 \geq (4t + 2d - w)/3$, $EU_1\left(\overline{p}_1, p_2\right) \geq EU_1\left(\underline{p_1}, p_2\right.$ When the opposing lobbyist bids high, the lobbyist obtains a higher expected utility by also bidding high, with the best response of $BR_1\left(p_2\right) = (w + p_2 - 2d)/2$.

When the opponent bids aggressively, the lobbyist's bid must at least be equal to his opponent's bias adjusted bid, otherwise his bid is not competitive. He can choose to bid conservatively, just enough to ensure that the game ends or match the opposing bid head on. Both bids depend on p_2, with $\overline{p}_1$ increasing and $\underline{p_1}$ decreasing in p_2. The more aggressive the opponent's bid is, the more likely he is to win the support of the legislator. Under bidding strategy $\underline{p_1}$ where the lobbyist bids conservatively, the lobbyist becomes less likely to win the support of the lobbyist but increases his possible take-home pay. Although the strategy made sense when the competing lobbyist was bidding conservatively, there is no clear incentive to do the same with a more aggressive competitor. The bids under the conservative strategy decrease as the opponent bid increases, essentially demanding for a higher tradeoff cost on the

probability of winning and the increase in the take home pay. Furthermore, the higher the opponent's bid gets, the less likely it is for the conservative bid to be eligible, so that he must bid aggressively in order to win the support of the legislator. The lobbysist bid should exceed the minimum competitive bid, $p_2 - 2d$, and is bounded above by the maximum bid w. In sum, by bidding according to $\overline{p}_1$, lobbyist 1 secures a significant chance of winning the legislator's support and enough utility once the support is won.

5 Equilibria

Proposition 1. *(Nash Equilibria)*

1. *If $d \geq w - t$, there exists a unique Nash equilibrium (p_1^*, p_2^*), where $p_1^* = p_2^* = (w + t - d)/2$.*
2. *If $(5/7)(w - t) < d < w - t$, there exists a continuum of Nash equilibria (p_1^*, p_2^*), where*
   - *$p_1^* = \frac{w+t-d}{2}$ and $p_2^* = \frac{3t-w+d}{2}$ or*
   - *$p_1^* = \frac{3t-w+d}{2}$ and $p_2^* = \frac{w+t-d}{2}$ or*
   - *$p_i^* \in \left(\frac{3t-w+d}{2}, \frac{4t+2d-w}{3}\right)$, $p_{-i}^* \in \left(\frac{3t+d-w}{2}, \frac{w+t-d}{2}\right)$ and $p_i^* = 2t - p_{-i}^*$.*
3. *If $d = (5/7)(w - t)$, there exist a continuum of Nash equilibria (p_1^*, p_2^*), where*
   - *$p_1^* = \frac{3w-5d+t}{4}$ and $p_2^* = \frac{w+t-d}{2}$ or*
   - *$p_1^* = \frac{w+t-d}{2}$ and $p_2^* = \frac{3w-5d+t}{4}$ or*
   - *$(p_1^*, p_2^*) \in \left(\frac{3t-w+d}{2}, \frac{4t+2d-w}{3}\right)$ and $p_2^* = 2t - p_1^*$.*
4. *If $(1/2)(w - t) < d < (5/7)(w - t)$, there exists a continuum of Nash equilibria (p_1^*, p_2^*), where*
   - *$p_1^* = \frac{w-2d+2t}{3}$ and $p_2^* = \frac{4t-w+2d}{3}$ or*
   - *$p_1^* = \frac{4t-w+2d}{3}$ and $p_2^* = \frac{w-2d+2t}{3}$ or*
   - *$p_i^* \in \left(\frac{3t-w+d}{2}, \frac{4t+2d-w}{3}\right)$, $p_{-i}^* \in \left(\frac{w-2d+2t}{3}, \frac{4t+2d-w}{3}\right)$ and $p_i^* = 2t - p_{-i}^*$.*
5. *If $d \leq (1/2)(w - t)$, there exists a unique Nash equilibrium (p_1^*, p_2^*), where $p_1^* = p_2^* = w - 2d$.*

6 Results and Discussion

The characterisation of equilibria is based on the relationship between the length of the bias interval, d, and the difference between the winning valuation, w, and the legislator's integrity threshold, t. We provide the full calculation of equilibria in Appendix C. The results of Proposition 1 are discussed point by point below.

In point one, when $d \geq w - t$, both lobbyists bid $p_1^* = p_2^* = (w + t - d)/2$ and keep the average bid below the legislator's bias integrity threshold, $(p_1^* + p_2^*)/2 \leq t$. The length of the bias interval measures the uncertainty of the legislator bias, with a longer interval indicating higher uncertainty over the legislator's bias. When the length of the bias interval is long enough ($d \geq w - t$), both lobbyists maximise their expected utilities by bidding conservatively. The best case scenario for a lobbyist would be to have the threshold adjusted heavily downwards by a favourable bias by the legislator. It follows that the worst

possible case for the lobbyist is when the bias is at its most favourable for his opponent, driving the bias adjusted threshold above the winning valuation and making it impossible for the lobbyist to win. Recall that both lobbyists are unaware of the actual legislator bias, but have the same information on its distribution. As it is equally likely for both of them to be at an advantageous position, but too costly for either one to secure a high enough probability of winning, both lobbyists bid conservatively. The bids, and consequently the utilities of lobbyists under this scenario do not depend on their opponents bid, and both lobbyists are equally likely to win the legislator's support. The lobbyist chooses to bid midway between his minimum possible bid $t - d$, and his maximum possible bid w, maximising his expected utility.

The uncertainty of the legislator bias has to be sufficiently higher than the difference between the lobbyist's winning valuation and the legislator's integrity threshold for the equilibrium bids to be valid. Without uncertainty, the minimum qualifying bid is just the integrity threshold of the legislator. An increase in the level of integrity of the legislator, holding the winning valuation constant, makes it more likely the bias interval to be sufficiently high. This may help explain why despite high levels of coverage on single issues in the United States (*e.g.* gun rights vs. gun control and pro-life vs. pro-choice), and the small degree of uncertainty on legislator preference on the issues, often earmarked by party memberships (*e.g.* Republicans for gun rights and Democrats for gun control), lobbyist spending in the single issue sector does not reach the top five sectors with the highest lobbying expenditure in 2018 (Center for Responsive Politics, 2018). Single issues also come with high legislator integrity thresholds. The positions of legislators are heavily publicised, with high costs on reputation if the legislator's integrity is questioned. With ideological issues, the rewards of the policy are often not as high as issues tied with industry interest. Lower winning valuations coupled with the the high integrity threshold can drive lobbyists to bid more conservatively.

Another way to look at the conservative bidding strategy of the lobbyist is through a budget perspective. If each lobbyist has an alloted amount to secure access to potential allies on a specific issue, it may be more effective to gather as many as possible. The results of point one indicate that the cheapest lobbyists are those who may have a strong preference for them or for their opponent. This supports an growing consensus identified by de Figueiredo and Richter (2013) where legislators, allied and marginal, from both sides of the issues are approached more often by lobbyists (Kollman, 1997; Holyoke, 2003; Heberlig, 2005; Hall and Deardorff, 2006; Bertrand et al., 2014; Gawande et al., 2012).[3] In particular, Hall and Miler (2008) and Hojnacki and Kimball (1998) argued that interest groups approach legislative allies first, followed by marginal legislators. Our results show that as long as there is a chance that a legislator has a strong preference for a lobbyist, it would be more cost-effective for the lobbyist to approach the legislator.

Moving on to points two to four, we find a continuum of asymmetric equilibria, with the characteristic that the average bid is equal to the legislator's threshold. When the interval is moderately long ($(5/7)(w-t) < d < w - t$), lobbyists can always bid aggressively to secure the win — however the increase in the winning probability must be worth the increased cost. As the uncertainty on the legislator is

[3]Marginal voters are accounted for in our model through the existence of uncertainty - however the magnitude of this uncertainty is what is explored in the analysis of equilibria.

still significant when the bias interval is moderately long, bidding aggressively causes the lobbyist to pay more than necessary to win the legislator's support. He is better off bidding conservatively. The other lobbyist bids enough to have him indifferent between bidding conservatively and aggressively, keeping the average bid at the threshold. On the other hand, when the bias interval is moderately short ($(1/2)(w-t) < d < (5/7)(w-t)$), bidding conservatively is risky as the interval is not long enough to ensure that the opponent does the same. One lobbyist tries and bids aggressively to increase the chance of securing the legislator's support, while the other bids just enough for the average bid to reach the threshold.

Lastly, we expound on point five. When $d \leq (w-t)/2$, both lobbyists bid $p_1^* = p_2^* = w - 2d$, in order to push the average bid above the legislator's bias integrity threshold. They favour more aggressive bidding strategies, as the information on the bias becomes more precise. A short bias interval implies that the benefit for each lobbyist of having the legislator's preferences in his favour is marginal. The winning bid needs to surpass both the threshold and the opposing bid. As the difference between the legislator's threshold and bias adjusted threshold is minimal under a short bias interval, the lobbyists both assume that the opposing bid has already surpassed the threshold. As the bids are the same, lobbyists are equally likely to win the legislator's support. Notice that the equilibrium bid is just the difference between the wealth valuation and the full length of the bias interval $2d$ [4]

If the uncertainty over the legislator's bias is sufficiently small relative to the difference between the lobbyist's winning valuation and the legislator's integrity threshold, then each lobbyist bids aggressively. A higher winning valuation makes the uncertainty of the legislator preference smaller in comparison. Similarly, a lower integrity threshold increases $(w-t)/2$, making it more likely for the uncertainty over the legislator's bias to satisfy $d \leq (w-t)/2$. Center for Responsive Politics (2018) reports that the top three sectors in terms of lobbying expenditure are Health, Finance, Miscellaneous Business, with over \$500 million spending for each sector in 2018. These sectors provide high payoffs for lobbyists. Alongside this, as the public is less likely to be aware of the legislator's preferences on various industry specific areas, the integrity threshold is significantly lower relative to hot button issues. The reputation of legislators are less likely to be questioned if lobbying requests are entertained for theses issues. Although the lobbyists may have less certainty in terms of the legislator's preferences, this is dwarfed by the substantial winning valuation and the lower integrity threshold. This echoes results from existing literature which finds that business interests, trade associations, and professional groups, are the groups with the highest level of lobbying expenditures (Baumgartner and Leech, 2001; de Figueiredo, 2004; McKay, 2012; de Figueiredo and Richter, 2013).

[4] With little to no uncertainty, the model may be reminscent of the Standard Tullock Contest (Tullock, 1967). Our model deviates from Tullock Games as we consider a simultaneous sealed-bid auction instead of an all-pay auction. Furthermore, the uncertainty here is on the price of the access rather than the opponent valuation found in the literature **please add lit**.

7 Conclusion

We explore how lobbying can proceed in an unregulated environment. A simultaneous lobbying structure is used to capture how lobbying proceeds behind closed doors. Under shadow lobbying, where lobbyist-legislator interactions are kept private, lobbyists may not be able to counter offer. The paper focuses on lobbyist interactions over one non-strategic legislator and explores the impact of uncertainty on the lobbyist's behaviour in isolation.

The model explains how the relationship between the uncertainty over the legislator's bias, the legislator's integrity threshold, and the lobbyist's winning valuation, influence the bidding strategies of lobbyists. When the uncertainty is low with respect to the winning valuation, lobbyists tend to bid aggressively in order to secure the support of the legislator. When the uncertainty is high, both lobbyists bid conservatively. In interim cases, we observe asymmetric equilibria, where the average bid is equal to the threshold.

The interactions discussed so far provide a snapshot of how lobbying may proceed behind closed doors. The results of this paper corroborate the growing consensus that lobbyists mostly approach legislators who are allied to them, and offer the additional insight that the possibility of a strong preference towards his policy stance is enough to make the legislator an attractive target audience. More generally, results indicate that for issues with high monetary rewards, lobbyists bid more aggressively. Most likely, these are issues where ideologies are not as clear cut. Political agents, however, may listen to constituent opinions and adjust their preferences accordingly, which, in turn, influence lobbyists behaviour.

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Appendices

A Deriving Winning Probabilities

A lobbyist only wins if the bid is considered sufficient and provides the most payoff to the legislator. The probabilities of winning for each lobbyist are:

$$P\left(1 \text{ wins}\right) = P\left(p_1 > t + b \bigcap p_1 > p_2 + 2b\right),$$
$$P\left(2 \text{ wins}\right) = P\left(p_2 > t - b \bigcap p_2 > p_1 - 2b\right).$$

We begin with looking at the probability of winning for lobbyist 1,

$$\begin{aligned} P(1 \text{ wins}) &= P\left(p_1 > t + b \bigcap p_1 > p_2 + 2b\right) \\ &= P\left(b < p_1 - t \bigcap b < (p_1 - p_2)/2\right) \\ &= P\left(b < \min\{p_1 - t, (p_1 - p_2)/2\}\right). \end{aligned}$$

The winning probabilities for lobbyist 2 are obtained in a similar manner. To sum up:

$$P(1 \text{ wins}) = \begin{cases} P(b < p_1 - t) & \text{if } \frac{p_1+p_2}{2} < t, \\ P(b < \frac{p_1-p_2}{2}) & \text{otherwise.} \end{cases} \tag{8}$$

$$P(2 \text{ wins}) = \begin{cases} P(b > t - p_2) & \text{if } \frac{p_1+p_2}{2} < t, \\ P(b > \frac{p_1-p_2}{2}) & \text{otherwise.} \end{cases} \tag{9}$$

When the average bid is equal to the integrity threshold, it follows that $P(b < p_1 - t) = P(b < (p_1 - p_2)/2)$, and $P(b > t - p_2) = P(b > (p_1 - p_2)/2)$. Subsequently, given that the bias is uniformly distributed, (8) and (9), reduce to

$$P(1 \text{ wins}) = \begin{cases} \dfrac{p_1 - t + d}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{p_1 - p_2 + 2d}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{10}$$

$$P(2 \text{ wins}) = \begin{cases} \dfrac{d - t + p_2}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{2d - p_1 + p_2}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{11}$$

B Best Responses

Each lobbyist compares the expected utility from bidding $\underline{p_i}$ and $\overline{p_i}$, given the opponent's bid, and chooses the bid that provides him with the highest utility. The best response of lobbyist i is equal to

$$BR_i\left(p_{-i}\right)=\begin{cases}\dfrac{w+t-d}{2} & \text{if } p_{-i}\leq\dfrac{3t-w+d}{2},\\ 2t-p_{-i} & \text{if } \dfrac{3t-w+d}{2}<p_{-i}<\dfrac{2d+4t-w}{3},\\ \dfrac{w-2d+p_{-i}}{2} & \text{otherwise.}\end{cases}\tag{12}$$

Below, we compute lobbyist's one best responses, given all possible bids of lobbyist two.

Case 1: $p_2\leq\frac{3t-w+d}{2}$

In that case, lobbyist one can choose to bid either $\underline{p_1}=(w+t-d)/2$, and keep the average bid below the threshold, or bid $\overline{p_1}=2t-p_2$, and push the average bid to the threshold. The expected utilities for each possible bid are compared below:

$$\begin{aligned}EU_1\left(\underline{p_1},p_2\right)&=EU_1\left(\overline{p}_1,p_2\right)\iff\\ \frac{(d-t+w)^2}{8d}&=\frac{(d-p_2+t)(p_2-2t+w)}{2d}\iff\\ d^2+9t^2+w^2+6dt-2dw-6tw&=4p_2(3t-w+d)-4p_2^2\iff\\ (3t-w+d)^2&=4p_2(3t-w+d)-4p_2^2\iff\\ (2p_2-(3t-w+d))^2&=0\iff\\ p_2&=\frac{3t-w+d}{2}.\end{aligned}\tag{13}$$

Whenever $p_2\leq(3t-w+d)/2$, it follows from (13) that $EU_1\left(\underline{p_1},p_2\right)\geq EU_1\left(\overline{p}_1,p_2\right)$, so that the best response of lobbyist one is $BR_1\left(p_2\right)=(w+t-d)/2$.

The intuition is as follows. When the opponent's bid is low enough, the lobbyist chooses to bid conservatively. The reason is that by increasing his bid, he increases the chances of winning the support of legislator, but at the expense of lowering his take-home win. As his opponent is not bidding aggressively, the lobbyist may end up paying more if he decides to bid just enough to ensure that the game ends, $\underline{p_1}=2t-p_2$. The lobbyist can afford to bid conservatively as the winning probability of the opponent is not high enough to warrant a price war. This is evident from the fact that his best response is independent of the opponent's bid.

Case 2: $\frac{3t-w+d}{2}<p_2<\frac{4t+2d-w}{3}$

For a given p_2 between the lower and upper bound, the best response of lobbyist's one is $BR_1\left(p_2\right)=2t-p_2$. As before, each lobbyist is faced with the trade-off between higher chances of winning the support of the legislator and lower take-home win. When the opposing bid is not low enough to justify bidding conservatively, but also not high enough to justify bidding aggressively, his best responds is to bid enough so that the game ends with a winning bid.

Case 3: $p_2 \geq \frac{4t+2d-w}{3}$

In that case, lobbyist one can choose to bid either $\underline{p_1} = 2t - p_2$, and keep the average bid at the threshold, or bid $\overline{p}_1 = (w + p_2 - 2d)/2$, and push the bid above the threshold. The expected utilities for the possible bids are compared below:

$$\begin{aligned} EU_1\left(\underline{p_1}, p_2\right) &= EU_1\left(\overline{p}_1, p_2\right) \iff \\ \frac{(d - p_2 + t)(p_2 - 2t + w)}{2d} &= \frac{(2d - p_2 + w)^2}{16d} \iff \\ -(4t + 2d - w)^2 &= -6p_2(4t + 2d - w) + 9p_2^2 \iff \\ 0 &= (3p_2 - (4t + 2d - w))^2 \iff \\ p_2 &= \frac{4t + 2d - w}{3}. \end{aligned} \tag{14}$$

Whenever $p_2 \geq (4t + 2d - w)/3$, it follows from (14) that $EU_1\left(\overline{p}_1, p_2\right) \geq EU_1\left(\underline{p_1}, p_2\right)$, so that the best response of lobbyist one is $BR_1\left(p_2\right) = (w + p_2 - 2d)/2$.

When the opponent bids aggressively, the lobbyist's bid must be at least his opponent's bias adjusted bid, otherwise his bid is not competitive. He can choose to bid conservatively in order to ensure that the game ends or match the opposing bid head on. Both bids depend on p_2, with $\overline{p}_1$ being increasing and $\underline{p_1}$ being decreasing in p_2. The more aggressive the opponent's bid is, the more likely he is to win the support of the legislator. The lobbyist must increase his bid in order to have a shot at winning the support of the legislator. This cannot be true under bidding strategy $\underline{p_1}$. By being conservative, the lobbyist becomes less likely to win the support of the lobbyist but increases his take-home pay. The higher the opponent's bid gets, the less likely it is for the conservative bid to be eligible, so that he must bid aggressively in order to win the support of the legislator. His bid exceeds the minimum competitive bid, $p_2 - 2d$, and is bounded above by the maximum bid w. To sum up, bidding according to $\overline{p}_1$, he secures a significant chance of winning the legislator's support and enough utility once the support is won.

C Computation of Nash Equilibria

In this section we compute all the Nash equilibria of the game. For ease of exposition, we present below the best response function of player i once more.

$$BR_i\left(p_{-i}\right) = \begin{cases} \dfrac{w + t - d}{2} & \text{if } p_{-i} \leq \dfrac{3t - w + d}{2}, \\ 2t - p_{-i} & \text{if } \dfrac{3t - w + d}{2} < p_{-i} < \dfrac{2d + 4t - w}{3}, \\ \dfrac{w - 2d + p_{-i}}{2} & \text{otherwise.} \end{cases}$$

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

The best response of lobbyist 1 is

$$BR_1\left(p_2\right) = \frac{w + t - d}{2}. \tag{15}$$

Let $p_1^* = (w + t - d)/2$. A Nash equilibrium exists if the best response of lobbyist two to p_1^* satisfies $p_2 \leq (3t - w + d)/2$. We solve for equilibria depending on where p_1^* is with respect to the lower and the upper bound.

Case 1.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response p_1^* satisfies

$$p_1^* \leq \frac{3t - w + d}{2} \iff w - t \leq d. \tag{16}$$

The best response of the second lobbyist is $p_2^* = (w+t-d)/2$ and satisfies $p_2^* \leq (3t-w+d)/2$ whenever $w - t \leq d$. Thus, the pair (p_1^*, p_2^*) is a Nash equilibrium. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = EU_2(p_2^*, p_1^*) = \frac{(d - t + w)^2}{8d}.$$

Case 1.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t+2d-w}{3}$

From (16), it follows that $p_1^* > (3t - w + d)/2$ whenever $d < w - t$. The restriction on parameters so that p_1^* is below the upper bound require

$$p_1^* < \frac{4t + 2d - w}{3} \iff d > \frac{5}{7}(w - t). \tag{17}$$

The best response of lobbyist's two reduces to

$$BR_2(p_1^*) = 2t - p_1^* = 2t - \frac{w + t - d}{2} = \frac{3t - w + d}{2},$$

which is consistent with the initial restriction on p_2. Thus, the pair (p_1^*, p_2^*) is a Nash equilibrium whenever $\frac{5}{7}(w - t) < d < w - t$. The expected utilities of both lobbyists are identical to those of the previous case.

Case 1.3: $p_1^* \geq \frac{4t+2d-w}{3}$

From (17), it follows that $p_1^* \geq (4t + 2d - w)/3$ whenever $d \leq \frac{5}{7}(w - t)$. The best response of lobbyist two is equal to

$$BR_2(p_1) = \frac{w - 2d + p_1}{2} = \frac{w - 2d + \frac{w+t-d}{2}}{2} = \frac{3w - 5d + t}{4}.$$

Let $p_2^* = (3w - 5d + t)/4$. The restriction on parameters so that p_2^* is below or equal to the lower bound require

$$p_2^* \leq \frac{3t - w + d}{2} \iff d \geq \frac{5}{7}(w - t). \tag{18}$$

Hence, as $p_2^* \leq (3t-w+d)/2$ whenever $d \geq \frac{5}{7}(w-t)$, and $p_1^* \geq (4t+2d-w)/3$ whenever $d \leq \frac{5}{7}(w-t)$, the pair (p_1^*, p_2^*) is a Nash equilibrium at $d = \frac{5}{7}(w - t)$. Substituting the latter restriction into the optimal

bids, yields $p_1^* = (6t + w)/7$ and $p_2* = (8t - w)/7$. The expected utilities of lobbyists are

$$EU_1\left(p_1^*, p_2^*\right) = \frac{18(w-t)}{35},\ EU_2\left(p_2^*, p_1^*\right) = \frac{16(w-t)}{35}.$$

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t+2d-w}{3}$

The best response of lobbyist one is

$$BR_1(p_2) = 2t - p_2. \tag{19}$$

Let $p_1^* = 2t - p_2$. As before, we analyse three cases depending on where p_1^* is with respect to the lower and upper bound.

Case 2.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response of lobbyist two is $p_2^* = (w + t - d)/2$. Combining (16),(17), it follows that p_2^* is between the lower and upper bound whenever $\frac{5}{7}(w-t) < d < w - t$. Substituting for p_2^* into p_1^*, yields $p_1^* = (3t - w + d)/2$. The pair (p_1^*, p_2^*) is a Nash equilibrium whenever $\frac{5}{7}(w-t) < d < w - t$ (compare with Case 1.2). The expected utilities of lobbyists are equal to

$$EU_1(p_1^*, p_2^*) = \frac{(3w - 3t - d)(3d + t - w)}{8d},\ EU_2(p_2^*, p_1^*) = \frac{(d - t + w)^2}{8d}.$$

Case 2.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t+2d-w}{3}$

The best response of lobbyist two is $BR_2\left(p_1^*\right) = 2t - p_1^*$. Rearranging the inequality restrictions of p_1^*, yields

$$\begin{gathered} \tfrac{3t-w+d}{2} < p_1^* < \tfrac{4t+2d-w}{3} \iff \\ 2t - \tfrac{4t+2d-w}{3} < 2t - p_1^* < 2t - \tfrac{3t-w+d}{2} \iff \\ \tfrac{w-2d+2t}{3} < p_2^* < \tfrac{w+t-d}{2}. \end{gathered} \tag{20}$$

Recall that p_2^* is restricted to lie within the lower and upper bounds. A sufficient condition requires the end points of (20) to lie within the lower and upper bounds (*i.e.* lower bound: $(3t - w + d)/2 \leq (w - 2d + 2t)/3$, and upper bound: $(4t + 2d - w)/3 \geq (w + t - d)/2$). To that end, we obtain

$$\frac{w-2d+2t}{3} \geq \frac{3t-w+d}{2} \iff \frac{5}{7}(w-t) \geq d \text{ and } \frac{w+t-d}{2} \leq \frac{4t+2d-w}{3} \iff \frac{5}{7}(w-t) \leq d. \tag{21}$$

At $d = \frac{5}{7}(w-t)$, the ends point of (20) coincide with the lower and upper bound. Hence, the pair $(p_1^*, p_2^*) \in \left(\frac{3t-w+d}{2}, \frac{4t+2d-w}{3}\right)$ is a Nash equilibrium. In turn, taking into account the restriction $d = \frac{5}{7}(w-t)$, it follows that $(p_1^*, p_2^*) \in (t - w/7, (6t + w)/7)$. The expected utilities of lobbyist are

$$EU_i(p_i^*, p_{-i}^*) = \frac{(d - p_{-i} + t)(p_{-i} - 2t + w)}{2d}.$$

However, there exist also multiple equilibria at $d \neq (5/7)(w-t)$. Suppose $d > (5/7)(w-t)$. Then, it

follows from (20) and (21) that the best response of lobbyist two lies in the interval $(3t - w + d)/2 < p_2 < (w + t - d)/2$. In turn, this interval is well-defined as long as $d < w - t$. Hence, the pair (p_1^*, p_2^*) that satisfies $(3t-w+d)/2 < p_1^* < (4t+2d-w)/3$, $(3t-w+d)/2 < p_2^* < (w+t-d)/2$ and $p_2^* = 2t - p_1^*$, is a Nash equilibrium whenever $(5/7)(w-t) < d < w - t$. Conversely, suppose $d < (5/7)(w-t)$. Then, it follows from (20) and (21) that the best response of lobbyist two lies in the interval $(w - 2d + 2t)/3 < p_2 < (4t + 2d - w)/3$. This interval is well-defined as long as $d > (1/2)(w-t)$. Hence, the pair (p_1^*, p_2^*) that satisfies $(3t - w + d)/2 < p_1^* < (4t + 2d - w)/3$, $(w - 2d + 2t)/3 < p_2^* < (4t + 2d - w)/3$ and $p_2^* = 2t - p_1^*$, is a Nash equilibrium whenever $(1/2)(w-t) < d < (5/7)(w-t)$.

Case 2.3: $p_1^* \geq \frac{4t+2d-w}{3}$

The best response of lobbyist two is

$$BR_2\left(p_1^*\right) = \frac{w - d + p_1^*}{2} = \frac{w - 2d + 2t - p_2}{2} = \frac{w - 2d + 2t}{3}.$$

Let $p_2^* = (w - 2d + 2t)/3$. It must lie within the lower and upper bound. To that end, we obtain

$$p_2^* > \frac{3t - w + d}{2} \iff \frac{5}{7}(w-t) > d \text{ and } p_2^* < \frac{4t + 2d - w}{3} \iff \frac{1}{2}(w-t) < d.$$

Moreover, substituting p_2^* into (19), yields $p_1^* = 2t - \frac{w-2d+2t}{3} = (4t - w + 2d)/3$. The pair (p_1^*, p_2^*) is a Nash equilibrium whenever $(w-t)/2 < d < (5/7)(w-t)$. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = \frac{(2w - 2t - d)(5d + t - w)}{9d},\ EU_2(p_2^*, p_1^*) = \frac{(d - t + w)^2}{9d}.$$

Case 3: $p_2 \geq \frac{4t+2d-w}{3}$

The best response of lobbyist 1 is

$$BR_1(p_2^*) = \frac{w - 2d + p_2^*}{2}. \tag{22}$$

Let $p_1^* = (w - 2d + p_2)/2$. As before, we split the argument into three cases.

Case 3.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response of lobbyist two is equal to $BR_2(p_1^*) = (w+t-d)/2$. Let $p_2^* = (w+t-d)/2$. Substituting the latter into (22), yields $p_1^* = (3w - 5d + t)/4$, which is below or equal to the lower bound whenever $d \geq \frac{5}{7}(w-t)$ (see Eq. 18). On the other hand, p_2^* is above or equal to the upper bound, whenever $d \leq \frac{5}{7}(w-t)$ (see Eq. 17).

At $d = (5/7)(w-t)$, the pair (p_1^*, p_2^*) is a Nash equilibrium. The expected utility of lobbyists are

$$EU_1(p_1^*, p_2^*) = \frac{16(w-t)}{35},\ EU_2(p_2^*, p_1^*) = \frac{18(w-t)}{35}.$$

Case 3.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t+2d-w}{3}$

The best response of lobbyist's two is equal to $BR_2\left(p_1^*\right) = 2t - p_1^*$. Let $p_2^* = 2t - p_1^*$. Substituting p_2^* into (22), yields $p_1^* = (w - 2d + 2t)/3$. In turn, substituting the latter into the best response of lobbyist two, yields $p_2^* = (4t + 2d - w)/3$.

Similar to Case 2.3, p_1^* lies within the lower and upper bound whenever $(w-t)/2 < d < (5/7)(w-t)$. Hence, the pair (p_1^*, p_2^*) is a Nash equilibrium whenever $(w-t)/2 < d < (5/7)(w-t)$. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = \frac{(d-t+w)^2}{9d},\ EU_2(p_2^*, p_1^*) = \frac{(2w-2t-d)(5d+t-w)}{9d}.$$

Case 3.3: $p_1^* \geq \frac{4t+2d-w}{3}$

The best response of lobbyists two is given by $BR_2\left(p_1^*\right) = (w - 2d + p_1^*)/2$. Let $p_2^* = (w - 2d + p_1)/2$. Substituting p_2^* into (22), yields $p_1^* = w - 2d$. In turn, substituting p_1^* into p_2^*, yields $p_2^* = w - 2d$. Note that $w - 2d \geq (4t + 2d - w)/3$ whenever $d \leq (w-t)/2$. Therefore, $p_1^* = p_2^* = w - 2d$ is a Nash equilibrium whenever $d \leq (w-t)/2$. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = EU_2(p_2^*, p_1^*) = d.$$