Multi-Energy Management to Facilitate Hydrogen Injection with Renewable Uncertainty

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Abstract—Power-to-gas (P2G) can convert excessive renewable energy into hydrogen via electrolysis, which can then be transported by natural gas systems to bypass constrained electricity systems. However, the injection of hydrogen could impact gas security since gas composition fundamentally changes, adversely affecting the combustion, safety and lifespan of appliances.

This paper develops a new gas security management scheme for hydrogen injection into natural gas systems produced from excessive wind power. It introduces four gas security indices for the integrated electricity and gas system (IEGS) measuring gas security, considering the coordinated operation of tightly coupled infrastructures. To maintain gas security under an acceptable range, the gas mixture of nitrogen and liquid petroleum gas with hydrogen is adopted to address the gas security violation caused by hydrogen injection. A distributionally robust optimization (DRO) modelled by Kullback-Leibler (KL) divergence-based ambiguity set is applied to flexibly control the robustness to capture wind uncertainty. The KL divergence-based ambiguity set defines uncertainties within a measured space which limits the shape of probability distributions. Case studies illustrate that wind power is maximally utilized and gas mixture is effectively managed, thus improving gas security and performance of IEGS. This work can bring many benefits: i) ensured gas security under hydrogen injection ii) low system operation cost and iii) high renewable energy penetration. It can be easily extended to manage injections of other green gases into IEGS.

Index Terms—Distributionally robust optimization, gas security management, integrated electricity and gas system, integrated energy system, power-to-gas, renewable uncertainty.

NOMENCLATURE

A. Indices and sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$t$, $T$</td>
<td>Index and set for time periods.</td>
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<tr>
<td>$n$, $N$</td>
<td>Index and set for nodes in gas system.</td>
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<tr>
<td>$i_e$, $l_e$</td>
<td>Index and set for traditional distributed generators (DG).</td>
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<td>$l_g$, $l_g$</td>
<td>Index and set for natural gas sources.</td>
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<td>$j$, $J$</td>
<td>Index and set for wind turbines.</td>
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<tr>
<td>$l_e$, $l_e$</td>
<td>Index and set for electric lines.</td>
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<td>$l_g$, $l_g$</td>
<td>Index and set for gas pipelines.</td>
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<td>$k_e$, $K_e$</td>
<td>Index and set for electric loads.</td>
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<td>$k_g$, $K_g$</td>
<td>Index and set for gas loads.</td>
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<td>$k_h$, $K_h$</td>
<td>Index and set for heating loads.</td>
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B. Parameters (P2G)

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<tr>
<td>$n$, $N$</td>
<td>Cost coefficients of traditional DG $i_e$.</td>
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<tr>
<td>$l_g$, $l_g$</td>
<td>Gas and heating load at time $t$.</td>
</tr>
<tr>
<td>$P_{e, max}$, $P_{e, min}$</td>
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<tr>
<td>$x_{i_e}$, $r_{i_e}$</td>
<td>Resistance, reactance of power line $l_e$.</td>
</tr>
<tr>
<td>$f_{i_e, max}$, $f_{i_e, max}$</td>
<td>Maximum active and reactive power flow of power line $l_e$.</td>
</tr>
<tr>
<td>$P_{k_g, t}$, $P_{k_g, t}$</td>
<td>Power and gas load at time $t$.</td>
</tr>
<tr>
<td>$\lambda^e_{i_e}$, $\lambda^e_{i_e}$</td>
<td>Cost coefficients of traditional DG $i_e$.</td>
</tr>
<tr>
<td>$\lambda^g_{i_g}$, $\lambda^g_{i_g}$</td>
<td>Power consumed by the electrolyser.</td>
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Variables (P2G)

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<th>Description</th>
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<tr>
<td>$P_{P2G}$, $P_{P2G}$</td>
<td>Power consumed by the electrolyser.</td>
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I. INTRODUCTION

The increasing penetration of renewable energy is effective for revolutionising energy mix and addressing the climate crisis. However, the abundant renewable generation poses operation and security challenges to power systems [1, 2]. At present, battery storage is the main technique for mitigating the over-penetration of renewables [3, 4]. In the U.S., 275 TWh wind power was generated in 2018 while 6 TWh wind energy was curtailed and wasted [5]. The main reason is that i) the fluctuating and uncertain characteristics of wind power cause unbalancing issues and ii) wind power cannot be fully consumed in local areas but cannot be transported to other areas due to network constraints.

As a promising solution to enable excessive renewable energy integration, power-to-gas (P2G) enables the conversion from electrical energy to hydrogen and synthetic natural gas. Accordingly, the bidirectional energy flow is achieved for tighter couplings between integrated energy systems (IES). P2G has been extensively investigated in existing research, particularly in network planning and operation problems [6-12].

One major research area is P2G planning in IES. A robust co-optimization model is presented in [6] to determine the optimal investment plan for installing investment candidates including P2Gs and gas compressors. Wind uncertainties and reliability are considered for economic and reliable solutions. Paper [7] proposes a bi-level multi-stage stochastic programming to minimize planning and operation cost of an integrated electricity and gas system (IEGS) with P2G. A real options model is designed for IEGS including P2Gs to determine the optimal investment timing and capacity of P2G [8]. The operating cost uncertainty is considered and the decision can be made immediately or postponed waiting for the operation opportunity based on real options.

P2G operation has also been well investigated to reduce operation cost and carbon emissions and maximise profits [9-12]. Paper [9] designs a decentralized IEGS with P2G technologies and wind energy to save daily operation cost. A linearized transient-state gas flow model is developed and the alternating direction multiplier method is used to solve the proposed problem. A stochastic optimization (SO) based day-ahead economic dispatch model for IEGS considering renewable uncertainties and contingencies is proposed in [10]. A second-order cone relaxation is developed to address the nonconvexity caused by uncertain gas flow direction.

Hydrogen is produced by electrolyzers of P2G and then injected into gas systems, which can inevitably affect gas composition. The variation in gas composition will impact the security of gas pipelines, gas engine performance, emissions as well as the gas security of end-users [13]. In gas distribution systems, Wobbe index (WI) is the most common parameter in the existing literature to measure gas security [14-17]. Paper [14] analyses gas interchangeability using WI on domestic appliances. The results demonstrate that WI associated with flashback and thermal output are important constraints to consider. A distributed injection of alternative gas with a steady-state method is presented in [15] and the paper also assesses the impact of utilizing various gas supply sources by WI. A small-scale renewable hydro methane production system is designed in [16] considering WI as a key security index.

The utilization of renewable as the source for P2G is influenced by the uncertain characteristics and existing research mainly uses SO [9, 10] and RO [17, 18]. SO assumes the decision making is either based on an explicit distribution knowledge or a large number of samples. The former solution is not always practical and the latter is prone to errors since it is difficult to estimate the accurate probability distribution when the dataset is not sufficiently large. Alternatively, RO finds the optimal solution under the worst-case scenario based on the uncertainty set, which is over-conservative. To overcome their shortcomings, distributionally robust optimization (DRO) is developed to balance the deficiencies of SO and RO with minor robustness guaranteed through partial distribution information [19-21].

A risk-based optimal gas flow is presented and solved by DRO [22]. Paper [23] designs an economic dispatch model for IEGS considering renewable and load uncertainty. An IES at the building level is proposed considering PV output uncertainty and DRO is used to mitigate the conservatism [24]. In summary, existing research has extensively assessed the gas security of hydrogen-gas admixture but the coordinated operation of energy infrastructures in IES is ignored. There is also a lack of an effective method to model renewable uncertainty.

Similar to the uncertainty set of RO, the ambiguity set of DRO is used to characterize uncertainties with certain known information of distributions. Constructing a proper ambiguity set is crucial to DRO, which must be sufficiently rich to accommodate the real distribution and small enough to exclude...
distributions that may cause over-conservatism. Generally, two main approaches to construct the ambiguity set are moment-based ambiguity set and discrepancy-based ambiguity set [25, 26]. Moment-based ambiguity set is the most common type due to its tractability and easy second-order cone program (SOCP) or semidefinite program (SDP) reformulations. For instance, Markov ambiguity set and Chebyshev ambiguity set rely on first and second-moment information from the historical data [27].

Discrepancy-based ambiguity set use more distributional information to shape real distributions compared with moment-based ambiguity set [28]. It measures the discrepancy between the candidate distribution and reference distribution. The discrepancy can be controlled to either decrease or increase the conservatism depending on the reliability requirement of the optimization. Kullback-Leibler (KL) divergence is a common divergence to measure the distance between two distributions. Estimation of uncertainty distributions can be obtained by statistical fitting [28, 29]. KL divergence-based ambiguity set models uncertainty requiring the candidate distribution within a predefined distance from the nominal distribution.

To fill the research gap, this paper designs new co-optimization for both gas security and system operation in an IEGS. Renewable uncertainty is captured by DRO approach with KL divergence-based ambiguity set to ensure both robustness and tractability. The key indices to quantify gas security, including gross calorific value (GCV), specific gravity (SG), WI, and CP, are included in the model. Apart from ensuring standard satisfaction, the injected gas from P2G is mixed with nitrogen and Liquid Petroleum Gas (LPG) to maintain overall gas security. The uncertainty of wind power output is handled by KL divergence-based DRO, which can be transformed into a tractable deterministic model.

The main contributions of this paper are as follows:

1) This is the first work to include four key indices in the economic operation of IEGS to ensure gas security with the injection of hydrogen generated from P2G, which can contribute to the combustion performance and lifespan of gas equipment.

2) This paper develops a novel co-optimization model to both minimize system operation costs and maintain gas security within an acceptable range, achieved by mixing with nitrogen and LPG.

3) A KL divergence based DRO is developed to model renewable uncertainties. Compared to SO and RO, it is less data-dependent and conservative. Compared to moment-based DRO, the robustness of the proposed ambiguity set can be controlled by adjusting divergence tolerance in the algorithm.

The remainder of this paper is organized as follows. Section II proposes the modelling for the gas security indices. Section III presents the objective function and constraints for IEGS including P2G facility modelling and gas security management. The KL divergence-based DRO methodology regarding and associated reformulations are given in Section IV. Section V demonstrates case studies and performance of the problem. Finally, section VI concludes the paper.

II. GAS SECURITY

To assess gas security, gas adaptability and interchangeability are the two most significant indexes. The adaptability of gas is referred to as the ability of the gas-fired appliances to work properly when the gas composition is changed due to gas injection. The gas interchangeability refers to that, during the mix of various gas compositions, the operational performance of gas equipment is still acceptable in terms of safety, efficiency and emissions. For gas turbines and pipelines, only limited change of gas composition is tolerated.

Calorific value is defined as the amount of released heat during combustion. GCV represents the amount of released heat by unit volume of fuel when the temperature of the gas is equal before and after the combustion, which means the water vapour is entirely condensed and heat recovered during the combustion. GCV must be within a range which determines the available amount of energy. The GCV for hydrogen is given in (1), where \( \Omega_g \) and \( \Omega_{hy} \) are the GCV for the mixed gas and hydrogen and \( \phi_{hy} \) is the volume of hydrogen.

\[
\Omega = \Omega_g + (\Omega_{hy} - \Omega_g)\phi_{hy}
\]

(SG) is the ratio of gas density to air density at the same pressure and temperature. It is used for limiting hydrocarbon content, given in (2), where \( \rho_{g} \), \( \rho_{hy} \) and \( \rho_{air} \) are gas density, hydrogen and air. A high hydrocarbon content will cause serious combustion problems, e.g., engine knock, carbon monoxide emissions and spontaneous ignition of gas turbines, etc.

\[
SG = \frac{\rho_{g} + (\rho_{hy} - \rho_{g})\phi_{hy}}{\rho_{air}}
\]

The WI for gas equipment can vary within a small range, which is defined by (3).

\[
WI = \frac{\Omega}{\sqrt{SG}}
\]

The most frequent used WI in the world is set within 5-10% of the standard setpoint. Otherwise, non-optimal gas combustion appears, which will lead to inefficient and unstable equipment working conditions and high greenhouse gas emissions. A significant change of WI can even result in emergency shutdowns of gas turbines due to the adverse impact on control issues, affecting the lifespan. In addition, the combustion performance is also influenced by the varying gas composition, e.g., flame stability, ignition properties and flashback. Ensuring equal WI can obtain the same energy input under the same gas pressure. CP is used to measure gas combustion stability, which can reflect combustion characteristics, including combustion flame and yellow flame, etc. CP is one important index for interchangeability of gas admixture that requires the CPs of mixed gases are close. Equation (4) defines CP.

\[
CP = 0.1\phi_{hy} + 0.6(\phi_{cm} + \phi_{hc}) + 0.3\phi_{me}
\]

where \( \phi_{cm} \), \( \phi_{hc} \) and \( \phi_{me} \) represent the volume of carbon monoxide, hydrocarbon except methane.

III. IEGS MODELLING

This section models P2G facility and IEGS, followed by the operation objective function. It is assumed that the entire IEGS is owned by a single system company, who has the full control
of DGs, power lines, wind generators, gas sources, pipelines, P2G facility, compressors and other equipment.

A. P2G Modelling

P2G facility enables redundant wind power to be recovered and transported by the gas system. Firstly, electrolyzers split the water (H2O) into hydrogen (H2) and oxygen (O2) by using excessive wind power. Then with the interaction with carbon dioxide (CO2), methane (CH4) can be obtained through methanation. Meanwhile, the produced H2 from the first step can be directly transported by the gas system. The relationship between the input and output of electrolyser is described in (5).

According to Sabatier reaction factors, equations (6)-(8) present the requirement of CO2 and production of CH4 in the process of methanation.

\[
G_{n,t}^{\text{hy}} = \eta_e \frac{P_{\text{P2G}}}{\bar{h}_{ny}} \\
G_{n,t}^{\text{hy,me}} + G_{n,t}^{\text{hy,d}} = G_{n,t}^{\text{hy}} \\
G_{n,t}^{\text{ca}} = \eta_{n-c} G_{n,t}^{\text{hy,me}} \\
G_{n,t}^{\text{n-m}} = \eta_{n-m} G_{n,t}^{\text{hy,me}}
\]

B. Modelling of Electricity and Gas Systems

The modelling of natural gas system is presented from (9) to (24). Equation (9) limits the gas production by natural gas source \( i_g \). Gas pressure is limited in (10) and (11). It is noted that the pressure of initial gas nodes is always higher than that of terminal nodes in distribution gas systems. Weymouth gas flow equation is used to describe the relationship between gas pressure and flow in (12). Equation (13) limits gas flow. The inlet and outlet gas pressures of the compressor are constrained in (14). Equations (1)-(4) describing gas security with hydrogen are modified considering the mix of methane, LPG and nitrogen, given in (15)-(18). Equation (19) is used to ensure all gas security indices are within a certain range for each gas node. The volume deviation between two consecutive time periods cannot be too big due to gas travelling speed in pipelines, which is presented in constraint (20). The total gas volume and its limit are given in (21) and (22). Constraint (23) presents the relationship between gas pressure and volume based on Boyle’s law [30]. The nodal gas balance constraint is presented in (24).

\[
G_{i_g,\text{min}} \leq G_{i_g,t} \leq G_{i_g,\text{max}} \\
Pr_{i_g,\text{min}} \leq Pr_{i_g,t} \leq Pr_{i_g,\text{max}} \\
p_{i_g,\text{ini}} \geq p_{i_g,t} \\
f_{i_g,t}^2 = y_{i_g} (p_{i_g,\text{ini}}^2 - p_{i_g,t}^2) \\
0 \leq f_{i_g,t} \leq f_{i_g,\text{max}} \\
Pr_{i_g,\text{ter}} \leq C_{F_i} Pr_{i_g,\text{ini}} \\
\Omega_{i_g,\text{mix}} = \Omega_{\text{hy}} (\psi_{i_g,\text{mix}} + \psi_{i_g,t}^\text{hy}) + \Omega_{\text{LPG}} \psi_{i_g,\text{LPG}} + \Omega_{\text{n}} \psi_{i_g,\text{n}} + \Omega_{\text{me}} \psi_{i_g,\text{me}} \\
SG_{i_g,\text{mix}} = \left[ \psi_{i_g,\text{hy}} + \psi_{i_g,\text{hy}}^\text{hy} + \psi_{i_g,\text{LPG}} + p_{i_g,\text{LPG}} + p_{i_g,\text{n}} \psi_{i_g,\text{n}} + p_{i_g,\text{me}} \psi_{i_g,\text{me}} + \psi_{i_g,\text{mix}}^\text{hy} \right] \\
W_{i_g,\text{mix}} = \Omega_{i_g,\text{mix}} / \sqrt{SG_{i_g,\text{mix}}} \\
CP_{i_g,\text{mix}} = O_i \left[ E_{i_g,\text{hy}} (\psi_{i_g,\text{mix}} + \psi_{i_g,t}^\text{hy}) + E_{i_g,\text{LPG}} \psi_{i_g,\text{LPG}} + E_{i_g,\text{n}} \psi_{i_g,\text{n}} + E_{i_g,\text{me}} \psi_{i_g,\text{me}} \right]
\]

The electricity distribution system is modelled from (25) to (30). Equation (25) is the constraint for the active and reactive power of substations. The generation limits for traditional DGs are presented in (26). In the distribution system, the DistFlow equation is used with the linearization as presented from (27) to (29). Equation (27) is obtained assuming (i) losses are negligible, (ii) bus voltage is close to 1.0 p.u. and (iii) reference bus voltage is 1.0 p.u. Voltage and flow constraints are given in (28) and (29), respectively. In (30) and (31), the power balance constraints for active and reactive power are given respectively.

\[
\{\psi_{i,t}\}_{\text{sub}} \leq \{\psi_{i,\text{sub, max}}\}_{\text{sub}} \leq \psi_{i,t} \\
P_{i,\text{min}} \leq P_{i,t} \leq P_{i,\text{max}} \\
V_{i,t}^\text{ini} - V_{i,t}^\text{ter} = (f_{i,t}^2 r_{i,t}^2 + f_{i,t}^2 x_{i,t}^2) / V_0 \\
V_{i,t}^\text{ini} \leq V_{i,t}^\text{sub, max} \leq V_{i,t}^\text{ini, ter} \\
0 \leq f_{i,t} \leq f_{i,\text{max}} \\
P_{\text{sub, max}} = \sum p_{i,t} + \sum q_{i,t} + \sum f_{i,t}^\text{ini} - \sum f_{i,t}^\text{ter} = \sum p_{i,t} \\
Q_{\text{sub, max}} = \sum q_{i,t} + \sum f_{i,t}^\text{ini} - \sum f_{i,t}^\text{ter} = \sum Q_{i,t}
\]

C. Objective function

The injection of hydrogen into natural gas pipelines will inevitably change gas compositions and might cause gas security issues, such as heat value, combustion potential, pressure. In order to maintain the 4 gas security indices within an acceptable statutory range, it is required to inject other gases with hydrogen into gas pipelines. Accordingly, the optimal gas mixture is required to determine the proper amount and timing of the injection of other gases. In this paper, LPG and nitrogen are used to blend with hydrogen to keep satisfied gas security. Nevertheless, the cost of purchase and injection of LPG are expensive compared with nitrogen. Accordingly, the key gas mixture process is to use the minimum LPG with gas security satisfied. The objective in (32) is to minimize system operation cost while ensuring gas security, considering the impact of uncertain wind power output.

\[
\min \Gamma = \min \sum_{i_g,\text{str} \in T} \lambda_{i_g,\text{str}}^\text{P, w} + \lambda_{i_g,\text{str}}^\text{L, w} + \lambda_{i_g,\text{sub, str}} \\
+ \lambda_{i_g,\text{str}}^\text{L, w} + \lambda_{i_g,\text{str}}^\text{L, w} + \lambda_{i_g,\text{LPG}} \psi_{i_g,\text{LPG}}
\]

The first three terms are the cost function for traditional DGs. The fourth one is electricity purchased from the upper electricity market. The gas production cost of natural gas sources is shown as the fifth term. The last two terms are the cost for purchase and injection of LPG and nitrogen during gas mixture process.
IV. METHODOLOGY

The uncertainty of wind power output is captured using DRO approach, reflected in the uncertain forecast error in (33). Equations (34) and (35) are the insecuity constraints of (29).

\[
\begin{align*}
\sum_{j \in J} P_{x,t} + \sum_{j \in J} \omega_{\tilde{j},t} + \sum_{i \in I} f_{a,i,t} - \sum_{i \in I} f_{a,i,t} & \geq 0 \\
\sum_{j \in J} P_{x,t} + \sum_{j \in J} \omega_{\tilde{j},t} + \sum_{i \in I} f_{a,i,t} - \sum_{i \in I} f_{a,i,t} & \leq 0
\end{align*}
\]

Constraint (34) is used as the representative of reformulations in the later section, which is transformed into (36) since DRO considers the worst distribution of uncertain forecast error.

\[
\begin{align*}
\min_{P} \sum_{j \in J} P_{x,t} + \sum_{j \in J} \omega_{\tilde{j},t} + \sum_{i \in I} f_{a,i,t} - \sum_{i \in I} f_{a,i,t} & \geq 0 \quad (36)
\end{align*}
\]

Equation (37) measures the discrepancy between two probability distribution \( P \) and reference distribution \( P_{ref} \) based on \( \phi \)-divergence through the divergence tolerance \( \eta \). Equation (38) defines the KL divergence between \( P \) and \( P_{ref} \).

\[
\begin{align*}
P &= \{ P \in D \mid D(P \parallel P_{ref}) \leq \eta \} \\
D(P \parallel P_{ref}) &= \int f(\xi) \log \frac{f(\xi)}{f_{ref}(\xi)} d\xi
\end{align*}
\]

DRO considers the worst distribution scenario and thus the expectation of constraint (36) is based on all the possible uncertainty distributions are considered, which is given in (39).

\[
\min_{P \in D} E_p [H(x, \xi)] \geq 0
\]

Based on the change-of-measure method, (40) is obtained according to [28], where \( L(\xi) = f(\xi)/f_{ref}(\xi) \). By applying the change-of-measure method to (39), (41) is obtained.

\[
\begin{align*}
D(P \parallel P_{ref}) &= \int f(\xi) \log \frac{f(\xi)}{f_{ref}(\xi)} d\xi \\
E_p [H(x, \xi)] &= \int H(x, \xi) \frac{f(\xi)}{f_{ref}(\xi)} f_{ref}(\xi) d\xi \\
E_p [H(x, \xi)] &= \int H(x, \xi) \frac{f(\xi)}{f_{ref}(\xi)} f_{ref}(\xi) d\xi \\
&= E_{P_{ref}}[H(x, \xi) L(\xi)]
\end{align*}
\]

To incorporate uncertainty within the constraint (36), it needs to be treated as an inner optimization problem with sub-objectives and constraints.

\[
\begin{align*}
\min_{P_{ref}} E_{P_{ref}} [H(x, \xi) L(\xi)] \\
\text{s.t. Constraints (5)-(31)}
\end{align*}
\]

The original optimization problem is reformulated into (43) as follows with the expectation of the constraints. Noted that the divergence tolerance

\[
\begin{align*}
\min_{\Gamma} \Gamma \\
\text{s.t. Constraints (5)-(31)}
\end{align*}
\]

\[
\begin{align*}
P &= \{ P \in D \mid D(P \parallel P_{ref}) \leq \eta \}
\end{align*}
\]

According to [28], when strong duality holds, (43) can be transformed into (44).

\[
\begin{align*}
\min_{\Gamma} \Gamma \\
\text{s.t. Constraints (5)-(31)}
\end{align*}
\]

Then, the explicit expression of constraints of (44) according to (30) can be obtained in (45).

\[
\begin{align*}
\max_{P_{ref}} \log E_{P_{ref}} e^\sum_{i \in I} f_{a,i,t}^\alpha - \sum_{i \in I} f_{a,i,t}^\alpha - \sum_{i \in I} P_{x,t} + \sum_{j \in J} \omega_{\tilde{j},t} \\
\alpha \eta & \geq 0
\end{align*}
\]

The logarithmic expression under expectation is a moment generating function with distribution \( P_{ref} \), which can be transformed into a deterministic formulation. In this paper, kernel density estimation (KDE) in (46) is used to estimate the reference distribution, where \( \xi_t \) represents error data, \( N \) is the number of error data, \( h_N \) is a positive smoothing parameter, and \( H(\cdot) \) is the kernel function (non-negative and the integral of the probability distribution is 1). Assuming \( H(\cdot) \) follows the normal distribution, (47) is formulated (46) with the mean value \( \xi \) and variance \( h_N \).

\[
\begin{align*}
f_N(\xi) &= \frac{1}{N h_N} \sum_{i=1}^{N} \frac{\xi - \xi_t}{h_N} \\
f_N(\xi) &= \frac{1}{N} \sum_{i=1}^{N} e^{-\frac{(\xi - \xi_t)^2}{2 h_N^2}} \\
\end{align*}
\]

Finally, (30) can be transformed into (48) based on [28].

\[
\begin{align*}
\max_{\alpha \geq 0} \left\{ \alpha \eta + \frac{h_N^2 \rho}{2 \alpha} + \alpha \ln \frac{1}{N \rho} \sum_{i=1}^{N} \frac{1}{N} e^{-\frac{(\xi_t/\alpha)^2}{2 h_N^2}} \right\} \geq 0
\end{align*}
\]

V. CASE STUDIES

The proposed gas security management for IEGS is demonstrated on a modified IEEE 33-bus system with a 10-node gas system [31]. The IEGS contains three traditional DGs, three renewable DGs and two natural gas sources. The wind DG at bus 10 is the power supply for the P2G facility with 1MW capacity. The parameters for natural gas sources and DGs are given in TABLE I and II respectively. In this paper, P2G efficiency is 50% [32]. The ambiguity set is controlled by a divergence tolerance (\( \eta=2.3026 \) and \( \beta=0.1 \)) for the DRO. TABLE III shows the limits of the considered four security indices. The GCV and combustion potential index (CPI) for hydrogen, methane, LPG and nitrogen are given in TABLE V. Four case studies in TABLE V are implemented based on optimization methods, hydrogen injection schemes, and gas mixture management strategies, which are presented.

A. Economic Performance

The economic results for all cases are investigated, including operation cost and gas mixture management cost, as is shown in TABLE VI. The IEGS operation cost is the sum of operation cost of power system and gas system. It shows that case 1 (\$601922) has the highest IEGS operation cost and case 3 (\$337889) has the lowest. The IEGS operation strategy for case 1 and 4 are the same which both consider hydrogen injection support for the gas system and gas mixture management for maintaining gas security. Case 1 derives \$135710 more operation cost in the power system since RO limits the uncertain wind power output with a higher degree of robustness, which, yields \$120445 less gas system operation cost. The reason is that the hydrogen injection is strictly limited, which reduces the need for additional LPG and nitrogen to maintain acceptable gas.
security indices. Overall, case 1 results in $15265 more IEGS operation cost compared with case 4.

Without considering hydrogen injection from the power system to the gas system, the two systems are operated separately in case 2. Accordingly, the power system only requires to supply electricity load in case 2 whose power system operation cost is 4.3% less than that of case 4. The purchase cost of nitrogen and LPG in case 2 are $1003 and $9760 respectively, which are $1421 and $230260 less than case 4. Since the original natural gas without hydrogen addition is more accessible to obtain acceptable gas security. Due to the disconnection between power and gas systems, the overall operation cost of case 2 is $246149 less than case 4. In case 3, hydrogen injection is considered without gas mixture. The gas system operation cost, i.e., $845, is purely the generation cost of natural gas sources. Without the blend of LPG and nitrogen, the gas volume is less than case 4 and the gas pressure is higher than case 4, which reduces the hydrogen injection from P2G facility. Thus, the wind power provides more supply to the power system and the power system operation cost is reduced.

The divergence tolerance \( \eta \) is used to characterize the size of the ambiguity set which contains all the possible uncertainty distributions and is associated with the conservatism of numerical performance. According to [28], the divergence tolerance influences the confidence interval, i.e., \( (\beta = e^{-\eta}) \), \( \beta \) is the confidence interval, which refers to the probability of the violation of constraint (42). The divergence tolerance \( \eta \) represents the radius of the ambiguity set, which affects the accuracy of estimating uncertainty distribution. The larger \( \eta \) leads to an ambiguity set with higher robustness while the smaller \( \eta \) leads to less conservative numerical results. When the confidence interval is set as 0 (\( \eta = 1 \)), the confidence interval turns into 100% and the candidate distribution is becoming the same as the reference distribution. Accordingly, the original DRO problem is equivalent to SO. With the variation of the confidence interval, the total operation cost for IEGS is depicted in TABLE VI. At the second column of the table, the divergence tolerance is determined based on selecting the confidence interval. Case 4 has the highest result with all the confidence intervals while case 3 remains the lowest. With the increase of the confidence interval, the total IEGS operation cost increases slowly. In case 4, when \( \beta = 0 \), the DRO degrades to SO and yields $583271 total cost. The considered largest ambiguity set results in $596454 with \( \beta = 1 \), which is 2.3% higher than the cost with the smallest ambiguity set.

### B. Gas Security under Gas Management

The resulting WI and CP with different P2G operation schemes are presented in this subsection. From Fig. 1 to Fig. 3, it can be seen that case 2 and 4 have a similar WI range and trend through the entire time period while case 3 shows a narrow range of WI. The WI of case 3 ranges from 32.65 to 32.75, which is 79% of the WI range of case 4. Besides, WI in case 3 does not fluctuate much while maintaining a smooth trend through the entire time period. The reason is that without the gas admixture of LPG and nitrogen, WI cannot be ensured in an acceptable range. In comparison with case 4, there is no hydrogen injection in case 2. Compared with hydrogen and methane, nitrogen and LPG have higher CGV, which lead to
### VI. Conclusion

A coordinated optimization for gas security management and operation of IEGS in the presence of wind uncertainty is proposed. The wind uncertainty is handled by DRO with KL divergence for controlling the conservatism of numerical performance. A tractable deterministic formulation is obtained and the resulted linear programming model can be efficiently solved. Through the extensive case studies, the key findings are:

- Gas security with hydrogen injection is not acceptable under international standard without gas security management, where GCV, SG, WI and CP should be considered.
- The P2G facility is useful for maximally utilizing the excessive wind power and economically effective for reducing the operation cost of IEGS.
- DRO provides less conservative results than RO in terms of economic performance.
- Through applying KL divergence, the size of the ambiguity set can be flexibly controlled based on confidence interval set by decision-makers for risk concerns.

The proposed co-optimization for IEGS ensures both economic performance and gas security via coordinating traditional DGs, natural gas resources and P2G facility. It can bring along many benefits: i) ensured gas security under hydrogen injection ii) low system operation costs and iii) high renewable energy penetration, thus facilitating high-security, affordable, and clean energy supply. The novel framework can be easily extended to cases for managing injections of other green gases into IEGS.

### References


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