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Zhao, Pengfei

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Optimization for Integrated Electricity and Gas Systems Considering Uncertainties

By
Pengfei Zhao

Thesis submitted for the degree of

Doctor of Philosophy

in

Department of Electronic and Electrical Engineering

University of Bath

December 2020

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Abstract

In recent years, there is an increasing need for the integration of multi-energy vectors with the traditional power systems due to energy decarbonization and the booming coupling technologies. Integrated electricity and gas system (IEGS) plays a vital part in the modern energy provision through coordinating supply, conversion, storage and consumption. Although the interaction between electricity and gas systems facilitates the economic performance and security, it raises computational and modelling challenges for analysis and accurate modelling of the emerging IEGS. The optimal operation of IEGS is one most significant research topic to ensure the economic and reliable perspectives of IEGS. Nevertheless, the uncertainties introduced from renewable energy resources (RES), integration of smart grid technologies and natural disasters will affect the economic operation and destroy the energy infrastructures. For instance, underestimated uncertain renewable generation could cause network congestion and overestimated renewable generation will lead to a lack of energy supply. Accordingly, non-optimal and even infeasible solutions will be yielded.

This thesis studies the centralized and coordinated operation of IEGS under different types of uncertainties which contributes to optimal operation schemes of IEGS for the economic, reliable, resilient and sustainable perspectives. The proposed studies will greatly contribute to efficient and economic-effective IEGS operation schemes and related industrial applications in the presence of inevitable uncertainties and disasters. The main achievements of this research can be summarized as follows:

(1) This work proposes a two-stage distributionally robust operation model for integrated water-energy systems in a distribution level considering wind uncertainty. The optimization aims to minimize the total operation cost of the overall system. The presence of wind uncertainty inevitably leads to risks in decision making. Accordingly, a coherent risk measure, i.e., conditional value-at-risk, is combined with the optimization objective to determine risk-aversion operation schemes.

(2) To alleviate the impacts of seismic events on both power lines and gas pipelines of IEGS. A two-stage distributionally robust optimization (DRO) model is proposed to enhance the resilience for an IEGS, where the damage on both power lines and gas pipelines are considered. The seismic activities are regarded as uncertain events and the random damage on power lines and pipelines are regarded as uncertainties, which are handled by DRO. A novel model to assess the performance of IEGS against seismic attacks is developed. This damage quantification builds a probabilistic model and estimated by damage scenarios. The proposed novel DRO framework avoids specifying uncertainty distributions but only uses moment information, which is more practical considering that it is normally not possible to gather a sufficiently large amount of distributional information for extreme events.

(3) To address the adverse impact caused by the high integration of intelligent data technologies in IEGS, a two-stage risk mitigation strategy to address the uneconomic operation of IEGS under false data injection attacks (FDIA) considering renewable
generation uncertainties. The FDIA mitigation scheme conducts the day-ahead and real-time operation, which is more powerful and convenient to be used by system operators to ensure the efficiency and security of the IEGS. FDIA is assumed to attack both electricity and gas meter readings, including i) load measurement of electricity and gas systems and ii) gas density measurement. Uncertainties of renewable resources are considered in the proposed model as they can worsen system operation conditions during FDIA.

(4) The high penetration of renewable generation poses severe challenges to Volt/VAR optimization because of its output uncertainties, leading to voltage deviation and fluctuation. To resolve unacceptable voltage deviation under energy system interdependency, a novel coordinated two-stage multi-objective optimization is proposed for voltage control in the operation of IEGS, considering uncertain renewable generation and multi-vector energy system integration. The optimal voltage is achieved through efficiently coordinating the operation of on-load tap changers (OLTC), photovoltaic systems, and shunt capacitor banks. A conic tractable form with the dual formulation is transformed from the original problem and solved by constraint generation algorithm (CGA).

(5) To investigate the optimal coordinated operation of energy infrastructures in IEGS meanwhile ensure the gas quality, a co-optimization for both gas quality and system operation in an IEGS is proposed. The renewable uncertainty is captured by DRO approach with Kullback-Leibler (KL) divergence-based ambiguity set to ensure both the system robustness and tractability. The key indices to quantify the gas quality include gross calorific value (GCV), specific gravity (SG), Wobbe Index (WI), and Combustion Potential (CP). Apart from ensuring the indices to meet the related standards, the injected gas from power-to-gas (P2G) facility to gas system is mixed with nitrogen and Liquid Petroleum Gas (LPG) for maintaining the overall gas quality.

Keywords: Cyber-attacks; distributionally robust optimization; integrated electricity and gas system; gas quality; renewable energy resources; resilience; uncertainty.
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<td>Renewable energy sources</td>
<td>RES</td>
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<td>Combustion potential</td>
<td>CP</td>
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<tr>
<td>Conditional value-at-risk</td>
<td>CVaR</td>
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<td>Coefficient of Performance</td>
<td>CoP</td>
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<td>Chance-Constrained Programming</td>
<td>CCP</td>
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<td>Combined Heat and Power</td>
<td>CHP</td>
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<td>Distributionally robust optimization</td>
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<td>Distributed generation</td>
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<td>Demand Response</td>
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<td>Distributed Energy Resources</td>
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<td>Energy Storage Systems</td>
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<td>False data injection attack</td>
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<td>Ground Source Heat Pump</td>
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<td>Gross calorific value</td>
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<td>Integrated energy system</td>
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<td>Integrated electricity and gas system</td>
<td>IEGS</td>
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<td>Information and communication technology</td>
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<td>Integrated water-energy nexus systems</td>
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<tr>
<td>Kullback-Leibler</td>
<td>KL</td>
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<td>Liquid Petroleum Gas</td>
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<td>Mixed Integer Linear Programming</td>
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<td>On-load tap changer</td>
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First-authored papers


Co-authored papers


Chapter 1

Introduction

This chapter introduces the background, motivation, objectives, and contributions of the research. The structure of the thesis is also outlined.
1.1 Research background

1.1.1 Transition to ‘Net Zero Emissions’ Target

The environmental challenges are becoming unignorable with the increasing usage and depletion of traditional energy resources. Global warming and greenhouse gas emissions have become the two most serious and concerning issues in the 21st century, which arouse the attention of the public and government all around the world. The average temperature of the global surface has increased by 0.74 degrees in the last 50 years according to [1]. As is shown in Fig. 1-1, global carbon emission has significantly increased by 90% since 1970 [2]. Paris Agreement [3] has been agreed and signed by many countries, which was the first full global agreement to deal with climate change, aiming at controlling the temperature increase below 2 degrees.

To limit the increasing trend of temperature, the UK has led the global effort and become the first major country to achieve the ‘Net Zero Emissions’ target by 2050 [4]. Net zero is referred as the emission can be balanced via offset measures [5], e.g., planting trees, applying advanced energy technologies and strategies. In 2020, the greenhouse gas emissions were 51% below the level of 1990. This milestone was achieved after a dramatic 11% decrease of greenhouse gas emissions compared with 2019, which is largely owing to the outbreak of COVID-19 [6]. Fig. 1-2 presents the opposite trend of the economy and emissions [4]. In Fig. 1-3, the progress of emission
reduction in different sectors is given. [5] To achieve the ‘Net Zero Emissions’ target, the UK government has taken the measures including:

**Reducing coal-fired power generation**: The UK government aims to eliminate all thermal power plants by 2025 to achieve carbon reduction targets [7]. Alternative energy sources include solar, wind, nuclear, natural gas and hydropower. An emission reduction test was designed in May 2019. During the two weeks, on average, nearly 40% of the electricity supply in the UK was from natural gas, 20% from nuclear power generation, 13% from wind power generation, and the rest came from other generations. It is estimated that the last coal-fired power plant will be closed by 2022 [8].

**Carbon capture technology**: The government provides strong support for the promotion of carbon capture technologies such as carbon capture and storage (CCS),

![Image 1](https://via.placeholder.com/150)

**Fig.1**- **2. UK realizes falling emissions with a growing economy** [4].

![Image 2](https://via.placeholder.com/150)

**Fig.1**- **3. The progress of emissions reduction in different sectors** [4].
carbon capture and utilisation (CCU) and carbon capture, utilisation and storage (CCUS). The industries have received government funding of 26 million pounds for carbon capture projects. CCUS aims to encourage industries to capture up to 70,000 tons of carbon dioxide each year, which is used for industrial applications [8].

**Renewable energy adoption**: In the last five years, one-third of the UK’s fossil fuel-based power generation capacity has been decommissioned. However, the installed capacity of wind, solar, biomass, and hydropower have been tripled to 42GW. Renewable energy accounts for the largest share of the UK’s installed power generation capacity, surpassing the 40.6GW installed capacity of fossil fuels. Wind power accounts for the largest share of renewable energy, with more than 20GW; solar power comes second with more than 13GW; biomass energy ranks third with 3.2GW [9].

1.1.2 Renewable Curtailment

Based on the global energy statistics [10], it anticipates that the share of renewable power generation will be nearly doubled globally, i.e., from 26% in the current level to 44% by 2040. And it is possible to overpass coal before 2026. Particularly, the share of combined wind and solar generation could potentially increase from 7% to 24% in the world. In comparison with the renewable generation share, the traditional generation share falls sharply from 67%. In particular, the share of coal’s generation, which surges 5 times from 1970 to 2013, would decline from the current 38% to 25% by 2040. And the gas-fired generation is predicted to increase rapidly to approximately 50% by 2040 due to the increasingly cheap price of gas sources [11].

In the UK, the large deployment of renewable energy resources causes curtailment problems, which causes the waste of renewable generation and economic losses. The wind curtailment is 3.6TWh in 2020 in the UK [12]. The potential reasons for renewable energy curtailment can be listed:

- The congestion of power systems affects the feed of power to users.
- The inflexible plants such as nuclear plants cannot be flexibly controlled to provide renewable energy.
- The growth of renewable capacity will lead to over-capacity issues when the demand is low.
1.1.3 Energy Internet

The concept of Energy Internet (EI) has been put forward by Jeremy Rifkin in 2011, which aims to make full use of the distributed renewable energy resources and thus improve the energy utilization efficiency [13]. The report pointed out that the final aim of the EI is to establish the realization of interdependent energy systems which facilitate the penetration of distributed renewable energy resources [14]. The achievement of EI requires the strong integration of power, gas, heating, cooling and water systems, with the supplement of energy storage technologies. The UK National Energy and Climate Plan strengthens the importance of developing and creating modern, integrated and low carbon energy systems [15]. In 2018, the Chinese government put forward a guideline for developing EI-based integrated energy systems and encouraged the applications and constructions of modern energy platforms [16]. Overall, the rising research and application attention on EI has motivated the associated modelling and operation research of EI.

1.1.4 Integrated Energy Systems

The power system is the most vital part of EI, facilitated by the smart grid and multi-energy technologies to achieve the coordination and complementation of each subsystem. Meanwhile, interdependent energy converters are also increasingly gaining attention, such as combined heat and power (CHP), power-to-gas (P2G) and gas turbine, etc. These conversion technologies enhance the couplings of subsystems and aggregate them into an efficient energy entity incorporating energy production, transmission, distribution, conversion, storage and usage. Integrate energy system (IES) is composed of multi-energy systems and renewable energy sources which are interconnected by coupling devices [17-19]. It improves energy efficiency compared with independent energy systems. Accordingly, the system economy, security, reliability and flexibility are strengthened. In addition, the excessive renewable generation can be accommodated within the IES. The aforementioned energy conversion technologies can tighten the couplings of different subsystems. For instance, CHP facilitates the interconnection between gas, power and heating systems [20]; P2G facilitates the interconnection between power and gas systems [21], and electric vehicles (EVs) enhance the interconnection between power and transportation systems [22]. Apart from the energy infrastructures, the optimal coordination between each energy vector also depends on
Chapter 1  
Introduction

the advanced energy management schemes. The operation algorithm of IES will further improve the system economic efficiency and reliability through comprehensively coordinating the energy infrastructures.

Gas-fired generation has gradually taken the place of coal-fired generation due to the lower price, lower emission and high reliability of natural gas [23]. In addition, the advancing techniques in the gas industry further impact the electric power industry. The high integration of renewable resources brings security issues to power systems due to its variability and fluctuation. However, the integration of a gas system counteracts the resulted negative effects thanks to the slow inertia characteristic of natural gas [24]. Accordingly, the high integration of natural gas is becoming the trend of IES, which introduces the concept of integrated electricity and gas system (IEGS). In the UK, granted by the government, modelling and building real IEGS is given sufficient attention by Supergen, HubNet and other research associations for the energy transition [25]. The Adaptation and Resilience in Energy Systems (ARIES) project conducted by the University of Edinburgh aims to develop new methods to model the system vulnerabilities under the climate change for IEGS [26]. The project Horizon 2020 MAGNITUDE is expected to identify flexibility options from synergies between power, gas, heating and cooling systems and also support the integration of renewable energy sources to achieve a cost-effective energy system [27]. CHP and gas turbines are widely utilized for coupling the power and gas systems in the existing IEGS projects. However, the energy flow is unidirectional, i.e., from a gas system to the power system. P2G is a promising technology, which enables to convert excessive renewable power output from the power system to the gas system and thus achieve the bidirectional energy flow in IEGS [28, 29]. However, most P2G projects are at the experimental stage and the application is few.

1.1.5 Research Motivation

Due to the energy crisis, pollution issues and the encouragement of governments’ policies, renewable industry witnesses a rapid development. The wide deployment of renewable energy sources leads to power systems facing more challenges on secure system operation and power-load balance. IEGS is considered and planned to resolve the new challenges based on the significant coordination and complementation between
power and gas systems. Accordingly, accurate and feasible mathematical modelling, strengthening energy integration, facilitating renewable integration are the key elements.

This thesis focuses on investigating the benefits and challenges of economic efficiency, security and sustainability due to the integration of power and gas systems. This thesis aims to resolve the following five problems:

i) The renewable uncertainty turns the operation of IEGS into a stochastic and complicated optimization problem. The economic performance of the IEGS operation needs to be guaranteed pertaining to the variation of renewable power generation.

ii) Natural disasters can cause huge power losses of energy systems that threaten the economy. Enhancing resilience to withstand seismic hazards and mitigate resulting damages is of great value for IEGS.

iii) With the extensive deployment of digital communication technologies, IEGS is under extensive exposure to information and communication technology (ICT). The adversary can launch false data injection attacks (FDIA) to tamper critical data and inject falsified data, which brings serious challenges to state estimators, indirectly affecting system operation and control.

iv) Due to the variable and intermittent nature renewable energy penetration, it poses operational and security challenges to voltage profile by affecting normal operations of on-load tap changer (OLTC) and capacity banks in IEGS.

v) P2G enables the conversion from electric energy to hydrogen and synthetic natural gas, accordingly achieving bidirectional energy flows for tighter couplings in IEGS. However, the injection of hydrogen could impact gas quality since gas composition fundamentally changes, adversely effecting the combustion, safety and lifespan of appliances.

1.2 Research Contributions

This thesis contains research contributions as follows:

- **Energy structure**: The innovative energy structures of IEGS and IWENS designed for economic effective, resilient, reliable and sustainable
functionalities are extensively modelled. It aggregates considerable interconnections and converters among subsystems, e.g., gas turbines, P2G facilities, CHP, GF, GSHP, water pumps and electric boilers. The enormous interdependencies and interactions between energy sectors are beneficial for improving economic efficiency and sustainability.

- **Uncertainty modelling:** Two-stage DRO models are applied to optimize operation and planning schemes. It is the first attempt to combine DRO with mean-risk optimization. The benefits are in threefold: i) it overcomes the shortages of SO and RO by using partial distributional information with moderate robustness, ii) the KL divergence-based ambiguity set can flexibly shape the considered candidate distributions compared with moment-based ambiguity sets and accordingly yields less-conservative results and iii) the trade-off between economic performance and risk can be realized based on the incorporation of Conditional Value at Risk (CVaR) on the objective function.

- **Resilience enhancement:** A two-stage earthquake-resilient co-optimization model is established, incorporating both planning and operation schemes, is for the first time proposed to enhance IES resilience, considering the worst-distributed seismic attack. A novel model to assess the performance of IES against seismic attacks is developed. This damage quantification builds a probabilistic model and estimated by damage scenarios. This assessment model can be easily combined with the proposed two-stage DRO model to determine the optimal enhancement plan for IES.

- **Cyber resilience:** This work models FDIA in an IEGS for the first time, particularly on natural gas load and density measurement, where existing research only focuses on FDIA on electricity systems.

- **Voltage regulation:** This paper is the first such effort to investigate VVO in an IEGS. The strong coupling of power and gas infrastructure and tight interdependency between the two systems are considered.
• *Gas quality improvement:* Four key indices are used in the economic operation of IEGS to quantify the impact of hydrogen injection from P2G on gas quality. A novel co-optimization model is developed to minimize system operation costs and maintain gas quality within an acceptable range, achieved by a mixture of nitrogen and LPG.

### 1.3 Thesis Layout

The rest of the thesis is organised as follows:

**Chapter two** is devoted to the DRO approach, where the moment-based DRO and discrepancy-based DRO are introduced, respectively. This chapter summarizes the methodology of handling uncertainties for chapters 3-7. The formulations and solution algorithms of each method are explicitly discussed.

**Chapter three** provides a two-stage economic operation scheme for an integrated electricity, gas and water system under the water-energy nexus with enormous interdependencies. The tight couplings and interactions between each subsystem enable the reliable and economic operation for the entire IES. Renewable uncertainty is captured by mean-risk DRO. The coherent risk measure, CVaR provides the trade-off to system operators with flexible alternatives on choosing between economic efficiency and risk. A tractable Bender’s decomposition is employed to solve the problem.

**Chapter four** proposes a two-stage DRO method to enhance the resilience of an IEGS under seismic attacks with combined planning and operation strategies. The proposed method provides optimal hardening plans for specific power lines and gas pipelines under different seismic intensity levels and investment budgets. This method can help system operators to make economical hardening and operation strategies to improve the resilience of the IES under seismic attacks.

**Chapter five** designs a risk mitigation scheme for IEGS against FDIA with a two-stage DRO model. The hierarchical two-stage framework can determine both day-ahead and real-time system optimal operation schemes considering the impact of FDIA and renewable uncertainties on electricity load, gas load and gas density. A tractable semidefinite programming formulation is built for the original problem, which is solved
by constraint generation algorithm (CGA) in an iterative manner. The proposed mitigation scheme ensures the economic performance of IEGS by providing a two-stage risk mitigation scheme by implementing efficient load shedding under FDIA and renewable uncertainty.

**Chapter six** proposes a multi-objective optimization for minimizing both operation cost and voltage deviation of IES considering renewable power uncertainty. A two-stage data-driven DRO approach is used to solve the voltage management with dual and semidefinite (SDP) formulations to ensure computational tractability. The reformulated two-stage voltage management is solved by CGA with master and subproblems. This work can benefit integrated system operators with powerful operation tool to manage the systems with fewer costs but integrate more renewable energy.

**Chapter seven** presents a coordinated optimization for gas quality management and operation of IEGS in the presence of wind uncertainty is proposed. The wind uncertainty is handled by DRO with Kullback-Leibler divergence for controlling the conservatism of numerical performance. A tractable deterministic formulation is obtained and the resulted linear programming model can be efficiently solved. The proposed co-optimization for IEGS ensures both the economic performance and gas quality via coordinating traditional DGs, natural gas resources and P2G facility, which can benefit system operators with economic benefits through saving operation cost and secure gas distribution with gas quality guaranteed.

**Chapter eight** concludes the main findings of the thesis and the major contributions.

**Chapter nine** presents some potential research topics in future work.
This chapter describes the application of distributionally robust optimization method on IEGS optimization problems. The problem formulations, mathematical reformulations and solution algorithms are explicitly discussed.
Distributionally robust optimization (DRO) is developed to bridge between stochastic optimization (SO) and robust optimization (RO) [30]. In terms of uncertainty modelling, SO either relies on a vast number of samples to approximate distributions or fits data into an empirical distribution, which could be overly optimistic. RO also considers the worst case via uncertainty sets, potentially resulting in over-conservative solutions. By contrast, DRO avoids assuming a specific uncertainty distribution and yields less-conservative results [31]. DRO has simpler requirements of uncertainties and is mathematically tractable, which accommodates distributions via an ambiguity set [30]. Additionally, DRO performs better in making the best use of limited statistical data and produces less-conservative results by considering the worst expectation over all possible distributions, compared to the traditional worst-case oriented RO.

Ambiguity set is used to characterize the distributional information of uncertainties. DRO can be generally categorized into moment-based DRO and discrepancy-based DRO with respect to different types of ambiguity sets. Moment-based DRO applies the moment information, e.g., mean vectors, covariance matrices, to shape the uncertain distribution [32]. Discrepancy-based DRO measures the closeness between the reference distribution and the candidate distributions [33]. This chapter presents i) the modelling of the moment-based DRO constructed via Markov and Chebyshev ambiguity sets; ii) the modelling of discrepancy-based DRO relies on the KL-divergence. The main technical chapters, i.e., chapters 3-7, utilize DRO to handle the uncertainty modelling.

2.1 Moment-Based DRO

2.1.1 Markov Ambiguity Set

Markov ambiguity set contains all distributions with known mean information. This section proposes the modelling and solution of a two-stage DRO problem based on the Markov ambiguity set. Assume that the abstract forms of matrices and vectors are used to describe the constraints and variables. Then the ambiguity set modelling is proposed followed by Bender’s decomposition as the solution algorithm.

The compact form of a two-stage optimization model can be reformulated as (2-1). The first-stage objective function is represented by \( c'x \). And the expected second-stage objective function is represented by \( Q(x, \xi) \). \( x \) and \( y \) are the variables of the first and second stages. The uncertain variable is denoted as \( \xi \). The ‘min-sup’ structure allows
the model to make decision under the worst-case scenario. The first and second stage constraints are represented by (2-1) and (2-4), respectively.

\[
\begin{align*}
\min_{x \in X} c' x + \sup_{P \in D} E_P [Q(x, \xi)] \\
\text{s.t. } Ax \leq b, \\
Q(x, \xi) = \min_y f'y \\
\text{s.t. } Ex + Fy + G\xi \leq h,
\end{align*}
\]

(2-1) (2-2) (2-3) (2-4)

The ambiguity set containing the mean vector information is presented as (2-5).

\[
D_{Markov} = \left\{ f(\xi) \mid P\{\xi \in \Xi\} = 1 \right\}
\]

(2-5)

In some occasions, the support \( \Xi \) is required for accommodating the uncertainties, i.e., the total number of a uncertain binary variable is fixed.

\[
\Xi = \left\{ \sum \xi = N \right\}
\]

(2-6)

The variables of the optimization problem are the probability densities in (2-5). The optimization problem contains a finite number of constraints and an infinite number of variables, which is non-tractable. By transforming it from the primal form into the dual form, the problem becomes tractable.

The second-stage objective function \( \sup_{P \in D_{EL,PL}} E_P [Q(x, \xi)] \) can be represented by \( S(x) \), where \( P(\xi) \) is the probability density function.

\[
S(x)^{primal} = \max_{P(\xi) \in D_{EL,PL}} \int_\Xi Q(x, \xi) P(\xi) d\xi
\]

(2-7)

\[
\text{s.t. } P(\xi) \geq 0, \forall \xi \in \Xi
\]

(2-8)

\[
\int_\Xi P(\xi) d\xi = 1
\]

(2-9)

\[
\int_\Xi \xi P(\xi) d\xi = \mu
\]

(2-10)

Based on dual theory [30], when the strong duality holds, the dual form of (2-7) is transformed to (2-11). Accordingly, the results of (2-11) are equal to those of (2-7) [34, 35]. Now, the dual form has an infinite number of constraints and a finite number of variables after the dual formulation, which is easier to solve. The dual variables \( \Psi \) and \( \psi_0 \) are associated with constraints (2-9) and (2-10).
Chapter 2  Distributionally Robust Optimization

\[ S(x)^{\text{dual}} = \min_{\Psi, \mu, \psi_0} \Psi \mu + \psi_0 \]

\[ \text{s.t. } \Psi \xi + \psi_0 \geq Q(x, \xi) \quad \forall \xi \in \Xi \]

Then, the dual formulation (2-11) is substituted into (2-1) and the following reformulations can be obtained:

\[ \min_{\Psi, \mu, \psi_0} c'x + \Psi \mu + \psi_0 \]

\[ \text{s.t. } Ax \leq b, \]

\[ \Psi \xi + \psi_0 \geq Q(x, \xi), \forall \xi \in \Xi \]

Bender’s decomposition is applied to solve the overall model in a master-subproblem framework, summarized in the flowchart in Fig. 2-1. Under the acceptable optimality gap, the master problem and subproblem are solved separately with the update of upper
and lower bounds. A Bender’s cut is formulated and added to the set in each iteration. The set of Bender’s cut becomes larger until the real gap between lower and upper bounds is smaller than the optimality gap. The master problem and primal subproblem are shown in (2-16)-(2-17) and (2-18).

\[
\min_{\psi, \psi_0} c^T x + \Psi \mu + \psi_0 \tag{2-16}
\]

\[
\text{s.t. } Ax \leq b, \tag{2-17}
\]

\[
\max_\psi Q(x^*, \xi) - \Psi^* \xi \tag{2-18}
\]

The new dual variable $\tau$ is introduced to represent the dual form of (2-3). Accordingly, (2-3) and (2-4) can be represented in the following closed form:

\[
\max \tau'(b - Ex - G\xi_s) \tag{2-19}
\]

\[
\text{s.t. } F'\tau \leq f, \tau \geq 0 \tag{2-20}
\]

Equation (2-18) is recast as (2-21) based on the new dual variable.

\[
\max \tau'(b - Ex - G\xi_s) - \Psi^* \xi \tag{2-21}
\]

\[
\text{s.t. } F'\tau \leq f, \tau \geq 0 \tag{2-22}
\]

It is worth noting that the term $\Psi^* \xi$ in (2-21) is nonlinear because it contains the product of dual variable $\Psi^*$ and binary variable $\xi$. Although it can be solved by some nonlinear solvers, linearization is still required to ensure a more efficient and global solution. McCormick inequality is used to relax this nonconvex problem, where $\vartheta_{ij}$ is used to represent $\tau_i G_{ij} \xi_j$.

\[
\vartheta_{ij} \geq G_{ij} \tau_i - M(1 - \xi_j) \tag{2-23}
\]

\[
\vartheta_{ij} \geq -M\xi_j \tag{2-24}
\]

Therefore, the dual subproblem can be written as:

\[
\max \tau'(b - Ex) - \vartheta - \Psi^* \xi \tag{2-25}
\]

\[
\text{s.t. } F'\tau \leq f, \tau \geq 0 \tag{2-26}
\]

\[
\vartheta_{ij} \geq G_{ij} \tau_i - M(1 - \xi_j) \tag{2-27}
\]

\[
\vartheta_{ij} \geq -M\xi_j \tag{2-28}
\]
Based on the obtained $\xi^*$, the primal and dual Bender’s cuts can be presented in (2-29) and (2-31) respectively.

$$\psi_0 \geq f' y^* - \Psi \xi^*$$  \hspace{1cm} (2-29)

$$Ex + F y^* + G \xi^* \leq h,$$  \hspace{1cm} (2-30)

$$\psi_0 \geq \tau'^*(b - Ex - G \xi^*) - \Psi \xi^*$$  \hspace{1cm} (2-31)

### 2.1.2 Chebyshev Ambiguity Set

Based on the Markov ambiguity set, Chebyshev ambiguity set models the distributional ambiguity utilizing the mean and covariance matrix. The original problem can be recast as a semidefinite programming (SDP). First of all, the linear DRO problem can be represented by a compact matrix form. Then, the family of possible uncertainty distributions is defined by an ambiguity set. Finally, the dual problem is formulated and solved by a column generation algorithm (CGA) efficiently.

Matrices and vectors are used to represent the original problem for notation abbreviation. The objective function (2-32) is to minimize the sum of the first-stage objective $c'x$ and the expected second-stage objective $E_{P_f}[Q(x, \xi)]$. The random parameter $\xi$ is sampled from a family of distributions $P_f$.

$$\min_{x \in X} c'x + \sup_{P_f \in D_\xi : \xi = \Delta P_k \xi_j} E_{P_f}[Q(x, \xi)]$$  \hspace{1cm} (2-32)

$$\text{s.t. } Ax \leq b,$$  \hspace{1cm} (2-33)

$$Q(x, \xi) = \min_y f'y$$  \hspace{1cm} (2-34)

$$\text{s.t. } Ex + Fy + G \xi \leq h,$$  \hspace{1cm} (2-35)

The first-stage constraints are shown in (2-33). Equations (2-34) and (2-35) represent the recourse function.

The uncertainties can be captured by ambiguity sets that define a family of distributions. Based on limited historical data, moment information, i.e., mean and covariance can be obtained for constructing empirical point-estimates. The proposed ambiguity set is given in (2-36) which guarantees i) the integral of distribution of $\xi$ is 1, and ii) the second moments are known.
The ambiguity set used to characterize uncertain variables is composed of mean and covariance information. Intuitively, a certain set of mean vector and covariance matrix contains all possible probability distributions. To obtain a ‘min’ form of the second-stage problem, dual reformulation is required for the inner problem ‘min sup \( \mathbb{E} P_f[Q(x, \xi)] \)’. The second-stage problem \( \mathbb{E} P_f[Q(x, \xi)] \) is an infinite-dimensional linear problem and the primal form is given in (2-37)-(2-41).

\[
D = \left\{ f(\xi) \left| \begin{array}{l}
P(\xi) = 1 \\
\mathbb{E}[\xi] = \mu \\
\mathbb{E}[\xi(\xi)'] = \Sigma + \mu(\mu)' \end{array} \right. \right\}
\] (2-36)

The ambiguity set used to characterize uncertain variables is composed of mean and covariance information. Intuitively, a certain set of mean vector and covariance matrix contains all possible probability distributions. To obtain a ‘min’ form of the second-stage problem, dual reformulation is required for the inner problem ‘min sup \( \mathbb{E} P_f[Q(x, \xi)] \)’. The second-stage problem \( \mathbb{E} P_f[Q(x, \xi)] \) is an infinite-dimensional linear problem and the primal form is given in (2-37)-(2-41).

\[
S(x)^{\text{primal}} = \max_{P_f \in D} \int_{\Xi} Q(x, \xi) P_f(\xi) d\xi
\] (2-37)

\[
\text{s.t. } P_f(\xi) \geq 0, \forall \xi \in \Xi
\] (2-38)

\[
\int_{\Xi} P_f(\xi) d\xi = 1
\] (2-39)

\[
\int_{\Xi} \xi^m P_f(\xi) d\xi = \mu_m, m=1,2, \ldots, \Xi
\] (2-40)

\[
\int_{\Xi} \xi^n \xi P_f(\xi) d\xi = \Sigma_{mn} + \mu_m \mu_n, m, n=1,2, \ldots, \Xi
\] (2-41)

Fig. 2-2. Flowchart of constrained generation algorithm.
For tractability, the primal form needs to be recast as (2-42) and (2-43), where \( \psi_0 \), \( \psi_j \) and \( \Psi_{jk} \) are dual variables associated with the second-stage constraints and \( \Theta \) represents \( \Sigma + \mu (\mu') \). When the weak duality holds, \( S(x)^{\text{prim}} \leq S(x)^{\text{dual}} \). However, (2-41) ensures that the strong duality holds when \( \Theta \) is strictly positive definite and thus \( S(x)^{\text{prim}} = S(x)^{\text{dual}} \) [30]. Accordingly, now the problem with an infinite number of variables is transformed into one with a finite number of variables (2-42)-(2-43), which is easier to solve.

\[
S(x)^{dual} = \min_{\psi, \psi_0} \langle \psi' \theta \rangle + \psi' \mu + \psi_0
\]  
\[
\text{s.t.} \ (\xi)'\psi \xi + \psi' \xi + \psi_0 \geq Q(x, \xi)
\]  
\[
\forall \xi \in \mathcal{Z}
\]  

The new compact form is:

\[
\min_{x \in X} c' x + S(x)^{dual}
\]  

Problem (2-44) is a semi-infinite-dimensional program which contains an infinite number of constraints. Thus, it is required to be transformed into a closed form [36]. By introducing the new dual variable \( \tau \), a positive quadratic function in (2-45) can be obtained from (2-34). \( VS \) denotes the polyhedral set of extreme points and \( N_v \) is the set of vertices of feasible region in \( VS \).

\[
\max_{u \in VS} \tau' (b - Ex - G \xi)
\]  
\[
VS = \{\tau | F' \tau = f, \tau \leq 0\}
\]  
\[
(\xi)'\psi \xi + (\psi + G' \tau^i)' \xi + \psi_0 - (h - Ex) \tau^i \geq 0
\]  
\[
\forall \xi \in \mathcal{Z}, i = 1, 2, ..., N_v
\]  

In summary, the SDP form is as follow, which is the master problem.

\[
\min_{x, \psi, \psi_0} c' x + \langle \psi' \theta \rangle + \psi' \mu + \psi_0
\]

\[
\begin{bmatrix}
\xi' \\
1
\end{bmatrix}
\begin{bmatrix}
\psi & \frac{1}{2} (\psi + G' \tau^i) \\
\frac{1}{2} (\psi + G' \tau^i)' & \psi_0 - (h - Ex)' \tau^i
\end{bmatrix}
\begin{bmatrix}
\xi \\
1
\end{bmatrix} \succeq 0
\]  
\[
\forall \xi \in \mathcal{Z}, i = 1, 2, ..., N_v, x \in X, \forall \tau^i \in VS
\]  

A large number of constraints with infinite cardinality of \( VS \) cause high computational burden. CGA initially enumerates a subset of vertices and incorporates
more vertices step by step. This relaxation method can efficiently solve the proposed problem, which is separated into a master and sub problem in (2-48) and (2-49). The flowchart of the CGA is given in Fig. 2-2.

\[(\xi_s)'\Psi \xi_s + \psi' \xi_s + \psi_0 - (h - Ex - G\xi_s)'\tau \geq 0 \quad (2-49)\]

s.t. \(\forall \xi \in \mathcal{E}, \tau \in \mathcal{V}S\)

### 2.2 KL-Divergence-Based DRO

Discrepancy-based ambiguity set use more distributional information to shape real distributions compared with moment-based ambiguity set [33]. It measures the discrepancy between the candidate distribution and reference distribution. The discrepancy can be controlled to either decrease or increase the conservatism depending on the reliability requirement of the optimization. KL divergence is a common \(\phi\)-divergence to measure the distance between two distributions. Estimation of uncertainty distributions can be obtained by statistical fitting [33, 37]. KL divergence-based ambiguity set models uncertainty requiring the candidate distribution within a predefined distance from the nominal distribution.

#### 2.2.1 Risk-Based Ambiguity Set

Firstly, the linear problem is represented by a compact form for notation brevity. Secondly, the KL divergence-based ambiguity set is used to define the uncertainty. Then, CVaR is derived. The final step incorporates the mathematical reformulation and decomposition methods for solving the problem.

The original problem can be represented by vectors and matrices to represent the objective function and constraints for notation simplicity. Compared to the proposed risk-averse model, the traditional risk-neutral DRO model does not consider risk factor, which is given in (2-50). In (2-50), the first-stage objective function is represented by \(c'x\). And the expected second-stage objective function is represented by \(Q(x, \xi)\). \(x\) and \(y\) are the variables of the first and second stages. The uncertain variable is denoted as \(\xi\). The ‘min-sup’ structure allows the model to make decision under the worst-case scenario.
Based on the traditional risk-neutral DRO model, the risk measure can be included in the second stage problem, shown below:

\[
\min_{x \in X} c'x + \sup_{p \in D_{\xi}} E_p[Q(x, \xi)]
\]

\[\text{s.t. } Ax \leq b,\]

\[Q(x, \xi) = \min_y f'y\]

\[\text{s.t. } Ex + Fy + G\xi \leq h,\]

Equation (2-51) presents the first-stage constraints. The recourse process is represented by (2-53) and (2-54), where \(f\) denotes the coefficient of (2-53).

The risk-averse objective function (2-51) is to minimize the sum of the first-stage objective \(c'x\), the weighted expected second-stage objective \((1 - \alpha)E_p[Q(x, \xi)]\), and the weighted risk measure \(\alpha R(Q(x, \xi))\). \(D_{\xi}\) denotes the ambiguity set, containing distribution \(p\). The weighting factor \(\alpha\) ranges between 0 and 1. When \(\alpha = 0\), (2-51) degrades to the traditional risk-neutral DRO. The discrepancy-based ambiguity set is constructed based on measuring the distance between probability distributions, i.e., the divergence tolerance \(\eta\) in (2-55). The true and reference probability distribution are represented by \(p\) and \(p_{\text{ref}}\), respectively. The KL divergence between \(p\) and \(p_{\text{ref}}\) is defined in (2-55), where \(p(\xi)\) and \(p_{\text{ref}}(\xi)\) are the probability density functions.

\[\text{Dis} = \{p \in D_{\xi} | D_{\xi}(p\|p_{\text{ref}}) \leq \eta\}\]

\[D_{\xi}(p\|p_{\text{ref}}) = \int f(\xi) \log \frac{p(\xi)}{p_{\text{ref}}(\xi)} d\xi\]

KL-divergence function of variable \(a\) is in (2-57), which will be used in the dual formulation to solve the inner maximization problem in section D.

\[\varphi_{KL}(a) = a \log a - a + 1\]

The probability of the second-stage objective function \(Q(x, \xi)\), i.e., the corrective operation cost including load shedding lost, is restricted by the threshold \(\zeta\).
emerging risk measure method, CVaR is a coherent risk measure, which is convex, transition-equivalent, and monotonic. The original expression of CVaR is in (2-58), which can be further approximated by (2-59) to avoid the computation of multiple integral [38]. \([Q(x, \xi) - \zeta]^+\) represent determining the larger value between \(Q(x, \xi) - \zeta\) and 0.

\[
CVaR_\beta(Q(x, \xi)) = \frac{1}{1-\beta} \int_{Q(x, \xi) \geq VaR_\beta(x, \xi)} Q(x, \xi)p(\xi)d\xi
\]  
(2-58)

\[
CVaR_\beta(Q(x, \xi)) = \min_{\zeta \in \mathbb{R}} \left\{ \zeta + \frac{1}{1-\beta} E_p[Q(x, \xi) - \zeta]^+ \right\}
\]  
(2-59)

The proposed FMS is formulated as (2-60) with weighted CVaR. Equation (2-61) can be derived by substituting CVaR in (2-62) with (2-61).

\[
\min_{x \in X} c'x + \sup_{p \in D, \xi = \Delta P_k, \xi_j} \{ (1 - \alpha)E_p[Q(x, \xi)] + \alpha CVaR_\beta(Q(x, \xi)) \}
\]  
(2-60)

\[
\min_{x \in X} \left\{ c'x + \sup_{p \in D, \xi = \Delta P_k, \xi_j} \min_{\zeta \in \mathbb{R}} \left\{ \alpha \zeta + E_p[G(x, \xi)] \right\} \right\}
\]  
(2-61)

\[
G(x, \xi) = (1 - \alpha)[Q(x, \xi)] + \frac{\alpha}{1 - \beta} \tilde{a}
\]

s.t. \(Q(x, \xi) - \tilde{a} - \zeta \leq 0, \tilde{a} \geq 0\)

Based on the proof in [39] on the strong duality, (2-61) can be reformulated to (2-62) and (2-63) with a further step, where

\[
\min_{x \in X} \left\{ c'x + \min_{\zeta \in \mathbb{R}} \sup_{p \in D, \xi = \Delta P_k, \xi_j} \{ \alpha \zeta + E_p[G(x, \xi)] \} \right\}
\]  
(2-62)

\[
\min_{x \in X} \left\{ c'x + \alpha \zeta + \max_{p \in D, \xi = \Delta P_k, \xi_j} \left\{ \sum_{i=1}^{m} p_i G_i(x, \xi) \right\} \right\}
\]  
(2-63)

The inner maximization problem can be handled by the Lagrange function (2-64) with its dual formulation (2-70).
\[ \mathcal{L}(p, \tau, \mu) = \sum_{i=1}^{m} p_i G_i(x, \xi) + \tau \left( 1 - \sum_{i=1}^{m} p_i \right) \]

\[ + \mu \left( \eta - \sum_{i=1}^{m} p_{ref,i} \varphi_{KL} \left( \frac{p_i}{p_{ref,i}} \right) \right) \]

\[ \max \mathcal{L}(p, \tau, \mu) = \tau + \eta \mu + \mu \sum_{i=1}^{m} p_{ref,i} \left[ \exp \left( \frac{G_i(x, \xi) - \tau}{\mu} \right) - 1 \right] \] (2-65)

According to Slater’s condition [40], when \( \eta \) is larger than 0, the below reformulation can be made:

\[ \max_{p \in \Delta p_k, \xi_j} \left\{ \sum_{i=1}^{m} p_i G_i(x, \xi) \right\} = \min_{\tau, \mu \geq 0} \max \mathcal{L}(p, \tau, \mu) \] (2-66)

\[ = \min_{\tau, \mu \geq 0} \left\{ \tau + \eta \mu + \mu \sum_{i=1}^{m} p_{ref,i} \left[ \exp \left( \frac{G_i(x, \xi) - \tau}{\mu} \right) - 1 \right] \right\} \] (2-67)

Substituting the inner maximization in (3-57) with (3-61), the below derivation can be obtained.

\[ \min_{\xi, \tau, \mu \geq 0} \left\{ c' x + a \xi + \tau + \eta \mu + \mu \sum_{i=1}^{m} p_{ref,i} \left[ \exp \left( \frac{G_i(x, \xi) - \tau}{\mu} \right) - 1 \right] \right\} \] (2-68)

s.t. \( x \in X, Q(x, \xi) - \bar{a} - \xi \leq 0, \bar{a} \geq 0, G(x, \xi) = (1 - \alpha)[Q(x, \xi)] + \frac{\alpha}{1-\beta} \bar{a} \)

However, the optimization problem (3-62) is nonlinear, which needs to be linearized before decomposition. For a given \( x = x^k \), when \( Q(x^k, \xi) < \infty \), then \( Q(x^k, \xi) \) is subdifferentiable [41] and equation (3-63) can be obtained, where \( \text{Dual}(x^k) = \arg \max \{ \pi'(h - E x^k) : F' \pi \leq f \} \) is the set of optimal solutions of dual problem for (3-47) and \( \pi^{k,i} \in \text{Dual}(x^k) \) is optimal solution for \( i \)th and \( k \)th iterations.

\[ \partial Q(x^k, \xi) = -E' \text{Dual}(x^k) \] (2-69)

Let \( s^k := \frac{G_i(x^k, \xi) - \tau^k}{\mu^k} \) and \( F^k_i := \mu^k[\exp(s^k) - 1] \), the subgradient of \( F^k_i \) can be described as:
Based on the subgradient inequality of convex function, the below equation can be obtained. The optimality cut can be defined in (3-66).

\[
\frac{\partial F_i^k}{\partial x} = \left[ (1 - \alpha) \exp(s^k) E^{k+1} x, (1 - s^k) \exp(s^k) \right.
\]
\[
-1, -\exp(s^k), \frac{\alpha}{1 - \beta} \exp(s^k) \right]
\]

A Bender’s decomposition is employed to solve the problem and the flowchart is given in Fig. 2-3.
2.2.2 Kernel Density Estimation-Based Ambiguity Set

The uncertainty in an equality constraint can be represented by two inequality constraints. For instance, the original equality constraint in (2-73) can be represented by (2-75) and (2-76), where \( \omega_{j,t}^{s} \) is the real-time wind power output and \( \xi \) is the uncertain wind power forecast error.

\[
\sum_{i_e \in I_e} P_{i_e,t} + \sum_{j \in J} \omega_{j,t}^{s} + \sum_{l_e \in L_e} f_{l_e,t}^{a,ini} - \sum_{l_e \in L_e} f_{l_e,t}^{a,ter} - \sum_{k_e \in K_e} P_{k_e,t} = 0 \quad (2-73)
\]

\[
\omega_{j,t}^{s} = \omega_{j,t}^{f} + \xi \quad (2-74)
\]

\[
\sum_{i_e \in I_e} P_{i_e,t} + \sum_{j \in J} \omega_{j,t}^{s} + \sum_{l_e \in L_e} f_{l_e,t}^{a,ini} - \sum_{l_e \in L_e} f_{l_e,t}^{a,ter} - \sum_{k_e \in K_e} P_{k_e,t} \geq 0 \quad (2-75)
\]

\[
\sum_{i_e \in I_e} P_{i_e,t} + \sum_{j \in J} \omega_{j,t}^{s} + \sum_{l_e \in L_e} f_{l_e,t}^{a,ini} - \sum_{l_e \in L_e} f_{l_e,t}^{a,ter} - \sum_{k_e \in K_e} P_{k_e,t} \leq 0 \quad (2-76)
\]

Constraint (2-75) is used as the representative of reformulations in the later section, which is transformed into (2-77) since DRO considers the worst distribution of uncertain forecast error.

\[
\min_{P} \sum_{i_e \in I_e} P_{i_e,t} + \sum_{j \in J} \omega_{j,t}^{s} + \sum_{l_e \in L_e} f_{l_e,t}^{a,ini} - \sum_{l_e \in L_e} f_{l_e,t}^{a,ter} - \sum_{k_e \in K_e} P_{k_e,t} \geq 0 \quad (2-77)
\]

Equation (2-78) measures the discrepancy between two probability distribution \( P \) and reference distribution \( P_{ref} \) based on \( \phi \)-divergence through the divergence tolerance \( \eta \). Equation (2-79) defines the KL divergence between \( P \) and \( P_{ref} \), where \( f(\xi) \) and \( f_{ref}(\xi) \) are the probability density functions.

\[
P = \{ P \in D | D(P \| P_{ref}) \leq \eta \} \quad (2-78)
\]

\[
D(P \| P_{ref}) = \int f(\xi) \log \left( \frac{f(\xi)}{f_{ref}(\xi)} \right) d\xi \quad (2-79)
\]

DRO considers the worst distribution scenario and thus the expectation of constraint (2-75) is based on all the possible uncertainty distributions are considered, which is given in (2-80).

\[
\min_{P \in D} E_{p} [H(x, \xi)] \geq 0 \quad (2-80)
\]
Based on the change-of-measure method, (2-81) can be obtained according to [33], where \( L(\xi) = f(\xi)/f_{ref}(\xi) \). By applying the change-of-measure method to constraint (2-80), equation (2-82) can be obtained.

\[
D(P\|P_{ref}) = \int f(\xi) \log \frac{f(\xi)}{f_{ref}(\xi)} \, d\xi = E_{P_{ref}}[L(\xi) \log L(\xi)] \tag{2-81}
\]

\[
E_p[H(x,\xi)] = \int H(x,\xi) \frac{f(\xi)}{f_{ref}(\xi)} \, d\xi = E_{P_{ref}}[H(x,\xi) L(\xi)] \tag{2-82}
\]

To incorporate uncertainty within the constraint (2-77), it needs to be treated as an inner optimization problem with sub-objectives and constraints.

\[
\min E_{P_{ref}}[H(x,\xi)L(\xi)] \tag{2-83}
\]

s.t. \( E_{P_{ref}}[L(\xi) \log L(\xi)] \leq \eta \)

The original optimization problem is reformulated into (2-84) as follows with the expectation of the constraints.

\[
\min \Gamma \tag{2-84}
\]

s.t. Constraints (5)-(31)

s.t. \( \min_{P \in P} E_{P_{ref}}[H(x,\xi)] \geq 0 \)

\[
P = \{ P \in D | D(P\|P_{ref}) \leq \eta \}
\]

According to [33], when strong duality holds, (2-84) can be transformed into (2-85).

\[
\min \Gamma \tag{2-85}
\]

s.t. Constraints (5)-(31)

s.t. \( \max_{P \in P} \alpha \log E_{P_{ref}}[e^{H(x,\xi)/\alpha} + \alpha \eta] \geq 0 \)

Then, the explicit expression of constraints of (2-85) according to (2-71) can be obtained in (2-86).

\[
\max_{P \in P} \alpha \log E_{P_{ref}}[e^{\Sigma_{i \in I} p_{i,t}^i + \Sigma_{j \in J} S_{j,t} + \Sigma_{e \in L} \hat{f}_{i,e,t}^{as,ini} - \Sigma_{e \in L} f_{i,e,t}^{ast,ter} - \Sigma_{k \in K} p_{k,e,t}/\alpha + \alpha \eta} \geq 0 \tag{2-86}
\]

The logarithmic expression under expectation is a moment generating function with distribution \( P_{ref} \), which can be transformed into a deterministic formulation. In this
paper, kernel density estimation (KDE) in (2-87) is used to estimate the reference distribution, where \( \xi^i \) represents error data, \( N \) is the number of error data, \( h_N \) is a positive smoothing parameter, and \( H(\cdot) \) is the kernel function (non-negative and the integral of the probability distribution is 1) [42]. Assuming \( H(\cdot) \) follows the normal distribution, (2-88) is formulated from (2-87) with mean value \( \xi^i \) and variance \( h_N^2 \).

\[
f_N(\xi) = \frac{1}{Nh_N} \sum_{i=1}^{N} H\left(\frac{\xi - \xi^i}{h_N}\right)
\]

(2-87)

\[
f_N(\xi) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{h_N \sqrt{2\pi}} e^{-\left(\frac{(\xi - \xi^i)^2}{2h_N^2}\right)}
\]

(2-88)

Finally, (2-73) can be transformed into (2-89) based on [33].

\[
\sum_{i \in I_e} P_{i,e,t}^s + \sum_{j \in J} \omega_{j,t}^f + \sum_{i \in I_e} f_{i,e,t}^{a,s,ini} - \sum_{l \in L_e} f_{l,e,t}^{a,s,ter} - \sum_{k \in L_e} P_{k,e,t}^s + \max_{\alpha \geq 0} \left\{ \alpha \eta + \frac{h_{N,P}^2}{2\alpha} + \alpha \ln \frac{1}{N_{P,t}} \sum_{i=1}^{N_{P,t}} e^{\left(\frac{\xi_{P,i}(t)/\alpha}{\alpha}\right)} \right\} \geq 0
\]
Chapter 3

Mean-Risk Distributionally Robust Co-Optimization of District Integrated Electricity-Gas-Water Systems

This chapter proposes a hierarchical mean-risk two-stage distributionally robust optimization (DRO) method to optimally coordinate the multi energy infrastructures in integrated power, gas and water systems. This two-stage mean-risk DRO is solved by Bender’s decomposition method.
**Statement of Authorship**

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<td>Pengfei Zhao</td>
</tr>
<tr>
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3.1 Abstract

Integrated energy system (IES) is a viable and effective solution for improving the energy utilization efficiency and promoting the renewable penetration via aggregating independent systems into an integrated management scheme. The water system management problem has been widely investigated. However, the interdependencies between water and energy systems are significant and the effective co-optimization is required considering strong interconnections. For instance, around 80% of the power consumed in water systems is used for pumping and distributing water. In power systems, surplus water resources significantly contribute to the generation and conversion in power systems. This paper proposes a two-stage distributionally robust operation model for integrated water-energy nexus systems (IWENS) including power, gas and water systems networked with energy hub systems in a distribution level considering wind uncertainty. The presence of the wind uncertainty inevitably leads to the risks in decision-making process. Accordingly, a coherent risk measure, i.e., conditional value-at-risk, is combined with the optimization objective to determine risk-aversion operation schemes. This two-stage mean-risk distributionally robust optimization is solved by Bender’s decomposition method. Case studies focus on investigating the strong interdependencies among the four interconnected energy systems. Numerical results validate the economic effectiveness of IES through optimally coordinating the multi-energy infrastructures. This work aims at jointly optimizing the IWENS in a distribution level considering the extensive water-energy nexus and renewable variation. To the end, the overall operational efficiency can be improved.

3.2 Nomenclature

Due to space limitation, the variables defined in section C represents both the scheduled variables in the first stage optimization and the regulated variables in the second stage. The superscript ‘s’ and ‘re’ representing ‘scheduled’ and ‘regulated’ respectively are omitted in section C. The full expression of variables is given in section II. In addition, the superscript ‘ini’ and ‘ter’ representing initial and terminal nodes of power bus, gas and water nodes are also omitted in this section to save space.

A. Indices and sets
\[ t, T \] Index and set of time periods.
Chapter 3  Mean-Risk DRO of District IES under Water-Energy Nexus

\[ b, B \]
Index and set of electricity buses.

\[ n, N \]
Index and set of gas nodes.

\[ w, W \]
Index and set of water nodes.

\[ i_e, I_e \]
Index and set of traditional distributed generators (DG).

\[ i_g, I_g \]
Index and set of natural gas sources.

\[ wr, WR \]
Index and set of water reservoirs.

\[ j, J \]
Index and set of renewable DGs.

\[ gt, GT \]
Index and set of gas turbines.

\[ wp, WP \]
Index and set of water pumps.

\[ l_e, L_e \]
Index and set of power lines.

\[ l_g, L_g \]
Index and set of gas pipelines.

\[ l_w, L_w \]
Index and set of water pipelines without pumps.

\[ l_{wp}, L_{wp} \]
Index and set of water pipelines with pumps.

\[ k_e, K_e \]
Index and set of power loads.

\[ k_g, K_g \]
Index and set of gas loads.

\[ k_w, K_w \]
Index and set of water loads.

### B. Parameters

\[ P_{k.e,t}, Q_{k.e,t}, G_{k.g,t}, \]
Demand of active power, reactive power, gas and water.

\[ P_{k.w.t}, P_{m.max}, P_{wr.max} \]
Maximum active power purchase from upper level market and water purchase from reservoir.

\[ R_{i.e}^{+}, R_{i.e}^{-}, R_{gt}^{+}, R_{gt}^{-}, R_{wp}^{+}, R_{wp}^{-} \]
Maximum up and down reserve capacity of traditional DGs, the gas turbine and water pumps.

\[ P_{e.max}, P_{e.min}, \]
Maximum and minimum limits for active power output of traditional DGs, gas turbine output and water pump power consumption.

\[ P_{wp.max}, P_{wp.min} \]
Maximum and minimum reactive power output of traditional DG \( i_e \).

\[ Q_{i.e.max}, Q_{i.e.min} \]
Maximum and minimum voltage limits.

\[ V_{b.max}, V_{b.min} \]
Reactance and resistance of power line \( l_e \).

\[ V_0 \]
Reference voltage magnitude.

\[ f_{l.e.max}, qf_{l.e.max} \]
Maximum active and reactive power flow of line \( l_e \).

\[ c_{eb}, c_{gt} \]
Conversion coefficient for electric boilers and the gas turbine.

\[ \omega^j(t) \]
Forecasted output of renewable DG \( j \) at time \( t \).

\[ G_{i.g.max}, G_{i.g.min} \]
Maximum and minimum output of gas source \( i_g \).
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\[ P_{\gamma_{g,\max}}, P_{\gamma_{g,\min}} \]

Maximum and minimum gas pressure of gas pipeline \( l_g \).

\[ \gamma_{lg} \]

Coefficient for Weymouth equation.

\[ f_{lg,\max}, \]

Maximum gas flow of line \( l_g \).

\[ C_{Fg} \]

Gas compressor coefficient.

\[ \eta_e \]

Electrical efficiency for electrolyser.

\[ \eta_{hy-ca}, \eta_{hy-me} \]

Reaction coefficients for required carbon dioxide and methanation output.

\[ h_{wp,\max}, h_{wp,\min}, h_{w,\max}, h_{w,\min} \]

Maximum and minimum limits for head pressure of water node connected with or without water pump.

\[ a_{wp}, b_{wp} \]

Water pump characteristic coefficients.

\[ R_{wp}, R_w \]

Head gain and loss coefficients.

\[ \pi_{wp} \]

Water pump efficiency.

\[ f_{wp,\max}, f_{wp,\min} \]

Water flow for water pipeline with and without pump.

\[ \eta_{cp}, \eta_{cp}^e \]

Electric and heating efficiency for combined heat and power (CHP).

\[ \eta_{COP}, \eta_{GF} \]

Coefficient of performance of ground source heat pump (GSHP) and efficiency of gas furnace (GF).

\[ p_{cp,\max}, p_{cp,\min}, p_{hp,\max}, p_{hp,\min} \]

Maximum and minimum input limits of CHP, GSHP and GF.

\[ p_{BS,\max}, p_{BS,\min}, p_{ch,\max}, p_{ch,\min} \]

Maximum and minimum charging and discharging power for battery storage.

\[ p_{HS,\max}, p_{HS,\min}, p_{dch,\max}, p_{dch,\min} \]

Maximum and minimum charging and discharging heat for heat storage.

\[ \eta_{BS,\max}, \eta_{BS,\min}, \eta_{ch,\max}, \eta_{ch,\min}, \eta_{dch,\max}, \eta_{dch,\min} \]

Charging and discharging efficiency for battery and heat storage.

\[ E_{BS,\max}, E_{BS,\min}, E_{HS,\max}, E_{HS,\min} \]

Maximum and minimum remaining energy limits of battery and heat storage.

\[ L_{e,t}, L_{h,t} \]

Electricity and heat load of energy hub system.

\[ \lambda^a_{te}, \lambda^b_{te}, \lambda^c_{te} \]

Cost coefficients for generation of traditional DG \( i_e \).

\[ \lambda_{lg} \]

Cost coefficient for output of natural gas source \( i_g \).

\[ \lambda_{m}, \lambda_{wr} \]

Cost coefficient of power and water purchase.

\[ \lambda^+_e, \lambda^-_e, \lambda^+_g, \lambda^-_g, \lambda^+_w, \lambda^-_w \]

Cost coefficient for up and down reserve of traditional DGs, the gas turbine and water pumps.
\( \lambda_{m}^{re}, \lambda_{i}^{re}, \lambda_{j}^{re}, \lambda_{g}^{re}, \lambda_{wr}^{re} \) Regulation cost coefficient of power purchase, traditional DGs \( i_e \), wind turbines, natural gas sources and water reservoir.

C. Variables

- \( P_{m,t}, P_{wr,t} \) Active power and water purchase.
- \( r_{i,e,t}^{+}, r_{i,e,t}^{-}, r_{gt,t}^{+}, r_{wp,t}^{+}, r_{wp,t}^{-} \) Up and down reserve capacity of traditional DGs, the gas turbine and water pumps.
- \( P_{i,e,t}, P_{gt,t}, P_{wp,t} \) Active power output of traditional DGs, gas turbine output and water pump power consumption.
- \( Q_{i,e,t} \) Reactive power output of traditional DGs.
- \( P_{i,g,t} \) Output of natural gas source.
- \( V_{b,t}^{s}, V_{b,t}^{re} \) Scheduled and regulated voltage of bus \( b \) at time \( t \).
- \( f_{i,e,t}, q_{i,e,t}, f_{i,g,t} \) Active and reactive power flow and gas flow.
- \( f_{i,e,eb,t}, P_{eb,t} \) Injected power flow and output of electric boiler.
- \( G_{i,g,t} \) Output of natural gas sources.
- \( P_{r,n,t} \) Pressure of gas node \( n \).
- \( f_{i,g,GT,t}, P_{gt,t} \) Injected gas flow and output of gas turbine.
- \( P_{P2G}^{n,t} \) Power consumed by the electrolyser.
- \( G_{n,t}^{hy}, G_{n,t}^{hy,me}, G_{n,t}^{hy,d}, G_{n,t}^{me} \) Gas output for overall P2G process, direct hydrogen injection, hydrogen during methanation process and methanation.
- \( G_{n,t}^{ca} \) Required gas of carbon dioxide during methanation process.
- \( h_{w,t}^{lwp}, h_{w,t}^{lw} \) Water pressure of pipe with and without water pump.
- \( h_{w,t}^{lwp}, h_{w,t}^{lw} \) Elevation of water node connected with and without pump.
- \( h_{w,t}^{lw}, h_{w,t}^{lwp} \) Head loss and gain of water node.
- \( f_{i,w,t}, f_{i,w,t} \) Water flow of pipe with and without water pump.
- \( f_{i,\text{inj}}, f_{i,\text{gk}} \) Power and gas flow injection to EHSs.
- \( P_{COP,t}^{i}, P_{\text{COP,t}}^{o} \) Power input and heat output of GSHP.
- \( P_{i}^{i}, P_{\text{GF,t}}^{o} \) Gas input and output of gas furnace.
- \( P_{i}^{s,i}, b_{i}^{s,o}, b_{i}^{p,e,t}, P_{i}^{s,o} \) Gas input and power and heat output of CHP.
- \( P_{\text{ch,b,s,t}}^{ch}, b_{\text{ch,b,s,t}}^{dach}, P_{\text{ch,b,s,t}}^{ch}, b_{\text{ch,b,s,t}}^{dach} \) Charging and discharging power and heat of battery and heat storage.
- \( E_{BS,t}, E_{HS,t} \) Remaining energy of battery and heat storage.
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\[ v_{e,t}, v_{g,t} \quad \text{Dispatch factors of power and gas.} \]
\[ x, y \quad \text{Vectors of first and second stage variables.} \]

3.3 Introduction

Integrated energy system (IES) is an interdependent configuration and management solution to coordinate multiple energy vectors. It can be realised by the utilization of energy converters, e.g., power-to-gas (P2G), combined heat and power (CHP), heat pumps and gas turbines, etc, further intensify the operational interdependency of IES. Through optimally coordinating multiple energy infrastructures, system efficiency can be significantly improved, renewable energy penetration can be highly facilitated, and environmental targets can be achieved.

Much effort has been focused on the optimization of IES, mainly achieving economic and environmental targets. A robust optimization (RO) model is proposed for an integrated power-gas-heat system in smart districts [43]. This model is demonstrated on a real multi-energy district and real-world physical limitations of energy infrastructures are examined. Paper [44] designs an optimal operation model for a regional IES considering energy price variations. Both system cost and environmental pollutions can be reduced through this optimization model. In [45], an energy sharing framework for multiple interconnected microgrids in an integrated power and heat system is proposed. This model comprehensively optimizes energy generation cost, trading cost with the utility grid and other microgrids, and discomfort cost. Paper [19] presents a decentralized optimization framework for an integrated power and gas system with networked energy hubs. A distributed algorithm based on Bender’s decomposition is used to solve this mixed-integer second-order cone programming problem. In [46], a consumption-based carbon pricing method is combined with an optimization model for IES. Accordingly, energy customers are given proper incentives to use low-carbon energy.

Traditionally, water and power systems are designed and operated separately. Nevertheless, water and energy systems are mutually interdependent [47]. According to [48], 3% of the U.S. electricity is facilitated by water distribution systems and approximately 80% of the water consumed electricity is used for distributing and pumping water. The abundant water resources largely contribute to power generation and conversion in power systems.
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The existing work on joint optimization of water and power systems mainly focus on reducing system operation cost and gas emissions. Paper [49] proposes an optimal water-power usage by controllable assets considering the couplings in an integrated water and power system (IWPS). A distributed algorithm based on the alternating direction method of multipliers helps pursue individual objectives. In [50], a coordinated day-ahead optimization model for IWPS is proposed considering the hydraulic constraints of water systems. An energy flexibility model for water systems is designed to offer the feasible energy flexibility capacity to the system operator. Paper [51] proposes an optimization model for the demand-side management of IWPS. The water system is treated as an effective resource to manage renewable generation. Stochastic programming (SP) based multi-stage fuzzy optimization is developed for a combined operation and planning problem in an IWPS considering uncertain power demand [52].

The inherent interdependencies between subsystems in IES have been promoted due to increasing energy demand growth, lower prices of gas resources, and emerging conversion technologies for interconnecting subsystems. The aforementioned literature review in the IES demonstrates the benefits of interdependencies. Moreover, the integration of multiple energy systems and water systems will further strengthen the couplings and interdependencies.

However, research on the integration of multi-energy systems and water system is very limited until very recent work in [53], which proposes a robust optimization model (RO) for a multi-energy water-energy nexus system (MEWENS) considering wind uncertainty based on a box-like uncertainty set. The multi-energy flow of power, gas, heat and water systems is analysed in a two-stage optimization framework and solved by column-and-constraint generation algorithm.

In the existing literature, the uncertainty pertaining to renewable generation in IES operation is commonly handled by SP [54, 55] and RO [43, 53]. SP assumes that the distribution of uncertain variables is known. However, obtaining explicit distributions is impractical and the scenario approach will lead to computational burden in optimization. RO copes with uncertainty considering all realizations, including the worst-case renewable fluctuation scenario, which ensures system robustness but sacrifices system cost effectiveness. Distributionally robust optimization (DRO), which employs partial distributional information to capture the ambiguous uncertainty
distributions, can overcome the limitations and deficiencies of SP and RO. Recently, DRO has been applied in the operation of IES. An optimal gas-power flow model is established in [56] and wind power uncertainty is characterized by Wasserstein-based ambiguity set. Paper [57] proposes distributionally robust scheduling for integrated electricity and gas systems considering demand response. The revenue from demand response is maximized and expected load shedding cost is minimized.

DRO employs ambiguity sets to capture the uncertainties pertaining to known distributional information. The optimization results will be intractable or over-conservative if the ambiguity set is not chosen appropriately. There are two common methods to characterize ambiguity sets, moment-based ambiguity set and discrepancy-based ambiguity set. The former one has simple tractable reformulations, e.g., semidefinite program (SDP) or second-order cone program (SOCP). Nevertheless, different distributions might have the same moment information, which introduces challenges for determining the worst-case distribution. Discrepancy-based ambiguity set measures the statistical distance between the reference distribution and candidate distributions. Kullback-Leibler (KL) divergence is widely applied in operation problems in the area of power systems [58, 59].

The uncertainties bring risks into economic operation. Intuitively, risks in the proposed IES operation model can lead to abnormal high operation cost. Mean-risk optimization considers a coherent trade-off between system economic performance and risk, which has been applied with SP on energy system operation [60-62]. Paper [60] develops a mean-risk stochastic programming model for unit commitment considering renewable energy uncertainty. A conditional value-at-risk (CVaR) is incorporated to assess the risk from renewable energy uncertainty. In [62], a day-ahead operational planning model for a regional energy service provider with electricity price uncertainty is proposed. The CVaR criterion is employed to hedge against the uncertainty.

This paper aims at constructing a two-stage mean-risk DRO model, which is helpful for providing system operators the trade-off operation scheme between operation cost and risk mitigation. Based on the common IES, this paper proposes a coordinated optimization for integrated water-energy nexus system (IWENS) with the connection of multiple energy hub systems (EHSs) containing power, gas, heat and water systems. This paper proposes a two-stage mean-risk distributionally robust optimization (TSMR-DRO) for IWENS considering the uncertainty of wind power generation. The two-stage
model includes day-ahead and real-time operation schemes, prior to and after wind uncertainty realization. The ambiguity set for capturing wind uncertainty is constructed using KL divergence. The coherent risk measure, i.e., CVaR is employed to model the trade-off between expected computational performance and risk. Bender’s decomposition is applied to solve the problem in an iterative manner. The proposed IWENS can provide system operators a two-stage operation scheme aiming at minimizing the system operation cost and a new perspective when dealing with enormous interdependencies.

Compared with the existing MEWENS [53], this paper makes further improvements: i) the proposed IWENS is in distribution level, which further considers the voltage at each bus, and both active and reactive power; ii) IWENS is connected with multiple EHSs, which supplies residential energy customers. It also brings about more conversions and interdependencies and accordingly enhances the overall energy utilization efficiency; iii) The proposed KL divergence-based DRO offers less-conservative results than RO; iv) compared with the risk-neutral optimization, TSMR-DRO offers the system operators with decision makings between the economic efficiency and the risk.

The main contributions of this paper are as follows:

1) Energy structure: It is the first attempt to model an innovative IWENS structure networked with EHSs and renewable distributed generators (DGs) in a distribution level. The intricate nexus between power, gas and water is extensively modelled. The high renewable penetration in the IWENS can be effectively facilitated by the energy conversions and thus the excessive power flow caused by renewable fluctuation can be compensated by other subsystems.

2) It aggregates considerable interconnections and converters among subsystems, e.g., gas turbines, P2G facilities, CHP, GF, GSHP, water pumps and electric boilers. The enormous interdependencies and interactions between energy sectors are beneficial for improving economic efficiency and sustainability.

3) A two-stage DRO model is applied to optimize both day-ahead and real-time operation schemes. The day-ahead stage determines the initial operation scheme with reserve capacity from traditional DGs and CHPs and water pumps.

4) Optimization method: It is the first attempt to combine DRO with mean-risk optimization. The benefits of the proposed DR-MRO is in threefold: i) it overcomes the
shortages of SO and RO by using partial distributional information with moderate robustness, ii) the KL divergence-based ambiguity set can flexibly shape the considered candidate distributions compared with moment-based ambiguity sets and accordingly yields less-conservative results and iii) the trade-off between economic performance and risk can be realized based on the incorporation of CVaR on the objective function.

The remainder of this paper is organized as follows. Section 3.4 presents the objective function and constraints for both day-ahead and real-time stages. Section 2.2.1 proposes the method for solving KL divergence-based TSMR-DRO considering the incorporation of CVaR. The case studies for demonstrating the advantages of IWENS and TSMR-DRO are given in Section 3.5. Finally, section 3.6 concludes the entire paper.

### 3.4 IWENS Structure

The enormous interdependencies among each subsystem are realized by the strong couplings for subsystems with multiple energy converters facilitated. CHP enables the conversion from gas to both heat and electricity to supply the heating and electricity loads of energy hubs. P2G facilities can convert excessive renewable power generation to synthetic natural gas; The conversion from gas to power is mainly realized by utilizing gas turbines; Ground source heat pump (GSHP) and gas furnace (GF) enable the heat conversion from power and gas respectively; The electrolyses in the P2G facilities consume the water from water system; The energy conversion from CHP relies on the water supply; Water pumps consume electricity from power system; The electricity boiler in the water system requests the electricity supply to convert the water to heat. Consequently, modelling and optimizing all the subsystems as an entity can facilitate the economy and security of IWENS.

The proposed IWENS structure is given in Fig. 3-1. The power and gas systems have three interconnection points: i) bus 6 and 15 in power system is connected with node 2 and 6 in gas system via gas turbines and P2G facility on bus 10 is connected with gas node 3. The two EHSs are sourced from both power and gas systems. The water distribution system interconnects with all the other subsystems: i) water node 11 is connected with the P2G facility for the water electrolysis process, ii) water node 2 connects with EHS 1 and 2 for CHP conversion; iii) water pump at node 1, 2 and 6 consume electricity from EHS 1 and iv) water system is connected with EHS via an electric boiler. The IWENS contains two EHSs. Each EHS contains a CHP, a GSHP, a
Chapter 3  Mean-Risk DRO of District IES under Water-Energy Nexus

GF. EHS 1 contains an energy storage system (ESS). The ESS is composed of a battery storage and a water tank for storing excessive electricity and heating respectively.

3.5 Problem Formulation

This section proposes the mathematical modelling for IWENS including both day-ahead and real-time operation schemes. Then the risk measure is given. Finally, the objective function is illustrated. The assumption is made that the entire IWENS is owned by a single entity who controls all the energy infrastructures and there is no trading between each subsystem.

3.5.1. Day-ahead Operation

The day-ahead optimization schedules power generation plan of traditional DGs and the reserve capacity dispatch from traditional DGs, gas turbines and water pumps considering the operation status of other energy infrastructures. The constraints are in (3-1)-(3-40). The power purchase from upper-level market is given in (3-1). The reserve capacity from traditional DGs, gas turbines and water pumps are shown in (3-2) and (3-3), followed by their output limits in (3-4) and (3-5). Constraint (3-6) limits the reactive power output of traditional DGs. In power distribution systems, linearized

![Fig. 3-1. Proposed structure of IWENS.](image-url)
DistFlow equations are commonly used in optimization problems, given in (3-7)-(3-9). Constraint (3-10) is the output of electric boiler. The balancing conditions for active and reactive power are in (3-11) and (3-12).

The output of natural gas source is constrained in (3-13). Constraints (3-14) and (3-15) are used to limit the gas pressure. Note that the gas pressures of initial nodes are always higher than terminal nodes due to the unidirectional gas flow. Initial and terminal nodes exist at the same pipeline. The initial node represents the origin of the gas flow and the terminal node represents the end of the gas flow. Accordingly, constraint (3-15) is used to ensure unidirectional gas flow. Equation (3-16) is the Weymouth gas flow equation that characterizes the relationship between gas pressure and flow. The gas flow of gas pipelines is constrained in (3-17). The output of gas turbine is in (3-18). Equation (3-19) presents the relationship between the gas pressure of initial and terminal nodes of gas compressors. The excessive renewable generation can be converted into gas via P2G. The electrolyser splits water into hydrogen and oxygen. The output of electrolyser is given in (3-20). The nodal gas balance is given in (3-21).

In water distribution systems, constraint (3-22) limits the output of reservoir. Equation (3-23) is the constraint of water pressure limit for pipes installed with and without water pumps. In (3-24)-(3-27), the hydraulic characteristics of water pipes are given for pipes installed with and without water pump in terms of head gain and loss. The pressure head gain of water pump is in (3-26). Equation (3-27) describes the hydraulic characteristic of pipes without pumps using Darcy-Weisbach equation [63]. The power consumption of water pump is in (3-28). Constraint (3-29) limits the water flow magnitude. The mass balance for the water system is in (3-30).

In EHSs, the energy conversion of CHP, GF and GSHP are in (3-31)-(3-33). The input limit for all converters is given in (3-34). Equation (3-35) is the constraint of the charging and discharging power and heat for ESSs. Constraint (3-36) and (3-37) limit the remaining energy for battery storage and water tank. Constraint (3-38) presents the coupling relationship for the EHSs, which is the energy balance constraint of EHSs.

\[
0 \leq P_{m_t}^s \leq P_{m,\text{max}} \tag{3-1}
\]

\[
0 \leq r_{\{i\},t}^+ \leq R_{\{i\},t}^+, \{\cdot\} = i_e, gt, wp \tag{3-2}
\]
Chapter 3  Mean-Risk DRO of District IES under Water-Energy Nexus

\[0 \leq \sum_{i_e \in I_e} P_{i_e,t}^s + \sum_{j \in J} q_{j,t}^s \leq \sum_{i_e \in I_e} f_{i_e,t}^{s,ini} - \sum_{i_e \in I_e} f_{i_e,t}^{s,ster} + \sum_{k_e \in K_e} P_{k_e,t}^s = \sum_{k_e \in K_e} P_{k_e,t}
\]

\[0 \leq \sum_{i_e \in I_e} Q_{i_e,t}^s + \sum_{i_e \in I_e} q_{i_e,t}^{s,ini} - \sum_{i_e \in I_e} q_{i_e,t}^{s,ster} = \sum_{k_e \in K_e} Q_{k_e,t}
\]

\[G_{i_g,min} \leq G_{i_g,t}^s \leq G_{i_g,max}
\]

\[P_{i_g,min} \leq P_{i_g,t}^s \leq P_{i_g,max}
\]

\[P_{i_g,t}^{s,ini} \geq P_{i_g,t}^{s,ster}
\]

\[f_{i_g,t}^s = \gamma_{i_g} \left( P_{i_g,t}^{s,ini} - P_{i_g,t}^{s,ster} \right)
\]

\[0 \leq f_{i_g,t}^s \leq f_{i_g,max}
\]

\[p_{g,t}^s = \gamma_{i_g} f_{i_g,t}^s
\]

\[P_{i_g}^{ster} \leq C_{F_g} P_{i_g}^{ini}
\]

\[G_{n,t}^s = \eta_{i_e} \frac{p_{n,t}}{p_{n,t}^{P2G}}
\]
\[
\sum_{l_g \in l_g} G_{g}^{s} + \sum_{n \in N} G_{n,t}^{s,hy} + \sum_{l_g \in l_g} f_{g,t}^{s,ini} - \sum_{l_g \in l_g} f_{g,t}^{s,ter} = \sum_{k_g \in K_g} G_{k_g,t} + \sum_{l_g \in l_g} f_{g,t}^{s} + \sum_{l_g \in l_g} f_{g,t}^{s,ini} \\
0 \leq P_{wr,t}^{s} \leq P_{wr,max}^{s} \tag{3-22}
\]

\[
h_{w,t}^{s,\min} \leq h_{w,t}^{s,\{\}} \leq h_{w,t}^{s,\max}, \{\} = l_w, l_{wp} \tag{3-23}
\]

\[
\tilde{h}_{l_{wp},t}^{s} = \left( \tilde{h}_{w,t}^{s,\{\} + \tilde{h}_{w,t}^{s,\{\}} \right) - \left( \tilde{h}_{w,t}^{s,\{\}} + \tilde{h}_{w,t}^{s,\{\}} \right), \{\} = l_w, l_{wp} \tag{3-24}
\]

\[
\tilde{h}_{l_{wp},t}^{s} + a_{l_{wp}} f_{l_{wp},t}^{s} + b_{l_{wp}} = R_{l_{wp}} f_{l_{wp},t}^{s} \tag{3-26}
\]

\[
p_{l_{wp},t}^{s} = \left( a_{l_{wp}} f_{l_{wp},t}^{s} + b_{l_{wp}} f_{l_{wp},t}^{s} \right) / \pi_{wp} \tag{3-28}
\]

\[
0 \leq f_{l_{wp},t}^{s} \leq f_{l_{wp},t}^{s,\text{max}}, \{\} = l_w, l_{wp} \tag{3-29}
\]

\[
\sum_{wr \in WR} P_{wr,t}^{s} + \sum_{l_{wp} \in l_{wp}} f_{l_{wp},t}^{s,ini} - \sum_{l_{wp} \in l_{wp}} f_{l_{wp},t}^{s,ter} = \sum_{k_{pg} \in K_{pg}} \sigma_{k_{pg}} p_{k_{pg},t}^{s,p2G} + \sum_{k_{cp} \in K_{cp}} \sigma_{k_{cp}} p_{k_{cp},t}^{s,i} + \sum_{eb \in EB} \sigma_{eb} p_{eb,t}^{s} \\
+ \sum_{k_{w} \in K_{w}} P_{k_{w},t} \tag{3-30}
\]

\[
p_{l_{wp},t}^{s,i} = \eta_{l_{wp}} p_{l_{wp},t}^{s,i}, \{\} = COP, GF \tag{3-31}
\]

\[
p_{k_{cp},t}^{s,i} = \eta_{k_{cp}} p_{k_{cp},t}^{s,i} \tag{3-32}
\]

\[
p_{k_{cp},t}^{s,i} = \eta_{k_{cp}} p_{k_{cp},t}^{s,i} \tag{3-33}
\]

\[
P_{l_{wp},t}^{i,\min} \leq P_{l_{wp},t}^{i,\{\} \leq P_{l_{wp},t}^{i,\max}, \{\} = CP, COP, GF \tag{3-34}
\]

\[
P_{l_{wp},t}^{s,\text{ch/dch}}, \{\} = P_{l_{wp},t}^{s,\text{ch/dch}}, \{\} = BS, HS \tag{3-35}
\]

\[
E_{l_{wp},t}^{s} = E_{l_{wp},t-1}^{s} + \sum_{1}^{t} P_{l_{wp},t}^{s,\text{ch/dch}} \eta_{l_{wp}}^{ch} - \frac{P_{l_{wp},t}^{s,\text{ch/dch}}}{\eta_{l_{wp}}^{dch}}, \{\} = BS, HS \tag{3-36}
\]
\[
E_{(\cdot),\min} \leq E_{(\cdot),t} \leq E_{(\cdot),\max}, \{\} = BS, HS
\]  
(3-37)
\[
\begin{bmatrix}
L_{e,t} + P_{BS,t}^s \\
L_{n,t} + P_{HS,t}^s
\end{bmatrix} = \\
\begin{bmatrix}
1 - v_{e,t}^s \\
v_{e,t}^s \eta_{CHP}(1 - v_{e,t}^s)
\end{bmatrix}
\times
\begin{bmatrix}
f_{l_{e,t}}^s \\
f_{l_{g,t}}^s
\end{bmatrix}
\]  
(3-38)

### 3.5.2. Real-time Operation

In the second stage, corrective operation schemes are deployed based on the realization of wind uncertainty. Equation (3-39) is the constraint for the regulated power output of traditional DGs and gas turbine. And (3-40) is the new power balance constraint considering wind uncertainty. Apart from (3-39) and (3-40), the rest second-stage constraints are the same as the first-stage constraints, where the superscript ‘s’ on each variable is changed to ‘re’. ‘s’ represents the scheduled decision variables in the first stage and ‘re’ represents the regulated decision variables in the second stage. The regulated decision variables are summarized in (3-41).

\[
\begin{aligned}
P_{(\cdot),t}^r - r_{(\cdot),t}^r &\leq P_{(\cdot),t}^re \leq P_{(\cdot),t}^r + r_{(\cdot),t}^+ \{\} = i_e, gt, wp
\end{aligned}
\]  
(3-39)
\[
\begin{aligned}
\sum_{i e \in i e} P_{i e,t}^re &+ \sum_{j \in j} \sum_{i e \in i e} f_{i e,t}^{re ini} - \sum_{i e \in i e} f_{i e,t}^{re ter} + P_{re G \cdot t}^r = \sum_{k e \in k e} P_{k e,t}^r \\
+ \sum_{l e \in l e} f_{l e,t}^{re ini} + \sum_{e b \in e b} P_{e b,t}^re + \sum_{n \in n} P_{n \cdot t}^re P_{2 g} &+ \sum_{w p \in w p} P_{w p \cdot t}^re
\end{aligned}
\]  
(3-40)
\[
y = \begin{cases}
\{P_{m,t}^{re}, P_{i e,t}^{re}, P_{g,t}^{re}, Q_{i e,t}^{re}, V_{i e,t}^{re}, f_{i e,t}^{re}, G_{i g,t}^{re}, P_{i g,t}^{re}, f_{i g,t}^{re}, P_{g,t}^{re P_{2 g}}, G_{n t}^{re hy}, G_{n t}^{re hy me}, G_{n t}^{re ca}, G_{n t}^{re me}, G_{n t}^{pre}, h_{i w t}^{re}, h_{i w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}, h_{w p t}^{re}
\end{cases}
\]  
(3-41)

### 3.5.3. Objective Function

In the first stage, the day-ahead objective in (3-42) is to minimize total operation cost, including i) generation cost of traditional DGs and natural gas sources, ii) power purchase cost from day-ahead upper-level market, iii) water purchase cost from water
reservoirs, iv) cost for reserve capacity from traditional DGs, gas turbines and water pumps.

\[
I_1 = \min_{i_e \in I_e, i_g \in I_g, t \in T, w \in WR, g_t \in GT} \sum_{m, t} \lambda_m^e P_{m,t}^s + \lambda_i^e P_{i_e,t}^s + \lambda_g^e P_{i_g,t}^s + \lambda_r^e
\]

\[
= \min_{i_e, g_t, w \in \{\}} (3-42)
\]

The second-stage problem considers real-time redispatch and corrective actions pertaining to wind uncertainty. The objective function contains the penalties due to the overestimation or underestimation of scheduling in the first stage. The first-stage generation decisions include the scheduled power and water purchase, wind generation forecast, scheduled output of traditional DGs and natural gas sources. The penalties for the renewable forecast error is calculated by the multiplication of the penalty coefficient and the forecast error. The renewable forecast error is calculated by the difference of the renewable forecast and the real-time realization, i.e., \( \omega_{j,t}^s - \xi_{j,t} \). The minimization of deviation between scheduled and regulated results promotes the utilization of renewable energy [64, 65].

\[
I_2 = \min_{i_e \in I_e, i_g \in I_g, t \in T, w \in WR} \sum_{m, t} \lambda_r^e |P_{m,t}^s - P_{m,t}^r| + \lambda_j^e |\omega_{j,t}^s - \xi_{j,t}|
\]

\[
= \min_{i_e, g_t, w \in \{\}} (3-43)
\]

### 3.6 Case Studies

The proposed DR-IWENS is verified on a district water-energy nexus system consisting of a modified IEEE 33-bus system, a 6-node gas system, two EHSs and a 11-node water system [53, 66-68], where generator information is given in TABLEs 3-1, 3-2 and 3-3. The gas pressure is regulated between 105 Psig and 170 Psig. The power system has two traditional DGs and four renewable DGs. The power system is connected with the gas system via two gas turbines and a P2G facility. Two EHSs are supplied by both electricity and gas from buses 20 and 25 of power system and nodes 2 and 5 from gas system. The water consumption from P2G and CHPs are supplied by node 11 of water system. It is assumed that the water is injected into the CHP with
heated outflow, which converts into the heating. The capacity of CHP is 0.35MW. The heat/power ratio is 1.73.

The electric boilers enable the heating conversion from power and water. The following cases are considered:

Case 1: Benchmark case.

Case 2: Risk-neutral optimization without considering CVaR in the objective function.

Case 3: The output of renewable DGs is twice of case 1.

Case 4: No P2G facility.

TABLE 3-1 PARAMETERS OF WATER RESERVOIRS

<table>
<thead>
<tr>
<th>Node No.</th>
<th>$P_{wr,max}$ (m$^3$/h)</th>
<th>$\lambda_{wr}$ ($/m^3$)</th>
<th>Elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>325</td>
<td>6.4</td>
<td>-252.5</td>
</tr>
<tr>
<td>2</td>
<td>700</td>
<td>2.6</td>
<td>-255</td>
</tr>
</tbody>
</table>

TABLE 3-2 PARAMETERS OF NATURAL GAS SOURCES

<table>
<thead>
<tr>
<th>Node No.</th>
<th>$P_{i,g,min}$ (kcf/h)</th>
<th>$P_{i,g,max}$ (kcf/h)</th>
<th>$\lambda_{i,g}$ ($) (kcf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>3000</td>
<td>2.2</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>6000</td>
<td>2</td>
</tr>
</tbody>
</table>

TABLE 3-3 GENERATOR PARAMETERS

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>$P_{i,e,max}$ (MW)</th>
<th>$P_{i,e,min}$ (MW)</th>
<th>$R_i^+ R_i^-$ (MW)</th>
<th>$a_i$ ($/MW^2$)</th>
<th>$b_i$ ($/MW$)</th>
<th>$c_i$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1.2</td>
<td>0.3</td>
<td>0.2</td>
<td>6000</td>
<td>7100</td>
<td>6200</td>
</tr>
<tr>
<td>28</td>
<td>1.0</td>
<td>0.1</td>
<td>0.2</td>
<td>4500</td>
<td>10500</td>
<td>4000</td>
</tr>
</tbody>
</table>

TABLE 3-4 ECONOMIC PERFORMANCE FOR ALL CASES

<table>
<thead>
<tr>
<th>Economic result</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power system operation cost ($)</td>
<td>22900</td>
<td>20400</td>
<td>13275</td>
<td>21472</td>
<td>28925</td>
</tr>
<tr>
<td>Gas system operation cost ($)</td>
<td>15512</td>
<td>14485</td>
<td>12044</td>
<td>16324</td>
<td>26140</td>
</tr>
<tr>
<td>Water system operation cost ($)</td>
<td>1962</td>
<td>1802</td>
<td>2310</td>
<td>1858</td>
<td>2352</td>
</tr>
<tr>
<td>System operation cost ($)</td>
<td>40374</td>
<td>36687</td>
<td>27629</td>
<td>39609</td>
<td>57417</td>
</tr>
</tbody>
</table>
Case 5: The gas price is twice of case 1.

The economic performance for all the cases is studied firstly in this section, followed by the optimal schedule of interdependent energy converters. The mathematical performance with different risk-aversion parameters is given in section 3.5.3.

**3.6.1. Economic Performance of Each Subsystem**

The economic performance for all the cases is given in TABLE 3-4, which incorporates the operation cost of power system, gas system, water system and entire IES. Overall, case 3 with twice output of the renewable DGs yields the lowest total operation cost whilst the total operation cost of case 5 is the highest when gas price is twice of case 1. Case 2 is the risk-neutral optimization without considering CVaR in
the objective function. It can be seen that the operation cost of each subsystem is lower than those of case 1. The total operation cost, i.e., $36687, is 91% of that of case 1. When the output of renewable DGs is doubled in case 3, the most distinct feature is the operation cost of power system, which is only $13275. Meanwhile, the gas system operation cost is also reduced by $3468 since there is more excessive renewable output injecting to the gas system via the P2G facility. However, the water system operation cost is $348 more than that of case 1. The reason is that the P2G and CHPs consume more water with increasing renewable output. In case 4, there is no supply from the power system to the gas system, which causes the higher operation cost of gas system since the excessive renewable generation cannot be fully utilized. The operation cost of all the subsystems and the overall system is the highest in case 5. Compared with case 3 with the lowest cost, the total operation cost is 107% higher. Particularly, the gas system operation cost is $26140, which is $10628 more than that of case 1.
Analysis of Energy Conversions

This section investigates the scheduling of coupling devices for interconnecting each system, i.e., gas turbines, P2G facility, electric boilers, CHP, gas furnace and GSHP. To begin with, the operation scheme of gas turbines and the P2G facility is given in Fig. 3-2. Note that it shows the input of gas turbines and output of P2G facility. It can be seen that the gas turbine at node 2 has higher gas consumption than node 5. The average gas consumption of node 2 is 1867kcf and that of node 5 is 591kcf. The potential reason of the higher gas consumption at node 2 are i) node 2 is connected to a natural gas source which has abundant gas supply and ii) the requirement of power transformation at bus 6 is higher as it is connected with more buses. As for P2G, it produces 549kcf averagely. The transformed gas from P2G can supply loads at nodes 3, 5 and 6. In addition, the abundant gas can be converted back to power system at node 5. The scheduling of water injection of electric boilers is shown in Fig. 3-3. The water injection is 3 at node 6m3 and 37m3 at node 1 averagely. Although the heating loads of EHS 1 and 2 have similar amount, the heating supplied by water system at water node 1 is more than 6 times of that at node 6. Since the gas supply of EHS 2 connected to gas node 5 is less but the water supply from water node 1 is sufficient connected to the water reservoir.

In Fig. 3-4, the heating output of converters in EHS 1 and 2 are given, respectively. Overall, the total heating output of converters in EHS 1 is 0.1MW higher than that of
EHS 2. The heating supply composition is different for EHS 1 and 2. The CHP is utilized around 0.15MW for each hour and takes up 50% of the total heating output of converters. While the CHP in EHS 2 outputs approximately 0.33MW, which is 81% of the total heating conversion. The reason is that the supply from power system is not sufficient, which affects the heat conversion of GSHP even though the heating conversion efficiency of GSHP is high. The insufficient electricity consumption needs to be satisfied by CHP conversion, which also increases heating conversion.

The water consumption of CHPs and P2G is in Fig. 3-5. As discussed for Fig. 3-4, the heating conversion from CHP in EHS 2 is higher than that of EHS 1. The water consumption of CHP in EHS 2 is also higher than that of EHS 1, i.e., the average water consumption of CHP in EHS 2 is 0.28m$^3$ and it is 0.77m$^3$ of CHP in EHS 1. Compared to CHP, P2G consumes less water and its average water consumption is 0.15 m$^3$.

### 3.6.3. The Impact of CVaR on Economic Performance

Through adjusting the confidence level and weighting factor for operation cost versus risk trade-off, the overall economic performance varies. TABLEs 3-5 and 3-6 present the economic performance with different beta and alpha, respectively. This paper considers 95% as the benchmark alpha used in TABLE IV. As shown in TABLE V, the total cost increases with the increase of alpha. For case 1, the highest total operation cost is $40652 with beta=0.9 and the lowest total operation cost is $35635 with beta=0.99. When beta is fixed, case 5 which considers twice of the original gas price has the highest total operation cost, followed by cases 4, 1 and 3, which is the same as discussed in section 3.5.1. In TABLE 3-6, the impact of changing beta on the economic performance for all cases is presented. It can be seen that the higher alpha causes higher priority on minimizing the risk, which leads to higher operation cost. When alpha=0, the mean-risk DRO degrades into the risk-neutral DRO. For case 5, the total operation cost is only $50767 compared with the $57417 solved by the benchmark mean-risk DRO.

### 3.7 Chapter Summary

A mean-risk coordinated optimization for an IES in the water-energy nexus with enormous interdependencies is proposed in this chapter. The tight couplings and interactions between each subsystem enable the reliable and economic operation for the entire IES. The renewable uncertainty is captured by mean-risk DRO. The coherent risk
measure, CVaR provides the trade-off to system operators with flexible alternatives on choosing between economic efficiency and risk. A tractable Bender’s decomposition is employed to solve the DR-IWENS problem.

Through the extensive case studies on the economic performance, scheduling of interdependent coupling devices and the risk management via adjusting parameters, the major contributions are tested:

▪ The coordination of each subsystem with the conversion technologies enhances the energy efficiency.

▪ The water system is highly required to consider in the IES operation as the water is extensively consumed by energy conversions.

The mean-risk DRO applied in IES operation problem provides system operators with not only economic but risk concerns.
This chapter proposes to utilise distributionally robust optimization (DRO) to analyse and resolve the resilience-oriented planning problem with uncertain seismic attacks in integrated energy systems (IES). The optimal system hardening plan optimal joint load shedding scheme can be eventually obtained.
## Statement of Authorship

<table>
<thead>
<tr>
<th><strong>This declaration concerns the article entitled:</strong></th>
<th>Resilience Enhancement and Emergency Response for Integrated Energy Systems against Seismic Attacks: A Data-Driven Approach</th>
</tr>
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<td>P. Zhao, Y. Shen, C. Gu, F. Teng, X. Xu and S. Li, &quot;Data-Driven Moment-Based Multi-Energy Investment and Management under Earthquakes&quot;, in <em>IEEE Transactions on Industrial Informatics</em></td>
</tr>
<tr>
<td><strong>Candidate’s contribution to the paper</strong></td>
<td>The candidate and the second author jointly proposed the idea of the paper, and they designed the methodology. The candidate executed all the coding to derive the experimental results. Other authors helped the candidate with the design of case studies, format of the paper, and improvement of academic writing. The percentage of the candidate did compared with the whole work is indicated as follows:</td>
</tr>
<tr>
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<td>Formulation of ideas: 70%</td>
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<td>Design of methodology: 80%</td>
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<td>Experimental work: 90%</td>
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<td>Presentation of data in journal format: 80%</td>
</tr>
<tr>
<td><strong>Statement from candidate</strong></td>
<td>This paper reports on original research I conducted during the period of my Higher Degree by Research candidature.</td>
</tr>
<tr>
<td><strong>Signed</strong></td>
<td>Pengfei Zhao</td>
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Chapter 4  Resilience Enhancement of IEGS against Seismic Attacks

4.1 Abstract

Seismic events can severely damage both electricity and natural gas systems, causing devastating consequences. Ensuring the secure and reliable operation of the integrated energy system (IES) is of high importance to avoid potential damage to the infrastructure and reduce economic losses. This paper proposes a new optimal two-stage data-driven optimization to enhance the resilience of IES planning and operation against seismic attacks. In the first stage, hardening investment on the IES is conducted, featuring in preventive measure for seismic attacks. The second stage minimizes the expected operation cost of emergency response. The random seismic attack is modelled as uncertainty, which is realized after the first stage. An explicit damage assessment model is developed to define the budget set of the uncertain seismic activity. Based on the survivability of transmission lines and gas pipelines of IES, an optimal system investment plan is developed. The problem is formulated as a two-stage data-driven distributionally robust optimization (DRO) model, which is tested on an integrated IEEE 30-bus system and 6-node gas network. Case studies demonstrate that the two-stage DRO outperforms robust optimization and single-stage optimization model in terms of minimizing the investment cost and expected economic loss.

4.2 Nomenclature

A. Sets

\( T \)  Set for time periods.
\( I \)  Set for power lines.
\( M \)  Set for gas pipelines.
\( G_E \)  Set for electricity distributed generators.
\( G_G \)  Set for gas-fired distributed generators.
\( D_E \)  Set for electricity loads.
\( D_G \)  Set for gas loads.

B. Parameters

\( \text{CL}_{c_o}, \text{CL}_{ax}, \text{CL}_{mo}, \text{CL}_{mi} \)  Connection loss for complete, extensive, moderate and minor seismic level.
\( \gamma \)  Failure rate constant for gas pipelines.
\( \pi_{ij}, \pi_{mn} \)  Unit hardening cost for power lines and gas pipelines.
\( EL_h, PL_h \)  Maximum number of hardening power lines and pipelines.
\( IC_{\text{max}} \)  Maximum monetary budget.
\( x_{ij} \)  Reactance of power line ij.
### Chapter 4  Resilience Enhancement of IEGS against Seismic Attacks

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{ij_{\text{max}}, f_{mn_{\text{max}}}} )</td>
<td>Maximum power flow of line ( ij ) and gas flow of pipeline ( mn ).</td>
</tr>
<tr>
<td>( P_{ge_{\text{min}}, P_{ge_{\text{max}}}} )</td>
<td>Minimum and maximum power output of electricity generators.</td>
</tr>
<tr>
<td>( \delta_{d_e_{\text{max}}, d_g_{\text{max}}} )</td>
<td>Maximum limit for electricity and gas load shedding.</td>
</tr>
<tr>
<td>( P_{d_e_{t}, P_{d_g_{t}}} )</td>
<td>Electricity and gas load demand at time ( t ).</td>
</tr>
<tr>
<td>( P_{gg_{\text{min}}, P_{gg_{\text{max}}}} )</td>
<td>Minimum and maximum output of gas-fired generators.</td>
</tr>
<tr>
<td>( P_{r_{\text{min}}, P_{r_{\text{max}}}} )</td>
<td>Minimum and maximum pressure.</td>
</tr>
<tr>
<td>( \gamma_{mn} )</td>
<td>Coefficient for Weymouth equation.</td>
</tr>
</tbody>
</table>

#### C. Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{co, P_{ex}, P_{mo}, P_{mi}} )</td>
<td>Probability of being complete, extensive, moderate and minor seismic attacks under a randomly chosen seismic level.</td>
</tr>
<tr>
<td>( P_{ED} )</td>
<td>Expected connection loss.</td>
</tr>
<tr>
<td>( h_{ij}, h_{mn} )</td>
<td>Binary variables indicate if power line ( (i,j)/\text{pipeline } (m,n) ) is hardened.</td>
</tr>
<tr>
<td>( \theta_{i,t}, \theta_{j,t} )</td>
<td>Phase angle at electricity bus ( i ) and ( j ).</td>
</tr>
<tr>
<td>( x_{ij} )</td>
<td>Reactance of power line ( ij ).</td>
</tr>
<tr>
<td>( f_{ij_{t}, f_{mn_{t}}} )</td>
<td>Power and gas flow of power line ( ij ) and gas pipeline ( mn ) at time ( t ).</td>
</tr>
<tr>
<td>( P_{ge_{t}, P_{gg_{t}}} )</td>
<td>Electricity and gas-fired distributed generation at time ( t ).</td>
</tr>
<tr>
<td>( \delta_{d_e_{t}, \delta_{d_g_{t}}} )</td>
<td>Electricity and gas load shedding at time ( t ).</td>
</tr>
<tr>
<td>( P_{r_{t}} )</td>
<td>Gas pressure at time ( t ).</td>
</tr>
<tr>
<td>( \xi_{ij}, \xi_{mn} )</td>
<td>Binary variable indicates if power line ( (i,j)/\text{pipeline } (m,n) ) is damaged.</td>
</tr>
<tr>
<td>( a_{ij}, a_{mn} )</td>
<td>Binary variable indicates if power line ( (i,j)/\text{pipeline } (m,n) ) is available.</td>
</tr>
</tbody>
</table>

#### 4.3 Introduction

Natural disasters can cause huge power losses of energy systems that threaten the economy. Earthquake is considered as one of the most disruptive natural disasters, which may cause large-scale blackouts without sufficient time for response due to the weak predictability. The Wenchuan earthquake in May 2008 damaged around 270 transmission lines and 900 substations, leaving 46 million people suffered without electricity. A massive power outage was caused in 2010 Chile earthquake.
Approximately 3GW generation capacity became unavailable, 26% of transmission network substations were damaged, where 93% residents suffered from a power outage with two weeks [69]. According to existing research, more than 90% of such damage can be avoided if upgraded seismic preventive measures are adopted [70]. The most powerful earthquake in 20 years damaged the gas transmission and distributions systems in July, 2019, which causes the unavailable gas usage of 13000 customers [71].

To enhance the resilience of power systems against natural disasters, resilience planning has been extensively investigated. Paper [72] proposes a multi-stage and multi-zone based robust optimization (RO) for a bi-level resilience problem considering power line hardening and distributed generation resource placement. Different grid enhancing strategies against extreme weather conditions are considered in [73] in a tri-level framework, which is transformed into an equivalent bi-level problem and solved by a greedy searching algorithm. Paper [74] develops a two-stage stochastic optimization (SO) for resilient planning in a large-scale transmission network to mitigate seismic risk. Optimal capacity expansion is considered as the planning strategy based on explicit damage distribution.

Meanwhile, energy infrastructures are becoming more complex and independent, especially with higher attention on the interdependence of different energy carriers. The rapid growth of gas consumption and mushrooming deployment of gas-fired generation and electrolysis have boosted the synergetic integration of electricity and natural gas systems. The integrated energy system (IES) can significantly increase energy utilization efficiency. Therefore, enhancing resilience to withstand seismic hazards and mitigate resulting damages is of great vitality for IES. A two-stage robust integrated planning of IES is proposed for enhancing the resilience in [75], which is implemented by replacing power lines by a gas transportation system. Paper [76] optimally minimizes the worst-case electricity and gas load shedding through a tri-level robust planning model with network hardening. Paper [77] proposes a resilience assessment for IES including heating, cooling and distributed generation systems. The proposed assessment is quantified through functionality loss and monetary costs.

However, recent IES planning studies consider the impacts of general natural disasters without specifying the types and ignore the attack assessment model, hardly targeting at mitigating seismic risks [75, 76]. This is impractical as the impacts of
natural disasters depend on the grid structure, disaster types, duration and intensity, etc. Different natural disasters could have a very different damage scale that needs specified preventive investment strategies. Thus, it is essential to develop models for assessing seismic risks on IES.

To accommodate the uncertainty of natural disasters, RO and SO have been extensively applied [73-75]. Nevertheless, RO ensures system robustness while inevitably leads to over conservativeness. SO requires the explicit distributions with a large number of samples, which not only produces high computational burden but also is not always practical. Distributionally robust optimization (DRO) bridges the strengths of RO and SO, which relaxes the assumption of specifying a certain distribution and considers the worst distributions compared to the worst-case oriented RO. An optimal gas-power flow is proposed in [78] by DRO with wind uncertainty. Wasserstein metric is used to select candidate distributions. A two-stage DRO model for IES scheduling is proposed in [79] and compared with traditional adjustable robust optimization, proving that DRO generates less conservative and more economical solutions.

This paper targets at alleviating the impacts of seismic events on both power lines and gas pipelines of IES. A two-stage DRO model is proposed to enhance the resilience for an IES, where the damage on both power lines and gas pipelines are considered. For simplicity, the keywords ‘distributionally robust’, ‘seismic’, ‘integrated’ and ‘planning’ are picked and this proposed model is referred to DR-SIP. The seismic activities are regarded as uncertain events and the random damage on power lines and pipelines are regarded as uncertainties, which are handled by DRO. The first stage minimizes the investment cost for hardening IES against seismic attacks. The hardening strategy incorporates strengthening power lines and gas pipelines with earthquake-resistant material and design. After the uncertainty of seismic activities is realized, the second stage minimizes the loss of emergency response through load shedding under the worst potential seismic risks. The Bender’s decomposition is utilized to solve this IES resilience optimization problem, which is then demonstrated through extensive case study. The merits of the proposed model are summarized in the case study section.

The main contributions of this paper are as follows:
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1) A two-stage DRO method, incorporating both planning and operation schemes, is for the first time proposed to enhance IES resilience, considering the worst-distributed seismic attack. Compared to existing research, such as [74], [75] and [76], this paper is specifically focused on resilience enhancement for IES.

2) A novel model to assess the performance of IES against seismic attacks is developed. This damage quantification builds a probabilistic model and estimated by damage scenarios. This assessment model can be easily combined with the proposed two-stage DRO model to determine the optimal enhancement plan for IES.

3) It utilizes the novel DRO in IES resilience assessment and enhancement with a tractable reformulation. The historical information of seismic events is efficiently used to reduce the conservativeness, thus producing more economical investment and operation decisions.

4) The proposed novel DRO framework avoids specifying uncertainty distributions but only uses moment information, which is more practical considering that it is normally not possible to gather a sufficiently large amount of distributional information for extreme events.

The rest of this paper is organized as follows. Section 4.4 proposes the damage assessment of seismic events. Section 4.5 designs resilience enhancement strategies. Section 4.6 presents the mathematical formulation for resilience planning and emergency operation. The methodology and solution algorithm are given in section 2.1.1. Section 4.7 demonstrates case studies and performance of DR-SIP. Conclusions are drawn in section 4.8.

4.4 Assessment of Seismic Damage on IES

This section provides the seismic damage modelling for both electricity and natural gas systems, which mainly considers the damages on power lines and gas pipelines. The relationship between damage consequence and seismic level is established in this section. The seismic intensity is described by peak ground acceleration (PGA) and peak ground velocity (PGV), which are mainly related to landslides and surface faulting [80]. The relationship between seismic level, PGA and PGV are summarized in TABLE 4-1 [81].

The concept of connection loss (CL) is adopted to quantify the line failures of
electricity systems [82] due to the damage on pylons and conductors from seismic events. They destroy conductors and shake pylons and consequently damage power lines. For simplicity, this paper considers the number of damaged power lines, which are modelled as CL. Paper [83] concludes four-line damage states, minor, moderate, extensive and complete, which refers to 4%, 12%, 50% and 80% of CL, indicating the number of damaged power lines in percentage. For clarity, the relationship between line damage state and the probability in each damage state is shown in TABLE 4-2.

A random intensity level can cause a certain range of PGA, presented in TABLE 4-1. Based on the fragility curve in Fig. 4-1, a random PGA can cause an earthquake in one or a combination of different line damage states. For example, when PGA is 0.6g, the line damage state of being complete is 0% and being extensive is 60%, while 100% for being moderate and minor. Thus, the expected number of line failures modelled as CL under a specific PGA is described in (4-1).

\[ N_{ED} = P_{co}CL_{co} + (P_{ex} - P_{co})CL_{ex} + (P_{mo} - P_{ex})CL_{mo} + (P_{mi} - P_{mo})CL_{mi} \] (4-1)

The steps to specify seismic damages on electricity lines are summarized as:

1. A random seismic intensity level is sampled from the Monte Carlo approach based on an empirical probability density function of seismic intensity level.
2. Based on TABLE 4-1 and the sampled intensity level, a certain range of PGA is given. Then Monte Carlo approach is used to pick a random PGA from the PGA range.
3. Based on the fragility curve in Fig. 4-1 and obtained PGA, the probability of 4 damage states is obtained.
4. Finally, according to TABLE 4-2 and equation (4-1), the expected number of damaged power lines, \( N_{ED} \), is obtained.

To quantify the damage of the natural gas system due to seismic events, similar to CL of the electricity system, failure rate (FR) is defined to represent the number of damage points on gas pipelines [84]. For simplicity, the natural gas system is assumed to operate in the steady state, ignoring the dynamic gas leakage characteristics of pipelines.

\[ FR = \gamma (PGV)^{2.25}L \] (4-2)

The steps to acquire \( FR \) of gas pipelines are as follows:
1. As described in step 1 of section A, the same seismic intensity level is obtained.
2. Based on TABLE 4-1 and obtained seismic intensity level, a certain range of PGV is given. Then Monte Carlo approach is used to pick a random PGV from the PGV range.
3. According to equation (4-2), FR is obtained.

### 4.5 Seismic Risk Oriented Resilience Enhancement

In terms of resilience-based IES planning, Fig. 4-2 presents the order of widely adopted enhancement steps [85], consisting of adaptation and recovery. The scope of this paper is on resilience planning and emergency response against seismic attacks. The preventive response is ignored as the preventive time after detecting seismic events...
is only up to a few seconds, which is too short for seismic preventive actions. In resilience planning, system operators make investment plans for seismic hardening on existing energy infrastructures. After the damage caused by seismic disasters, emergency response is implemented by system operators to mitigate the impact. There are two steps in the proposed DR-SIP: the first stage - resilience planning and the second stage - emergency response:

In the first stage, hardening on power lines is considered as the proposed resilience planning measure. Grid hardening, including constructing new lines and facilities, upgrading damaged poles and burying power lines underground, etc, is the most widely-proposed and effective measure to protect systems against natural disasters [86]. In this paper, reconstruction and upgrade to be earthquake-resistant are considered as the hardening measures for power lines and gas pipelines. It is assumed that the hardened lines will survive from seismic attacks [72, 74, 76]. In the second stage, the daily operation is implemented in the pre-hardened IES. The daily operation is designed instantly after the emergence of the seismic attack with a 24-hour time horizon. To maintain the system balance while suffered from uncertain seismic attacks, load shedding is considered.

4.6 Mathematical Formulation for Resilience Enhancement

Resilience planning and emergency response modelling are illustrated in this section. The seismic hardening strategy is considered for power lines and gas pipelines. Linearized DC power flow and Weymouth gas flow equations are employed with load shedding to meet flow constraints. Finally, a two-stage optimization is proposed to minimize hardening costs in the first stage and operation cost in the second stage.

4.6.1. Overall Objective

It is assumed that the system operator curtails electricity and gas load under the worst-distributed seismic attacks to mitigate damage. The first and second-stage objectives are in (4-3) and (4-4). $\Gamma_P$ represents hardening investment cost. $\Gamma_R$ is
emergency response cost, including: 1) shedding costs of electricity and gas loads and 2) generation costs of electricity and gas-fired generators. In the second stage objective, in addition to supplying electricity by gas, gas generation in the gas system is also considered, which is given in ‘ \( \sum_{t \in T, ge \in GG} \lambda_{gg} P_{gg,t} \).’

\[
\Gamma_p = \sum_{i,j \in I} h_{ij} \pi_{ij} + \sum_{m,n \in M} h_{mn} \pi_{mn} \tag{4-3}
\]

\[
\Gamma_R = \sum_{t \in T, d_e \in DE} \delta_{d_e, t} \pi_{d_e} + \sum_{t \in T, d_e \in DE} \delta_{d_g, t} \pi_{d_g} + \sum_{t \in T, ge \in GE} \lambda_{ge} P_{ge, t}^2 + \lambda_{ge} P_{ge, t} + \lambda_{ge} c + \sum_{t \in T, gg \in GG} \lambda_{gg} P_{gg, t} \tag{4-4}
\]

The overall objective of the proposed DR-SIP is to minimize the planning cost in the first stage and the emergency response cost in the second stage in (4-5).

\[
\min \Gamma_p + \sup E_p [\Gamma_R] \tag{4-5}
\]

### 4.6.2. Seismic Risk Oriented Resilience Planning

Here, the feasibility set of hardening strategies for electricity network, gas network and the overall IES are shown in (4-6) to (4-8) respectively. The maximum number of power lines and pipelines to be hardened are constrained by (4-6) and (4-7). Constraint (4-8) means the total hardening investment cannot exceed the maximum monetary budget.

\[
\sum_{i,j \in I} h_{ij} \leq E L_h \tag{4-6}
\]

\[
\sum_{m,n \in M} h_{mn} \leq P L_h \tag{4-7}
\]

\[
\sum_{i,j \in I} h_{ij} \pi_{ij} + \sum_{m,n \in M} h_{mn} \pi_{mn} \leq I C_{\text{max}} \tag{4-8}
\]

### 4.6.3. Emergency Response

Seismic events can have disruptive damage on power lines and gas pipelines, where the damages are considered as uncertainty, represented by binary variables \( d_{ij} \) and \( d_{mn} \). In the second stage of DR-SIP, to mitigate the loss of seismic events, electricity and gas...
load shedding is identified and implemented accordingly to keep the system balance. The sub-objective is to minimize the cost of load shedding and generation as an emergency response. When considering the effects of hardening strategies against seismic damage, the availability of lines needs to be identified and multiplied with the power and gas flow. Reference [76] considers the availability as $1 - \xi + \xi h$, but the term $\xi h$ is nonlinear. This paper presents the availability as $\xi + h$ in (4-14) and (4-15), which is linear but may cause $\xi + h = 2$ and thus leads to overloading on power lines. For instance, in (4-14), $(\xi_{ij} + h_{ij})f_{ij,max}$ can be $2f_{ij,max}$. Accordingly, additional constraints are added to ensure the original limits, i.e., $-f_{ij,max} \leq f_{ij,t} \leq f_{ij,max}$.

The DC linearized power flow and Weymouth gas flow are utilized for modelling power flow and gas flow respectively, which are shown in (4-9) and (4-10).

\[
\begin{align*}
    x_{ij}f_{ij,t} &= (\theta_{i,t} - \theta_{j,t}) \\
    \frac{f_{mn,t}}{|f_{mn,t}|} &= y_{mn}(P_{r,m,t}^2 - P_{r,n,t}^2)
\end{align*}
\]  

However, this paper considers the availability of asset that will inevitably lead to the nonlinear term $(\xi + h)(\theta_{i,t} - \theta_{j,t})$ and $(\xi + h)(P_{r,m,t}^2 - P_{r,n,t}^2)$. This nonlinearity is linearized based on sufficiently large constants $M_{ij}$ and $M_{mn}$. Thus, the linearized DC power flow and Weymouth gas flow constraints are used in (4-11)-(4-12) and (4-22)-(4-23).

The constraints of electricity and gas-fired generation output are in (4-16) and (4-17). Equations (4-18) and (4-19) show the constraints for electricity and gas load shedding. Equation (4-20) shows the upper and lower bounds for the pressure square of pipelines. Weymouth gas flow is presented in (4-21) and (4-22). The power and gas balance constraints are in (4-23) and (4-24).

\[
\begin{align*}
    x_{ij}f_{ij,t} &\leq (\theta_{i,t} - \theta_{j,t}) + (1 - \xi_{ij} - h_{ij})M_{ij} \\
    x_{ij}f_{ij,t} &\geq (\theta_{i,t} - \theta_{j,t}) - (1 - \xi_{ij} - h_{ij})M_{ij} \\
    -\pi &\leq \theta_{ij,t} \leq \pi \\
    -(\xi_{ij} + h_{ij})f_{ij,max} &\leq f_{ij,t} \leq (\xi_{ij} + h_{ij})f_{ij,max} \\
    0 &\leq f_{mn,t} \leq (\xi_{mn} + h_{mn})f_{mn,max}
\end{align*}
\]

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\[ P_{ge,\text{min}} \leq P_{ge,t} \leq P_{ge,\text{max}} \quad (4-16) \]
\[ P_{gg,\text{min}} \leq P_{gg,t} \leq P_{gg,\text{max}} \quad (4-17) \]
\[ 0 \leq \delta_{d_e,t} \leq \delta_{d_e,\text{max}} \quad (4-18) \]
\[ 0 \leq \delta_{d_g,t} \leq \delta_{d_g,\text{max}} \quad (4-19) \]
\[ P_{r_{\text{min}}}^2 \leq P_{r_t}^2 \leq P_{r_{\text{max}}}^2 \quad (4-20) \]

\[ f_{mn,t} \leq \gamma_{mn} \left( \left( P_{r_m,t}^2 - P_{r_n,t}^2 \right) + \left( 1 - \xi_{mn} - h_{mn} \right) M_{mn} \right) \quad (4-21) \]
\[ f_{mn,t} \geq \gamma_{mn} \left( \left( P_{r_m,t}^2 - P_{r_n,t}^2 \right) - \left( 1 - \xi_{mn} - h_{mn} \right) M_{mn} \right) \quad (4-22) \]
\[ \sum_{d_e \in \mathcal{D}_E} P_{d_e,t} - \delta_{d_e,t} = \sum_{ge \in \mathcal{G}_E} P_{ge,t} + \sum_{ij \in (\text{ter})} f_{ij,t} - \sum_{ij \in (\text{ini})} f_{ij,t} \quad (4-23) \]
\[ \sum_{d_g \in \mathcal{D}_G} P_{d_g,t} - \delta_{d_g,t} + \sum_{gg \in \mathcal{G}} P_{gg,t} = \sum_{mn \in M(\text{ter})} f_{mn,t} - \sum_{g \in M(\text{ini})} f_{mn,t} \quad (4-24) \]

The Weymouth gas flow is nonlinear, but \( P_{r_t}^2 \) is modelled in the squared form in this paper, which does not require linearization. The nonlinear term \( f_{mn,t}^2 \) can be linearized by piecewise linear approximation to convert DR-SIP into a MILP problem [23], which is presented as follows. \( h(f_{mn,t}) \) represents the nonlinear function, \( \Delta f_{mn,k} \) is the segment of gas pipeline \( mn \), and \( \chi_{mn,t,k} \) is the auxiliary continuous variable.

\[ h(f_{mn,t}) \approx h(\Delta f_{mn,1}) \]
\[ + \sum_{k \in K} \left( h(\Delta f_{mn,k+1}) - h(\Delta f_{mn,k}) \right) \chi_{mn,t,k} \quad (4-25) \]
\[ f_t = \Delta f_{mn,1} + \sum_{k \in K} (\Delta f_{mn,k+1} - \Delta f_{mn,k}) \chi_{mn,t,k} \quad (4-26) \]
\[ 0 \leq \chi_{mn,t,k} \leq 1 \quad (4-27) \]

4.7 Case Studies

This section presents the numerical case studies of the proposed DR-SIP on an integrated electricity and gas system. The IES consists of the standard IEEE 30 busbars
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electricity system and a 6-node gas network, shown in Fig. 4-3 [57]. The gas network includes a gas-fired generator, 4 gas demands and 7 pipelines. A gas turbine is connected between bus 2 of the electricity network and bus 6 of the gas network. Three cases are performed to optimize DR-SIP. The only difference between cases 2 and 1 is the addition of the planning stage before the seismic attacks.

- Case 1: Single-stage emergency response without considering hardening investment for IES.
- Case 2: Two-stage model including resilience enhancement planning and emergency response for IES.
- Case 3: Two-stage model for IEEE 30-bus system.

Section 4.7.1 presents comparisons of each case. Sections 4.7.2 and 4.7.3 illustrate the optimal hardening plan and computational results for case 2. Section 4.7.4 compares
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4.7.1 Case Comparisons

TABLE 4-3 OPTIMAL HARDENING PLAN UNDER DIFFERENT PLANNING BUDGET

<table>
<thead>
<tr>
<th>Intensity level</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Expected operation cost (10^3$)</td>
<td>Total cost (10^3$)</td>
</tr>
<tr>
<td>I</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>II–III</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>IV</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>V</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>VI</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>VII</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>VIII</td>
<td>263</td>
<td>263</td>
</tr>
<tr>
<td>IX</td>
<td>402</td>
<td>402</td>
</tr>
</tbody>
</table>

TABLE 4-4 OPTIMAL HARDENING PLAN UNDER DIFFERENT PLANNING BUDGET

<table>
<thead>
<tr>
<th>Planning budget</th>
<th>Optimal hardening plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-2</td>
</tr>
<tr>
<td>2</td>
<td>1-2, 1-3</td>
</tr>
<tr>
<td>3</td>
<td>1-3, 3-4, 2-6</td>
</tr>
<tr>
<td>4</td>
<td>1-2, 2-4, 2-5, 6-9</td>
</tr>
<tr>
<td>5</td>
<td>1-2, 2-5, 2-6, 12-13, 12-15</td>
</tr>
<tr>
<td>6</td>
<td>1-2, 2-4, 4-12, 12-13, 12-15, 25-27</td>
</tr>
<tr>
<td>7</td>
<td>1-2, 2-4, 4-12, 12-13, 12-15, 25-27, 2-g1</td>
</tr>
<tr>
<td>8</td>
<td>1-2, 2-4, 2-5, 5-7, 4-12, 12-13, 12-15, 8-28</td>
</tr>
<tr>
<td>9</td>
<td>1-2, 2-4, 2-5, 5-7, 4-12, 12-13, 12-15, 8-28, 2-g1</td>
</tr>
<tr>
<td>10</td>
<td>1-2, 2-4, 2-5, 5-7, 4-12, 12-13, 12-15, 8-28, 2-g1, g1-g4</td>
</tr>
<tr>
<td>15</td>
<td>1-2, 2-4, 2-5, 5-7, 4-12, 12-13, 12-15, 12-16, 16-17, 10-17, 10-21, 27-30, 8-28, 2-g1, g1-g4</td>
</tr>
<tr>
<td>20</td>
<td>1-2, 2-5, 2-6, 4-6, 6-7, 6-10, 4-12, 12-13, 12-15, 10-21, 10-22, 15-23, 22-24, 23-24, 28-27, 8-28, 6-28, 2-g1, g1-g4</td>
</tr>
</tbody>
</table>

the numerical performance of case 2 between DRO and RO. Section 4.7.5 highlights the merits of the proposed model.
139% increase of investment cost when intensity is up to level IX. When the intensity is above level VII, the investment cost remains constant as the 10 most vital lines to be hardened are found.

It is to be noted that the operation cost in case 1 is always higher than that in case 2, i.e., the gap is from 10% under level IV to 220% under level IX, since more load shedding is made without hardening investment to protect lines. With the increasing intensity level, the total cost of case 1 is increasing faster than that of case 2. When the intensity level is I, the total cost of case 1 is only 8% of case 2, which reflects that only implementing load shedding without hardening investment is much more economical. However, under level IX, the ratio is up to 44%, which shows that emergency response under seismic attacks without hardening planning causes huge economic loss.

When generation cannot satisfy all original load due to broken lines caused by seismic attacks, load shedding is made to maintain the system balance, which inevitably

---

Fig. 4-4. Load shedding cost of three cases.

Fig. 4-5. Frequency of line hardened.
leads to huge load shedding cost. Fig. 4-4 depicts the load shedding cost for three cases under different seismic intensity levels. Higher load shedding cost is regarded as the system is more vulnerable when facing seismic attacks. It shows that case 1 yields the highest load shedding cost in all intensity levels, which reaches 350×10^3$. This figure also shows that case 2 requires less load shedding than case 3, with the cost in case 3:169×10^3$ and in case 2: 90×10^3$. The potential reason is that: i). Gas-fired generators provide more supplies to the IES; ii) The IES network is more complex that can defend more severe seismic attacks; iii) Underground pipelines are more reliable than power lines, which is pre-set in the mean value vector in ambiguity set.

With increasing intensity level, the cost difference between each case is becoming larger. When intensity level is IX, the cost of case 2 and 3 are only 26% and 48% of case 1 respectively. Overall, it can be observed that the integration of electricity and gas networks makes the system more reliable against seismic attacks.

### 4.7.2 Optimal Hardening Plan

The impact of increasing the budget on both planning and operation is given in TABLE 4-4 and the intensity level is set as level V. However, when a large number of damaged lines with few hardening lines considered, e.g., budget=1, may probably cause an infeasible solution. Since there are still many lines damaged without previously hardened, causing the congestion and unbalance among transmission lines. Line 1-2 is
almost considered in all the budget ranges since both buses 1 and 2 contain the two largest generators and the power can be transmitted to buses 3, 5, 6 and gas bus 1. Line 2-4 is the second most frequent line to harden as the second largest generation of bus 2 needs to be securely transmitted to bus 4, which is connected with buses 3, 6 and 12. Fig. 4-5 shows the times for most frequent lines being hardened when planning budget is under 20. It is concluded that buses 2 and 12 are the most significant buses that involved most frequently. As observed in Fig. 4-3, bus 2 connects with 5 buses with a generator and load. Bus 12 connects with 5 buses with a load.

4.7.3 Computational Results Under Different Planning Budgets

In TABLE 4-5, it shows that a larger planning budget directly leads to more investment cost, but also improves the resilience and accordingly reduces load shedding cost. When the budget increases from 1 to 2, the investment cost increases dramatically by $66 \times 10^3$. The reason is that when the budget is extremely insufficient, e.g., budget=1 or 2, the two most important line 1-2 and 1-3 are chosen. Lines 1-2 and 1-3 are not only connected with large generators but also have large line ratings, and thus the hardening
cost is high. Due to the hardening of line 1-3, it survives from seismic attack by protecting 2980MWh electricity load and 10Sm$^3$ gas load from shedding. When the budget increases above 2, the investment cost and load shedding change slowly. Fig. 4-6 shows the load shedding curve for both electricity and gas with respect to different investment cost. When the investment cost increases, both electricity and gas load shedding decreases. The additional investment cost increasing from $50\times10^3$ to $116\times10^3$ helps to prevent load shedding greatly for both electricity and gas network. When the investment cost is above $116\times10^3$, the impact on load shedding is less effective.

### 4.7.4 Comparison with Robust Optimization

RO as a benchmark method is compared with the proposed DRO method in terms of mathematical performance in cases 1, 2 and 3. From TABLE 4-6, it can be observed that RO yields higher costs in all the three cases. Overall, the total cost and operation cost of DRO are 13% and 18% less than those of RO respectively. The reason is that RO always considers line damages that cause the most severe load shedding. Therefore, in the first stage, RO makes decisions on hardening investment to protect the system before the potential worst damage. In the second stage, under the worst seismic attack, load shedding is made while the whole system flow balance is ensured.

### 4.7.5 Discussion on Numerical Results

Three cases are extensively investigated and the resulting difference shows that case 2, i.e., the two-stage model including resilience enhancement planning and emergency response for IES, outperforms cases 1 and 3. Compared with case 1 which only contains emergency response to maintain system power balance, case 2 provides a more secure and reliable operation scheme with 54% less load shedding cost. In comparison with case 3 which is implemented on the electricity system without gas system integration, case 2 produces better solution with less load shedding and investment cost. Since the interdependency between electricity and gas systems enables coordinated energy flow to maintain both power and gas balance, it is effective to reduce load shedding and ensure the security of IES. The benefits of using the newly developed DRO over RO for all three cases are also analyzed, which results in reduced expected operation cost and total cost. The reason is that DRO captures the uncertainty of seismic attacks through partial distribution information of uncertainties via ambiguity sets, i.e., moment
information. It considers more specific uncertainty information than RO and accordingly produces less-conservative results.

The extensive case study and analysis illustrate the advantages of DR-SIP, which are:

1) DR-SIP is a hybrid optimization model containing both planning and operation schemes. It not only provides an optimal hardening plan to enhance resilience against seismic attacks but takes actions to maintain supply and demand balance via load shedding, ensuring the security for IES in two stages.

2) The impact of seismic attacks on IES is assessed by considering historical data of seismic events in a probabilistic manner. Thus, the impact of seismic attacks can be easily included in the optimization model for resilience enhancement.

3) The developed data-driven DRO method is less conservative than RO because the statistical information of the uncertainty of seismic attacks is utilized.

4) This paper demonstrates that the integrated planning model can further improve the resilience of electricity systems, following the trend multi-vector energy system integration.

4.8 Chapter Summary

In this paper, a two-stage DRO method is developed to enhance the resilience of an IES under seismic attacks with combined planning and operation strategies. The proposed method provides optimal hardening plans for specific power lines and gas pipelines under different seismic intensity levels and investment budgets. Through extensive case study demonstrations, the key findings are as follows: i) In the first stage, DR-SIP effectively determines the most vital lines to harden. In the second stage, DR-SIP optimally shed loads in order to keep system balance and minimize the system operation cost; ii) With RO that considers the most extreme event and serious damage, the proposed DRO provides less conservative results for both planning and operation stages with 13% less cost; iii) Investment plan with higher budget is more likely to yield a reliable IES with high resilient performance; iv) The optimal hardening plan is effective for protecting transmission lines and loads and IES is more resilient than electricity network against seismic attacks. This method can help system operators to make economical hardening and operation strategies to improve the resilience of integrated energy system under seismic attacks.
This chapter proposes to apply two-stage distributionally robust optimization to analyse and resolve the integrated electricity and gas system (IEGS) optimisation under false data injection attacks. This work provides system operators with a powerful model to operate the IEGS with enhanced cyber security and high penetrated renewable energy.
## Statement of Authorship

<table>
<thead>
<tr>
<th><strong>This declaration concerns the article entitled:</strong></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Coordinated Risk Mitigation Strategy For Integrated Energy Systems Under Cyber-Attacks</td>
<td></td>
</tr>
<tr>
<td>Publication status: Published</td>
<td></td>
</tr>
</tbody>
</table>
| **Candidate’s contribution to the paper** | **The candidate and the second author jointly proposed the idea of the paper, and they designed the methodology. The candidate executed all the coding to derive the experimental results. Other authors helped the candidate with the design of case studies, format of the paper, and improvement of academic writing. The percentage of the candidate did compared with the whole work is indicated as follows:**  
  
  - Formulation of ideas: 70%
  - Design of methodology: 80%
  - Experimental work: 90%
  - Presentation of data in journal format: 80% |  |
| **Statement from candidate** | **This paper reports on original research I conducted during the period of my Higher Degree by Research candidature.** |  |
| **Signed** | **Date** | **01/06/2020** |
5.1 Abstract

The dramatic increase of cyber-attacks on energy systems can cause huge losses, which has drawn extensive attention due to the fast integration of information communication technologies (ICTs). This issue is becoming worse with the integration of electricity and gas systems (IEGS), facilitated by gas generation and new coupling technologies.

This paper investigates the risk and mitigation strategies for IEGS under false data injection attacks (FDIA) in a hierarchical two-stage framework. The FDIA on both electricity and gas systems are modelled through injecting falsified data by adversaries. To mitigate the adverse impacts, a novel two-stage distributionally robust optimization (DRO) is proposed: i) day-ahead operation to determine initial operation scheme and ii) real-time corrective operation with the realization of FDIA and renewable generation uncertainties. A semidefinite programming is formulated for the original problem and it is then solved by a convex optimization-based algorithm. A typical IEGS is used for case demonstration, which shows that the proposed model is effective in mitigating the risks caused by potential FDIA and renewable uncertainties, by optimal coordinating energy infrastructures and load shedding. This work provides system operators with a powerful model to operate the IEGS with enhanced cyber security and high penetrated renewable energy. It can be easily extended to mitigate other natural and malicious attacks for IEGS.

5.2 Nomenclature

A. Indice and sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t, T$</td>
<td>Index and set for time periods.</td>
</tr>
<tr>
<td>$b, B$</td>
<td>Index and set for electricity buses.</td>
</tr>
<tr>
<td>$i_e, I_e$</td>
<td>Index and set for electricity generators.</td>
</tr>
<tr>
<td>$i_g, I_g$</td>
<td>Index and set for gas wells.</td>
</tr>
<tr>
<td>GT</td>
<td>Index for gas turbine.</td>
</tr>
<tr>
<td>$j, J$</td>
<td>Index and set for renewable generators.</td>
</tr>
<tr>
<td>$l_e, L_e$</td>
<td>Index and set for power lines.</td>
</tr>
<tr>
<td>$l_g, L_g$</td>
<td>Index and set for gas pipelines.</td>
</tr>
<tr>
<td>$k_e, K_e$</td>
<td>Index and set for electric loads.</td>
</tr>
<tr>
<td>$k_g, K_g$</td>
<td>Index and set for gas loads.</td>
</tr>
</tbody>
</table>

B. Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIL</td>
<td>Attack injection level for FDIA.</td>
</tr>
</tbody>
</table>
$\lambda_{i_e}^a, \lambda_{i_e}^b, \lambda_{i_e}^c$ Cost coefficients for of electricity generator $i_e$.

$\lambda_{i_g}$ Cost coefficient for gas well $i_g$.

$\lambda_{i_e}^+, \lambda_{i_e}^-$ Cost coefficient for up and down reserve of electricity generator $i_e$.

$\lambda_{i_e}^{re}, \lambda_{j}^{re}$ Regulation cost coefficient for electricity generator $i_e$ and renewable generator $j$.

$\lambda_{k_e}^{ls}, \lambda_{k_g}^{ls}$ Penalty cost coefficient for electricity and gas load shedding.

$\omega_{j,t}^s$ Forecasted output of renewable generator $j$ at time $t$.

$R_{i_e}^+, R_{i_e}^-$ Maximum up and down reserve capacity of electricity generator $i_e$ at time $t$.

$R_{GT}^+, R_{GT}^-$ Maximum up and down reserve capacity of gas turbine GT at time $t$.

$P_{ie,\text{max}}, P_{ie,\text{min}}$ Maximum and minimum output of electricity generator $i_e$.

$P_{i_g,\text{max}}, P_{i_g,\text{min}}$ Maximum and minimum output of gas well $i_g$.

$P_{GT,\text{max}}, P_{GT,\text{min}}$ Maximum and minimum output of gas turbine GT.

$x_{i_e}$ Resistance of power line $l_e$.

$f_{i_e,\text{max}}$ Maximum power flow of line $l_e$.

$f_{i_g,\text{max}}$ Maximum gas flow of line $l_g$.

$\omega_{j,t}^s$ Forecasted renewable generation at time $t$.

$P_{r_{i_g,\text{max}}}, P_{r_{i_g,\text{min}}}$ Maximum and minimum gas pressure of gas pipeline $l_g$.

$\gamma_{i_g}$ Coefficient for Weymouth equation.

$D_{i_g}, L_{i_g}$ Diameter and length of pipeline $l_g$.

$F_{i_g}$ Pipeline friction coefficient.

$R$ Specific gas constant.

$Z$ Compression factor of pipeline $l_g$.

$\rho_{i_g}$ Gas density.

$\text{Temp}$ Temperature.

$P_{k_e,t}, P_{k_g,t}$ Electricity and gas load at time $t$.

$p_{k_e,\text{max}}, p_{k_g,\text{max}}^{ls}$ Maximum electricity and gas load shedding at time $t$. 
\( \eta_{i_e,t}, \eta_{GT,t} \) Participation factor for reserves of electricity generator and gas turbine at time \( t \).

C. Variables and functions

\( P_{i_e,t}^s, P_{i_e,t}^{re} \) Scheduled and regulated output of electricity generator \( i_e \) at time \( t \).

\( P_{i_g,t}^s, P_{i_g,t}^{re} \) Scheduled and regulated output of gas well \( i_g \) at time \( t \).

\( P_{GT,t}^s, P_{GT,t}^{re} \) Scheduled and regulated output of gas turbine GT at time \( t \).

\( r_{i_e,t}^+, r_{i_e,t}^- \) Up and down reserve of electricity generator \( i_e \) at time \( t \).

\( r_{GT,t}^+, r_{GT,t}^- \) Up and down reserve of gas turbine GT at time \( t \).

\( f_{i_e,t}^s, f_{i_e,t}^{re} \) Scheduled and regulated power flow.

\( f_{i_g,t}^s, f_{i_g,t}^{re} \) Scheduled and regulated gas flow.

\( f_{i_g,t}^{ini}, f_{i_g,t}^{ter} \) Gas flow from initial node and to terminal node of pipeline \( l_g \) at time \( t \).

\( P_{r_{i_g,t}}^s, P_{r_{i_g,t}}^{re} \) Scheduled and regulated gas pressure of gas pipeline \( l_g \) at time \( t \).

\( P_{r_{i_g,t}}^{s,ini}, P_{r_{i_g,t}}^{s,ter} \) Scheduled gas pressure of initial and terminal nodes of pipeline \( l_g \) at time \( t \).

\( P_{r_{i_g,t}}^{re,ini}, P_{r_{i_g,t}}^{re,ter} \) Regulated gas pressure of initial and terminal nodes of pipeline \( l_g \) at time \( t \).

\( P_{k_e,t}^{ls} \) Electricity load shedding at time \( t \).

\( P_{k_g,t}^{ls} \) Gas load shedding at time \( t \).

\( x, y \) Vectors of first and second stage variables.

\( Pf() \) Probability function.

\( E_P[] \) Expectation over distribution.

\( \langle \rangle \) Trace of matrix.

\( \psi_0, \psi_j, \psi_{jk}, \tau \) Dual variables.

D. Uncertainty

\( \xi_{j,t} \) Real-time renewable power output of \( j \) at time \( t \).

\( \Delta P_{k_e,t}, \Delta P_{k_g,t}, \rho_{l_g}^{l} \) FDIA on electricity load, gas load and gas density at time \( t \).

\( D \) Ambiguity set for FDIA and renewable uncertainty.
\( \mu, \Sigma \)  
Mean vector and covariance matrix for FDIA and renewable uncertainty.

\( \Theta \)  
Second moment matrix.

\( VS \)  
Polyhedral set of extreme points.

### 5.3 Introduction

Power systems have evolved to be more intelligent, efficient and reliable with the increasing dependence on data communication infrastructures [87]. The information and communication technology (ICT) supports bidirectional information flows and thus enhances the optimal control of the physical power system with better observability and controllability. However, high integration and modernization of ICT can naturally raise threats to power system security [88]. The adversary can launch false data injection attacks (FDIA) to tamper critical data and inject falsified data, which brings serious challenges to state estimators, indirectly affecting system operation and control. In 2015, three Ukrainian regional power distribution companies were attacked by FDIA which caused power outages for 225,000 customers [89].

Existing research of FDIA most focuses on investigating i) maximally launching FDIA to cause damages, ii) detection algorithm against FDIA and iii) mitigation and protection schemes against FDIA. As for attack modelling and detection, paper [90] models how an adversary can trigger sequential outages on targeted branches by identifying critical branches. A stochastic model is proposed to design FDIA that affects electricity market by adopting imperfect grid information [91]. Paper [92] proposes an FDIA that can be launched through the approximation on system states based on injection measurements. An online anomaly detection algorithm is used to detect FDIA based on load forecasting and generation scheduling [93], where the minimum attack magnitudes and detection thresholds are determined. A detection and isolation scheme is proposed in [94] by using interval observer based on the physical dynamics of grids. State estimation is of significance in FDIA detection. However, malicious FDIA can be masked and hidden through judiciously designing residue of bad data detection [95, 96]. Therefore, mitigation strategy is the final barrier for protecting power systems provided that detection is failed. A corrective scheme is proposed to address overloading and uneconomic dispatch in [97] against the worst-case FDIA. Paper [98] proposes a unit commitment model by using a trilevel optimization model, which is converted into a
bilevel mixed integer programming problem.

The increasing demand growth of both power and gas systems, low price of natural gas resources, and conversion technologies between the two systems, e.g., gas-fired units and power-to-gas facilities, have promoted the interdependency between power and gas systems. Consequently, modelling and optimizing the two independent systems as an entity can facilitate the economy and security for both systems. Integrated electricity and gas systems (IEGS) realizes the coordination between energy infrastructures in both power and gas systems.

The electricity and natural gas systems are increasingly independent, interconnected by many coupling technologies to form IEGS. Conversion technologies between the two systems include gas-fired units and power-to-gas facilities. Combined heat and power (CHP) enables the conversion of gas to both heat and electricity [99, 100]. Power-to-gas (P2G) facilities can convert excessive renewable energy to synthetic natural gas [21, 29]. The conversion from gas to power is mainly realized by utilizing gas turbines [101, 102]. In some compressor stations of natural gas systems, electricity is used to drive compressors. The interdependency produces many benefits, including enhanced security of supply, more absorption of renewable energy, but there are also many adverse impacts. Cascading failures in one system can propagate to the other and the cyber-attacks on one system can affect the security of the other.

Existing IEGS literatures on making use of its unique interdependency generally concentrates on i) operation under normal conditions, ii) security-based operation under reliability issues and iii) resilience enhancement and operation strategies under natural disasters. Paper [24] proposes an optimal operation scheme for IEGS considering electricity demand response and the impact on the entire system is profound due to the strong interdependency of power and gas systems. A low carbon IEGS community with heat delivery system is proposed in [103], where the uncertainties of renewable generation and demand are handled by stochastic optimization (SO). Paper [104] models a security-constrained unit commitment against $N - k$ outages by using robust optimization (RO). Nonlinear gas flow is relaxed into a second-order cone problem for convexity. To enhance the resilience of IEGS, a robust network hardening strategy is proposed in [76], considering the uncertainties of natural disasters. Paper [105] proposes a minimax-regret robust unit commitment model for enhancing the resilience of IEGS against the extreme weather obtained by spatial dynamic method.

Leveraging between RO and SO, distributionally robust optimization (DRO) is
widely applied in power systems to handle uncertainties [56, 57, 64, 106, 107]. SO either assumes specific knowledge of probability distributions or requires a large number of uncertainty samples, which is not always practical and can cause high computational burden. RO accommodates uncertainties in predefined uncertainty sets and considers the worst-case scenario, which could have extremely low probability and thus produces over-conservative results. DRO, taking the advantage of distributional information, e.g., moment information, deals with uncertainties within a feasible set, called ambiguity set. Therefore, compared with RO, DRO determines expected results over all possible distributions, which are less-conservative. Compared with SO, DRO avoids intensive computation, thus improving calculation efficiency. Paper [56] applies DRO to a risk-based optimal gas-power flow. An optimal scheduling of IEGS considering electricity and gas load uncertainties is proposed in [57].

Due to the strong interdependency between electricity and gas systems in IEGS, the FDIA on either electricity or gas system can propagate to each other. The adverse impact can be exaggerated when there is large volume of uncertain renewables in the electricity system. But limited effort is dedicated to studying the impact of FDIA on IEGS. This paper proposes a two-stage risk mitigation strategy to address the uneconomic operation of IEGS under FDIA considering renewable generation uncertainties. FDIA is assumed to attack both electricity and gas meter readings, including i) load measurement of electricity and gas systems and ii) gas density measurement. In the first stage, the day-ahead optimization determines an optimal IEGS scheduling scheme based on forecasted renewable generation without considering potential FDIA. In the second stage, both FDIA and renewable uncertainties are revealed, a real-time corrective optimization is built to minimize the attack impacts through load shedding and adjusting generation output. The original problem is converted into equivalent semidefinite programming (SDP) and a constraint generation algorithm (CGA) is adopted to solve the SDP problem. For conciseness and simplicity, the proposed distributionally robust FDIA mitigation scheme is represented by DR-FMS.

The major contribution of this paper is as follows:

1) It models FDIA in an IEGS for the first time, particularly on natural gas load and density measurement, where existing research only focuses on FDIA on electricity systems.
2) Uncertainties of renewable resources are considered in the proposed model as they can worsen system operation conditions during FDIA, compared to existing FDIA papers that ignore the impact of renewable uncertainties.

3) Compared to SO and RO for modelling FDIA, the ambiguous distribution of DRO developed in this paper, which is less data-dependent and conservative, can better characterize uncertain variables.

4) A two-stage FDIA mitigation scheme is proposed for the first time to conduct the day-ahead and real-time operation, which is more powerful and convenient to be used by system operators to ensure the efficiency and security of the IEGS.

The rest of this paper is organized as follows. Section 5.4 models FDIA for both electricity and gas sides. Section 5.5 presents problem formulation of the DR-FMS. The DRO methodology regarding and associated reformulations are given in Section 2.1.2. Section 5.6 demonstrates case studies and performance of the DR-FMS. Finally, the conclusion is given in Section 5.7.

5.4 Attack Modelling

This section presents the attack modelling for electricity and gas system. State estimation is a powerful tool to detect FDIA by processing raw data measurements, but a successful FDIA can be undetectable by adversary’s stealthy design.

5.4.1 Attacks on Electricity System

The nonlinear relationship between state variable $x$ and measurement $z$ is given in (5-1), where $h(x)$ denotes the nonlinear vector function of $x$ and $e$ is the error measurement. Based on DC state estimation, equation (5-1) can be transformed into (5-2), where $H$ represents the Jacobian matrix.

\[
\begin{align*}
    z &= h(x) + e \tag{5-1} \\
    z &= Hx + e \tag{5-2}
\end{align*}
\]

After the realization of FDIA, the measurement vector $z$ becomes $z_{bad} = z + a$, and the estimated state vector can be represented as $\hat{x}_{bad} = \hat{x} + c$ where $a$ is attack vector...
and \( c \) is the resulted deviation vector of state variable after FDIA. Accordingly, to determine the estimated state variable, \( \hat{x}_{bad} \) can be formulated as:

\[
\hat{x}_{bad} = (H'WH)^{-1}H'Wz_{bad}
\]  

(5-3)

The largest normalized residual (LNR) can be used to detect and identify measurement errors by (5-4). If the residual is less than a threshold \( \varepsilon \), then the state estimate is valid without FDIA.

\[
LNR = \|z - H\hat{x}\| \leq \varepsilon
\]  

(5-4)

Then, equation (5-5) representing LNR is given based on (5-3) and (5-4). Finally, equation (5-6) is obtained.

\[
LNR = \|z + a - H((H'WH)^{-1}H'Wz_{bad})\|
\]  

(5-5)

\[
LNR = \|z - H\hat{x} + (a - Hc)\|
\]  

(5-6)

If \( a \) is the linear combination of \( H \) and \( c \), i.e., \( a = Hc \), then \( LNR = \|z - H\hat{x}\| \) has no change of residual. Therefore, a successful FDIA is launched which can evade detection. As a special case of FDIA, load measurement can be attacked according to [95, 96] by enforcing the sum of load attack vector to be zero in (5-7). For simplicity, the FDIA on electricity load is represented by EL-FDIA. Equation (5-8) constraints the attack deviation by attack injection level (AIL).

\[
\sum \Delta P_{ke,t} = 0
\]  

(5-7)

\[
-AIL_{ke} P_{ke,t} \leq \Delta P_{ke,t} \leq AIL_{ke} P_{ke,t}
\]  

(5-8)

### 5.4.2 Attack on Gas System

The FDIA on a gas system is considered on both gas load and density measurement. Similar to FDIA on electricity load measurement, the FDIA on gas load measurement and density are given in (5-9) and (5-10), namely GL-FDIA and GD-FDIA. The changed gas density measurement results in a change of Weymouth coefficient \( \gamma_{lg} \). Accordingly, the initial coefficient in (5-11) is changed to (5-12) under FDIA.

\[
0 \leq \Delta P_{kg,t} \leq AIL_{kg} P_{kg,t}
\]  

(5-9)

\[
\rho_{lg} \leq \rho_{lg}^f \leq (1 + \beta_d)\rho_{lg}
\]  

(5-10)


**Chapter 5  
Risk Mitigation of IEGS under False Data Injections**

<table>
<thead>
<tr>
<th>TABLE 5-1 TWO-STAGE MITIGATION FRAMEWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stage I</strong></td>
</tr>
<tr>
<td>Decision variables</td>
</tr>
<tr>
<td>$p^e_{i,t}, p^e_{i,\text{ter}}, p^g_{i,t}, r^e_{i,t}, r^g_{i,t}, r^e_{i,\text{ter}}, r^g_{i,\text{ter}}, \theta^e_{i,t}, \theta^e_{i,\text{ter}}, f_{i,t}, P_{i,t}$</td>
</tr>
<tr>
<td>Objective</td>
</tr>
<tr>
<td>Generation and reserve cost for electricity and gas</td>
</tr>
<tr>
<td>Uncertainty treatment</td>
</tr>
<tr>
<td>Renewable generation forecast</td>
</tr>
</tbody>
</table>

\[
\gamma^e_{t,g} = \left(\frac{\pi}{4}\right)^2 \frac{\alpha^2 D_{t}^5}{L_{t,g} F_{t,g} R Z \rho_{t,g}^2 \text{Temp}} \tag{5-11}
\]

\[
\gamma^F_{t,g} = \left(\frac{\pi}{4}\right)^2 \frac{\alpha^2 D_{t}^5}{L_{t,g} F_{t,g} R Z \rho_{t,g}^2 \text{Temp}} \tag{5-12}
\]

### 5.5 Two-Stage Risk Mitigation Scheme

The risk mitigation for IEGS under potential FDIA consists of: i) day-ahead operation without considering FDIA or renewable uncertainties and ii) real-time operation actions for a corrective mitigation scheme under potential FDIA with the realization of renewable uncertainties. The objective functions and associated constraints are presented in this section.

#### 5.5.1 DR-FMS Objective Function

A summary of decision variables, objective functions and uncertainty modelled in the two stages is presented in **TABLE 5-1**. Equation (5-13) presents the day-ahead operation objective function in the first stage. The first four terms represent the generation cost of electricity and gas respectively. Reserve costs of electricity generators are shown in the rest. It is noted that the reserve capacity is prepared for FDIA and uncertainties from renewable resources.

\[
\ell_1 = \min \sum_{i, e, t, g, e, t} \lambda^e_{i,t} p^e_{i,t} + \lambda^g_{i,t} p^g_{i,t} + \lambda^e_{i,t} r^e_{i,t} + \lambda^g_{i,t} r^g_{i,t} + \lambda^e_{i,t} r^e_{i,\text{ter}} + \lambda^g_{i,t} r^g_{i,\text{ter}} \tag{5-13}
\]

The real-time objective function in the second stage is given in (5-14), which mitigates the impact against the presence of FDIA and renewable uncertainty. The first three terms represent the penalty cost for renewable generators, electricity generators and gas wells when regulated generation deviates from scheduled generation. The final two terms represent electricity and gas load shedding cost.
\[ F_2 = \min_{i_e,t_e,\cdots,\epsilon_g,t_g} \sum_{i_e,t_e \in T_e, \epsilon_g,t_g \in T_g} \left[ + \lambda_r^{i_e} |\omega_e^{i_e} - \xi_e| + \lambda_z^{\epsilon_g} |\epsilon_g^{\epsilon_g} - \delta^{\epsilon_g}| + \lambda_r^{r_e} |s_e^{r_e} - s^{r_e}| 
+ \lambda_z^{\epsilon_g} |p_e^{\epsilon_g} - p^{\epsilon_g}| + \lambda_z^{\epsilon_g} |x_e^{\epsilon_g} - x^{\epsilon_g}| \right] \] (5-14)

### 5.5.2 Proposed Coordinated Modelling of IEGS

The IEGS is a tight coupling entity due to the strong interdependency between electricity and gas systems. Accordingly, the two systems should be modelled together by one decision maker. The modelling of IEGS in the existing literatures can be generally categorized into three types: i) Modelling from the perspective of electricity system operators, which overlooks the operational and security constraints of gas system [108, 109]. This ignorance will cause the physical gas flow violation due to the renewable power fluctuation and load variability; ii) Sequential optimization for IEGS [110, 111], which firstly solves the power system model for determining the optimal schedule for generators while neglects the operational constraints of gas system. Based on the obtained solution from power system, the gas system model can be solved and iii) Co-optimization for IEGS which optimizes the comprehensive objective for both electricity and gas systems simultaneously. Consequently, the optimal solution for the entire IEGS can be obtained.

This paper provides a simultaneous coordinated model for the electricity and gas systems. Due to the different characteristics of electricity and gas systems, the operational constraints of two systems are nonrelevant. However, the two systems are solved interdependently with the gas turbine interconnected between the two systems. The gas flow through the gas turbine can be used to generate power flow, which is considered as the supplement for electricity system.

### 5.5.3 Day-ahead Operation

The day-ahead operation scheme is implemented based on renewable generation forecast without considering FDIA risks, whose constraints are shown in (5-15)-(5-28). Constraint (5-15) and (5-16) limit the reserve capacity for electricity generators and gas turbine. The scheduled output of electricity generators and gas turbine are enforced within limits in (5-17) and (5-18). The linearized DC power flow is given in (5-19) and (5-20). Constraint (5-21) ensures the power balance. Gas well output is limited in (5-22). Gas pressure is limited in (5-23). Constraint (5-24) means the pressure at the initial node is larger than the terminal node since the proposed gas system has a radial
topology. Weymouth gas equation for describing gas flow is shown in (5-25) and (5-26), where the coefficient is defined in (5-11). Gas turbine connects two interdependent systems as a coupled infrastructure. Constraint (5-27) presents the transformation from gas flow injection to power generation. The gas balancing condition is given in (5-28).

$$0 \leq r_{(\cdot),t}^+ \leq R_{(\cdot)}^+ \{ \cdot \} = i_e, \text{GT} \tag{5-15}$$

$$0 \leq r_{(\cdot),t}^- \leq R_{(\cdot)}^- \{ \cdot \} = i_e, \text{GT} \tag{5-16}$$

$$P_{(\cdot),t}^s + r_{(\cdot),t}^+ \leq P_{(\cdot),max} \{ \cdot \} = i_e, \text{GT} \tag{5-17}$$

$$P_{(\cdot),min} \leq P_{(\cdot),t}^s - r_{(\cdot),t}^- \{ \cdot \} = i_e, \text{GT} \tag{5-18}$$

$$x_t f_{e,t} = \left( \theta_{i_e}^{s,\text{ini}} - \theta_{i_e}^{s,\text{ter}} \right) \tag{5-19}$$

$$-f_{i_e,\text{max}} \leq f_{i_e,t}^s \leq f_{i_e,\text{max}} \tag{5-20}$$

$$\sum_{i_e \in I_e} P_{i_e,t}^s + P_{GT,t}^s + \sum_{j \in J} \omega_{j,t}^s + \sum_{i_e \in I_e} f_{i_e,t}^{s,\text{ini}} - \sum_{i_e \in I_e} f_{i_e,t}^{s,\text{ter}} = \sum_{k \in K_g} p_{g,k,t} \tag{5-21}$$

$$P_{i_g,\text{min}} \leq P_{i_g,t}^s \leq P_{i_g,\text{max}} \tag{5-22}$$

$$P_{i_g,\text{min}} \leq P_{i_g,t}^{s,\text{ter}} \leq P_{i_g,\text{max}} \tag{5-23}$$

$$P_{i_g,t}^{s,\text{ini}} \geq P_{i_g,t}^{s,\text{ter}} \tag{5-24}$$

$$f_{i_g,t}^s = \gamma_{i_g} \left( P_{i_g,t}^{s,\text{ini}}^2 - P_{i_g,t}^{s,\text{ter}}^2 \right) \tag{5-25}$$

$$0 \leq f_{i_g,t}^s \leq f_{i_g,\text{max}} \tag{5-26}$$

$$P_{GT,t}^s = c_{GT} f_{i_g,\text{GT}}^s \tag{5-27}$$

$$\sum_{l_g \in L_g} P_{l_g,t}^g + \sum_{l_g \in L_g} f_{l_g,t}^{s,\text{ini}} - \sum_{l_g \in L_g} f_{l_g,t}^{s,\text{ter}} = \sum_{k \in K_g} p_{l_g,t} \tag{5-28}$$

### 5.5.4 Real-time Risk Mitigation

Considering potential FDIA and uncertainties of renewable resources, real-time risk mitigation is presented in the second stage to mitigate uneconomic dispatch. The approach is distributionally robust against FDIA and renewable uncertainty. The regulated generator and gas turbine output are shown in (5-29). Constraint (5-30) represents the electricity and gas load shedding limits. The limits considered in the
model is based on the existing research [112]. The regulated power flow is constrained in (5-31) and (5-32). Constraint (5-33) presents the limits for gas wells. The regulated gas pressure and flow are limited in (5-34)-(5-37). Based on (5-12), the new Weymouth coefficient $\gamma^F_i$ influenced by the attacked gas density is applied. Constraint (5-38)-(5-40) show that the power imbalance caused by renewable uncertainties should be offset by adjusting the reserves of generators and gas turbine. Specifically, constraint (5-38) ensures the deviation of renewable generation is within the range of up and down reserve limits. In (5-39) and (5-40), the adjustment factor $\eta_{i,t}$ and $\eta_{i,g,t}$ are the regulation commitment from generators and gas turbine to mitigate renewable uncertainties. Constraint (5-41) presents the regulated power generation of gas turbine. Constraint (5-42) and (5-43) ensure power and gas balance in the second stage.

$$P_{i,t}^{re} - r_{i,t}^- \leq P_{i,t}^{re} \leq P_{i,t}^{re} + r_{i,t}^+, \{i\} = i_e,GT$$  \hspace{1cm} (5-29)

$$0 \leq P_{i,t}^{is} \leq P_{i,t}^{is} \max, \{i\} = k_e, k_g$$  \hspace{1cm} (5-30)

$$x_i f_{i,t}^{re} = (\theta_i^{re,ini} - \theta_i^{re,ter})$$  \hspace{1cm} (5-31)

$$-f_{i,t}^{re,\max} \leq f_{i,t}^{re} \leq f_{i,t}^{re,\max}$$  \hspace{1cm} (5-32)

$$P_{i,g,\min} \leq P_{i,g,t}^{re} \leq P_{i,g,\max}$$  \hspace{1cm} (5-33)

$$P_{i,g,\min}^{re2} \leq P_{i,g,t}^{re2} \leq P_{i,g,\max}^{re2}$$  \hspace{1cm} (5-34)

$$P_{i,g,t}^{re,\ini} \geq P_{i,g,t}^{re,\ter}$$  \hspace{1cm} (5-35)

$$f_{i,g,t}^{re,\max} = \gamma_i^F (P_{i,g,t}^{re,\ini2} - P_{i,g,t}^{re,\ter2})$$  \hspace{1cm} (5-36)

$$0 \leq f_{i,g,t}^{re} \leq f_{i,g,\max}$$  \hspace{1cm} (5-37)

$$r_{i,t}^- \leq \eta_i \sum_{j \in I} (\omega_{j,t} - \xi_{j,t}) \leq r_{i,t}^+, \{i\} = i_e,GT$$  \hspace{1cm} (5-38)

$$0 \leq \eta_i \leq 1, \{i\} = i_e,GT$$  \hspace{1cm} (5-39)

$$\sum_{i \in I_e} \eta_{i,t} + \sum_{i \in I_g} \eta_{i,g,t} = 1$$  \hspace{1cm} (5-40)

$$P_{GT,t}^{re} = c_{GT} f_{i,g,t}^{re}$$  \hspace{1cm} (5-41)
\[
\sum_{i \in I_e} p_{i_e,t} + \sum_{j \in J} \xi_{j,t} + P_{G'\tau,t} = \sum_{k \in K_e} P_{k_e,t} + \Delta P_{k_e,t} - P_{k_e,t}^{\text{ls}}
\]

(5-42)

\[
\sum_{i \in I_g} p_{i_g,t} + \sum_{l_g \in L_g} f_{l_g,t}^{\text{re,ini}} - \sum_{l_g \in L_g} f_{l_g,t}^{\text{re,ter}} = \sum_{k \in K_g} P_{k_g,t} + \Delta P_{k_g,t} - P_{k_g,t}^{\text{ls}}
\]

(5-43)

Constraint (5-25) contains one nonlinear term, i.e., \( f_{l_g,t}^{\text{re,2}} \), and constraint (5-36) contains two nonlinear terms, i.e., \( f_{l_g,t}^{\text{re,2}} \) and \( \gamma_{l_g}^{F}(P_{l_g,t}^{\text{ini}2} - P_{l_g,t}^{\text{ter}2}) \). They need to be linearized for obtaining convex functions and guaranteeing global optimal solutions. A sufficiently large constant \( M_{l_g} \) can be used to linearize \( \gamma_{l_g}^{F}(P_{l_g,t}^{\text{ini}2} - P_{l_g,t}^{\text{ter}2}) \), shown in (5-44) and (5-45). The bilinear term \( f_{l_g,t}^{\text{re,2}} \) can be linearized by piecewise linear approximation by separating nonlinear function into pieces. Readers are referred to [23] for details. It should be noted that \( P_{l_g,t}^{\text{ini}2} \) does not require linearization since it is regarded as squared form throughout the paper.

\[
f_{l_g,t}^{\text{re,2}} \leq (P_{l_g,t}^{\text{re,ini}2} - P_{l_g,t}^{\text{re,ter}2}) + (1 - \gamma_{l_g}^{F}) M_{l_g}
\]

(5-44)

\[
f_{l_g,t}^{\text{re,2}} \geq (P_{l_g,t}^{\text{re,ini}2} - P_{l_g,t}^{\text{re,ter}2}) + (1 - \gamma_{l_g}^{F}) M_{l_g}
\]

(5-45)

### 5.6 Case Studies

A combined IEEE 30-bus electricity system and a 6-node gas system is used to test the effectiveness of the DR-FMS through the extensive case studies [57]. In the case studies, three types of FDIA are considered, namely EL-FDIA, GL-FDIA and GD-FDIA, which represent FDIA on electricity load, gas load and gas density, respectively. For EL-FDIA, the total load is unchanged, which is the fundamental condition of FDIA.
for evading the detection. The modelling of EL-FDIA can be also found in [95, 97, 113].

The following 10 cases are considered:

Case 1: Single-stage deterministic optimization for IEGS without considering FDIA or renewable uncertainty.

Case 2: RO based FMS with three types of FDIA ($AIL = 5\%$).

---

### Table 5-2 Parameters of Gas Wells

<table>
<thead>
<tr>
<th>Node No.</th>
<th>$P_{ig,\text{max}}$ (kcf/h)</th>
<th>$P_{ig,\text{min}}$ (kcf/h)</th>
<th>$\lambda_{ig}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>35</td>
<td>10</td>
<td>2.2</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table 5-3 Generator Parameters

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>$P_{ie,\text{min}}$ (MW)</th>
<th>$P_{ie,\text{max}}$ (MW)</th>
<th>$R_{ie}^+, R_{ie}^-$ (MW)</th>
<th>$a_{ie}$</th>
<th>$b_{ie}$</th>
<th>$c_{ie}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>200</td>
<td>20</td>
<td>0.004</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>80</td>
<td>16</td>
<td>0.002</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>50</td>
<td>10</td>
<td>0.006</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>35</td>
<td>7</td>
<td>0.008</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>0.025</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>40</td>
<td>16</td>
<td>0.025</td>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

---

Fig. 5-1. Modified IEEE 30-bus system.
Case 3: Case 2 considering renewable uncertainty ($AIL = 5\%$).

Case 4: Two-stage DR-FMS considering FDIA on electricity load ($AIL = 5\%$).

Case 5: Case 4 considering FDIA on both gas and electricity load ($AIL = 5\%$).

Case 6: Case 5 considering FDIA on gas density ($AIL = 5\%$).

Case 7: Case 6 considering renewable uncertainty ($AIL = 5\%$).

Case 8-10: Case 7 with $AIL = 10\%$, 15\% and 20\%.

In case 3 and 7, the addition of renewable uncertainty is considered and the $AIL$ is still 5\% as case 2 and 4. The proposed test network is shown in Fig. 5-1, which contains 30 buses, 6 electricity generators, 2 renewable generators, 2 gas wells, 21 electricity loads and 3 gas loads. The renewable generators are connected to bus 22 and 25 with 60MW for each output. Parameters of electricity generators, gas wells are given in TABLEs 5-2 and 5-3, which can be found in [66].

The deterministic method for case 1 is a deterministic global optimization algorithm for solving linear programming. The reasons for not using metaheuristic optimization methods are: i) The deterministic linear programming problem solved by deterministic global optimization and metaheuristic optimization methods have similar results [114-116]; ii) The focus of this paper is to address FDIA and design mitigation schemes. The deterministic optimization method in case 1 is only used for comparison; iii) In practice, system operators implement economic dispatch after the data-filtering by state estimators, which requires high computational efficiency;; and iv) Metaheuristic methods, such as genetic algorithm and particle swarm optimization, easily converge prematurely and could be trapped into a local minimum, particularly with complex problems [117].

It should be noted that the DR-FMS considers the worst-case uncertainty distribution for both FDIA and renewable energy from all candidate distributions. Based on the partial distributional information, i.e., mean value vector and covariance matrix, DR-FMS can test all possible distributions modelled by moment information. Accordingly, this worst-distribution oriented mitigation scheme is a data-driven approach and actually tests a variety of scenarios.
5.6.1 Studies on Economic Performance

Firstly, the economic performance for all 10 cases under different combinations of uncertainties is shown in TABLE 5-4. Case 10 has the highest total cost, i.e., $132000, which is 21% higher than that of case 1, since EL-FDIA and GD-FDIA are comprehensively considered with the highest AIL. Case 1 has the lowest economic result since the deterministic model is applied. When considering the FDIA handled by RO in case 2 and DRO in case 6, the total cost increases by 11% and 10% respectively. In addition to the only consideration of FDIA by case 2, in case 3, when renewable...
uncertainty is further considered, the expected total cost increases by 4%. The total economic cost of case 3 is $13080, higher than that in case 4. The main reasons are: i) case 3 considers all three types of FDIA while case 4 only considers EL-FDIA; ii) case 3 considers renewable uncertainty while case 4 does not, iii) case 3 is implemented under RO, which provides more conservative solutions even in the single-stage framework. For case 4, the second stage of DR-FMS considers corrective actions for the day-ahead operation, which accounts for a small portion of the total cost. The big portion of cost is from the first stage, because generation costs for electricity generators and gas wells are considered; and iv) the two-stage framework is a combination of stochastic programming, which derives more flexible second-stage decisions to adjust first-stage decisions and hedges against uncertain FDIA and renewable generation after their realization in the second stage [118, 119]. From case 4 to 10, two-stage DRO is applied, where different types of FDIA are considered in cases 4-7 and sensitivity analysis is studied for cases 7-10. Both the first-stage and second-stage expected costs are increasing for cases 4-7 from only considering EL-FDIA to considering all three types of FDIA with renewable uncertainties. It can be found that GL-FDIA has the largest impact on economic performance, i.e., the total cost of case 5 has 4.7% more cost than case 4. On the contrary, GD-FDIA has the least impact on economic performance with a 1.4% rise of total cost from case 5 to case 6. From case 7 to 10, 15% more AIL causes an increase of total cost from $124252 to $132000. It should be noted that although EL-FDIA does not increase the overall load increase since some loads are increasing while the rest are decreasing, FDIA aims at attacking critical loads for causing economic losses. Accordingly, under these three types of FDIA with high risks, DR-FMS is more suitable for risk assessment and mitigation considering the worst-distribution. This advantage with less-conservative solutions is reflected in the comparison between cases 2 and 6 as wells as cases 3 and 7, where cases 6 and 7 reduce $1215 and $1558 compared with cases 2 and 3, respectively.

**TABLE 5-4 ECONOMIC PERFORMANCE FOR CASES 1-10**

<table>
<thead>
<tr>
<th>Economic result</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>Case 9</th>
<th>Case 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-stage cost ($)</td>
<td>108930</td>
<td>120955</td>
<td>125810</td>
<td>108835</td>
<td>113922</td>
<td>115465</td>
<td>119250</td>
<td>120880</td>
<td>122140</td>
<td>123480</td>
</tr>
<tr>
<td>Expected Second-stage cost ($)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3895</td>
<td>4132</td>
<td>4275</td>
<td>592</td>
<td>7862</td>
<td>8043</td>
<td>8520</td>
</tr>
<tr>
<td>Total cost ($)</td>
<td>108930</td>
<td>120955</td>
<td>125810</td>
<td>118054</td>
<td>119740</td>
<td>124252</td>
<td>128742</td>
<td>130183</td>
<td>132000</td>
<td></td>
</tr>
</tbody>
</table>
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5.6.2 Studies on Load Shedding

To maintain the feasibility of optimization and system balance under FDIA and renewable uncertainty, it is necessary to implement load shedding. The electricity load shedding (ELS) and gas load shedding (GLS) for 24 hours under FDIA are given in Figs. 5-2 and 5-3. In Fig. 5-2, ELS is up to 140MWh when EL-FDIA and GL-FDIA are both 20%. ELS is not sensitive to increase when only increasing GL-FDIA level, but sensitive when increasing EL-FDIA. The reason is that the scale of electricity load is much larger than the gas load. Therefore, the GL-FDIA has a minor effect on ELS. In
Fig. 5-3, GLS reaches up to 4.7kcf when FDIA is at the maximum level. GLS increases smoothly when AIL of EL-FDIA is under 17% while increases significantly when it is over 17%.

The ELS and GLS under EL-FDIA and GD-FDIA are shown in Fig. 5-4 and Fig. 5-5. Compared with GL-FDIA in Fig. 5-2, 84 MWh more ELS is made when considering the GD-FDIA. Since the wrong gas density can directly influence the gas flow. When there is no EL-FDIA, ELS caused by GD-FDIA can still reach up to 3MWh. In Fig. 5-5, GLS reaches 13kcf at the maximum AIL compared with the 4.7kcf in Fig. 5-3, which again proves the significant impact of gas density on GLS. EL-FDIA and GL-FDIA show the similar impact on GLS, i.e., GLS increases by 10kcf when fixing GL-FDIA and increasing EL-FDIA while GLS increases by 13kcf when fixing EL-FDIA and only increasing GL-FDIA. Since as observed from Fig. 5-2, the scale magnitude of electricity load is much larger than gas load, which largely influences on both ELS and GLS.

As observed from Figs. 5-4 and 5-6, the impact of GL-FDIA and GD-FDIA on ELS is minor when EL-FDIA is 0%. It shows that when one type of FDIA is manipulated, the impact on ELS is minor. However, when multiple types of FDIA are conducted in,
and 21% increase respectively. Compared to pipeline N4, it will lead to massive load shedding.

When GD-FDIA is 0%, EL-FDIA has a low impact on GLS even when the AIL is 20%. The EL-S ranges from 4.7kcf to 13kcf, and when AIL of GD-FDIA is at 20%, GLS ranges from 4.7kcf to 13kcf. This indicates that the security interdependency between electricity and gas systems is minor when attackers only conduct one type of FDIA, but when multiple types of FDIA is attacking the IEGS, it will lead to massive load shedding.

### 5.6.3 Studies on Flow-Capacity Ratio

To study the FDIA impact on power and gas flow, in TABLE 5-5, flow-capacity ratio (FCR) for three power lines and three gas pipelines at the peak load time period are studied, which is defined as the percentage of flow divided by the line capacity. The FCR of line N1-N2 and pipeline N4-N3 and N1-2 all reach 100% when maximum AIL is considered since these three lines play vital parts for interconnecting buses and transmitting flow. From cases 4 to 7, there is a general increase for FCR of power lines and gas pipelines since types of FDIA are gradually incorporated. From cases 7 to 10, the FCR still monotonically increases when AIL is increasing from 5% to 20%. It should be noted that line 1-2 and line 27-28 are more sensitive to increase of AIL with a 24% and 21% increase respectively. Compared to pipeline N4-N3 which is prone to

<table>
<thead>
<tr>
<th>TABLE 5-6 EL-FDIA ON GAS LOAD SHEDDING</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIL</td>
</tr>
<tr>
<td>GLS (kcf)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 5-7 GL-FDIA ON ELECTRICITY LOAD SHEDDING</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIL</td>
</tr>
<tr>
<td>ELS (MWh)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 5-8 GD-FDIA ON ELECTRICITY LOAD SHEDDING</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIL</td>
</tr>
<tr>
<td>ELS (MWh)</td>
</tr>
</tbody>
</table>

the impact on ELS could be large. In Fig. 5-2, with the AIL increase of both GL-FDIA and EL-FDIA, the ELS is massive, which is 99MWh when the AIL of GL-FDIA is 0% and 139MWh when AIL of GL-FDIA is 20%. The EL-FDIA is fixed but there is a 40MWh increase of GLS. In Fig. 5-5, GLS under EL-FDIA and GD-FDIA is given. When GD-FDIA is 0%, EL-FDIA has a low impact on GLS even when the AIL is 20%. However, when AIL of GD-FDIA is at 20%, GLS ranges from 4.7kcf to 13kcf, and when AIL of GD-FDIA is at 20%, GLS ranges from 4.7kcf to 13kcf. This indicates that the security interdependency between electricity and gas systems is minor when attackers only conduct one type of FDIA, but when multiple types of FDIA is attacking the IEGS, it will lead to massive load shedding.

5.6.3 Studies on Flow-Capacity Ratio

To study the FDIA impact on power and gas flow, in TABLE 5-5, flow-capacity ratio (FCR) for three power lines and three gas pipelines at the peak load time period are studied, which is defined as the percentage of flow divided by the line capacity. The FCR of line N1-N2 and pipeline N4-N3 and N1-2 all reach 100% when maximum AIL is considered since these three lines play vital parts for interconnecting buses and transmitting flow. From cases 4 to 7, there is a general increase for FCR of power lines and gas pipelines since types of FDIA are gradually incorporated. From cases 7 to 10, the FCR still monotonically increases when AIL is increasing from 5% to 20%. It should be noted that line 1-2 and line 27-28 are more sensitive to increase of AIL with a 24% and 21% increase respectively. Compared to pipeline N4-N3 which is prone to
overloading, the FCR of pipeline N6-N5 ranges only reaches 69%, indicating that gas flow is mainly sourced from the gas well connected to node 4.

5.6.4 Discussion on System Interdependency under FDIA

The interdependency between electricity and gas systems worsen the system security, i.e., the FDIA on electricity system has adverse impact on gas system and vice versa. TABLE 5-6, 5-7 and 5-8 present the impact of EL-FDIA on GLS, GL-FDIA on ELS and GD-FDIA on ELS, respectively. The EL-FDIA has minor impact on gas system compared with GL-FDIA on gas system and GL-FDIA or GD-FDIA have also minor impact on electricity system compared with EL-FDIA on electricity system. In order to observe a more obvious result, TABLE 5-6 is under the fixed 10% of GD-FDIA and TABLEs 5-7 and 5-8 are under the fixed 10% of EL-FDIA.
A smooth increase of GLS is shown in TABLE 5-6, i.e., the increase of GLS with the 20% increase of EL-FDIA is only 3kcf. The GLS accounts for 1% when EL-FDIA is at 20%. The potential reasons for this result are i) the electricity system is resilient enough based on multiple electricity generators and renewable generators which does not require massive supply from gas system and ii) the overall operation cost and load shedding of IEGS will increase significantly provided that gas system provides more supply to electricity system while implements more GLS. In TABLE 5-7, GL-FDIA causes 48.5MWh ELS when AIL is at 20%. The result is on the contrary of TABLE 5-6 since GL-FDIA causes gas wells supply more on gas system itself, which can be also viewed from Fig. 5-3. The increase of GL-FDIA does not result in significant increase of GLS, which indicates that there is far less gas to power flow. In TABLE 5-8, GD-FDIA shows a more severe impact on ELS. The ELS when GD-FDIA is 0% is only 34.5MWh while it increases dramatically and reaches 111.7 when GD-FDIA is 20%. The reason is that GD-FDIA directly increases the gas density, which accordingly limits the gas flow and the gas turbine production is restricted.

5.6.5 Comparison with RO Based Mitigation Scheme

State estimation calculates the states of all buses and filters raw measurements based on the data from the SCADA system. Then reliable measurement will be provided to system operators to conduct system operation, e.g., economic dispatch, optimal power flow and contingency analysis, etc. In practice, there are three possible scenarios under FDIA: i) FDIA is detected by state estimator, thus launched unsuccessfully; ii) FDIA is launched successfully and the system operator takes immediate corrective measures according to the falsified measurements; iii) FDIA is launched successfully but the risk mitigation scheme proposed in this paper can be deployed.

The first scenario can be handled by deterministic optimization shown in case 1 based on the ‘clean’ load measurement with false data filtered out. The second scenario needs to be modelled with the expectation of optimization solutions. The realization for the second scenario requires huge computational burden since the computational dimension is large, i.e., i) EL-FDIA can be manipulated on 20 electricity loads, ii) GL-FDIA can be manipulated on 3 gas loads, iii) GD-FDIA can be manipulated on 4 gas pipelines and iv) this mitigation scheme considers 24 time periods. For the sake of computational efficiency, the mitigation scheme based on scenario approach is ignored in this study. The third scenario is computational efficient and can be modelled by either DR-FMS.
or robust optimization based FDIA mitigation scheme (R-FMS). Furthermore, the real-
time mitigation can be implemented prior to the real attacks by considering reasonable 
and relatively reliable attack scenarios. Note that the FDIA includes EL-FDIA, GL-
FDIA and GD-FDIA. DR-FMS which mimics moderate-conservative FDIA scenarios 
is compared with R-FMS representing the worst-case FDIA scenario on the system.

The comparison made between DR-FMS and R-FMS is in Figs. 5-8 and 5-9. In 
general, the (flow-capacity ratio) FCR from R-FMS is higher than that from DR-FMS 
for all power lines and gas pipelines. In Fig. 5-6, the FCR in pipeline N1-2 has the 
highest level. The FCR solved by DR-FMS is 90% and 98% by R-FMS. Pipeline N4-
N3 has the second-highest FCR and the difference between DR-FMS and R-FMS is 9%. 
Fig. 5-7 depicts the FCR at the lowest load period, which shows lower FCR for all 
power lines and gas pipelines. The largest FCR difference from DR-FMS and R-FMS 
is for line 27-28, which is 13%. For line 6-7, the FCR difference modelled by two 
schemes is the lowest for both peak and lowest load period. To summarize, DR-FMS 
not only provide more economic operation scheme but yields lower FCR since the 
FDIA is considered in a moderate robust manner.

5.7 Chapter Summary

A risk mitigation scheme for IEGS against FDIA is proposed in this paper with a 
two-stage DRO model. The hierarchical two-stage framework is able to determine both 
day-ahead and real-time system optimal operation schemes considering the impact of 
FDIA and renewable uncertainties on electricity load, gas load and gas density. A 
tractable SDP formulation is built for the original DR-FMS, which is solved by CGA 
in an iterative manner. Through the extensive case studies, the key findings are listed 
below:

▪ Considering all three types of FDIA, i.e., EL-FDIA, GL-FDIA and GD-FDIA, 
leads to higher economic results and more load shedding than considering two types or 
one type of FDIA.

▪ Load shedding is more sensitive to EL-FDIA than GD-FDIA or GL-FDIA.

▪ DRO provides less-conservative results than RO in terms of economic 
performance and load shedding.

▪ Renewable generation uncertainty is necessary to consider, which leads to 3.7% 
more operation cost.
The proposed DR-FMS ensures the economic performance of IEGS by providing a two-stage risk mitigation scheme via implementing efficient load shedding under FDIA and renewable uncertainty. The beneficiaries of this work include: network operators can have powerful operation models, end customers will enjoy better supply security, and renewable can penetrate to the maximum level without much curtailment.
Chapter 6

Voltage Management in Integrated Energy Systems Considering Interdependency and Renewable Uncertainty

This chapter investigates the volt-VAR optimization within the integrated electricity and gas systems. The optimal voltage management is achieved through efficiently coordinating the operation of on-load tap changers, photovoltaic systems, and shunt capacitor banks.
## Statement of Authorship

<table>
<thead>
<tr>
<th>This declaration concerns the article entitled:</th>
<th>Voltage Management in Integrated Energy Systems Considering Interdependency and Renewable Uncertainty</th>
</tr>
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<tr>
<td>Publication status: Accepted</td>
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<tr>
<td>Candidate’s contribution to the paper</td>
<td>The candidate and the second author jointly proposed the idea of the paper, and they designed the methodology. The candidate designed the case studies, and executed most of the coding to derive the experimental results. The third author helped the candidate with the format of the paper, and improvement of academic writing. The percentage of the candidate did compared with the whole work is indicated as follows: Formulation of ideas: 70% Design of methodology: 80% Experimental work: 90% Presentation of data in journal format: 70%</td>
</tr>
<tr>
<td>Statement from candidate</td>
<td>This paper reports on original research I conducted during the period of my Higher Degree by Research candidature.</td>
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<tr>
<td>Signed</td>
<td>Pengfei Zhao Date 03/04/2020</td>
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</table>
6.1 Abstract

Volt/VAR optimization (VVO) is one important operation in distribution systems to maintain acceptable voltage profiles. However, the high penetration of renewable generation poses severe challenges to VVO, leading to voltage deviation and fluctuation. This is further complicated by the growing coupling between electricity and natural gas systems. To resolve unacceptable voltage deviation under energy system interdependency, this paper proposes a co-optimization of VVO for an integrated electricity and gas system (IEGS) with uncertain renewable generation. A two-stage data-driven distributionally robust optimization (DRO) is developed to model the coordinated optimization problem, which determines two-stage VVO and operation schemes with dispatch and corrective adjustment through active power regulation and reactive power support in both day-ahead and real-time stage. A semidefinite programming is reformulated to ensure the tractability and the proposed problem is solved by a constraint generation framework. Simulation studies are conducted on a 33-bus-6-node and a 69-bus-20-node IEGS. Case studies demonstrate that the interdependency between electricity and gas systems reduces 4.7% of operation cost and a significant rise in the voltage profile.

6.2 Nomenclature

*Indices and sets*

- $t, T$: Index and set for time periods.
- $b, B$: Index and set for electricity buses.
- $i_e, I_e$: Index and set for traditional distributed generators (DG).
- $i_g, I_g$: Index and set for natural gas sources.
- $g_T, GT$: Index and set for gas turbines.
- $j, J$: Index and set for renewable power generators.
- $l_e, L_e$: Index and set for power lines.
- $l_g, L_g$: Index and set for gas pipelines.
- $k_e, K_e$: Index and set for electric loads.
- $k_g, K_g$: Index and set for gas loads.

*Parameters*
### Chapter 6  
**Voltage Management in IES with Renewable Uncertainty**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^V, w_1^E$</td>
<td>Weighting coefficients for voltage regulation and economic dispatch oriented sub-objectives in the first stage.</td>
</tr>
<tr>
<td>$w_2^V, w_2^E$</td>
<td>Weighting coefficients for voltage regulation and economic dispatch oriented sub-objectives in the second stage.</td>
</tr>
<tr>
<td>$\pi_v$</td>
<td>the penalty cost coefficient for penalizing the voltage deviation.</td>
</tr>
<tr>
<td>$V_b^{ref}$</td>
<td>Nominal voltage magnitude.</td>
</tr>
<tr>
<td>$\lambda^a_{sub}, \lambda^r_{sub}$</td>
<td>Unit cost for active and reactive power supplied from upper market.</td>
</tr>
<tr>
<td>$\lambda_i^a, \lambda_i^b, \lambda_i^c$</td>
<td>Cost coefficients for generation of traditional DG $i_e$.</td>
</tr>
<tr>
<td>$\lambda_i^+$, $\lambda_i^-$</td>
<td>Cost coefficient for up and down reserve of traditional DG $i_e$.</td>
</tr>
<tr>
<td>$\lambda_i^e, \lambda_j^e$</td>
<td>Regulation cost coefficient for traditional DG $i_e$ and renewable power generator $j$.</td>
</tr>
<tr>
<td>$\lambda_i^{ls}, \lambda_g^{ls}$</td>
<td>Penalty cost coefficient for electricity and gas load shedding.</td>
</tr>
<tr>
<td>$P_{sub,max}$</td>
<td>Maximum active power transfer of substation.</td>
</tr>
<tr>
<td>$R_{i_e}^+, R_{i_e}^-$</td>
<td>Maximum up and down reserve capacity of traditional DG $i_e$ at time $t$.</td>
</tr>
<tr>
<td>$R_{gt}^+, R_{gt}^-$</td>
<td>Maximum up and down reserve capacity of gas turbine $gt$ at time $t$.</td>
</tr>
<tr>
<td>$P_{i_e,max}, P_{i_e,min}$</td>
<td>Maximum and minimum output of traditional DG $i_e$.</td>
</tr>
<tr>
<td>$P_{i_g,max}, P_{i_g,min}$</td>
<td>Maximum and minimum output of natural gas source $i_g$.</td>
</tr>
<tr>
<td>$P_{gt,max}, P_{gt,min}$</td>
<td>Maximum and minimum output of gas turbine $gt$.</td>
</tr>
<tr>
<td>$V_b^{max}, V_b^{min}$</td>
<td>Maximum and minimum voltage limit.</td>
</tr>
<tr>
<td>$\delta^{OLT C}$</td>
<td>Size of change for each step in OLTC tap position.</td>
</tr>
<tr>
<td>$nTP_{max}^{OLT C}$</td>
<td>Maximum allowed number of switching operations of OLTC.</td>
</tr>
<tr>
<td>$\omega_{j,t}^{PS}$</td>
<td>Forecasted active power output of renewable power generator $j$ at time $t$.</td>
</tr>
</tbody>
</table>
Chapter 6 Voltage Management in IES with Renewable Uncertainty

\( u_{PV} \)  Associated coefficient for connecting active and reactive PV power.

\( P_F^{PV,\text{min}} \)  Minimum power factor of PV system \( pv \).

\( Q_{cap} \)  Reactive power capability for capacitor bank \( cb \).

\( V_0 \)  Reference voltage magnitude.

\( f_{le,\text{max}}, f_{le,\text{max}}^r \)  Maximum active and reactive power flow of line \( le \).

\( P_{ke,t}, Q_{ke,t}, P_{kg,t} \)  Active and reactive electricity load and gas load at time \( t \).

\( Pr_{ig,\text{max}}, Pr_{ig,\text{min}} \)  Maximum and minimum gas pressure of gas pipeline \( lg \).

\( \gamma_{ig} \)  Coefficient for Weymouth equation.

\( f_{ig,\text{max}} \)  Maximum gas flow of pipeline \( lg \).

\( p_{ke,\text{max}}, p_{kg,\text{max}}^{ls} \)  Maximum electricity and gas load shedding at time \( t \).

\( \eta_{ie,t}, \eta_{gt,t} \)  Participation factor for reserves of traditional DG \( ie \) and gas turbine \( gt \) at time \( t \).

**Variables and functions**

\( P_{sub,t}^s, Q_{sub,t}^s \)  Scheduled active and reactive power supply from upper market.

\( P_{sub,t}^r, Q_{sub,t}^r \)  Regulated active and reactive power supply from upper market.

\( P_{ie,t}^s, P_{ie,t}^r \)  Scheduled and regulated output of traditional DG \( ie \) at time \( t \).

\( P_{ig,t}^s, P_{ig,t}^r \)  Scheduled and regulated output of natural gas source \( ig \) at time \( t \).

\( r_{ie,t}^+, r_{ie,t}^- \)  Up and down reserve of traditional DG \( ie \) at time \( t \).

\( r_{gt,t}^+, r_{gt,t}^- \)  Up and down reserve of gas turbine \( gt \) at time \( t \).

\( V_{b,t}^s, V_{b,t}^r \)  Scheduled and regulated voltage of bus \( b \) at time \( t \).

\( V_{sub,t}^s, V_{sub,t}^r \)  Scheduled and regulated voltage of substation at time \( t \).

\( T_{OLTC}^s, T_{OLTC}^r \)  Scheduled and regulated tap position of OLTC at time \( t \).

\( \omega_{j,t}^s, \omega_{j,t}^{r,\text{re}} \)  Scheduled and regulated reactive power output of renewable power generator \( j \) at time \( t \).

\( u_{cb,t}^s, u_{cb,t}^r \)  Scheduled and regulated switch status for capacitor bank \( cb \) at time \( t \).
Chapter 6 Voltage Management in IES with Renewable Uncertainty

\( Q_{cb,t}^s, Q_{cb,t}^r \)

Scheduled and regulated reactive power output for capacitor bank \( cb \) at time \( t \).

\( V_{b,t}^{s,ini}, V_{b,t}^{s,ter} \)

Scheduled voltage magnitude for initial and terminal nodes.

\( V_{b,t}^{re,ini}, V_{b,t}^{re,ter} \)

Regulated voltage magnitude for initial and terminal nodes.

\( f_{le,t}^{a,s}, f_{le,t}^{r,s} \)

Scheduled active and reactive power flow at time \( t \).

\( f_{le,t}^{a,re}, f_{le,t}^{r,re} \)

Regulated active and reactive power flow at time \( t \).

\( f_{lg,t}^{ini}, f_{lg,t}^{ter} \)

Gas flow from initial node and to terminal node of pipeline \( l_g \) at time \( t \).

\( P_{lg,t}^s, P_{lg,t}^r \)

Scheduled and regulated gas pressure of gas pipeline \( l_g \) at time \( t \).

\( P_{lg,t}^{s,ini}, P_{lg,t}^{s,ter} \)

Scheduled gas pressure of initial and terminal nodes of pipeline \( l_g \) at time \( t \).

\( P_{lg,t}^{re,ini}, P_{lg,t}^{re,ter} \)

Regulated gas pressure of initial and terminal nodes of pipeline \( l_g \) at time \( t \).

\( p_{k_e,t}^{ls}, p_{k_g,t}^{ls} \)

Electricity and gas load shedding at time \( t \).

\( x, y \)

Vectors of first and second stage variables.

**Uncertainty**

\( \xi_{j,t} \)

Uncertainty of renewable power forecast of \( j \) at time \( t \).

\( D_{\xi_{j,t}} \)

Ambiguity set for renewable power uncertainty.

\( \mu_{\xi_{j,t}}, \Sigma_{\xi_{j,t}} \)

Mean vector and covariance matrix for renewable forecast uncertainty.

\( \Theta \)

Second moment matrix.

\( VS \)

Polyhedral set of extreme points.

### 6.3 Introduction

Volt-VAR optimization (VVO) is one primary function in the distribution management system to maintain voltage in an acceptable range by optimally coordinating equipment, e.g., capacitor banks, on-load tap changers (OLTC) and voltage regulators [120-130]. Paper [121] proposes a deterministic VVO as mixed-integer quadratic programming to control voltage and VAR devices for day-ahead operation. Considerable loss reduction and total demand reduction are achieved through
the proposed VVO. To achieve energy savings and peak demand reduction through voltage reduction, a multi-objective VVO is proposed in [122] using a non-dominated sorting genetic algorithm. Paper [123] proposes a three-phase distribution system considering unbalancing with battery storage providing reactive power. Both power loss and energy purchase cost are minimized. In [124], to extend the life of distribution system transformers, a detailed model of life loss is proposed to estimate the ageing reduction of transformers under VVO.

The penetration of renewable energy has dramatically increased over the past decade. However, due to its variable and intermittent nature, it poses operational and security challenges to VVO by affecting normal operations of OLTCs and capacity banks [127, 131, 132]. Existing literature has considered renewable power uncertainties in designing economic and reliable periodical equipment scheduling plans. A two-stage chance-constrained VVO is employed to handle the uncertainties of distributed generation and load demand [127]. Paper [131] proposes a chance-constrained optimization to model the randomness of renewable energy and minimize feeder power losses while avoiding voltage violations, solved by a gradient descent based algorithm. A hierarchical robust optimization (RO) is adopted for coordinating reactive compensators to guarantee voltage magnitudes [132]. This reactive power optimization is formulated to a mixed-integer convex-based programming based on the conic relaxation of the branch flow. And a modified column-and-constraint generation algorithm based on the second-order cone programming is employed to solve the problem.

The interdependency of multi-energy systems, electricity, natural gas, and heating/cooling, is also becoming markedly common, which has many implications for VVO. Research on integrated electricity and gas systems (IEGS) is widely investigated in traditional problems, e.g., modelling, operation and planning [133-135]. However, high renewable integration brings security and operational challenges to managing IEGS, particularly to system voltage, due to intermittency and fluctuation, [104, 110, 135]. To hedge against renewable power uncertainty in IEGS, deterministic optimization, two-stage and multi-stage stochastic optimization (SO) have been widely used [110]. For example, paper [104] proposes a robust security-constrained unit commitment in IEGS considering distributed natural gas storage to enhance operational reliability.
Overall, RO and SO are the two main approaches to handle uncertainties from growing renewable energy resources for both VVO and IEGS operation problems. As for SO, it either assumes an explicit distribution for random variable or requires a large number of data samples. It is prone to causing errors when the historical data is not adequately sufficient to represent true distributions and inevitably leads to high computational burden if a large number of data samples are used. As for RO, it does not require an exact probability distribution but constrains uncertain variables in a predefined uncertainty set. RO considers the worst-case scenario against all realizations characterized by uncertainty sets, which could have extremely low probability and thus produces over-conservative results. In practice, it is rare that the uncertainty realization appears on the bounds of uncertainty set. The interval-based RO strictly ensures that the optimization is feasible even considering the worst-case solution. The uncertainty is treated as variables bounded within the predetermined set without the association with any probability distribution.

Therefore, it is of necessity to handle the uncertainty through a relative less conservative optimization technique without requiring large datasets. As a promising optimization method to handle uncertainties, distributionally robust optimization (DRO) inherits the advantages of both RO and SO, overcoming the explicit assumptions on probability distributions of SO and over-conservatism of RO [65, 136-138]. For DRO, the ambiguity set is constructed by statistical information, such as moment, to restricting possible distributions. Based on more valuable distribution information, research finds that the best estimate of the distribution can be obtained through the statistical fitting. Accordingly, statistical distance information can be added in the ambiguity set and thus the size of the ambiguity set can be controlled. In addition, compared with RO, DRO determines expected results over all possible distributions, which are less-conservative.

Paper [78] proposes a risk-based optimal gas power flow by using DRO. It considers the zonal linepack and linepack reserve to distinguish fuel suppliers and ensure the security of gas systems. A security-constrained two-stage ED with renewable power uncertainty modelled by DRO is designed in [64]. The segregated linear decision rule is used to affinely approximate the decision variables after the first time period which reduces the computational burden. Paper [57] investigates a DRO for IEGS considering the uncertainties of electricity and gas loads. Price-Based demand response is
considered to improve energy efficiency and economic benefits. DRO can provide a less conservative solution for VVO problem when capturing renewable uncertainties and thus mitigate the impact on voltage deviations caused by renewable uncertainties. In addition, the two-stage framework contains both day-ahead and real-time framework, which provides flexible measures for system operators with adjustment capability.

This paper proposes a novel coordinated two-stage multi-objective optimization for voltage control in economic dispatch (ED), considering uncertain renewable generation and multi-vector energy system integration. The two-stage voltage constrained optimization is referred to as TS-VCO for simplicity. The optimal voltage is achieved through efficiently coordinating the operation of OLTCs, photovoltaic (PV) systems, and shunt capacitor banks. In the first stage, based on historical PV output, an initial day-ahead operation plan for traditional DGs, natural gas sources, OLTCs and capacitor banks is produced to maintain voltage and minimize daily operation cost. In the second stage, after the realization of uncertain PV output, the recourse action is developed to control voltage controlling those devices, meanwhile minimizing system operation cost in real time. The original TS-VCO is transformed into a conic tractable form with the dual formulation and solved by constraint generation algorithm (CGA). Case studies demonstrate that voltage control devices and dispatchable generators can be optimally controlled and coordinated to realise the designed objectives.

The main contributions of this paper are as follows:

1) To the best of authors’ knowledge, none existing work has investigated VVO in an IEGS, where this paper is the first such effort to fill the research gap. The strong coupling of power and gas infrastructure and tight interdependency between two systems are considered.

2) It develops a multi-objective coordinated optimization for maintaining acceptable voltage while considering system operation cost of IEGS, which ensures system security and economic performance.

3) The two-stage DRO approach is first applied in VVO, which provides less-conservative results using RO and requires fewer data samples. It can handle renewable uncertainties effectively, providing flexible measures for IEGS operators.

The rest of this paper is organized as follows. Section 6.4 presents objective function and system constraints of TS-VCO. The DRO method and associated reformulations are presented in section 2.1.2. Section 6.5 demonstrates case studies and the
performance of the TS-VCO. Section 6.6 concludes this paper.

6.4 Problem formulation

The proposed TS-VCO contains i) day-ahead co-optimization that restricts voltages for all buses while scheduling traditional DGs and natural gas sources, and ii) real-time recourse action that regulates voltage and redispatches generators considering PV output uncertainty. When implementing economic operation in distribution systems, the voltage profile of each bus can also be improved by optimally coordinating voltage regulating equipment [139-142]. In this paper, maintaining the voltage deviation within an acceptable range is considered as voltage profile improvement.

6.4.1 TS-VCO Objective Function

The first-stage problem in (6-1) is to simultaneously minimize i) the voltage deviation for all buses, $\Gamma_1^V$ in (6-2) and ii) the cost of generation and reserve capacity, $\Gamma_1^E$ in (6-3), respectively. $\Gamma_1^V$ represents the total voltage deviation for all buses in the entire time horizon. $\Gamma_1^E$ includes i) generation cost of traditional DGs and natural gas sources and ii) reserve cost of traditional DGs. The generation cost function of traditional DGs is quadratic with coefficients $a_i$, $b_i$, and $c_i$. The weighting coefficients $w_1^V$ and $w_1^E$ represent the priorities that TS-VCO have on $\Gamma_1^V$ and $\Gamma_1^E$, respectively. It should be noted that the penalty cost coefficient $\pi_v$ is applied for penalizing the voltage deviation on each bus. The penalty cost coefficient enables to transform voltage deviation to monetary lost which can be combined with operation cost.

$$\Gamma_1 = \min \left( w_1^V \pi_v \Gamma_1^V + w_1^E \Gamma_1^E \right)$$  \hspace{1cm} (6-1)

$$\Gamma_1^V = \min \sum_{b \in B, t \in T} | V_{b,t} - V_{b,ref}^r |$$  \hspace{1cm} (6-2)

$$\Gamma_1^E = \min \sum_{i_e \in I_e, i_g \in I_g, t \in T} \lambda_{sub}^g P_{sub,t}^s + \lambda_{sub}^r Q_{sub,t}^s + \lambda_{i_e}^a p_{i_e,t}^2 + \lambda_{i_e}^b p_{i_e,t}^s$$
$$+ \lambda_{i_e}^i P_{i_g,t}^s + \lambda_{i_e}^r r_{i_e,t}^+ + \lambda_{i_e}^r r_{i_e,t}^-$$  \hspace{1cm} (6-3)

Similar to (6-1)-(6-3), (6-4)-(6-6) are the second-stage overall objective and sub-objectives. The second-stage optimization considers load shedding to keep the system balance under fluctuation caused by renewable uncertainties. It should be noted that $\Gamma_2^E$ contains i) the penalty cost for PV curtailment, ii) regulated generation cost of traditional DGs and natural gas sources and iii) electricity and gas load shedding cost.
\[
\Gamma_2 = \min w_2^V \pi_v \Gamma_2^V + (1 - w_2^V) \Gamma_2^E
\]

\[
\Gamma_2^V = \min \sum_{b \in B, t \in T} |V_{b,t}^{re} - V_{b}^{ref}|
\]

\[
\Gamma_2^E = \min \sum_{i_e \in I_e, i_g \in I_g, t \in T, k_e \in K_e, k_g \in K_g} \lambda_{sub}^{a} P_{sub,t}^{re} + \lambda_{sub}^{r} Q_{sub,t}^{re}
\]

\[
+ \lambda_{j}^{e} \left( a_{j,t}^{e} - \xi_{j,t} \right) + \lambda_{i_e}^{re} \left( P_{i_e,t}^{re} - P_{i_e}^{re} \right) + \lambda_{i_e}^{rez} \left( P_{i_e,t}^{re} - P_{i_e}^{re} \right)
\]

\[
+ \lambda_{k_e}^{is} P_{k_e,t}^{ls} + \lambda_{k_g}^{ls} P_{k_g,t}^{ls}
\]

6.4.2 Day-ahead VVO

In the first stage, the day-ahead optimization is based on the forecasted renewable output before its uncertainty realised. Equations (6-7)-(6-29) represent the first-stage constraints. The active and reactive power for substation injected from the upper level is limited in (6-7) and (6-8). The up and down reserve capacity for traditional DGs and gas turbines is constrained in (6-9) and (6-10). In the distribution network, the proposed day-ahead reserve capacity is for compromising the real-time renewable power uncertainties. Constraints (6-11)-(6-12) ensure the generation of traditional DGs and gas turbines within the predefined limits considering reserve capacity. The voltage magnitude for all buses is regulated in (6-13) by setting minimum and maximum limits.

In (6-14), the substation voltage can be determined by OLTC tap position and the step size of each tap position. Constraint (6-15) regulates the total operation number of OLTC tap since too many operations will accelerate the wear process of the transformer [128, 143, 144]. In (6-16), the reactive power of PV is described by the forecasted active PV power output and the power factor as defined in (6-17). The reactive power from capacity banks is given in (6-18). The linearized DistFlow for distribution systems is presented in (6-19) and (6-20). The power balance constraints for active and reactive power are in (6-21) and (6-22). The output of natural gas sources is constrained in (6-23). Equations (6-24) and (6-25) are the constraints on gas pressure, where, in distribution systems, the pressure of initial nodes is always higher than terminal nodes.

In (6-26), Weymouth equation is used to characterize the relationship between gas flow and pressure. The gas flow is constrained in (6-27). The relationship between the gas turbine output and injected gas flow is given in (6-28). And equation (6-29) models nodal gas balancing.
\[ 0 \leq P^s_{\text{sub},t} \leq P_{\text{sub,max}} \]  
(6-7)

\[ 0 \leq Q^s_{\text{sub},t} \leq Q_{\text{sub,max}} \]  
(6-8)

\[ 0 \leq r^+_{(\cdot),t} \leq R^+_{(\cdot)}, \{\} = i_e, g_t \]  
(6-9)

\[ 0 \leq r^-_{(\cdot),t} \leq R^-_{(\cdot)}, \{\} = i_e, g_t \]  
(6-10)

\[ P^s_{(\cdot),t} + r^+_{(\cdot),t} \leq P_{(\cdot),\text{max}}, \{\} = i_e, g_t \]  
(6-11)

\[ P_{(\cdot),\text{min}} \leq P^s_{(\cdot),t} - r^-_{(\cdot),t}, \{\} = i_e, g_t \]  
(6-12)

\[ V_{b,\text{min}} \leq V_{b,t} \leq V_{b,\text{max}} \]  
(6-13)

\[ V^s_{\text{sub},t} = V^\text{ref}_{\text{sub}} + \delta_{O LTC} T_P^{s,O LTC} \]  
(6-14)

\[ \sum_{t \in T} \left| T_P^{s,O LTC} - T_P^{s,O LTC}_{t-1} \right| \leq n T_P^{max,O LTC} \]  
(6-15)

\[ 0 \leq \omega^q_{j,t} \leq u_{PV} \omega^p_{j,t} \]  
(6-16)

\[ u_{PV} = \sqrt{\frac{1 - P_{PF,\text{min}}^2}{P_{PF,\text{min}}^2}} \]  
(6-17)

\[ Q^s_{cb,t} = u^s_{cb,t} Q^\text{cap}_{cb} \]  
(6-18)

\[ V^s_{b,t} - V_{b,t} = \left( f^a_{s,t} r_{e} + f^r_{s,t} x_{l_e} \right) / V_0 \]  
(6-19)

\[ 0 \leq f^l_{s,t} \leq f^l_{s,\text{max}}, \{\} = a, r \]  
(6-20)

\[ \sum_{i_e \in I_e} P^s_{i_e,t} + \sum_{j \in J} \omega^p_{j,t} + \sum_{i_e \in I_e} f^a_{s,\text{ini}} = \sum_{l_e \in L_e} f^s_{a,\text{ini}} \]  
(6-21)

\[ \sum_{l_e \in L_e} f^a_{s,\text{ter}} + \sum_{g_t \in G_T} P^s_{g_t,t} = \sum_{k_e \in K_e} P_{k_e,t} \]  
(6-21)

\[ \sum_{i_e \in I_e} Q^s_{i_e,t} + \sum_{j \in J} \omega^q_{j,t} + \sum_{cb \in CB} Q^s_{cb,t} + \sum_{l_e \in L_e} f^r_{s,\text{ini}} + \sum_{l_e \in L_e} f^r_{s,\text{ter}} = \sum_{k_e \in K_e} Q_{k_e,t} \]  
(6-22)

\[ P_{i_g,\text{min}} \leq P^s_{i_g,t} \leq P_{i_g,\text{max}} \]  
(6-23)
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\[ P_{ri,g,min}^2 \leq P_{ri,g,t}^2 \leq P_{ri,g,max}^2 \]  
\[ p_{ri,g,t}^{s,ini} \geq p_{ri,g,t}^{s,ter} \]  
\[ f_{lg,t}^s = y_{lg} \left( p_{ri,g,t}^{s,ini} - p_{ri,g,t}^{s,ter} \right) \]  
\[ 0 \leq f_{lg,t}^s \leq f_{lg,max} \]  
\[ p_{gt,t}^s = c_{gt} f_{lg,gt}^s \]  
\[ \sum_{l_g \in L_g} p_{lg,t}^s + \sum_{l_g \in L_g} f_{lg,t}^s,ini - \sum_{l_g \in L_g} f_{lg,t}^s,ter = \sum_{k_g \in K_g} P_{kg,t} + \sum_{g \in G} P_{gt,t} \]

6.4.3 Real-time VVO

The real-time corrective dispatch is in the second stage considering renewable power uncertainty, which regulates voltage and generation output of traditional DGs and natural gas sources. Equations (6-30) and (6-31) limit the power transfer of substations. The regulated output of traditional DGs and gas turbines is given in (6-32). Electricity and gas load shedding constraint is in (6-33). Equation (6-34) is the constraint for voltages for all buses. The substation voltage is defined in (6-35). In (6-36), the regulated total operation number of OLTC tap is constrained. The reactive power of PV generators and capacity banks are ensured in (6-37) and (6-38). DistFlow is applied again in the second stage describing power flow in (6-39) and (6-40). The active and reactive power balance constraints are in (6-41) and (6-42), respectively. The regulated output of natural gas sources is given in (6-43). For modelling gas flow, the Weymouth equation is presented from (6-44)-(6-47) with gas pressure constrained. Constraint (6-48) describes the relationship between the gas flow injection and gas turbine output.

The aim of the regulation of traditional DGs and natural gas sources is to mitigate adverse effects from renewable output deviation, which is achieved by adjusting reserves for power capacity from (6-49) to (6-51). In (6-49), renewable output deviation should be within the up and down reserve limits. To address renewable power uncertainty, the participation factors \( \eta_{i_e,t} \) and \( \eta_{i_g,t} \) are defined in (6-50) and (6-51) to represent the regulation commitment by traditional DGs and natural gas sources. Finally, (6-52) presents the balancing condition of the gas system.
\begin{align*}
0 & \leq P^r_{sub,t} \leq P^r_{sub,max} \quad (6-30) \\
0 & \leq Q^r_{sub,t} \leq Q^r_{sub,max} \quad (6-31) \\
p^r_{\{i\},t} - r^-_{\{i\},t} & \leq p^r_{\{i\},t} \leq p^r_{\{i\},t} + r^+_{\{i\},t}, \{i\} = i_e, g_t \\
0 & \leq p^ls_{\{i\},t} \leq p^ls_{\{i\},max}, \{i\} = k_e, k_g \quad (6-33) \\
V_{b,\text{min}} & \leq V^r_{b,t} \leq V_{b,\text{max}} \quad (6-34) \\
V^r_{sub,t} & = V^r_{\text{ref}} + \delta^{\text{OLTIC}} T^r_{t} \quad (6-35) \\
\sum_{t \in T} |T^r_{t} - T^r_{t-1}| & \leq n T^{\text{OLTIC}}_{\text{max}} \quad (6-36) \\
0 & \leq \omega^{Q,\text{re}}_{j,t} \leq u_{\text{pv}} \omega^{P,\text{re}}_{j,t} \quad (6-37) \\
Q^r_{cb,t} & = u^{r^f}_{cb,t} V^r_{cb,t} \quad (6-38) \\
v^{r,\text{init}}_{b} - v^{r,\text{ter}}_{b} & = (f^{\text{are}}_{t_e,t} r_{t_e} + f^{r,\text{re}}_{t_e,t} x_{t_e})/V_0 \quad (6-39) \\
0 & \leq f^{\{,\text{re}}_{t_e,t} \leq f^{\{,\text{max}}_{t_e,t} \{,\} = a, r \quad (6-40) \\
\sum_{t_e \in L_e} p^r_{t_e,t} + \sum_{j \in j} \xi_{j,t} + \sum_{g \in G_T} P^s_{g,t} & = \sum_{k \in K_e} P_{k_e,t} + \Delta P_{k_e,t} - P^ls_{k_e,t} \quad (6-41) \\
\sum_{t_e \in L_e} Q^s_{t_e,t} + \sum_{j \in j} \omega^{Q,s}_{j,t} + \sum_{cb \in CB} Q^s_{cb,t} + \sum_{t_e \in L_e} f^{r,s,\text{init}}_{t_e,t} - \sum_{t_e \in L_e} f^{r,s,\text{ter}}_{t_e,t} & = \sum_{k \in K_e} Q_{k_e,t} \quad (6-42) \\
P_{g,\text{min}} & \leq P^r_{t_g,t} \leq P_{g,\text{max}} \quad (6-43) \\
P^r_{t_g,\text{min}} & \leq P^{r^2}_{t_g,t} \leq P^r_{t_g,\text{max}} \quad (6-44) \\
P^{r,\text{re,\text{init}}}_{t_g,t} & \geq P^{r,\text{re,\text{ter}}}_{t_g,t} \quad (6-45) \\
f^{r,\text{re}}_{t_g,t} \bigg| f^{r,\text{re}}_{t_g,t} & = \gamma_{t_g} \left( P^{r,\text{re,\text{init}}}_{t_g,t} - P^{r,\text{re,\text{ter}}}_{t_g,t} \right) \quad (6-46) \\
0 & \leq f^{r,\text{re}}_{t_g,t} \leq f_{t_g,\text{max}} \quad (6-47) \\
P^r_{g,t} & = c_{gt} f^{r,\text{re}}_{t_g,t} \quad (6-48)
\end{align*}
\[ r_{i_e,t}^{-} \leq \eta_{i_e,t} \sum_{j \in J} (\omega_{j,t}^{e} - \xi_{j,t}) \leq r_{i_e,t}^{+}, \{ \cdot \} = i_e, gt \] (6-49)

\[ 0 \leq \eta_{\cdot,t} \leq 1, \{ \cdot \} = i_e, gt \] (6-50)

\[ \sum_{i_e \in I_e} \eta_{i_e,t} + \sum_{g \in G_T} \eta_{i_g,t}^{gt} = 1 \] (6-51)

\[ \sum_{i_g \in I_g} p_{i_g,t}^{re} + \sum_{g \in G_T} j_{i_g,t}^{re,ini} - \sum_{l_g \in L_g} j_{i_g,t}^{re,ter} \] (6-52)

\[ = \sum_{k_g \in K_g} p_{k_g,t} + \sum_{g \in G_T} p_{gt,t}^{re} - p_{lt}^{ls} \]

### 6.5 Case Studies

As shown in Fig. 6-1, the proposed TS-VCO and CGA are firstly verified on a modified IEEE 33-bus system connected with a 6-node gas system in radial topology from [145] and [66]. A gas turbine connects the two separated systems, which generates electricity using natural gas. To testify the effectiveness of the TS-VCO in different conditions, comparison between 8 scenarios is considered and the details are given in

![Fig. 6-1. A modified IEEE 33-bus system and a 6-node gas system.](image-url)
TABLE 6-1 CASE ILLUSTRATION

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Weighting coefficients</th>
<th>Optimization method</th>
<th>PV system capacity (kVA)</th>
<th>Connected to gas system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$w^V = 0.5$</td>
<td>Deterministic</td>
<td>200</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>$w^V = 0.5$</td>
<td>Robust</td>
<td>200</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>$w^V = 0.5$</td>
<td>DRO</td>
<td>200</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>$w^V = 0.25$</td>
<td>DRO</td>
<td>200</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>$w^V = 0.75$</td>
<td>DRO</td>
<td>200</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>$w^V = 0.5$</td>
<td>DRO</td>
<td>400</td>
<td>Yes</td>
</tr>
<tr>
<td>7</td>
<td>$w^V = 0.5$</td>
<td>DRO</td>
<td>800</td>
<td>Yes</td>
</tr>
<tr>
<td>8</td>
<td>$w^V = 0.5$</td>
<td>DRO</td>
<td>200</td>
<td>No</td>
</tr>
</tbody>
</table>

TABLE 6-1. Cases 1-3 are used to compare mathematical performance of different optimization methods. Cases 3-5 compare the impact of varying optimization priorities on voltage deviation and economic performance. The impact of PV penetration and capacitor bank capacity are analysed in cases 3, 6 and 7. Case 8 studies the scenario without natural gas connection. In addition, a modified IEEE 69-bus system with a 20-node gas system is used for the test with 8 cases.

6.5.1 Studies on The 33-bus-6-node IEGS

In the electricity system shown in Fig. 6-1, it contains i) 3 traditional DGs connected with bus 13, 21 and 28, ii) 3 PV systems connected with bus 11, 16 and 22 with each capacity of 200kVA, and iii) 7 capacitor banks which have the same capacity of 30kVar. The substation transformer has 32 tap positions with a step size of 0.003 which ranges from -16 to 16. The maximum allowed operation number between two continuous-time slots for OLTC is set as 3. The voltage limit on each bus is set between 0.95 p.u. and 1.05 p.u.. The gas system has 6 nodes, containing 2 natural gas sources and 3 gas loads. The conversion factor 3.313 is used to convert $/kcf to $/MWh when simultaneously considering electricity and gas load shedding in IEGS. This paper uses 100$/MWh and 120$/kcf (400$/$MWh) as the shedding cost for electricity and gas load, respectively. The detailed parameters of natural gas sources and traditional DGs are given in TABLEs 6-2 and 6-3.
Chapter 6  Voltage Management in IES with Renewable Uncertainty

A. Studies on Economic Performance

The economic performance for all cases is analysed first, presented in TABLE 6-4. The cost of the first and second stages are presented in TABLE 6-4. Case 5 has the highest total cost ($30111) while case 4 has the lowest cost ($25398) since the optimization priority of case 5 focuses on minimizing the voltage deviation while gives less focus on economic performance. Case 4 considers 75% of objective weighting on economic performance, which leads to $4713 less total cost than case 5. Cases 1-3 have the same optimization priority on sub-objectives and the same IEGS configuration. Case 1 provides a total cost of $27343, which is 5.8% and 2.8% less than case 2 and 3. The advantage of DRO is the less conservatism when modelling PV output uncertainty compared with RO, which is reflected in the 2.9% less cost of case 3 compared with case 2. Compared with case 3, the PV capacity is doubled and quadrupled in cases 6 and 7, which address the high generation of traditional DGs and natural gas sources. However, compared with case 3, case 6 and 7 cause higher second-stage cost due to the penalty cost of PV output deviation in the first and second stage. Without support from the gas system, case 8 yields 4.7% more cost than case 3. It is because the generation cost from natural gas sources is lower than that of traditional DGs and the pipeline capacity is set enough for large gas flow.

<table>
<thead>
<tr>
<th>Node No.</th>
<th>$P_{ig,min}$ (kcf/h)</th>
<th>$P_{ig,max}$ (kcf/h)</th>
<th>$\lambda_{ig}$ ($/kcf$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1000</td>
<td>6000</td>
<td>2.2</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>3000</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### Table 6-3 Generator Parameters

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>$P_{te,max}$ (MW)</th>
<th>$P_{te,min}$ (MW)</th>
<th>$R^+_t$, $R^-_t$ (MW)</th>
<th>$a_i$ ($/MW$)</th>
<th>$b_i$ ($/MW$)</th>
<th>$c_i$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1.2</td>
<td>0.3</td>
<td>0.2</td>
<td>6000</td>
<td>7100</td>
<td>6200</td>
</tr>
<tr>
<td>21</td>
<td>1.2</td>
<td>0.3</td>
<td>0.2</td>
<td>4500</td>
<td>10500</td>
<td>4000</td>
</tr>
<tr>
<td>28</td>
<td>1.0</td>
<td>0.1</td>
<td>0.2</td>
<td>4500</td>
<td>10500</td>
<td>4000</td>
</tr>
</tbody>
</table>
B. Studies on Voltage Profile

The voltage profiles of 33 buses in 24 time periods for cases 3, 6 and 7 with different capacity of PV systems are shown in Fig. 6-2. The red dotted curve is the mean voltage profile among all buses for clearer presentation and comparison. In case 3, with the

Fig. 6-2. Expected real-time voltage profiles for case 3.

Fig. 6-3. Expected real-time profiles for case 6.

Fig. 6-4. Expected real-time profiles for case 7.
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least PV capacity connected, the voltage level ranges from 0.963 p.u. to 1.016 p.u.
With the higher PV connection in case 6, i.e., 200kVA more capacity of each PV, the voltage profile has been improved by 1%. In case 7, the voltage level reaches up to 1.032 p.u. when the PV capacity is 800kVA, which causes voltage improvement by 1.5% compared with case 3. The comparison between cases 3, 6 and 7 shows the increase of voltage level for all buses with the increasing PV penetration. The PV systems not only provide active power support but reactive power support, which will lead to the reduction of power losses in the real world. In Fig. 6-5, without connecting the gas system, the voltage profile of case 8 ranges between 0.96 p.u. and 1.00 p.u., which is lower than in case 3.

Cases 3, 6, 7 and 8 are used to study the impact of PV uncertainty on voltage deviations with different methods. In Fig. 6-6, voltage profile from the deterministic optimization is the highest, whilst that from RO is the lowest. The voltage difference is approximately 0.01 p.u., i.e., 0.13kV. Compared with the deterministic approach, DRO
leads to a more conservative result. Compared with RO, DRO mitigates the conservatism by 0.008 p.u., which considers the worst uncertainty distribution instead of the worst-case PV uncertainty of RO.

C. Studies on OLTC

The OLTC tap position for 24 time periods is given in Fig. 6-7. With the highest PV capacity connected, the OLTC tap position remains the highest in case 7, which starts from +10 position and maintains at +11 position from the 8th hour to 24th hour. With lower PV capacity, OLTC tap position in cases 3 and 6 are at relatively lower level since the system voltage level is low and can be maintained in an acceptable range without a large deviation from nominal voltage. In case 8, the tap position ranges from +1 to +3 and the maximum tap position deviation is 1, i.e., from +1 to +2 or +2 to +3. The reason is without the gas system connected, there is no power support converted from the gas flow, which will not raise the voltage level.

6.5.2 Studies on the 69-bus-20-node IEGS

The scalability study is conducted in the modified IEEE 69-bus system as given in Fig. 6-8. There are 6 PV systems connected at buses 9, 23, 26, 34, 44 and 58, respectively. The 12 capacitor banks and transformer are used to compensate the reactive power. The 20-node gas system contains two natural gas sources and two gas turbines, which are connected between the gas and electricity system.

A. Studies on Voltage Profile

In Fig. 6-9, the voltage profiles of cases 1, 6 and 8 are studied to investigate the impact from gas system connection. In case 1, it can be seen that the voltage level is decreasing along the main branch from bus 5 to bus 28. And the voltage level remains approximately the same value between bus 28 and bus 50 at 1.02 p.u.. With two gas turbines connected, the voltage level ranges from 0.952 p.u. to 1.020 p.u.. With only one connection with gas system, the voltage level is lower than that of case 1, which ranges from 0.952 p.u. to 1.007 p.u.. Compared with case 1, when no gas turbines equipped, the voltage profile decreases by 0.8% in case 8. The comparison between cases 1, 6 and 8 shows the increase of the voltage level for all buses with the addition of gas system connection.


**B. Studies on Reactive Power Compensators**

This subsection studies the scheduling and impact of capacitor banks and PV systems when regulating the voltage magnitude through reactive power support. The reactive power output of capacitor banks in cases 1, 4, 5 and 8 on selected buses is given in Fig. 6-10. The reactive power of case 4 is the highest since the capacity of capacitor banks...
is twice compared with case 1. At buses 25, 29, 34, 39 and 42, the reactive power output is scheduled in the relatively low level. Since the load level is low with the less requirement of voltage regulation.

The reactive power output of PV systems is given in Figs. 6-11 - 6-14. Overall, the total PV reactive power output follows the trend of the reactive power load curve, where it peaks at 20:00 and the lowest level occurs at 4:00. The reactive power output from PV at bus 23 contributes the most while the PV at bus 34 contributes the least. The potential reason is that bus 34 is connected to both a PV system and a capacitor bank. Accordingly, the requirement of reactive power from PV system is not much. Case 5 has the highest level of PV reactive power output due to the case setting. The reactive power output ranges from 0.14MVar to 0.21MVar. By contrast, the reactive power output from PV systems in case 4 is the lowest since the total capacity of capacitor banks is the highest in all cases, which reduces the reactive power support from PV systems.

6.6 Chapter Summary

This paper proposes a multi-objective optimization for minimizing both operation cost and voltage deviation of IES considering renewable power uncertainty. A two-stage data-driven DRO approach is used to solve the TS-VCO with dual and SDP formulations to ensure computational tractability. The reformulated TS-VCO is solved by CGA with master and subproblems. The key findings from the case studies are:

- Based on a large amount of moment information, DRO produces less conservative results compared with RO, more effective for maintaining voltage deviation and reducing operation cost considering renewable power uncertainty.
- The interdependency between electricity and gas systems reduces 4.7% of operation cost and a significant rise in the voltage profile.
- The proposed TS-VCO is effective in maintaining voltage and saving operation cost considering PV uncertainty.

This work can benefit integrated system operators with powerful operation tool to manage the systems with fewer costs but integrate more renewable energy.
Chapter 7

Distributionally Robust Operation for Integrated Energy Systems with Hydrogen Injection and Gas Quality Improvement

This chapter develops a new gas quality management scheme for hydrogen injection into natural gas systems produced from excessive wind power. It introduces four gas quality indices for the integrated electricity and gas system measuring gas quality, considering the coordinated operation of tightly coupled infrastructures.
# Statement of Authorship

**This declaration concerns the article entitled:**
Distributionally Robust Operation for Integrated Energy Systems with Hydrogen Injection and Gas Quality Improvement

**Publication status:** Accepted

|---------------------------------|--------------------------------------------------------------------------------------------------|

**Candidate’s contribution to the paper**

The candidate and the second author jointly proposed the idea of the paper, and they designed the methodology. The candidate designed the case studies, and executed most of the coding to derive the experimental results. The third author helped the candidate with the format of the paper, and improvement of academic writing. The percentage of the candidate did compared with the whole work is indicated as follows:

- Formulation of ideas: 70%
- Design of methodology: 80%
- Experimental work: 90%
- Presentation of data in journal format: 70%

**Statement from candidate**

This paper reports on original research I conducted during the period of my Higher Degree by Research candidature.

**Signed** Pengfei Zhao  **Date** 04/04/2020
Chapter 7  Distributionally Robust Operation of IEGS with Hydrogen Injection

7.1 Abstract

Power-to-gas (P2G) can convert excessive renewable energy into hydrogen via electrolysis, which can then be transported by natural gas systems to bypass constrained electricity systems. However, the injection of hydrogen could impact gas quality since gas composition fundamentally changes, adversely affecting the combustion, safety and lifespan of appliances.

This paper develops a new gas quality management scheme for hydrogen injection into natural gas systems produced from excessive wind power. It introduces four gas quality indices for the integrated electricity and gas system (IEGS) measuring gas quality, considering the coordinated operation of tightly coupled infrastructures. To maintain gas quality under an acceptable range, the gas mixture of nitrogen and liquid petroleum gas with hydrogen is adopted to address the gas quality violation caused by hydrogen injection. A distributionally robust optimization (DRO) modelled by Kullback-Leibler (KL) divergence-based ambiguity set is applied to flexibly control the robustness to capture wind uncertainty. The KL divergence-based ambiguity set defines the uncertainties within a measured space which limits the shape of probability distributions. Case studies demonstrate that wind power is maximally utilized and gas mixture is effectively managed, thus improving both gas quality and performance of IEGS. The work can benefit system operators with i) ensured gas quality under hydrogen injection ii) low system operation cost and iii) high renewable energy penetration.

7.2 Nomenclature

Indices and sets

\(t, T\)  
Index and set for time periods.

\(n, N\)  
Index and set for nodes in gas system.

\(i_e, I_e\)  
Index and set for traditional distributed generators (DG).

\(i_g, I_g\)  
Index and set for natural gas sources.

\(j, J\)  
Index and set for wind turbines.

\(l_e, L_e\)  
Index and set for power lines.

\(l_g, L_g\)  
Index and set for gas pipelines.

\(k_e, K_e\)  
Index and set for electric loads.

\(k_g, K_g\)  
Index and set for gas loads.

\(k_h, K_h\)  
Index and set for heating loads.
Chapter 7  Distributionally Robust Operation of IEGS with Hydrogen Injection

Parameters

\( \eta_e \)  
Electrical efficiency for electrolyser.

\( \eta_{hy-ca}, \eta_{hy-me} \)  
Reaction coefficients for required carbon dioxide and methanation output.

\( G_{i_g,\text{max}}, G_{i_g,\text{min}} \)  
Maximum and minimum output of natural gas source \( i_g \) at time \( t \).

\( Pr_{i_g,\text{max}}, Pr_{i_g,\text{min}} \)  
Maximum and minimum gas pressure of gas pipeline \( l_g \).

\( \gamma_{l_g} \)  
Weymouth constant for pipeline \( l_g \).

\( f_{l_g,\text{max}} \)  
Maximum gas flow of pipeline \( l_g \).

\( CF_{l_g} \)  
Compressor’s compression factor at pipeline \( l_g \).

\( \eta_{GT} \)  
Conversion efficiency of gas turbine.

\( \Omega_{hy}, \Omega_{LPG}, \Omega_{ni}, \Omega_{me}, \Omega_{\text{mix}} \)  
Gross calorific value (GCV) for hydrogen, liquid petroleum gas (LPG), nitrogen, methane and mixed natural gas.

\( \rho_{hy}, \rho_{LPG}, \rho_{ni}, \rho_{me} \)  
Gas density of hydrogen, liquid petroleum gas, nitrogen and methane.

\( E_{hy}, E_{LPG}, E_{ni}, E_{me} \)  
Combustion potential index (CPI) of hydrogen, liquid petroleum gas, nitrogen and methane.

\( O_l \)  
Oxygen index.

\( \Omega_{\text{mix}}, S_{\text{mix}} \)  
Maximum limit for GCV, specific gravity, wobbe index (WI) and Combustion Potential (CP) of mixed gas.

\( W_{\text{mix}}, CP_{\text{mix}}, \Omega_{\text{mix}}, S_{\text{mix}}, W_{\text{mix}}, CP_{\text{mix}} \)  
Minimum limit for GCV, specific gravity (SG), WI and CP of mixed gas.

\( \phi_{n,\text{max}}, \phi_{n,\text{min}}, \phi_{hy,\text{max}}, \phi_{hy,\text{min}}, \phi_{n,\text{max}}, \phi_{n,\text{min}}, \phi_{LPG,\text{max}}, \phi_{LPG,\text{min}}, \phi_{n,\text{max}}, \phi_{n,\text{min}}, \phi_{\text{mix}} \)  
Maximum volume deviation for hydrogen producing methane, direct used hydrogen, LPG, nitrogen and methane.

\( \varphi_{n,\text{max}}, \varphi_{n,\text{min}} \)  
Maximum and minimum volume for mixed gas at node \( n \).

\( \Theta \)  
Constant in Boyle’s law.

\( G_{k_g,t}, G_{kh,t} \)  
Gas and heating load at time \( t \).

\( P_{\text{sub, max}}, Q_{\text{sub, max}} \)  
Maximum active and reactive power injection at substation from upper level market.

\( P_{i_e,\text{max}}, P_{i_e,\text{min}} \)  
Maximum and minimum output of traditional DG \( i_e \).

\( x_{l_e}, r_{l_e} \)  
Resistance and reactance of power line \( l_e \).

\( f_{l_e,\text{max}}, f_{l_e,\text{max}} \)  
Maximum active and reactive power flow of power line \( l_e \).

\( P_{k_e,t}, P_{k_g,t} \)  
Power and gas load at time \( t \).

\( \lambda^a_{i_e}, \lambda^b_{i_e}, \lambda^c_{i_e} \)  
Cost coefficients for of traditional DG \( i_e \).

\( \lambda_{\text{sub}}, \lambda_{i_g} \)  
Cost coefficients for electricity purchase at substation and natural gas source \( i_g \).

\( \lambda_N, \lambda_{LPG} \)  
Cost coefficients for nitrogen and liquid petroleum gas.
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\[ \omega_{j,t} \]  
Forecasted output of wind turbine \( j \) at time \( t \).

**Variables**

\[ p_{n,t}^{P2G} \]  
Power consumed by the electrolyser.

\[ G_{n,t}^{hy}, G_{n,t}^{hy,me}, G_{n,t}^{hy,d}, G_{n,t}^{me} \]  
Gas output for overall P2G process, direct hydrogen injection, hydrogen during methanation process and methanation.

\[ G_{n,t}^{ca} \]  
Required gas of carbon dioxide during methanation process.

\[ G_{i_g,t} \]  
Output of natural gas source \( i_g \) at time \( t \).

\[ P_{l_{i_g,t}}, P_{l_{t_{e_r,t}}} \]  
Gas pressure of initial and terminal nodes of pipeline \( l_g \) at time \( t \).

\[ f_{l_{i_g,t}}, f_{l_{t_{e_r,t}}} \]  
Gas flow of pipeline \( l_g \) at time \( t \).

\[ f_{l_{i_g,t}}, f_{l_{t_{e_r,t}}} \]  
Gas at initial and terminal nodes of gas turbine.

\[ f_{l_{i_g,t}}, f_{l_{t_{e_r,t}}} \]  
Volume for hydrogen with methanation process, direct use, LPG, nitrogen, methane and mixed natural gas.

\[ \phi_{n_{mix}}, S_{G_{n_{mix}}}, W_{I_{mix}}, C_{P_{mix}} \]  
CGV, SG, WI and CP for mixed gas of node \( n \) at time \( t \).

\[ f_{l_{i_g,t}}, f_{l_{t_{e_r,t}}} \]  
Gas flow from initial node and to terminal node of pipeline \( l_g \) at time \( t \).

\[ P_{sub,t}, Q_{sub,t} \]  
Electricity purchase of substation at time \( t \).

\[ P_{i_e,t}, Q_{i_e,t} \]  
Traditional DG active and reactive power output of \( i_e \) at time \( t \).

\[ f_{i_e,t}^a, f_{i_e,t}^r \]  
Active and reactive power flow of power line \( l_e \) at time \( t \).

\[ f_{l_{i_g,t}}, f_{l_{t_{e_r,t}}} \]  
Gas flow from initial node and to terminal node of pipeline \( l_g \) at time \( t \).

#### 7.3 Introduction

The increasing penetration of renewable energy is effective for revolutionising energy mix and addressing the climate crisis. In the U.S., 275 TWh wind power was generated in 2018 while 6 TWh wind energy was curtailed and wasted [146]. The main reason is that i) the fluctuating and uncertain characteristics of wind power cause unbalancing issues and ii) wind power cannot be fully consumed in local areas but cannot be transported to other areas due to network constraints.

As a promising solution to enable excessive renewable energy integration, power-to-gas (P2G) enables the conversion from electrical energy to hydrogen and synthetic natural gas. Accordingly, the bidirectional energy flow is achieved for tighter couplings between integrated energy systems (IES). P2G has been extensively investigated in
Distributionally Robust Operation of IEGS with Hydrogen Injection

existing research, particularly in network planning and operation problems [29, 54, 55, 147-150].

One major research area is P2G planning in IES. A robust co-optimization model is presented in [147] to determine the optimal investment plan for installing investment candidates including P2Gs and gas compressors. Wind uncertainties and reliability are considered for economic and reliable solutions. Paper [148] proposes a bi-level multi-stage stochastic programming to minimize planning and operation cost of an integrated electricity and gas system (IEGS) with P2G. A real options model is designed for IEGS including P2Gs to determine the optimal investment timing and capacity of P2G [149]. The operating cost uncertainty is considered and the decision can be made immediately or postponed waiting for the operation opportunity based on real options.

P2G operation has also been well investigated to reduce operation cost and carbon emissions and maximize profits [29, 54, 55, 150]. Paper [54] designs a decentralized IEGS with P2G technologies and wind energy to save daily operation cost. A linearized transient-state gas flow model is developed and the alternating direction multiplier method is used to solve the proposed problem. A stochastic optimization (SO) based day-ahead economic dispatch model for IEGS considering renewable uncertainties and contingencies is proposed in [55]. A second-order cone relaxation is developed to address the nonconvexity caused by uncertain gas flow direction. Paper [29] aims to reduce CO₂ emissions and optimally utilize surplus renewable energy. To maximize the expected profit of P2G facilities to a gas grid, a distributed supply coordination is proposed [150], which is a two-layer optimization problem and solved by a model predictive control method.

Hydrogen is produced by electrolysers of P2G and then injected into gas systems, which can inevitably affect gas composition. The variation in gas composition will impact the security of gas pipelines, gas engine performance, emissions as well as the gas quality of end-users [151]. The changes in the safety and performance of domestic gas appliances are assessed in [152] with natural gas-hydrogen mixtures. Paper [153] proposes a steady-state analysis method with the injection of alternative gases at different locations. Both centralized and decentralized injection of hydrogen and biogas are studied and results show that the optimal management of diversified gas components can help reduce carbon emissions. Paper [154] investigates the impact on gas quality standards in terms of heating value indices, Wobbe indices and relative gas
density when hydrogen is injected to gas systems with variation between 1 and 10 vol%. In gas distribution systems, Wobbe index (WI) is the most common parameter in the existing literature to measure gas quality [21, 152-156]. Paper [152] analyses gas interchangeability using WI on domestic appliances. The results demonstrate that WI associated with flashback and thermal output are important constraints to consider. A distributed injection of alternative gas with a steady-state method is presented in [153] and the paper also assesses the impact of utilizing various gas supply sources by WI. A small-scale renewable hydro methane production system is designed in [155] considering WI as a key security index. Paper [156] investigates the effect of different hydrogen injection levels on gas quality based on WI for both distribution and transmission gas networks. Paper [21] studies P2G operation in IEGS considering using WI and Combustion Potential (CP). Through optimally managing gas mixture, hydrogen injection is maximized by using robust optimization (RO), effectively hedging wind power uncertainties.

The utilization of renewable as the source for P2G is influenced by the uncertain characteristics and existing research mainly uses SO [54, 55] and RO [21, 157]. SO assumes the decision making is either based on an explicit distribution knowledge or a large number of samples. The former solution is not always practical and the latter is prone to errors since it is difficult to estimate the accurate probability distribution when the dataset is not sufficiently large. Alternatively, RO finds the optimal solution under the worst-case scenario based on the uncertainty set, which is over-conservative. To overcome their shortcomings, distributionally robust optimization (DRO) is developed to balance the deficiencies of SO and RO with minor robustness guaranteed through partial distribution information. A risk-based optimal gas-power flow is presented and solved by DRO [56]. Paper [57] designs an economic dispatch model for IEGS considering renewable and load uncertainty. An IES at the building level is proposed considering PV output uncertainty and DRO is used to mitigate the conservatism [158]. In summary, existing research has extensively assessed the gas quality of hydrogen-gas admixture but the coordinated operation of energy infrastructures in IES is ignored. There is also a lack of an effective method to model renewable uncertainty.

Similar to the uncertainty set of RO, the ambiguity set of DRO is used to characterize uncertainties with certain known information of distributions. Constructing a proper ambiguity set is crucial to DRO, which must be sufficiently rich to accommodate the real distribution and small enough to exclude distributions that may cause over-
conservatism. So far, moment-based ambiguity set is the most common type due to its tractability and easy second-order cone program (SOCP) or semidefinite program (SDP) reformulations, e.g., Markov ambiguity set and Chebyshev ambiguity set that depend on first and second-moment information. Nevertheless, the moment information is not abundant enough to shape the real distribution compared with discrepancy-based ambiguity set which measures the discrepancy between the candidate distribution and the reference distribution. The discrepancy can be controlled by the decision-maker to either decrease or increase the conservatism depending on the reliability requirement of the optimization.

To fill the research gap, this paper designs new co-optimization for both gas quality and system operation in an IEGS. Kullback-Leibler (KL) divergence is used to measure the distance between two distributions. Renewable uncertainty is captured by DRO approach with KL divergence-based ambiguity set to ensure both robustness and tractability. The key indices to quantify gas quality, including gross calorific value (GCV), specific gravity (SG), WI, and CP, are included in the model. Apart from ensuring standard satisfaction, the injected gas from P2G is mixed with nitrogen and Liquid Petroleum Gas (LPG) to maintain overall gas quality. The uncertainty of wind power output is handled by KL divergence-based DRO, which can be transformed into a tractable deterministic model.

The main contributions of this paper are as follows:

1) Four key indices are used in the economic operation of IEGS to quantify the impact of hydrogen injection from P2G on gas quality.

2) A novel co-optimization model is developed to both minimize system operation costs and maintain gas quality within an acceptable range, achieved by a mixture of nitrogen and LPG.

3) A KL divergence based DRO is developed to model renewable uncertainties. Compared to SO and RO, it is less data-dependent and conservative. Compared to moment-based DRO, the robustness of the proposed ambiguity set can be controlled by adjusting divergence tolerance in the algorithm.

The remainder of this paper is organized as follows. Section 7.4 proposes the modelling for the gas quality indices. Section 7.5 presents the objective function and constraints for IEGS including P2G facility modelling and gas quality management. The KL divergence-based DRO methodology regarding and associated reformulations
are given in Section 2.2.2. Section 7.6 demonstrates case studies and performance of the problem. Finally, section 7.7 concludes the paper.

7.4 Gas Quality

To assess gas quality, gas adaptability and interchangeability are the two most significant indexes. The adaptability of gas is referred to as the ability of the gas-fired appliances to work properly when the gas composition is changed due to gas injection. The gas interchangeability refers to that, during the mix of various gas compositions, the operational performance of gas equipment is still acceptable in terms of safety, efficiency and emissions. For gas turbines and pipelines, only limited change of gas composition is tolerated.

Calorific value is defined as the amount of released heat during combustion. GCV represents the amount of released heat by unit volume of fuel when the temperature of the gas is equal before and after the combustion, which means the water vapour is entirely condensed and heat recovered during the combustion. GCV must be within a range which determines the available amount of energy. The GCV for hydrogen is given in (7-1), where \( \Omega_g \) and \( \Omega_{hy} \) are the GCV for the mixed gas and hydrogen and \( \phi_{hy} \) is the volume of hydrogen.

\[
\Omega = \Omega_g + (\Omega_{hy} - \Omega_g)\phi_{hy} \tag{7-1}
\]

\( SG \) is the ratio of gas density to air density at the same pressure and temperature. It is used for limiting hydrocarbon content, which is given in (7-2), where \( \rho_g, \rho_{hy} \) and \( \rho_{air} \) denote the density of gas, hydrogen and air. A high hydrocarbon content will cause serious combustion problems, e.g., engine knock, carbon monoxide emissions and spontaneous ignition of gas turbines, etc.

\[
SG = \frac{\rho_g + (\rho_{hy} - \rho_g)\phi_{hy}}{\rho_{air}} \tag{7-2}
\]

The WI for gas equipment can vary within a small range, which is defined by (7-3).

\[
WI = \frac{\Omega}{\sqrt{SG}} \tag{7-3}
\]

The most frequent used WI in the world is set within 5-10% of the standard setpoint. Otherwise, non-optimal gas combustion appears, which will lead to inefficient and unstable equipment working conditions and high greenhouse gas emissions. A
significant change of WI can even result in emergency shutdowns of gas turbines due to the adverse impact on control issues, affecting the lifespan. In addition, the combustion performance is also influenced by the varying gas composition, e.g., flame stability, ignition properties and flashback. Ensuring equal WI can obtain the same energy input under the same gas pressure. CP is used to measure gas combustion stability, which can reflect combustion characteristics, including combustion flame and yellow flame, etc. CP is one important index for interchangeability of gas admixture that requires the CPs of mixed gases are close. Equation (7-4) defines CP.

\[
CP = O_l \phi_{hy} + 0.6(\phi_{cm} + \phi_{hc}) + 0.3 \phi_{me} \sqrt{SG}
\]

Where \(\phi_{cm}\), \(\phi_{hc}\) and \(\phi_{me}\) represent the volume of carbon monoxide, hydrocarbon except methane and methane.

### 7.5 IEGS Modelling

This section models P2G facility and IEGS, followed by the operation objective function. It is assumed that the entire IEGS is owned by a single system company, who has the full control of DGs, power lines, wind generators, gas sources, pipelines, P2G facility, compressors and other equipment.

#### 7.5.1 P2G Modelling

P2G facility enables redundant wind power to be recovered and transported by the gas system. The P2G process is given in Fig. 7-1. Firstly, electrolysers split the water (H\(_2\)O) into hydrogen (H\(_2\)) and oxygen (O\(_2\)) by using excessive wind power. Then with...
the interaction with carbon dioxide (CO$_2$), methane (CH$_4$) can be obtained through methanation. Meanwhile, the produced H$_2$ from the first step can be directly transported by the gas system. The relationship between the input and output of electrolyser is described in (7-5). According to Sabatier reaction factors [159], equations (7-6)-(7-8) present the requirement of CO$_2$ and production of CH$_4$ in the process of methanation.

$$G_{n,t}^{hy} = \eta_e \frac{P_{G_n}^{zi}}{G_{hy}}$$ (7-5)

$$G_{n,t}^{hy,me} + G_{n,t}^{hy,d} = G_{n,t}^{hy}$$ (7-6)

$$G_{n,t}^{ca} = \eta_{hy-ca} G_{n,t}^{hy,me}$$ (7-7)

$$G_{n,t}^{me} = \eta_{hy-me} G_{n,t}^{hy,me}$$ (7-8)

### 7.5.2 Modelling of Electricity and Gas Systems

The modelling of natural gas system is presented from (7-9) to (7-24). Equation (7-9) limits the gas production by natural gas source $i_g$. Gas pressure is limited in (7-10) and (7-11). It is noted that the pressure of initial gas nodes is always higher than that of terminal nodes in distribution gas systems. Weymouth gas flow equation is used to describe the relationship between gas pressure and flow in (7-12). Equation (7-13) limits gas flow. The inlet and outlet gas pressures of the compressor are constrained in (7-14). Equations (7-1)-(7-4) describing the gas quality with hydrogen are modified considering the mix of methane, LPG and nitrogen, which are given in (7-15)-(7-18). Equation (7-19) is used to ensure all gas quality indices are within a certain range for each gas node. The volume deviation between two consecutive time periods cannot be too big due to gas travelling speed in pipelines, which is presented in constraint (7-20). The total gas volume and its limit are given in (7-21) and (7-22). Constraint (7-23) presents the relationship between gas pressure and volume based on Boyle’s law [160]. The nodal gas balance constraint is presented in (7-24).

$$G_{i_g,\text{min}} \leq G_{i_g,t} \leq G_{i_g,\text{max}}$$ (7-9)

$$P_{r_{i_g,t}}^{2,\text{min}} \leq P_{r_{i_g,t}}^{2} \leq P_{r_{i_g,t}}^{2,\text{max}}$$ (7-10)

$$P_{r_{i_g,t}}^{\text{ini}} \geq P_{r_{i_g,t}}^{\text{ter}}$$ (7-11)
\[ f_{i,g,t}^2 = \gamma_{tg} \left( P_{i,g,t}^{ini} - P_{i,g,t}^{ter} \right)^2 \] (7-12)

\[ 0 \leq f_{i,g,t} \leq f_{i,g,\text{max}} \] (7-13)

\[ P_{i,g,t}^{ter} \leq C_{tg} P_{i,g,t}^{ini} \] (7-14)

\[ \Omega_{n,t}^{mix} = \Omega_{hy}(\varphi_{n,t}^{hy,me} + \varphi_{n,t}^{hy,d}) + \Omega_{LPG} \varphi_{n,t}^{LPG} + \Omega_{ni} \varphi_{n,t}^{ni} + \Omega_{me} \varphi_{n,t}^{me} \] (7-15)

\[ SG_{n,t}^{mix} = \rho_{hy}(\varphi_{n,t}^{hy,me} + \varphi_{n,t}^{hy,d}) + \rho_{LPG} \varphi_{n,t}^{LPG} + \rho_{ni} \varphi_{n,t}^{ni} \]

\[ + \rho_{me} \varphi_{n,t}^{me} \left( \varphi_{n,t}^{hy,me} + \varphi_{n,t}^{hy,d} + \varphi_{n,t}^{LPG} + \varphi_{n,t}^{ni} + \varphi_{n,t}^{me} \right) \] (7-16)

\[ W_{n,t}^{mix} = \Omega_{n,t}^{mix} \sqrt{SG_{n,t}^{mix}} \] (7-17)

\[ CP_{n,t}^{mix} = O_i \frac{E_{hy}(\varphi_{n,t}^{hy,me} + \varphi_{n,t}^{hy,d}) + E_{LPG} \varphi_{n,t}^{LPG} + E_{ni} \varphi_{n,t}^{ni} + E_{me} \varphi_{n,t}^{me}}{\sqrt{SG_{n,t}^{mix}}} \] (7-18)

\[ \{\} \text{min} \leq \{\} \leq \{\} \text{max}, \] (7-19)

\[ \{\} = \Omega_{n,t}^{mix}, SG_{n,t}^{mix}, W_{n,t}^{mix}, CP_{n,t}^{mix} \]

\[ -\Delta \varphi_{n,t}^{(r)} \leq \varphi_{n,t}^{(r)} - \varphi_{n,t-1} \leq \Delta \varphi_{n,t}^{(r)} \] (7-20)

\[ \{\} = \text{hy}_\text{me}, \text{hy}_d, \text{LPG}, \text{ni}, \text{me} \]

\[ \varphi_{n,t}^{hy,me} + \varphi_{n,t}^{hy,d} + \varphi_{n,t}^{LPG} + \varphi_{n,t}^{ni} + \varphi_{n,t}^{me} = \varphi_{n,t}^{mix} \] (7-21)

\[ \varphi_{n,t}^{mix} \leq \varphi_{n,t}^{mix} \leq \varphi_{n,t}^{mix} \] (7-22)

\[ \varphi_{n,t}^{mix} = \frac{\theta}{P_{n,t}} \] (7-23)

\[ \sum_{l_g \in L_g} G_{l_g,t} + \sum_{n \in N} G_{n,t}^0 + \sum_{l_g \in L_g} f_{l_g,t}^{s,ini} - \sum_{l_g \in L_g} f_{l_g,t}^{s,ter} \]

\[ = \sum_{k_g \in K_g} G_{k_g,t} + \sum_{k_h \in K_h} G_{k_h,t} \] (7-24)

The electricity distribution system is modelled from (7-25) to (7-30). Equation (7-25) is the constraint for the active and reactive power of substations. The generation limits for traditional DGs are presented in (7-26). In the distribution system, the DistFlow equation is used with the linearization as presented from (7-27) to (7-28). In (7-29) and
(7-30), the power balance constraints for active and reactive power are given respectively.

\[ 0 \leq \{\cdot\}_{sub,t} \leq \{\cdot\}_{sub,max}, \{\cdot\} = P, Q \tag{7-25} \]

\[ P_{i,e,min} \leq P_{i,e,t} \leq P_{i,e,max} \tag{7-26} \]

\[ V_{b,t}^{ini} - V_{b,t}^{ter} = (f_{i,e,t}^{\alpha} n_{i,e} + f_{i,e,t}^{r} x_{i,e})/V_0 \tag{7-27} \]

\[ 0 \leq f_{i,e,t}^{(\cdot)} \leq f_{i,e,max}^{(\cdot)}, \{\cdot\} = a, r \tag{7-28} \]

\[ \sum_{i,e \in I_e} P_{i,e,t} + \sum_{j \in J} \omega_{j,t}^{P} + \sum_{l,e \in L_e} f_{l,e,t}^{a,ini} - \sum_{k,e \in K_e} f_{l,e,t}^{a,ter} = \sum_{k,e \in K_e} P_{k,e,t} \tag{7-29} \]

\[ \sum_{l,e \in L_e} Q_{l,e,t} + \sum_{i,e \in I_e} f_{i,e,t}^{r,ini} - \sum_{k,e \in K_e} f_{l,e,t}^{r,ter} = \sum_{k,e \in K_e} Q_{k,e,t} \tag{7-30} \]

### 7.5.3 Objective function

The injection of hydrogen into natural gas pipelines will inevitably change gas compositions and might cause gas quality issues, such as heat value, combustion potential pressure. In order to maintain the 4 gas quality indices within an acceptable statutory range, it is required to inject other gases with hydrogen into gas pipelines. Accordingly, the optimal gas mixture is required to determine the proper amount and timing of the injection of other gases. In this paper, LPG and nitrogen are used to blend with hydrogen to keep a satisfied gas quality. Nevertheless, the cost of purchase and injection of LPG are expensive compared with nitrogen. Accordingly, the key gas mixture process is to use the minimum LPG with gas quality satisfied. The objective in (7-31) is to minimize system operation cost while ensuring gas quality, considering the impact of uncertain wind power output.

\[ \min \Gamma = \min \sum_{i,e \in I_e, g \in G, t \in T} \lambda_{i,e}^a P_{i,e,t}^2 + \lambda_{i,e}^b P_{i,e,t} + \lambda_{i,e}^c + \lambda_{sub} P_{sub,t} \tag{7-31} \]

\[ + \lambda_{lg} P_{lg,t} + \lambda_{N} P_{N,t} + \lambda_{LPG} P_{LPG,t} \]

The first three terms are the cost function for traditional DGs. The fourth term represents electricity purchased from the upper electricity market. The gas production cost of natural gas sources is shown as the fifth term. The last two terms are the cost for purchase and injection of LPG and nitrogen during gas mixture process.
7.6 Case Studies

The proposed gas quality management for IEGS is demonstrated on a modified IEEE 33-bus system with a 10-node gas system in Fig. 7-2. The gas system data is obtained from [14]. The temperature and pressure in the gas pipelines are of the standard level. The IEGS contains three traditional DGs, three renewable DGs and two natural gas sources. The wind DG at bus 10 is the power supply for the P2G facility with 1MW capacity. The parameters for natural gas sources and DGs in TABLEs 7-1 and 7-2 respectively. The ambiguity set is controlled by a divergence tolerance ($\eta=2.3026$ and $\beta=0.1$) for the DRO. The GCV and combustion potential index (CPI) for hydrogen, methane, LPG and nitrogen are given in TABLE 7-3. The gas composition of original natural gas and LPG is provided in TABLE 7-4, mainly consisting of methane, ethane,

![Fig. 7-2. The proposed IEGS test system.](image)

<table>
<thead>
<tr>
<th>TABLE 7-1 Parameters of natural gas sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node No.</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 7-2 Generator parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus No.</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>21</td>
</tr>
<tr>
<td>28</td>
</tr>
</tbody>
</table>
propane and butane. LPG has high GCV but low CPI, which is used to increase WI and decrease CP. By contrast, the GCV and CPI of nitrogen are both zero, which enables more flexible gas mixture. Four case studies in TABLE 7-5 are implemented based on optimization methods, hydrogen injection schemes, and gas mixture management strategies, which are presented.
7.6.1 Economic Performance

The economic results for all cases are investigated, including operation cost and gas mixture management cost, as is shown in TABLE 7-6. The IEGS operation cost is the sum of operation cost of power system and gas system. It shows that case 1 ($601922) has the highest IEGS operation cost and case 3 ($337889) has the lowest. The IEGS operation strategy for case 1 and 4 are the same which both consider hydrogen injection support for the gas system and gas mixture management for maintaining gas quality. Case 1 derives $135710 more operation cost in the power system since RO limits the uncertain wind power output with a higher degree of robustness, which, yields $120445 less gas system operation cost. The reason is that the hydrogen injection is strictly limited, which reduces the need for additional LPG and nitrogen to maintain acceptable gas quality indices. Overall, case 1 results in $15265 more IEGS operation cost compared with case 4.

Without considering hydrogen injection from the power system to the gas system, the two systems are operated separately in case 2. Accordingly, the power system only requires to supply electricity load in case 2 whose power system operation cost is 4.3% less than that of case 4. The purchase cost of nitrogen and LPG in case 2 are $1003 and $9760 respectively, which are $1421 and $230260 less than case 4. Since the original natural gas without hydrogen addition is more accessible to obtain acceptable gas quality. Due to the disconnection between power and gas systems, the overall operation cost of case 2 is $246149 less than case 4. In case 3, hydrogen injection is considered without gas mixture. The gas system operation cost, i.e., $845, is purely the generation cost of natural gas sources. Without the blend of LPG and nitrogen, the gas volume is less than case 4 and the gas pressure is higher than case 4, which reduces the hydrogen injection from P2G facility. Thus, the wind power provides more supply to the power system and the power system operation cost is reduced.

The divergence tolerance \( \eta \) is used to characterize the size of the ambiguity set which contains all the possible uncertainty distributions and is associated with the conservatism of numerical performance. According to [33], the divergence tolerance influences the confidence interval, i.e., \( (\beta = e^{-\eta}) \). The divergence tolerance \( \eta \) represents the radius of the ambiguity set, which affects the accuracy of estimating uncertainty distribution. The larger \( \eta \) leads to an ambiguity set with higher robustness.
while the smaller $\eta$ leads to less conservative numerical results. When the divergence tolerance is set as 0, the confidence interval turns into 100% and the candidate distribution is becoming the same as the reference distribution. Accordingly, the original DRO problem is equivalent to SO. With the variation of the confidence interval, the total operation cost for IEGS is depicted in TABLE 7-7. At the second column of the table, the divergence tolerance is determined based on selecting the confidence interval. Case 4 has the highest result with all the confidence intervals while case 3 remains the lowest. With the increase of the confidence interval, the total IEGS operation cost increases slowly. In case 4, when $\beta = 0$, the DRO degrades to SO and yields $583271$ total cost. The considered largest ambiguity set results in $596454$ with $\beta = 1$, which is 2.3% higher than the cost with the smallest ambiguity set.

**7.6.2 Gas Quality Performance under Gas Quality Management**

The resulting WI and CP with different P2G operation schemes are presented in this subsection. From Fig. 7-3 to Fig. 7-5, it can be seen that case 2 and 4 have a similar WI range and trend through the entire time period while case 3 shows a narrow range of WI. The WI of case 3 ranges from 32.65 to 32.75, which is 79% of the WI range of case 4. Besides, WI in case 3 does not fluctuate much while maintaining a smooth trend through the entire time period. The reason is that without the gas admixture of LPG and nitrogen, WI cannot be ensured in an acceptable range. In comparison with case 4, there is no hydrogen injection in case 2. Therefore, the gas quality management is implemented on the original environment based on the natural gas composition component given in TABLE 7-4. Compared with hydrogen and methane, nitrogen and LPG have higher CGV, which lead to higher WI in case 2 since case 2 has no hydrogen injected and the other gas components contribute to a higher WI. For case 2 and 4, the WI remains at a high level from 7:00 to 17:00 and peak at 49 in both cases. The WI of N3 and N8 are maintained smoothly around 42. For N6, the WI remains around 41 before 15:00 and then decreases dramatically.

The CP for all cases at 12:00 is provided in TABLE 7-8. Case 3 yields the highest CP and case 4 has the lowest. Since nitrogen and LPG have low CPI, which results in low CP. Without the blend of nitrogen and LPG, the CP in case 3 is relatively high considering only hydrogen and methane as the gas composition. On the contrary, without the hydrogen injection, case 2 has low CP. Compared with case 4, the CP of
N1 in case 2 is 30% less. In case 2, CP is slightly lower than case 4 when solved by RO since the higher degree of robustness leads to less P2G power output. Accordingly, the hydrogen is produced less and the resulting CP is lower.

### 7.6.3 Pressure Performance under Gas Quality Management

The nodal gas pressure for case 2-4 is given from Fig. 7-6 to Fig. 7-8. Overall, it can be seen that the pressure of N1 is the highest and N6 is the lowest. When the node is
close to the gas source, e.g., N1 and N6, the gas pressure is relatively high. Passing through the gas compressor between N2 and N3, the gas pressure of N3 is pumped up. As the base case with both hydrogen injection and gas mixture management considered, case 4 shows an increasing trend from 1:00 to 10:00 and a decrease from 11:00 to 14:00, followed by another pressure peak at 16:00. Connected to the main gas source, the pressure of N1 peaks at 1.2 Psig and the lowest is 0.6 Psig. At N3, with the hydrogen
injection from P2G facility and pumped pressure by from N2, the gas pressure of N3 reaches 1.0 Psig at 10:00.

Compared with case 4, case 2 shows similar pressure profile. However, the average pressure level of N1 in case 2 is 6% higher than in case 4. The reason is that without the power supply by hydrogen, the natural gas source on N1 needs to provide more gas supply to the gas system which results in higher gas pressure. In case 3, without gas
quality management, the gas volume is not limited. This leads to the large pressure deviation between nodes which are close to the sources and the nodes far from the sources. For instance, the average 0.4 Psig pressure deviation between N3 and N6 can provide large gas flow. Taking advantage of the unlimited gas quality and volume, case 3 enables large gas flow with different gas pressure profile compared with other cases.

7.7 Chapter Summary

A coordinated optimization for gas quality management and operation of IEGS in the presence of wind uncertainty is proposed. The wind uncertainty is handled by DRO with KL divergence for controlling the conservatism of numerical performance. A tractable deterministic formulation is obtained and the resulted linear programming model can be efficiently solved. Through the extensive case studies, the key findings are:

- Considering the four gas quality indices, i.e., GCV, SG, WI and CP, are necessary. The gas quality is not acceptable under the relative international standard without gas quality management.
- The P2G facility is useful for maximally utilizing the excessive wind power and economically effective for reducing the operation cost of IEGS.
- DRO provides less conservative results than RO in terms of economic performance.
- Through applying KL divergence, the size of the ambiguity set can be flexibly controlled based on confidence interval set by decision-makers for risk concerns.

The proposed co-optimization for IEGS ensures both the economic performance and gas quality via coordinating traditional DGs, natural gas resources and P2G facility, which can benefit system operators with economic benefits through saving operation cost and secure gas distribution with gas quality guaranteed.
Chapter 8

Conclusion

This chapter summarises the thesis by outlining the major contributions and findings from the research.
IEGS provides a powerful concept to advise system operators in terms of planning, dispatching and converting multi-energy infrastructures to achieve a coordinated optimal condition among power and gas systems. However, the renewable integration, natural disasters, cybersecurity issues, voltage fluctuation and gas quality issues have unignorable impacts on the secure and economic operation of IEGS. In consequence, this paper extensively investigates the solutions to counteract the aforementioned problems, which attempts to fill the research gaps of the existing literature.

**Mean-Risk Distributionally Robust Co-Optimization of District Integrated Electricity-Gas-Water Systems**

A mean-risk coordinated optimization for an IES in the water-energy nexus with enormous interdependencies is proposed in this paper. The tight couplings and interactions between each subsystem enable the reliable and economic operation for the entire IES. Renewable uncertainty is captured by mean-risk DRO. The coherent risk measure, CVaR provides the trade-off to system operators with flexible alternatives on choosing between economic efficiency and risk. A tractable Bender’s decomposition is employed to solve the DR-IWENS problem.

Through the extensive case studies on the economic performance, scheduling of interdependent coupling devices and the risk management via adjusting parameters, the major contributions are tested:

- The coordination of each subsystem with the conversion technologies enhances energy efficiency.
- The water system is highly required to consider in the IES operation as the water is extensively consumed by energy conversions.

The mean-risk DRO applied in IES operation problem provides system operators with not only economic but risk concerns.

**Resilience Enhancement and Emergency Response for Integrated Energy Systems against Seismic Attacks: A Data-Driven Approach**

In this paper, a two-stage DRO method is developed to enhance the resilience of an IES under seismic attacks with combined planning and operation strategies. The proposed method provides optimal hardening plans for specific power lines and gas
pipelines under different seismic intensity levels and investment budgets. Through extensive case study demonstrations, the key findings are as follows: i) In the first stage, DR-SIP effectively determines the most vital lines to harden. In the second stage, DR-SIP optimally shed loads to keep system balanced and minimize the system operation cost; ii) With RO that considers the most extreme event and serious damage, the proposed DRO provides less conservative results for both planning and operation stages with 13% less cost; iii) Investment plan with a higher budget is more likely to yield a reliable IES with high resilient performance; iv) The optimal hardening plan is effective for protecting transmission lines and loads and IES is more resilient than electricity network against seismic attacks.

This method can help system operators to make economical hardening and operation strategies to improve the resilience of the integrated energy system under seismic attacks.

**Two-Stage Coordinated Risk Mitigation Strategy for Integrated Electricity and Gas Systems under Malicious False Data Injections**

A risk mitigation scheme for IEGS against FDIA is proposed in this paper with a two-stage DRO model. The hierarchical two-stage framework can determine both day-ahead and real-time system optimal operation schemes considering the impact of FDIA and renewable uncertainties on electricity load, gas load and gas density. A tractable SDP formulation is built for the original DR-FMS, which is solved by CGA in an iterative manner. Through the extensive case studies, the key findings are listed below:

- Considering all three types of FDIA, i.e., EL-FDIA, GL-FDIA and GD-FDIA, leads to higher economic results and more load shedding than considering two types or one type of FDIA.

- Load shedding is more sensitive to EL-FDIA than GD-FDIA or GL-FDIA.

- DRO provides less-conservative results than RO in terms of economic performance and load shedding.

- Renewable generation uncertainty is necessary to consider, which leads to 3.7% more operation cost.
The proposed DR-FMS ensures the economic performance of IEGS by providing a two-stage risk mitigation scheme via implementing efficient load shedding under FDIA and renewable uncertainty. The beneficiaries of this work include: network operators can have powerful operation models, end customers will enjoy better supply security, and renewable can penetrate to the maximum level without much curtailment.

**Voltage Management in Integrated Energy Systems Considering Interdependency and Renewable Uncertainty**

This paper proposes a multi-objective optimization for minimizing both operation cost and voltage deviation of IES considering renewable power uncertainty. A two-stage data-driven DRO approach is used to solve the TS-VCO with dual and SDP formulations to ensure computational tractability. The reformulated TS-VCO is solved by CGA with master and subproblems. The key findings from the case studies are:

- Based on a large amount of moment information, DRO produces less conservative results compared with RO, more effective for maintaining voltage deviation and reducing operation cost considering renewable power uncertainty.
- The interdependency between electricity and gas systems reduces 4.7% of operation cost and a significant rise in the voltage profile.
- The proposed TS-VCO is effective in maintaining voltage and saving operation cost considering PV uncertainty.

This work can benefit integrated system operators with powerful operation tool to manage the systems with fewer costs but integrate more renewable energy.

**Distributionally Robust Operation for Integrated Energy Systems with Hydrogen Injection and Gas Quality Improvement**

A coordinated optimization for gas quality management and operation of IEGS in the presence of wind uncertainty is proposed. The wind uncertainty is handled by DRO with KL divergence for controlling the conservatism of numerical performance. A tractable deterministic formulation is obtained and the resulted linear programming model can be efficiently solved. Through the extensive case studies, the key findings are:
Considering the four gas quality indices, i.e., GCV, SG, WI and CP, are necessary. The gas quality is not acceptable under the relative international standard without gas quality management.

The P2G facility is useful for maximally utilizing the excessive wind power and economically effective for reducing the operation cost of IEGS.

DRO provides less conservative results than RO in terms of economic performance.

Through applying KL divergence, the size of the ambiguity set can be flexibly controlled based on confidence interval set by decision-makers for risk concerns.

The proposed co-optimization for IEGS ensures both the economic performance and gas quality via coordinating traditional DGs, natural gas resources and P2G facility, which can benefit system operators with economic benefits through saving operation cost and secure gas distribution with gas quality guaranteed.
Future Work

This chapter presents the potential future work to enrich the optimisation methodologies for multi-carrier energy system, as well as the interaction with other smart grid technologies or frameworks.
Future work will continue to investigate the optimal operation of IEGS. Instead of progressing on the optimization approach, more effort will be made on the optimization scenarios and models, aiming to develop an economic, reliable and resilient IEGS framework. With the increasing integration of DGs, the application of microgrids and energy hubs are becoming wider. Accordingly, the first future work will establish a co-optimization framework for IEGS connected with multiple energy hubs. Moreover, based on the existing works of VVO and gas quality management, the combination of them is also practical and innovative. In addition, when cyber-attacks are launched on IES with more energy vectors and interdependencies, the cyber-resiliency faces more challenges as the attack on each sub-system will propagate to other ones. Thus, it is critically essential to investigate the impact and mitigation scheme against cyber-attacks in IES with more energy systems. Detailed future work is illustrated as follows.

**IEGS Operation with Networked Energy Hubs**

To facilitate energy efficiency, energy interdependency and conversion are also essential at the customers’ side. Energy hubs feature micro-energy systems with multi-vector energy inputs and outputs. An IEGS in distribution level networked with multiple energy hubs further enhances the energy efficiency and energy system reliability. A hierarchical optimization framework for IEGS networked with energy hubs will be considered. A two-stage DRO model will be utilized to handle extensive renewable uncertainty. This work will benefit the system operators of both IEGS and energy hubs on minimizing the daily operation cost with enriched energy flexibility.

**VVO with Gas Quality Management**

VVO is a powerful management scheme in distribution systems to manage the voltage magnitudes within acceptable limits under the high renewable penetration era. This is further complicated by the growing interdependencies between power and gas systems. Meanwhile, the emerging P2G poses gas quality problems with hydrogen injection into gas pipelines.

To overcome the voltage and gas quality problems, this future work will propose a coordinated volt-VAR-pressure (VVP) optimization of IEGS for limiting voltage
magnitudes and gas pressure considering renewable uncertainties. Due to the effective energy conversion to handle surplus renewable generation, P2G operation can be utilized as a mitigation strategy for VVO problems. As a consequence, the optimal set of management actions of voltage regulating devices and P2G facilities is determined. Meanwhile, the gas quality issues caused by the hydrogen injection via P2G is handled by the proposed gas quality management scheme. A two-stage mean-risk distributionally robust optimization (TSMR-DRO) is developed to model the coordinated optimization problem considering risk measures for the objective function. The proposed VVP provides a guideline to IEGS operators with an efficient and economic operation scheme to effectively manage the voltage profile and gas quality with fewer operation costs and higher integration of renewable energy.

**Mitigate Cyber-Attacks in IES**

The wide implementation of ICT causes power system operations exposed to cyber-attacks. Meanwhile, the tendency of integrated multi-energy vectors has worsened this issue with multiple energy coupled.

This future work will propose a two-stage risk-averse mitigation strategy for IES, incorporating power, natural gas and water systems against FDIA under water-energy nexus. The FDIA on individual sub-systems is modelled through hampering false data integrity to the systems. An innovative two-stage risk-averse distributionally robust optimization (RA-DRO) is proposed to mitigate uneconomic operation and provides a coordinated optimal load shedding scheme for the nexus system security. A coherent risk measure, Conditional Value-at-Risk is incorporated into the RA-DRO to model risk. A Benders decomposition method is used to solve the original NP-hard RA-DRO problem. This research provides IES operators with an economic system operation tool by optimally coordinating energy infrastructures and implementing reasonable load shedding to enhance cybersecurity.
Reference


[27] "MAGNITUDE – Bringing flexibility provided by multi energy carrier integration to a new MAGNITUDE." https://www.magnitude-project.eu/ (accessed.


