Vying for Support: Lobbying a Legislator with Uncertain Preferences*

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Abstract

We consider a model where two opposing lobbyists bid for the support of a legislator with an integrity threshold and an uncertain bias towards either lobbyist. The results show that the cheapest legislators are the ones with higher levels of bias uncertainty. Furthermore, under moderate bias uncertainty, the average bid is equal to the legislator's integrity threshold. In particular, at low levels of bias uncertainty, lobbyists bid aggressively in order to stay competitive and secure the support of the legislator. Conversely, at high levels of bias uncertainty, lobbyists are aware that the legislator's bias may lean strongly for or against them and bid lower. Overall, we find a non-monotonic relationship between the uncertainty of legislator's bias and the bids of lobbyists in equilibrium.

Keywords: lobbying; uncertainty; integrity threshold; legislatures

JEL Classification: D72, D80

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1 Introduction

Lobbying is ubiquitous in most legislative systems. It is also one of the more controversial features of democratic governance. The persistent negative reputation of lobbying can be attributed to its close association with corruption. In an effort to shift this negative image, there has been a marked increase in national and supranational lobbying regulations from five in 2000 to eighteen in 2020 (McKay and Wozniak, 2020). This also means that a significant amount of lobbying done in the rest of the world is still largely unregulated and with little to no information on the process, its inputs, and the corresponding outcomes.

Uncertainty over preferences can provide politicians with more options when interacting with lobbyists. Politicians are less likely to seek rent where there is increased scrutiny. For example, the sectors with the highest levels of lobbying spending in the United States in the past five years do not include hot button issues such as abortion and gun laws (The Center for Responsive Politics, 2017). Schneider (2012), in his study on the role of the agenda-setter and lobbying, found that for issues with low salience, committee chairs have more incentive to propose more extreme policies and reap monetary rewards. Low salience sectors, including finance and health care, have the highest levels of lobbyist spending for 2018 (The Center for Responsive Politics, 2018). Business interests, trade associations, and professional groups have been shown to employ more lobbyists per issue and spend more (Baumgartner and Leech, 2001; de Figueiredo and Richter, 2013; McKay, 2012), accounting for 84% and 86% of total lobbying expenditures at the U.S. Federal and state level (de Figueiredo, 2004). The views of legislators are often undisclosed in these sectors, and this uncertainty over preferences provides politicians with opportunities for gain at the expense of the collective good. Increased information on lobbying activities to the public, and the reflection of public sentiment by non-profit associations, may influence legislator's preferences, and consequently the outcomes of the lobbying process to improve public welfare.

The paper studies the effect of this opacity and uncertainty on lobbying behaviour. Uncertainty, specifically in legislator's preferences, is key to understanding unregulated lobbying environments as the uncertainty over a legislator's preference provides lobbyists with a unique opportunity to obtain this once "unaccounted" support. The model captures this by looking at the uncertainty over a legislator's preference and her known level of integrity. We approach opacity in the lobbying process through a simultaneous lobbying structure. Where lobbyist-legislator interactions are kept private, the opportunity for lobbyists to counteroffer may not exist. The simultaneous lobbying approach takes this into account and retains focus on how interactions center on the uncertainty of legislator's preferences.

Our results show that the uncertainty over the legislator's preferences directly affects the bidding strategy of lobbyists. A non-monotonic relationship is observed between equilibrium lobbyist's bids and uncertainty over legislator's preferences. With the exception of moderate levels of uncertainty, we find that increasing uncertainty leads to a decrease in bid - with the cheapest legislators having higher levels of bias uncertainty. At low levels of uncertainty, lobbyists bid aggressively. The potential bias of the legislator towards a lobbyist's policy is not high enough to risk losing the legislator's support and each lobbyist bids high to stay competitive. Conversely, at high levels of bias uncertainty, lobbyists take into account the possibility that the legislator has a strong preference towards a lobbyist's policy position and bid lower. Our results suggest that as long as there is a possibility that a given legislator has strong preference for the lobbyist's position, it would be more cost efficient to vie for them. This is in line with empirical evidence that lobbying is often done towards legislators who already agree with them (Hojnacki and Kimball, 1998; Hall and Miler, 2008; de Figueiredo and Richter, 2013).

Empirical evidence suggests that the effects of cash-for-favour lobbying activities on policy are marginal (Grossman and Helpman, 1994; Ansolabehere et al., 2003; de Figueiredo and Richter, 2013). Despite this, public awareness on lobbying is centered largely on the perception of transactional lobbyist-legislator interactions. The impact of lobbying activities from non-profit associations on legislation is significant enough for the US congress to attempt to legislate restrictions on their participation in lobbying in the mid-1990s (Balassiano and Chandler, 2010). An article from the New York Times in 2013 reported that the influence of Wall Street in Washington has grown substantially (Lipton and Protess, 2013). In one of the bills passed by the House Financial Services committee in May 2013 exempting a large portion of financial trades in new regulation, the recommendations of Citigroup were reflected in seventy of the eighty five lines of the bill (Lipton and Protess, 2013).This alludes to a prevalence of shadow lobbying - one where lobbying proceeds without any regulatory oversight, within one of the strictest lobbying environments in the world.

Even in the UK, which has been deemed as one of the most transparent governments in the world, shadow lobbying can also be observed. According to Transparency International Report - Lifting the Lid on Lobbying *" The vast majority of lobbying in the UK occurs behind closed doors and is not disclosed ... great deal of policy-making and lobbying takes place elsewhere, as our interviews with both policy-makers and lobbyists confirmed."*(David-Barrett, 2015, p.15). In the same 2015 report, Transparency International UK outlined lobbying and public sector scandals and its most recent occurrence. Scandals where 'money is exchanged for access to politicians and party policy committees', 'local councilors acting as paid lobbyists', and 'former government and military officers selling access and influence for money' have been recorded up to a year before the review was written. The proliferation of cash-for-favour exchange scandals in the UK government indicate that this practice is still commonplace despite its illegality.

One can understand shadow lobbying better through the study of legislator-lobbyist interactions beyond initial introductions. In his continuum access model, Wright (1996) called this interaction the 'messaging' stage. Heberlig (2005) explored Wright's continuum model further and found that instead of a straight path regarding legislator interaction from introduction ('positioning') to exchange ('messaging'), the lobbyists appeared to gather information on legislators first, with varied target legislators at the 'positioning' and 'messaging' stages. We take this into account by incorporating uncertainty over the legislator's preferences in our model - focus on the transition from the 'positioning' to the 'messaging' stage - where the lobbyist secures the support of the legislator. We move away from the sequential lobbying structure introduced by Groseclose and Snyder (1996), instead opting for a simultaneous sealed bid lobbying structure to better recreate shadow lobbying. When lobbyists and legislators meet behind closed doors, the opportunity for competing lobbyists to counteroffer may not exist. Che and Gale (1998) looked at lobbying under capped expenditures through an all pay auction structure. We also move away from using an all pay auction as it considers all the legwork done prior to the discussions to gain access to the legislator. We argue that any interactions prior to the actual exchange, such as the dinners and perks received, although helpful in securing access do not always come into play when the decision making is done. The legislator needs to consider whether the "loot is commensurate to the punishment" and can only feasibly do so using a forward-looking approach (*i.e.* if I do this for you, how much will I get in return?). In our model, the legislator accepts the first offer that meets her expectations. The decision to support the lobbyist has to be made quickly as the risk of discovery through holding repeated negotiations will be high and the consequences substantial.

Preferences of legislators are often private and unknown to the lobbyists (Heberlig, 2005). We explore this uncertainty through studying lobbyist behaviour given uncertain legislator's preferences similar to Buzard and Saiegh (2016) and Dekel et al. (2006).There has been a significant amount of work on the effects of lobbying on policy outcomes, either through information or transactional exchanges (Austen-Smith, 1993; Groseclose and Snyder, 1996; de Figueiredo, 2002; Hall and Deardorff, 2006), although less have explored uncertainty in lobbying. Austen-Smith and Wright (1992) looked at lobbying as information transmission at the agenda-setting and voting stages and found that when there is occasional uncertainty on how informed the lobbyist is, more information transmission can occur. Buzard and Saiegh (2016) looked at sequential vote buying models, and specifically, at the allocation of bribes amongst three legislators. Dekel et al. (2006) looked at vote buying and explored as an extension the presence of uncertainty in legislatures. They found that with a large enough body of legislators, one can predict who the winning lobbyist is. Tyutin and Zaporozhets (2017) focus on the uncertainty over legislator types in a legislature and the interaction of the legislature and a single lobbyist. Uncertainty is rarely explored on its own, we address this by looking at lobbyist interactions over one non-strategic legislator. We remove the dimension of budget allocation and instead assume that the lobbyists are willing to pay at most the value of the legislator support. To the best of our knowledge, we are the first to study how uncertainty on the position of a single legislator can affect the behaviour of competing lobbyists.

Our model also features a measure of the legislator's level of integrity. The politician does not hold an open sale for her support given restrictions on vote-buying. Instead, the politician chooses the lobbyist that promises to contribute enough, and if there is more than one lobbyist that offers enough, she goes for the lobbyist offering the most value to her. We can think about this from the perspective of a reputation conscious politician. A reputation conscious politician may not want to be associated with more than one lobbying group for an issue and would like to be viewed as consistent and honest by her constituents. The more controversial an issue is, the more likely it is for the politician's threshold to be higher. Lobby's revolving-door phenomenon, whereby former staffers turned lobbyists use their connections to incumbents to push for legislation (Blanes i

Vidal et al., 2012; LaPira and Thomas, 2014; Lazarus et al., 2016), explains why the integrity of the legislator is known to lobbyists. As politicians move from public service to the private sector, they take their expertise on both the workings of the political system and the proclivities of their former colleagues with them. Politicians would know who among their colleagues can be wooed for support, and who are strong in their convictions from previous interactions. Moving through the revolving door as lobbyists, they provide lobbies with information on the level of integrity of each legislator. This does not mean that the specific preferences of each legislator are known to the lobbyists. As the preferences vary across issues, the reliability of the information on preferences would be less than that of the integrity of the legislator, making a range of possible legislator's preference values more likely than a clear cut one.

The rest of the paper is organized as follows. Section 2 introduces the model. Sections 3 and 4 show the expected utility and the computation of best responses. The equilibrium of the game is characterized in full in Section 5. The results are discussed in Section 6, and the paper is concluded in Section 7. The proofs are shown in full in the appendices.

2 The Model

Consider a legislator and two lobbyists. The legislator has a policy bias b distributed uniformly $b \sim \mathcal{U}(-d, d)$, with $d > 0$, and an integrity threshold $t > 0$. A positive b indicates a legislator bias towards policy two, while a negative b indicates a preference for policy one. At $b = 0$ the legislator is unbiased. The legislator's utility is given by,

$$U_L = \begin{cases} p_1 - t - b & \text{if Lobbyist 1 wins,} \\ p_2 - t + b & \text{if Lobbyist 2 wins,} \\ 0 & \text{otherwise,} \end{cases}$$

where p_i is the bid submitted by lobbyist i.[1]

The lobbyists support opposing policy positions and try to sway the legislator. The game can be viewed through the lens of a policy dimension, with each lobbied policy on the opposite end of a one-dimensional policy space. The preference of each lobbyist is fully aligned with their policy of choice and lies on the extremes of the policy space. The lobbyist preferences are fully revealed from the onset with each lobbyist supporting only their policy of choice.

The lobbyists know the distribution of the legislator's bias and the integrity threshold. Both bid simultaneously $p_i \geq 0$ to win the legislator's support. Bids are only considered by the legislator when they are above the bias-adjusted threshold (i.e. $t \pm b$). The bid that provides the legislator with the highest utility to the legislator wins. Only the winning bid is collected. Lobbyist i gains $w \in \mathbb{R}_+$ upon winning.

The utilities of each lobbyist i are given below:

$$U_1 = \begin{cases} w - p_1 & \text{if } p_1 > t + b \text{ and } p_1 > p_2 + 2b, \\ 0 & \text{otherwise.} \end{cases}$$

$$U_2 = \begin{cases} w - p_2 & \text{if } p_2 > t - b \text{ and } p_2 > p_1 - 2b, \\ 0 & \text{otherwise.} \end{cases}$$

The game is summarized in the symmetric simultaneous-move game (N, P, b, U), defined formally as follows:

1. Three players ($N = 3$), two lobbyists and one legislator;

[1]Note that we assume that in the absence of lobbying the legislator finds it optimal to not implement a new policy and this gives the legislator an outside option of 0 utility. When either policy is approved, we assume the legislator suffers a loss of t, the integrity threshold.

2. Lobbyist's bids belong to $p_i \in P = [0, w]$;

3. b is the legislator's realised type, $b \sim \mathcal{U}(-d, d)$;

4. U_i, for $i \in \{1, 2\}$, is the lobbyist's utility, and U_L the legislator's utility.

Note that the legislator is a non-strategic player. The legislator's utility function can be viewed as the rules of the game. The winner is determined not by any strategic decision making on the legislator's side. As we are looking at shadow lobbying, it stands to reason that the legislator will accept the first offer that meets her expectations, as the risk of discovery through holding repeated negotiations will be high and the consequences substantial.

A lobbyist only wins if the bid is considered sufficient and provides the most payoff to the legislator. The probabilities of winning are given as follows:

$$P(1 \text{ wins}) = \begin{cases} \dfrac{p_1 - t + d}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{p_1 - p_2 + 2d}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{1}$$

$$P(2 \text{ wins}) = \begin{cases} \dfrac{d - t + p_2}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{2d - p_1 + p_2}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{2}$$

The probability of lobbyist i winning must be within $[0, 1]$. The probabilities in (1) and (2) can be written in full as $min\{max\{0, P(i \text{ wins})\}, 1\}$. In the subsequent arguments, the parameter restrictions are consistent with well-defined probabilities. The probabilities of winning are derived in full in Appendix A.

3 Expected Utilities

The expected utility of lobbyist i is given by the probability of winning, expressions (1) and (2), and the utility of the lobbyist, U_i. As with the probabilities, the computation of the expected utility depends on the relationship of the average bid to the integrity threshold of the legislator. We compute the expected utility of each lobbyists when the average bid is below the threshold, $(p_1 + p_2)/2 \leq t$, and when it is above the threshold, $(p_1 + p_2)/2 \geq t$. The expected utility is continuous at the point where the average bid is equal to the threshold. At each scenario, we compute the optimal bids, with $\underline{p}_i$ and $\overline{p}_i$ denoting the optimal bids when the average bid is below and above the threshold, respectively. The expected utilities are derived in full in Appendix B.

Given the symmetric environment, it is sufficient to compute the expected utility of one lobbyist, say lobbyist one. The expected utility of lobbyist 1 is

$$EU_1(p_1, p_2) = \begin{cases} \dfrac{p_1 - t + d}{2d}(w - p_1) & \text{if } \frac{p_1+p_2}{2} \leq t, \quad (3) \\ \dfrac{p_1 - p_2 + 2d}{4d}(w - p_1) & \text{if } \frac{p_1+p_2}{2} \geq t. \quad (4) \end{cases}$$

Scenario 1: Average bid below the threshold ($\frac{p_1+p_2}{2} \leq t$)

Given an opposing bid p_2, we solve for the maximum of the expected utility (3). The optimal bid is $p_1 = (w+t-d)/2$. The function (3) is concave, and the second derivative, $-1/d$, us always negative. The lobbyist's bids must be feasible, $p_1 \in [t - d, 2t - p_2]$, at the given p_2, in order to keep the average bid below the threshold, $(p_1 + p_2)/2 \leq t$, and exceed the effective lower bound, $t - d$. Note that the feasible bids of lobbyist one belong to a well-defined interval as long as $p_2 < t + d$.[2]

[2]When $p_2 \geq t + d$, lobbyist one does not choose to keep the average below the threshold. All feasible bids will neither reach the threshold nor beat the opposing bid. Lobbyist two wins for sure if lobbyist's one bidding strategy keeps the average bid below the threshold.

The lower bound, $t-d$, is the minimum possible bid for the lobbyist. The lower bound is the lowest possible bias adjusted threshold—the legislator's integrity threshold, t, adjusted to account for the maximum possible bias the legislator can have for the lobbyist, $-d$. As the legislator only considers bids that are at least the bias adjusted threshold, bidding below the lower bound renders lobbyist's bid irrelevant. The winning valuation, w, always exceeds the minimum possible bid for the lobbyists and is the lobbyist's upper bound. In effect, the optimal bid of lobbyist one in this scenario $p_1 = (w+t-d)/2$ is just the average of the minimum and maximum possible bids. If $(w+t-d)/2$ is beyond the set of permissible values $[t-d, 2t-p_2]$, the optimal bid will be the upper bound $2t-p_2$; This is true when $p_2 > \frac{3t-w+d}{2}$.

The optimal bid of lobbyist 1 when the average bid is below the threshold, $\frac{p_1+p_2}{2} \leq t$, is given by:

$$\underline{p_1} = \begin{cases} \dfrac{w+t-d}{2} & \text{if } p_2 \leq \dfrac{3t-w+d}{2}, \\ 2t-p_2 & \text{otherwise.} \end{cases} \tag{5}$$

The expected utilities when the average bid is below the threshold are outlined in (18).

$$EU_1\left(\underline{p_1}, p_2\right) = \begin{cases} \dfrac{(d-t+w)^2}{8d} & \text{if } p_2 \leq \dfrac{3t-w+d}{2}, \\ \dfrac{(d-p_2+t)(p_2-2t+w)}{2d} & \text{otherwise.} \end{cases} \tag{6}$$

Scenario 2: Average bid above the threshold ($\frac{p_1+p_2}{2} \geq t$)

We follow the same process in the identification of expected utilities for scenario 2. Given an opposing bid p_2, we solve for the maximum of the expected utility (4). The optimal bid is $p_1 = (w+p_2-2d)/2$. The function (4) is always concave with a negative second derivative, $-1/2d$. For the average bid to be greater than or equal to the threshold, for any given p_2, lobbyist one has to bid at least $2t-p_2$, and the feasible set of bids reduce to $p_1 \in [max\{p_2-2d, 2t-p_2\}, w]$.

Under this scenario, the lobbyist must bid at least $p_2 - 2d$ for the expected utility to be positive. When the average bid is above or equal to the threshold, it is certain that one of the lobbyists wins the support of the legislator. The lobbyist shifts from making sure that the minimum eligibility condition, the bias adjusted threshold, is met to ensure that his bid is competitive enough to remain in the game. If both bids exceed their respective bias adjusted thresholds, lobbyist one only wins once his bid, adjusted with the bias, exceeds that of his opponent (*i.e.* $p_1 - b > p_2 + b$). Recall that a negative bias is advantageous for lobbyist 1, obtaining the maximum possible advantage when $b = -d$. For lobbyist one to have an eligible bid when the average bid is above or equal to the threshold, the bid must at least be greater than $p_2 - 2d$. The bid is capped by the winning valuation w. Similar to the previous scenario, the optimal bid of lobbyist two in this scenario, $p_1 = (w + p_2 - 2d)/2$, is the average of the minimum and maximum possible bids. Note that the optimal bid $p_1 = (w + p_2 - 2d)/2$ only satisfies the condition $(p_1 + p_2)/2 \geq t$ when $p_2 \geq (4t - w + 2d)/3$.

The optimal bid of lobbyist 1 when the average bid is above the threshold, $\frac{p_1+p_2}{2} \geq t$, is given by:

$$\overline{p_1} = \begin{cases} \dfrac{w - 2d + p_2}{2} & \text{if } p_2 \geq \dfrac{4t - w + 2d}{3}, \\ 2t - p_2 & \text{otherwise.} \end{cases} \tag{7}$$

The expected utility of lobbyist one when the average bid is above the threshold are outlined in (20).

$$EU_1\left(\overline{p}_1, p_2\right) = \begin{cases} \dfrac{(2d - p_2 + w)^2}{16d} & \text{if } p_2 \geq \dfrac{4t - w + 2d}{3}, \\ \dfrac{(d - p_2 + t)(p_2 - 2t + w)}{2d} & \text{otherwise.} \end{cases} \tag{8}$$

Note that the optimal bids alternate between $\underline{p_1}$ and $\overline{p}_1$, depending on where the lobbyist's two bid p_2 is, in relation to thresholds $(3t - w + d)/2$ and $(4t - w + 2d)/3$. As the minimum possible bid cannot exceed the winning valuation, $t - d \leq w$, it follows that $(3t - w + d)/2 \leq (4t - w + 2d)/3$ for all feasible values of t, d, w.

4 Best Responses

Each lobbyist compares the expected utility from bidding $\underline{p_i}$ and $\overline{p_i}$, given the opponent's bid, and chooses the bid that provides him with the highest utility. The best response of lobbyist i is equal to

$$BR_i\left(p_{-i}\right) = \begin{cases} \dfrac{w+t-d}{2} & \text{if } p_{-i} \leq \dfrac{3t-w+d}{2}, \\ 2t - p_{-i} & \text{if } \dfrac{3t-w+d}{2} < p_{-i} < \dfrac{4t-w+2d}{3}, \\ \dfrac{w-2d+p_{-i}}{2} & \text{otherwise.} \end{cases} \tag{9}$$

We determine the best responses of lobbyist 1 from the sets of possible bids of lobbyist 2 identified in (9). The best responses are derived in full in Appendix C. For the purposes of discussion, we refer to the strategy of lobbyist i as conservative when $p_i \leq (3t-w+d)/2$ and aggressive when $p_i \geq (4t - w + 2d)/3$.

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

If lobbyist two bids conservatively at or below $\frac{3t-w+d}{2}$, lobbyist one can choose to bid either $\underline{p_1} = (w+t-d)/2$ and keep the average bid below the threshold, or bid $\overline{p_1} = 2t-p_2$ and push the average bid to the threshold.

Whenever $p_2 \leq (3t-w+d)/2$, a comparison of expected utilities shows that $EU_1\left(\underline{p_1}, p_2\right) \geq EU_1\left(\overline{p}_1, p_2\right)$. This means that the utility of lobbyist 1 is higher if he bids to keep the average below the legislator's integrity threshold and the best response of lobbyist one is $BR_1\left(p_2\right) = (w+t-d)/2$.

When the opponent's bid is low enough, the lobbyist chooses to bid conservatively ($p_i \leq$

$(3t-w+d)/2)$. It may appear counterintuitive at first as the lobbyist can potentially secure the legislator's support immediately. By increasing his bid, he increases the chances of winning the support of legislator, but this is at the expense of lowering his take-home win. The lobbyist may end up paying more if he decides to bid just enough to ensure that the game ends, $\underline{p_1} = 2t - p_2$. The lobbyist can afford to bid conservatively as the winning probability of the opponent is not high enough to warrant a price war. This is evident from the fact that his best response stays constant regardless of the opponent's bid.

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t-w+2d}{3}$

For a given p_2 between the lower and upper bound, the best response of lobbyist's one is $BR_1(p_2) = 2t - p_2$. As before, each lobbyist is faced with the trade-off between a higher chance of winning the support of the legislator and a lower take-home win. When the opposing bid is not low enough to justify bidding conservatively ($p_i \leq (3t-w+d)/2$), but also not high enough to justify bidding aggressively ($p_i \geq (4t-w+2d)/3$), his best response is to bid enough so that the game ends with a winning bid.

Case 3: $p_2 \geq \frac{4t-w+2d}{3}$

When lobbyist 2 bids aggressively at or above $\frac{4t-w+2d}{3}$, lobbyist one can choose to bid either $\underline{p_1} = 2t - p_2$ to keep the average bid at the threshold or bid $\overline{p}_1 = (w + p_2 - 2d)/2$ to push the bid above the threshold.

Whenever $p_2 \geq (4t-w+2d)/3$, $EU_1(\overline{p}_1, p_2) \geq EU_1\left(\underline{p_1}, p_2\right)$. When the opposing lobbyist bids high, the lobbyist obtains a higher expected utility by also bidding high, with the best response of $BR_1(p_2) = (w + p_2 - 2d)/2$.

When the opponent bids aggressively, the lobbyist's bid must at least be equal to his opponent's bias adjusted bid, otherwise his bid is not competitive. He can choose to bid conservatively ($p_i \leq (3t-w+d)/2$), just enough to ensure that the game ends or match the opposing bid head on and bid aggressively ($p_i \geq (4t - w + 2d)/3$). Both bids depend on p_2, with $\overline{p}_1$ increasing and $\underline{p_1}$ decreasing in p_2. The more aggressive the opponent's bid is, the more likely he is to win the support of the legislator. Under bidding strategy $\underline{p_1}$ where the lobbyist bids conservatively, the lobbyist becomes less likely to win the support of the lobbyist but increases his possible take-home pay. Although the strategy made sense when the competing lobbyist was bidding conservatively, there is no clear incentive to do the same with a more aggressive competitor. The bids under the conservative strategy decrease as the opponent bid increases, essentially demanding for a higher tradeoff cost on the probability of winning and the increase in the take home pay. Furthermore, the higher the opponent's bid gets, the less likely it is for the conservative bid to be eligible, so that he must bid aggressively in order to win the support of the legislator. The lobbyist bid should exceed the minimum competitive bid, $p_2 - 2d$, and is bounded above by the maximum bid w. In sum, by bidding according to $\overline{p}_1$, lobbyist 1 secures a significant chance of winning the legislator's support and enough utility once the support is won.

5 Equilibria

The Nash Equilibria are identified from the best responses outlined in (9).

Proposition 1. *(Nash Equilibria)*

1. *If $d \leq (1/2)(w - t)$, there exists a unique Nash equilibrium (p_i^*, p_{-i}^*), where $p_i^* = p_{-i}^* = w - 2d$.*

2. *If $(1/2)(w - t) < d < (5/7)(w - t)$, there exists a continuum of Nash equilibria (p_i^*, p_{-i}^*), where $p_i^* \in \left(\frac{w-2d+2t}{3}, \frac{4t-w+2d}{3}\right)$ and $p_{-i}^* = 2t - p_i^*$.*

3. *If $d = (5/7)(w - t)$, there exists a continuum of Nash equilibria (p_i^*, p_{-i}^*), where $p_i^* \in \left(\frac{8t-w}{7}, \frac{6t+w}{7}\right)$ and $p_{-i}^* = 2t - p_i^*$.*

4. *If* $(5/7)(w-t) < d < w-t$, *there exists a continuum of Nash equilibria* (p_i^*, p_{-i}^*), *where* $p_i^* \in \left(\frac{3t-w+d}{2}, \frac{w+t-d}{2}\right)$ *and* $p_{-i}^* = 2t - p_i^*$.

5. *If* $d \geq w-t$, *there exists a unique Nash equilibrium* (p_i^*, p_{-i}^*), *where* $p_i^* = p_{-i}^* = (w+t-d)/2$.

The results of Proposition 1 are illustrated in Figure 1 and discussed in sections 6.1. We further summarize the results from Proposition 1 and identify the total bid of the lobbyists in equilibrium in Proposition 2 below.

Proposition 2. *(Sum of Lobbyists' Bids)*

$$p_1^* + p_2^* = \begin{cases} 2w - 4d & \text{if} \quad d \leq \dfrac{w-t}{2}, \\ 2t & \text{if} \quad \dfrac{w-t}{2} < d < w-t, \\ w+t-d & \text{otherwise} \end{cases} \tag{10}$$

and $p_1^* + p_2^*$ *is non-increasing in* d.

The cases under Proposition 2 are used to guide the remaining sections, 6.2, 6.3, and 6.4, of the results and discussion.

6 Results and Discussion

The characterisation of equilibria is based on the relationship between the length of the bias interval, d, and the difference between the winning valuation, w, and the legislator's integrity threshold, t. The full calculation of equilibria can be found in Appendix D.

Recall that the length of the bias interval, d, measures the uncertainty around the legislator bias: a higher d indicates higher uncertainty over the legislator's bias.[3] We

[3]Without uncertainty, the model may be reminiscent of the Standard Tullock Contest (Tullock, 1967).

use the values of d identified in Proposition 2 to determine the levels of bias uncertainty used in the subsequent discussion: where "high uncertainty over the bias" refers to $d \geq w - t$, "moderate uncertainty over the bias" refers to $(w - t)/2 < d < w - t$, and "low uncertainty over the bias" refers to $d \leq (w - t)/2$. Note that uncertainty over the bias and bias uncertainty are both used to refer to d.

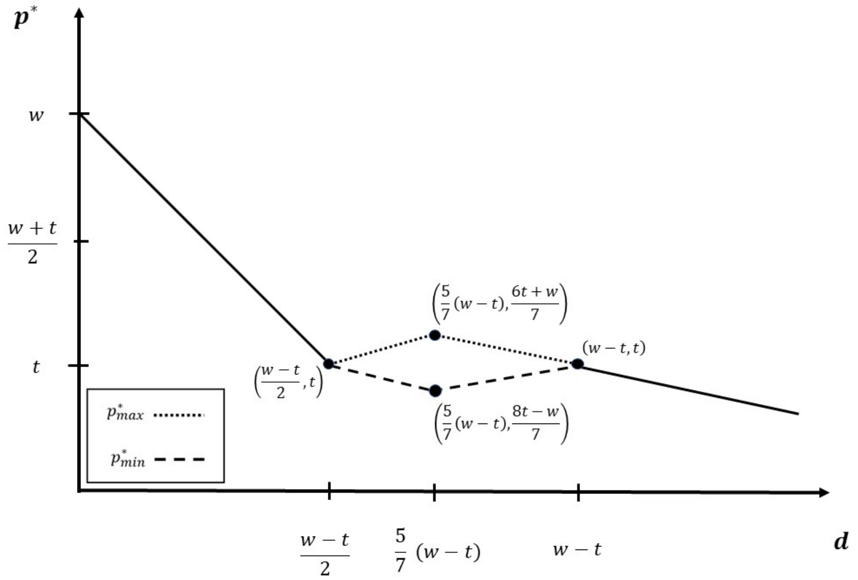

Figure 1: Summary of Equilibrium Bids over Legislator Preference Uncertainty

6.1 Effect of Bias Uncertainty on Lobbyists' Bids

The results of Proposition 1 are illustrated in Figure 1. Figure 1 shows the relationship between the equilibrium bid, p^*, and the uncertainty over the legislator's bias, d. The values of p^* for each of the five points in Proposition 1 are shown in the figure. Note that as the possible p^* values change across the identified points in Proposition 1, we use p^*_{min} and p^*_{max} to represent the minimum and maximum possible bids under each point

We deviate from Tullock Games as we consider a simultaneous sealed-bid auction instead of an all-pay auction. Uncertainty in our model is on the price of access rather than opponent valuation per the Tullock literature.

of the proposition. In general, we can see from Figure 1 that there is a non-monotonic relationship between the uncertainty over the legislator's bias and the equilibrium bids.

It can be observed in Figure 1 that the slopes of p^*_{min} and p^*_{max} vary over three main intervals: (i) $d \leq (w-t)/2$, (ii) $(w-t)/2 < d < w-t$, and (iii) $d \geq w-t$. The intervals identified here are in line with the cases observed under Proposition 2.

Let's start with the intermediate interval: (ii) $(w-t)/2 < d < w-t$. In (ii), we find a continuum of asymmetric equilibria. Within this interval, we also find the symmetric equilibrium $p^*_i = p^*_{-i} = t$ at $d = (5/7)(w-t)$. Under moderate bias uncertainty, $(w-t)/2 < d < w-t$, the sum of equilibrium bids is constant at $2t$ as indicated in case two of Proposition 2. The interval $(w-t)/2 < d < w-t$ reflected in Figure 1 is comprised of points 2, 3 and 4 of Proposition 1. In Figure 1, p^*_{min} decreases at the same rate p^*_{max} increases over $(w-t)/2 < d < (5/7)(w-t)$, while , p^*_{min} increases at the same rate p^*_{max} decreases over $(5/7)(w-t) < d < w-t$. The equilibrium bids under moderate bias uncertainty are explored more closely through points 2 to 4 of Proposition 1.

From point 2 of Proposition 1, the continuum of equilibrium bids identified for $(w-t)/2 < d < (5/7)(w-t)$ is $p^*_i \in ((w-2d+2t)/3, (4t-w+2d)/3)$ and $p^*_{-i} = 2t - p^*_i$. The change in the lower bound of p^*_i given a unit change in d is $-2/3$ (the slope of p^*_{min} in Figure 1), while the change in the upper bound of p^*_i given a unit change in d is $2/3$ (the slope of p^*_{max} in Figure 1). Note that at $d = (w-t)/2$, both the upper and lower bounds of $p^*_i \in ((w-2d+2t)/3, (4t-w+2d)/3)$ are equal to the legislator integrity threshold t.

From point 4 of Proposition 1, the continuum of equilibrium bids identified for $(5/7)(w-t) < d < w-t$ is $p^*_i \in ((3t-w+d)/2, (w+t-d)/2)$ and $p^*_{-i} = 2t - p^*_i$. The change in the lower bound of p^*_i in the above interval given a unit change in d is $1/2$ (the slope of p^*_{min} in Figure 1 for $(5/7)(w-t) < d < w-t$), while the change in the upper bound of p^*_i given a unit change in d is $-1/2$ (the slope of p^*_{max} in Figure 1 for $(5/7)(w-t) < d < w-t$). Note that at $d = w-t$, both the upper and lower bounds of

$p_i^* \in ((3t - w + d)/2, (w + t - d)/2)$ are equal to the legislator integrity threshold t.

At $d = (5/7)(w - t)$, the values of p_{min}^* and p_{max}^* do not vary according to d. From point 3 of Proposition 1, the continuum of equilibrium bids identified for $d = (5/7)(w - t)$ is $p_i^* \in ((8t - w)/7, (6t + w)/7)$ and $p_{-i}^* = 2t - p_i^*$. The equilibrium where both lobbyists bid the legislator integrity threshold, $p_i^* = p_{-i}^* = t$, can also be observed here.

In sum, for moderate bias uncertainty where $(w - t)/2 < d < w - t$, the average equilibrium bid is equal to the integrity threshold t. Asymmetric equilibria are observed over the interval $(w - t)/2 < d < w - t$, all of which have its sum of equilibrium bids at $2t$. We also observe the symmetric equilibrium $p_i^* = p_{-i}^* = t$ at $d = (5/7)(w - t)$. Under moderate bias uncertainty, the lobbyists bid just enough to ensure that an offer is accepted by the legislator.

Now let's compare the two remaining intervals: (i) $d \leq (w - t)/2$ and (iii) $d \geq w - t$. Unique symmetric equilibrium can be observed for (i): $p_i^* = p_{-i}^* = w - 2d$ and (iii): $p_i^* = p_{-i}^* = (w + t - d)/2$. The equilibrium bids p_i^* and p_{-i}^* are equal to the legislator's integrity threshold , t, at $d = (w - t)/2$ and $d = w - t$. From Proposition 2, we find that the sum of equilibrium bids, $p_i^* + p_{-i}^*$, decrease as the uncertainty over legislator bias increases when there is low uncertainty over the bias ($d \leq (w - t)/2$) or high uncertainty over the bias ($d \geq w - t$). Furthermore, we note from Figure 1 that the slope of p^* is different for $d \leq (w - t)/2$ and $d \geq w - t$. More specifically, holding the winning valuation w and integrity threshold t constant, we find that:

- (From Proposition 1.1) When $d \leq \frac{w-t}{2}$, $\frac{\partial p^*}{\partial d} = -2$.
- (From Proposition 1.5) When $d \geq w - t$, $\frac{\partial p^*}{\partial d} = -\frac{1}{2}$.

The effect of an increase in bias uncertainty d is bigger when the level of uncertainty is low, $d \leq (w - t)/2$. As the position of the legislator becomes more certain, lobbyists will try to win over the legislator by offering the highest bid. Under no bias uncertainty

($d = 0$), the lobbyists will bid their valuation w at equilibrium. The higher drop in bids under low bias uncertainty can be explained by the increased pressure to bid closer to the lobbyist's valuation as uncertainty nears zero. When the bias uncertainty is high, $d \geq w - t$, this pressure disappears, and the equilibrium bids, although still decreasing over d, are less affected by changes in the bias uncertainty.

It also follows that the lowest equilibrium bids are observed at high values of d at $d \geq w-t$ (see Proposition 1.5). Our results show that the cheapest legislators are those whose preferences are highly uncertain. The high level of uncertainty over the legislator bias is risky and in effect discounts the price the lobbyist is willing to bid in order to win access to the legislator. Although there is a chance that the legislator may have a very strong bias towards the lobbyist's position, the friction brought about by the uncertainty is reflected in the price. Our results indicate that as long as there is a chance that a legislator has a strong preference for a lobbyist, it would be more cost-effective for the lobbyist to approach the legislator. If this result is observed, the legislators approached by lobbyists should vary depending on the issue on the table. This supports the growing consensus in the empirical literature identified by de Figueiredo and Richter (2013) where legislators, allied and marginal, from both sides of the issues are approached more often than staunch opposition by lobbyists (Kollman, 1997; Holyoke, 2003; Heberlig, 2005; Hall and Deardorff, 2006; Bertrand et al., 2014; Gawande et al., 2012). [4]

The equilibrium bidding strategies are explored further in the rest of the discussion below.

6.2 Strategies under Low Bias Uncertainty: $d \leq (w - t)/2$

When there is low uncertainty over the legislator's bias ($d \leq (w - t)/2$), both lobbyists bid $p_1^* = p_2^* = w - 2d$, with an average bid above the legislator's bias integrity threshold

[4] Marginal legislators are accounted for in our model through the existence of uncertainty - however the magnitude of this uncertainty is what is explored in the analysis of equilibria.

(see point 1 in Proposition 1). They favour more aggressive bidding strategies, as the information on the bias becomes more precise. A short bias interval implies that the benefit for each lobbyist of having the legislator's preferences in his favour is marginal. The winning bid needs to surpass both the threshold and the opposing bid. As the difference between the legislator's threshold and bias adjusted threshold is minimal under a short bias interval, the lobbyists both assume that the opposing bid has already surpassed the threshold. As the bids are the same, lobbyists are equally likely to win the legislator's support. Notice that the equilibrium bid is just the difference between the wealth valuation and the full length of the bias interval $2d$.

If the uncertainty over the legislator's bias is sufficiently small relative to the difference between the lobbyist's winning valuation and the legislator's integrity threshold, then each lobbyist bids aggressively. A higher winning valuation makes the uncertainty of the legislator's preferences smaller in comparison. Similarly, a lower integrity threshold increases $(w - t)/2$, making it more likely for the uncertainty over the legislator's bias to satisfy $d \leq (w - t)/2$. The Center for Responsive Politics (2018) reports that the top three sectors in terms of lobbying expenditure are Health, Finance, Miscellaneous Business, with over \$500 million spending for each sector in 2018. These sectors provide high payoffs for lobbyists. Alongside this, as the public is less likely to be aware of the legislator's preferences on various industry specific areas, the integrity threshold is significantly lower relative to hot button issues. The reputation of legislators are less likely to be questioned if lobbying requests are entertained for these issues. Although the lobbyists may have less certainty in terms of the legislator's preferences, this is dwarfed by the substantial winning valuation and the lower integrity threshold. This echoes results from existing literature which finds that business interests, trade associations, and professional groups, are the groups with the highest level of lobbying expenditures (Baumgartner and Leech, 2001; de Figueiredo, 2004; McKay, 2012; de Figueiredo and Richter, 2013).

6.3 Strategies under Moderate Bias Uncertainty: $(w-t)/2 < d < w-t$

When there is moderate uncertainty over the legislator's bias ($(w-t)/2 < d < w-t$), we find a continuum of equilibria, with the characteristic that the average bid is equal to the legislator's threshold (see case 2 in Proposition 2). Specifically, under point 4 in Proposition 1, when the bias interval is $(5/7)(w-t) < d < w-t$, that is the uncertainty over the legislator's bias is quite high, lobbyists can always bid aggressively ($p_i \geq (4t - w + 2d)/3$) to secure the win — however the increase in the winning probability must be worth the increased cost. Bidding aggressively may cause the lobbyist to pay more than necessary to win the legislator's support. He is better off bidding conservatively. The other lobbyist bids enough to have him indifferent between bidding conservatively and aggressively, keeping the average bid at the threshold. On the other hand, under point 2 in Proposition 1, when the uncertainty over the legislator's bias is quite low with $(1/2)(w-t) < d < (5/7)(w-t)$, bidding conservatively is risky as the interval is not long enough to ensure that the opponent does the same. One lobbyist tries and bids aggressively to increase the chance of securing the legislator's support, while the other bids just enough for the average bid to reach the threshold. We also find the special case where one lobbyist bids aggressively and another bids conservatively at $d = (5/7)(w-t)$ (see point 3 in Proposition 1). The symmetric equilibrium $p_1^* = p_2^* = t$ is also observed at $d = (5/7)(w-t)$.

Both lobbyists can also choose to bid just enough to ensure that the average bid is equal to the legislator's threshold in equilibrium at $(w-t)/2 < d < w-t$. A continuum of equilibria (p_i^*, p_{-i}^*) exists where $p_i^* = 2t - p_{-i}^*$ and $p_{-i}^* = 2t - p_i^*$ under moderate bias uncertainty. As the cost to the probability of winning tradeoff is not high enough, both bid just enough to ensure that the legislator's support is won.

6.4 Strategies under High Bias Uncertainty: $d \geq w - t$

When there is high uncertainty over the legislator's bias ($d \geq w - t$), both lobbyists maximise their expected utilities by bidding conservatively ($p_i \leq (3t - w + d)/2$). Under point 5 in Proposition 1, when $d \geq w - t$, both lobbyists bid $p_i^* = p_{-i}^* = (w + t - d)/2$. The average bid here is less than legislator's bias integrity threshold, $(p_1^* + p_2^*) \leq 2t$. Both lobbyists are unaware of the actual legislator bias but have the same information on its distribution. The best case scenario for a lobbyist would be to have the threshold adjusted heavily downwards by a favourable bias by the legislator; and the worst possible case for the lobbyist is when the bias is at its most favourable for his opponent, driving the bias adjusted threshold above the winning valuation and making it impossible for the lobbyist to win. The bids, and consequently the utilities of lobbyists under this case do not depend on their opponents bid. As it is equally likely for both of them to be at an advantageous position, but too costly for either one to secure a high enough probability of winning, both lobbyists choose to bid midway between the minimum possible bid $t-d$, and the maximum possible bid w, maximising their expected utility.

The uncertainty of the legislator bias has to be sufficiently higher than the difference between the lobbyist's winning valuation and the legislator's integrity threshold for the equilibrium bids , $w - t$, to be valid. Without uncertainty, the minimum qualifying bid is just the integrity threshold of the legislator. An increase in the level of integrity of the legislator t, holding the winning valuation w constant, makes it more likely the bias interval to be sufficiently high. For highly salient issues, t is very high; the positions of legislators are heavily publicized, with high costs on reputation if the legislator's integrity is questioned. This may help explain why despite high levels of coverage on single issues in the United States (*e.g.* gun rights vs. gun control and pro-life vs. pro-choice), and the low uncertainty over legislator's preferences on the issues, often earmarked by party memberships (*e.g.* Republicans for gun rights and Democrats for gun control), lobbyist spending in the single issue sector does not reach the top five sectors with the highest lobbying expenditure in 2018 (The Center for Responsive Politics, 2018). With ideological issues, the rewards of the policy are often not as high as issues tied with industry interest. Lower winning valuations coupled with a high integrity threshold can drive lobbyists to bid more conservatively.

7 Conclusion

We explore how lobbying can proceed in an unregulated environment. A simultaneous lobbying structure is used to capture how lobbying proceeds behind closed doors. Under shadow lobbying, where lobbyist-legislator interactions are kept private, lobbyists may not be able to counteroffer. The paper focuses on lobbyist interactions over one non-strategic legislator and explores the impact of uncertainty on the lobbyist's behaviour in isolation.

The model explains how the relationship between the uncertainty over the legislator's bias, the legislator's integrity threshold, and the lobbyist's winning valuation, influence the bidding strategies of lobbyists. Overall, we find a non-monotonic relationship between equilibrium bids and uncertainty over the legislator's bias. We find the average equilibrium bid of the lobbyists given moderate bias uncertainty is equal to the integrity threshold of the legislator. In cases where bias uncertainty is low or high, we find that increased uncertainty over the legislator's bias tend to decrease the bids of lobbyists, although this is more evident at lower level of bias uncertainty. In particular, when the bias uncertainty is low with respect to the winning valuation, lobbyists tend to bid aggressively in order to secure the support of the legislator. When the bias uncertainty is high, both lobbyists bid conservatively, taking into account the possibility that the legislator may strongly prefer their policy of choice. The results also indicate that for issues with high monetary rewards and lower public salience, lobbyists bid more aggressively.

Most likely, these are issues where ideologies are not as clear cut.

The results of this paper corroborate the growing consensus that lobbyists mostly approach allied or marginal legislators and offer the additional insight that the possibility of a strong preference towards the policy lobbied for makes the legislator more attractive to lobbyists. Political agents, however, may listen to constituent opinions and adjust their preferences accordingly, which, in turn, influence lobbyist's behaviour.

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Appendix

A Deriving Winning Probabilities

A lobbyist only wins if the bid is considered sufficient and provides the most payoff to the legislator. The probabilities of winning for each lobbyist are:

$$P\,(1 \text{ wins}) = P\left(p_1 > t + b \bigcap p_1 > p_2 + 2b\right),$$
$$P\,(2 \text{ wins}) = P\left(p_2 > t - b \bigcap p_2 > p_1 - 2b\right).$$

We begin with looking at the probability of winning for lobbyist 1,

$$\begin{aligned} P(1 \text{ wins}) &= P\left(p_1 > t + b \bigcap p_1 > p_2 + 2b\right) \\ &= P\left(b < p_1 - t \bigcap b < (p_1 - p_2)/2\right) \\ &= P\left(b < \min\{p_1 - t, (p_1 - p_2)/2\}\right). \end{aligned}$$

The winning probabilities for lobbyist 2 are obtained in a similar manner. To sum up:

$$P(1 \text{ wins}) = \begin{cases} P(b < p_1 - t) & \text{if } \frac{p_1+p_2}{2} < t, \\ P(b < \frac{p_1-p_2}{2}) & \text{otherwise.} \end{cases} \tag{11}$$

$$P(2 \text{ wins}) = \begin{cases} P(b > t - p_2) & \text{if } \frac{p_1+p_2}{2} < t, \\ P(b > \frac{p_1-p_2}{2}) & \text{otherwise.} \end{cases} \tag{12}$$

When the average bid is equal to the integrity threshold, it follows that $P(b < p_1 - t) = P(b < (p_1 - p_2)/2)$, and $P(b > t - p_2) = P(b > (p_1 - p_2)/2)$. Subsequently, given that the bias is uniformly distributed, (11) and (12), reduce to

$$P(1 \text{ wins}) = \begin{cases} \dfrac{p_1 - t + d}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{p_1 - p_2 + 2d}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{13}$$

$$P(2 \text{ wins}) = \begin{cases} \dfrac{d - t + p_2}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{2d - p_1 + p_2}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases} \tag{14}$$

B Expected Utilities

Scenario 1: Average bid below the threshold ($\frac{p_1+p_2}{2} \leq t$)

Given an opposing bid p_2, we solve for the bid p_1 that will yield maximum expected utility:

$$EU_1(p_1, p_2) = \begin{cases} \dfrac{p_1 - t + d}{2d}(w - p_1) & \text{if } \frac{p_1+p_2}{2} \leq t, \quad (15) \\ \dfrac{p_1 - p_2 + 2d}{4d}(w - p_1) & \text{if } \frac{p_1+p_2}{2} \geq t. \quad (16) \end{cases}$$

At $\frac{p_1+p_2}{2} \leq t$, the expected utility is given by $\frac{p_1-t+d}{2d}(w - p_1)$.

Taking the first and second derivatives over p_1:

$$\frac{\partial EU_1}{\partial p_1} = \frac{d - 2p + t + w}{2d} = 0$$
$$p_1 = \frac{w + t - d}{2}$$

$$\frac{\partial^2 EU_1}{\partial^2 p_1} = -\frac{1}{d}$$

The function is concave and its expected utility is maximized at $p_1 = (w + t - d)/2$. We will refer to the $(w + t - d)/2$ as $\underline{p_1}$.

The minimum bid for the lobbyist to stay in the game is $t - d$. To fulfill the assumption where the average bid is below the threshold ($(p_1 + p_2)/2 \leq t$), only feasible bids are considered: $p_1 \in [t - d, 2t - p_2]$, given p_2. Note that the feasible bids of lobbyist one belong to a well-defined interval as long as $p_2 < t + d$. When $p_2 \geq t + d$, lobbyist one does not choose to keep the average below the threshold. All feasible bids will neither reach the threshold nor beat the opposing bid. Lobbyist two wins for sure if lobbyist's one bidding strategy keeps the average bid below the threshold.

For the average bid to be below the threshold t, $\underline{p_1} \leq 2t - p_2$.

$$\underline{p_1} \leq 2t - p_2$$
$$\frac{w + t - d}{2} \leq 2t - p_2$$
$$p_2 \leq \frac{3t - w + d}{2}$$

when $p_2 > \frac{3t-w+d}{2}$, $\underline{p_1} = 2t - p_2$.

From the above, we obtain the following optimal bids for the case where $((p_1+p_2)/2 \leq t)$

$$\underline{p_1} = \begin{cases} \dfrac{w+t-d}{2} & \text{if } p_2 \leq \frac{3t-w+d}{2}, \\ 2t-p_2 & \text{otherwise.} \end{cases} \tag{17}$$

Substituting (17) into (3), we obtain

$$EU_1\left(\underline{p_1}, p_2\right) = \begin{cases} \dfrac{(d-t+w)^2}{8d} & \text{if } p_2 \leq \dfrac{3t-w+d}{2}, \\ \dfrac{(d-p_2+t)(p_2-2t+w)}{2d} & \text{otherwise.} \end{cases} \tag{18}$$

Scenario 2: Average bid above the threshold ($\frac{p_1+p_2}{2} \geq t$)

Given an opposing bid p_2, we solve for the bid p_1 that maximizes the expected utility $EU_1(p_1, p_2) = \frac{p_1-p_2+2d}{4d}(w-p_1)$ if $\frac{p_1+p_2}{2} \geq t$.

Taking the first and second derivatives over p_1:

$$\frac{\partial EU_1}{\partial p_1} = \frac{d(-2d-2p_1+p_2+w)}{4} = 0$$
$$p_1 = \frac{w+p_2-2d}{2}$$

$$\frac{\partial^2 EU_1}{\partial^2 p_1} = -\frac{d}{2}$$

The optimal bid is $p_1 = (w+p_2-2d)/2$. The function is concave with a negative second derivative, $-1/2d$. Let $\overline{p_1} = (w+p_2-2d)/2$.

For the average bid to be greater than or equal to the threshold, for any given p_2, lobbyist one has to bid at least $2t-p_2$, and the feasible set of bids reduce to $p_1 \in$

$[max\{p_2 - 2d, 2t - p_2\}, w]$.

Note that the average bid has to be above the threshold t, $\overline{p_1} \geq 2t - p_2$.

$$\begin{aligned} \overline{p_1} &\geq 2t - p_2 \\ \frac{w + p_2 - 2d}{2} &\geq 2t - p_2 \\ p_2 &\geq \frac{4t - w + 2d}{3} \end{aligned}$$

when $p_2 < \frac{4t-w+2d}{3}$, $\overline{p_1} = 2t - p_2$.

From the above, we summarize the following optimal bids and expected utilities for the case where $((p_1 + p_2)/2 \geq t)$:

$$\overline{p_1} = \begin{cases} \dfrac{w + p_2 - 2d}{2} & \text{if } p_2 \geq \frac{4t-w+2d}{3}, \\ 2t - p_2 & \text{otherwise.} \end{cases} \tag{19}$$

$$EU_1\left(\overline{p}_1, p_2\right) = \begin{cases} \dfrac{(2d - p_2 + w)^2}{16d} & \text{if } p_2 \geq \dfrac{4t - w + 2d}{3}, \\ \dfrac{(d - p_2 + t)(p_2 - 2t + w)}{2d} & \text{otherwise.} \end{cases} \tag{20}$$

C Best Responses

We begin with looking at the expected utilities of each lobbyist from bidding $\underline{p_i}$ and $\overline{p_i}$, given their opponent's bid. We show the determination of best responses by computing the bids that maximizes lobbyist's one utility, given all possible bids of lobbyist two. Note that as the game is symmetric, the same derivation applies to lobbyist two's best responses.

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

Lobbyist one can choose to bid either $\underline{p_1} = (w+t-d)/2$, and keep the average bid below the threshold, or bid $\overline{p_1} = 2t - p_2$, and push the average bid to the threshold.

The expected utilities for each possible bid are compared below:

$$\begin{aligned}
EU_1\left(\underline{p_1}, p_2\right) &= EU_1\left(\overline{p}_1, p_2\right) \iff \\
\frac{(d-t+w)^2}{8d} &= \frac{(d-p_2+t)(p_2-2t+w)}{2d} \iff \\
d^2+9t^2+w^2+6dt-2dw-6tw &= 4p_2(3t-w+d)-4p_2^2 \iff \\
(3t-w+d)^2 &= 4p_2(3t-w+d)-4p_2^2 \iff \\
(2p_2-(3t-w+d))^2 = 0 &\iff \\
p_2 &= \frac{3t-w+d}{2}.
\end{aligned} \tag{21}$$

Whenever $p_2 \leq (3t-w+d)/2$, it follows from (21) that $EU_1\left(\underline{p_1}, p_2\right) \geq EU_1\left(\overline{p}_1, p_2\right)$, so that the best response of lobbyist one is $BR_1(p_2) = (w+t-d)/2$.

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t-w+2d}{3}$

For a given p_2 between the lower and upper bound, the best response of lobbyist's one is $BR_1(p_2) = 2t - p_2$. As before, each lobbyist is faced with the trade-off between higher chances of winning the support of the legislator and lower take-home win. When the opposing bid is not low enough to justify bidding conservatively, but also not high enough to justify bidding aggressively, his best responds is to bid enough so that the game ends with a winning bid.

Case 3: $p_2 \geq \frac{4t-w+2d}{3}$

Lobbyist one bid either $\underline{p_1} = 2t - p_2$, and keep the average bid at the threshold, or bid

$\overline{p}_1 = (w + p_2 - 2d)/2$, and push the bid above the threshold.

The expected utilities for the possible bids are compared below:

$$\begin{aligned} EU_1\left(\underline{p_1}, p_2\right) &= EU_1\left(\overline{p}_1, p_2\right) \iff \\ \frac{(d - p_2 + t)(p_2 - 2t + w)}{2d} &= \frac{(2d - p_2 + w)^2}{16d} \iff \\ -(4t - w + 2d)^2 &= -6p_2(4t - w + 2d) + 9p_2^2 \iff \\ 0 &= (3p_2 - (4t - w + 2d))^2 \iff \\ p_2 &= \frac{4t - w + 2d}{3}. \end{aligned} \tag{22}$$

Whenever $p_2 \geq (4t - w + 2d)/3$, it follows from (22) that $EU_1\left(\overline{p}_1, p_2\right) \geq EU_1\left(\underline{p_1}, p_2\right)$, so that the best response of lobbyist one is $BR_1\left(p_2\right) = (w + p_2 - 2d)/2$.

To summarize, the best response of lobbyist i is given by:

$$BR_i\left(p_{-i}\right) = \begin{cases} \dfrac{w + t - d}{2} & \text{if } p_{-i} \leq \dfrac{3t - w + d}{2}, \\ 2t - p_{-i} & \text{if } \dfrac{3t - w + d}{2} < p_{-i} < \dfrac{4t - w + 2d}{3}, \\ \dfrac{w - 2d + p_{-i}}{2} & \text{otherwise.} \end{cases} \tag{23}$$

D Computation of Nash Equilibria

In this section we compute all the Nash equilibria of the game. For ease of exposition, we present below the best response function of player i from (9) once more.

$$BR_i\left(p_{-i}\right)=\begin{cases}\dfrac{w+t-d}{2} & \text{if } p_{-i}\leq\dfrac{3t-w+d}{2},\\ 2t-p_{-i} & \text{if } \dfrac{3t-w+d}{2}<p_{-i}<\dfrac{4t-w+2d}{3},\\ \dfrac{w-2d+p_{-i}}{2} & \text{otherwise.}\end{cases}$$

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

The best response of lobbyist 1 is

$$BR_1\left(p_2\right)=\frac{w+t-d}{2}. \tag{24}$$

Let $p_1^* = (w+t-d)/2$. A Nash equilibrium exists if the best response of lobbyist two to p_1^* satisfies $p_2 \leq (3t-w+d)/2$. We solve for equilibria depending on where p_1^* is with respect to the lower and the upper bound.

Case 1.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response p_1^* satisfies

$$p_1^* \leq \frac{3t-w+d}{2} \iff w-t\leq d. \tag{25}$$

The best response of the second lobbyist is $p_2^* = (w+t-d)/2$ and satisfies $p_2^* \leq (3t-w+d)/2$ whenever $w-t\leq d$. Thus, the pair (p_1^*, p_2^*) is a Nash equilibrium. The expected utilities of lobbyists are

$$EU_1(p_1^*,p_2^*)=EU_2(p_2^*,p_1^*)=\frac{(d-t+w)^2}{8d}.$$

Case 1.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t-w+2d}{3}$

From (25), it follows that $p_1^* > (3t - w + d)/2$ whenever $d < w - t$. The restriction on parameters so that p_1^* is below the upper bound require

$$p_1^* < \frac{4t - w + 2d}{3} \iff d > \frac{5}{7}(w - t). \tag{26}$$

The best response of lobbyist's two reduces to

$$BR_2(p_1^*) = 2t - p_1^* = 2t - \frac{w + t - d}{2} = \frac{3t - w + d}{2},$$

which is consistent with the initial restriction on p_2. Thus, the pair (p_1^*, p_2^*) is a Nash equilibrium whenever $(5/7)(w-t) < d < w-t$. The expected utilities of both lobbyists are identical to those of the previous case.

Case 1.3: $p_1^* \geq \frac{4t-w+2d}{3}$

From (26), it follows that $p_1^* \geq (4t-w+2d)/3$ whenever $d \leq \frac{5}{7}(w-t)$. The best response of lobbyist two is equal to

$$BR_2(p_1) = \frac{w - 2d + p_1}{2} = \frac{w - 2d + \frac{w+t-d}{2}}{2} = \frac{3w - 5d + t}{4}.$$

Let $p_2^* = (3w - 5d + t)/4$. The restriction on parameters so that p_2^* is below or equal to the lower bound require

$$p_2^* \leq \frac{3t - w + d}{2} \iff d \geq \frac{5}{7}(w - t). \tag{27}$$

Hence, as $p_2^* \leq (3t-w+d)/2$ whenever $d \geq \frac{5}{7}(w-t)$, and $p_1^* \geq (4t-w+2d)/3$ whenever $d \leq \frac{5}{7}(w-t)$, the pair (p_1^*, p_2^*) is a Nash equilibrium at $d = \frac{5}{7}(w-t)$. Substituting the latter restriction into the optimal bids, yields $p_1^* = (6t + w)/7$ and $p_2* = (8t - w)/7$.

The expected utilities of lobbyists are

$$EU_1\left(p_1^*, p_2^*\right) = \frac{18(w-t)}{35},\ EU_2\left(p_2^*, p_1^*\right) = \frac{16(w-t)}{35}.$$

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t-w+2d}{3}$

The best response of lobbyist one is

$$BR_1(p_2) = 2t - p_2. \tag{28}$$

Let $p_1^* = 2t - p_2$. As before, we analyse three cases depending on where p_1^* is with respect to the lower and upper bound.

Case 2.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response of lobbyist two is $p_2^* = (w+t-d)/2$. Combining (25),(26), it follows that p_2^* is between the lower and upper bound whenever $(5/7)(w-t) < d < w-t$. Substituting for p_2^* into p_1^*, yields $p_1^* = (3t-w+d)/2$. The pair (p_1^*, p_2^*) is a Nash equilibrium whenever $(5/7)(w-t) < d < w-t$ (compare with Case 1.2). The expected utilities of lobbyists are equal to

$$EU_1(p_1^*, p_2^*) = \frac{(3w-3t-d)(3d+t-w)}{8d},\ EU_2(p_2^*, p_1^*) = \frac{(d-t+w)^2}{8d}.$$

Case 2.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t-w+2d}{3}$

The best response of lobbyist two is $BR_2(p_1^*) = 2t - p_1^*$. Rearranging the inequality restrictions of p_1^*, yields

$$\begin{aligned} \tfrac{3t-w+d}{2} < p_1^* < \tfrac{4t-w+2d}{3} \iff \\ 2t - \tfrac{4t-w+2d}{3} < 2t - p_1^* < 2t - \tfrac{3t-w+d}{2} \iff \\ \tfrac{w-2d+2t}{3} < p_2^* < \tfrac{w+t-d}{2}. \end{aligned} \tag{29}$$

Recall that p_2^* is restricted to the Case 2 assumption:

$$p_2^* \in \left(\frac{3t-w+d}{2}, \frac{4t-w+2d}{3}\right) \tag{30}$$

There are three outcomes here:

1. The upper and lower bounds of (29) coincide with the bounds of (30).
2. The upper bound of (29) is in (30), while its lower bound is outside (30).
3. The lower bound of (29) is in (30), while its upper bound is outside (30).

Outcome 1. When all possible values of p_2^* identified in (29) lie within (30), the following conditions must be satisfied: $(3t-w+d)/2 \leq (w-2d+2t)/3$ for the lower bound, and $(4t-w+2d)/3 \geq (w+t-d)/2$ for the upper bound.

$$\frac{w-2d+2t}{3} \geq \frac{3t-w+d}{2} \iff \frac{5}{7}(w-t) \geq d \text{ and } \frac{w+t-d}{2} \leq \frac{4t-w+2d}{3} \iff \frac{5}{7}(w-t) \leq d. \tag{31}$$

At $d = (5/7)(w-t)$, the bounds of (29) coincide with the bounds of (30). Hence, the pair $(p_1^*, p_2^*) \in ((3t-w+d)/2, (4t-w+2d)/3)^2$ is a continuum of Nash equilibria. Taking into account the restriction $d = (5/7)(w-t)$, the Nash equilibrium can be rewritten as $p_i^* \in ((8t-w)/7, (6t+w)/7)$ and $p_{-i}^* = 2t - p_i^*$. Here, we also find the symmetric equilibrium, $p_i^* = p_{-i}^* = t$.

Outcome 2. Suppose $d > (5/7)(w-t)$. From (31), the upper bound of (29) is in (30), while its lower bound is less than the lower bound of (30). Hence, $p_2^* \in ((3t-w+d)/2, (w+t-d)/2)$. The interval $((3t-w+d)/2, (w+t-d)/2)$ is well defined as long as $d < w-t$. Hence, the pair (p_1^*, p_2^*) where $p_2^* \in ((3t-w+d)/2, (w+t-d)/2)$ and $p_1^* = 2t - p_2^*$, is a continuum of Nash equilibria whenever $(5/7)(w-t) < d < w-t$.

Outcome 3. Suppose $d < (5/7)(w-t)$, then, the lower bound of (29) is in (30), while its upper bound is greater than the upper bound of (30). The best response of lobbyist two lies in the interval $(w-2d+2t)/3 < p_2^* < (4t-w+2d)/3$. This interval is well-defined as long as $d > (1/2)(w-t)$. Hence, the pair (p_1^*, p_2^*) where $p_2^* \in ((w-2d+2t)/3, (4t-w+2d)/3)$ and $p_1^* = 2t - p_2^*$ is a continuum of Nash equilibria whenever $(1/2)(w-t) < d < (5/7)(w-t)$.

Case 2.3: $p_1^* \geq \frac{4t-w+2d}{3}$

The best response of lobbyist two is

$$BR_2(p_1^*) = \frac{w-d+p_1^*}{2} = \frac{w-2d+2t-p_2}{2} = \frac{w-2d+2t}{3}.$$

Let $p_2^* = (w-2d+2t)/3$. It must lie within the lower and upper bound. To that end, we obtain

$$p_2^* > \frac{3t-w+d}{2} \iff \frac{5}{7}(w-t) > d \text{ and } p_2^* < \frac{4t-w+2d}{3} \iff \frac{1}{2}(w-t) < d.$$

Moreover, substituting p_2^* into (28), yields $p_1^* = 2t - \frac{w-2d+2t}{3} = (4t-w+2d)/3$. The pair (p_1^*, p_2^*) is a Nash equilibrium whenever $(w-t)/2 < d < (5/7)(w-t)$. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = \frac{(2w - 2t - d)(5d + t - w)}{9d}, \; EU_2(p_2^*, p_1^*) = \frac{(d - t + w)^2}{9d}.$$

Case 3: $p_2 \geq \frac{4t-w+2d}{3}$

The best response of lobbyist 1 is

$$BR_1(p_2^*) = \frac{w - 2d + p_2^*}{2}. \tag{32}$$

Let $p_1^* = (w - 2d + p_2)/2$. As before, we split the argument into three cases.

Case 3.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response of lobbyist two is equal to $BR_2(p_1^*) = (w + t - d)/2$. Let $p_2^* = (w + t - d)/2$. Substituting the latter into (32), yields $p_1^* = (3w - 5d + t)/4$, which is below or equal to the lower bound whenever $d \geq \frac{5}{7}(w - t)$ (see Eq. 27). On the other hand, p_2^* is above or equal to the upper bound, whenever $d \leq \frac{5}{7}(w - t)$ (see Eq. 26).

At $d = (5/7)(w - t)$, the pair (p_1^*, p_2^*) the pair (p_1^*, p_2^*) where $p_1^* = (6t + w)/7$ and $p_2* = (8t - w)/7$ is a Nash equilibrium. The expected utility of lobbyists are

$$EU_1(p_1^*, p_2^*) = \frac{16(w - t)}{35}, \; EU_2(p_2^*, p_1^*) = \frac{18(w - t)}{35}.$$

Case 3.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t-w+2d}{3}$

The best response of lobbyist's two is equal to $BR_2\,(p_1^*) = 2t - p_1^*$. Let $p_2^* = 2t - p_1^*$. Substituting p_2^* into (32), yields $p_1^* = (w - 2d + 2t)/3$. In turn, substituting the latter into the best response of lobbyist two, yields $p_2^* = (4t - w + 2d)/3$.

Similar to Case 2.3, p_1^* lies within the lower and upper bound whenever $(w - t)/2 < d <$

$(5/7)(w-t)$. Hence, the pair (p_1^*, p_2^*) is a Nash equilibrium whenever $(w-t)/2 < d < (5/7)(w-t)$. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = \frac{(d-t+w)^2}{9d},\ EU_2(p_2^*, p_1^*) = \frac{(2w-2t-d)(5d+t-w)}{9d}.$$

Case 3.3: $p_1^* \geq \frac{4t-w+2d}{3}$

The best response of lobbyists two is given by $BR_2(p_1^*) = (w-2d+p_1^*)/2$. Let $p_2^* = (w-2d+p_1)/2$. Substituting p_2^* into (32), yields $p_1^* = w-2d$. In turn, substituting p_1^* into p_2^*, yields $p_2^* = w-2d$. Note that $w-2d \geq (4t-w+2d)/3$ whenever $d \leq (w-t)/2$. Therefore, $p_1^* = p_2^* = w-2d$ is a Nash equilibrium whenever $d \leq (w-t)/2$. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = EU_2(p_2^*, p_1^*) = d.$$