Aggregation in economies with search frictions*

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Abstract

We derive an aggregation result in economies with indivisible labor supply choices and frictional labor markets, obtaining a tractable model of gross worker flows in aggregate labor markets with search frictions. Our result explores the fact that economies with non-convex choice sets and idiosyncratic shocks allow for sunspot equilibria à la Kehoe et al. (2002). We use comparative steady state analysis to demonstrate the applicability of our aggregation result. Our framework reconciles the neoclassical growth model with search frictions with a mildly procyclical participation rate and matches the gross worker flows underpinning those dynamics.

Keywords: indivisibilities, sunspots, search frictions, gross worker flows.

JEL Classification: D50, D60, D91, J22.

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1 Introduction

Since the seminal work of Merz (1995) and Andolfatto (1996), dynamic stochastic general equilibrium (DSGE) models with labor market search frictions have been widely used to study unemployment fluctuations. However, these two approaches place different restrictions on individual choices. In turn, Merz (1995) assumes the existence of a representative "large family", constrained by budget sets and an employment law of motion, while Andolfatto (1996) assumes a game of "musical chairs" (exogenous shocks), that randomly allocate individuals to labor market states, with perfect insurance against idiosyncratic risk.[1] We take our cue from the latter approach and make the following contribution: we generalise the musical chairs' approach to a model with gross worker flows and individual participation choices, using results from the literature on sunspots and lotteries, along the lines of Kehoe et al. (2002). To the best of our knowledge, ours is the first paper to offer an aggregation result in economies with three state labor markets, indivisible labor and search frictions, based on individual choices (without having to impose the assumption of the "large family"), that yields a constrained efficient competitive equilibrium.

Our approach delivers a tractable characterization of equilibrium in economies combining indivisible labor supply choices (participation margin), and labor market frictions. The literature has often restricted attention to two-state labor markets, ignoring participation and focusing on the margin between employment and unemployment.[2] However, recent empirical work attributes an important role to the participation margin for labor market transitions. Elsby et al. (2015) showed that the participation margin accounts for one-third of the cyclical variation in the unemployment rate. Moreover, unlike the Merz (1995) large family set-up,

[1] Also, Merz (1997) used a randomisation device analogous to Andolfatto to decentralise the constrained optimum in a two-state labor market.

[2] Several papers consider participation in DSGE models with search frictions. Some early examples are Veracierto (2008), Ravn (2008) and Shimer (2013). This notwithstanding, the inclusion of an intertemporal labor supply margin in economies with indivisibilities and search frictions remains a difficult task. All examples above use the Merz (1995) "large family" model to achieve aggregation, and deliver stark counterfactual predictions about the labor market (for example, procyclical unemployment).

which only identifies net worker flows, our model yields a characterization of equilibrium gross worker flows. This is important, since Krusell et al. (2010, 2011) and Krusell et al. (2017) stressed the importance of gross worker flows and developed models with missing insurance markets and indivisibilities in labor supply choice to account for these transitions.

We develop a general equilibrium model of gross worker flows with complete markets, where individuals face heterogeneous employment histories and idiosyncratic risk. Following Andolfatto (1996), musical chairs allocate individuals to different labor market states each period; conditional on this, individuals face an indivisible participation choice, in labor market with search frictions. To overcome indivisibilities, individuals play lotteries over participation as in Rogerson (1988) and Hansen (1985). The decision of each individual is based on the joint outcome of public ("musical chairs") and contrived randomness (lotteries) and the realisation of idiosyncratic shocks. This hybrid decision process may seem unusual, but we argue it can be microfounded as follows. We demonstrate that one can mimic the joint effects of musical chairs and lotteries by indexing on the basis of two naturally occurring random variables (sunspots) prior and after the realisation of the idiosyncratic shocks. Such an arrangement is consistent with the existence of the usual Arrow-Debreu contingent commodities.

Subsequently, similarly to Christiano et al. (2020) we use comparative steady state analysis as a short-cut for analyzing model dynamics. Two main insights emerge from this analysis. First, our model reconciles the neoclassical growth model with search frictions with a mildly procyclical participation rate. This result is particularly important given the tendency for models featuring intertemporal substitution in frictional labor markets to deliver excessively procyclical participation and, thus, procyclical unemployment (a problem stressed by Ravn, 2008; Veracierto, 2008; Shimer, 2013, for example). Second, we show using a calibrated example that the model accounts well for the observed flows. In particular, it is able to match the high transition rate from unemployment to inactivity, which early papers by Garibaldi and Wasmer (2005) and Krusell et al. (2010, 2011) have shown to be challenging for equilibrium models of gross worker flows, under either complete or incomplete markets.

The literature on sunspots and lotteries in economies with non-convexities and complete markets includes, among others, Prescott and Townsend (1984), Shell and Wright (1993), Garratt (1995), Garratt et al. (2002), Kehoe et al. (2002), and Garratt et al. (2004). Our results generalise models with indivisibilities to include idiosyncratic risk arising from frictional labor markets. To achieve that, we introduce the distinction between public randomisations prior and after the realisation of idiosyncratic shocks in each period—although this distinction is already discussed by Kehoe et al. (2002), it is not important for their analysis.

Our paper contributes to a recent literature that combines indivisible labor supply choices in models with intertemporal substitution (what Krusell et al., 2008, call "non-trivial labor supply choices"), together with search frictions in the labor market. Krusell et al. (2008) show that in a set-up with indivisibility and incomplete markets (similar to Chang and Kim, 2006, 2007), search frictions avoids indeterminacy in labor supply choices.[3] Building on this framework, Krusell et al. (2010, 2011) study individual transitions across employment, unemployment and non-participation, in a three-states labor market model, with incomplete markets, search frictions and non-trivial labor supply choices.[4] They show that whilst the benchmark model is unable to match the persistence of the employment and non-participation states found in the data, a version of the same model with persistent idiosyncratic productivity shocks affecting the individual value of work is able to match the transition flows well.

Further, Krusell et al. (2011) study a version of their model with complete markets (with insurable idiosyncratic shocks), but do not discuss decentralization and, instead, consider the solution to the social planner's problem, in which each individual receives equal weight. The

[3]Krusell et al. (2008) show how an economy with indivisible labor and incomplete markets, current labor supply is indeterminate for individuals with intermediate levels of wealth. They subsequently suggest labor market frictions, à la Lucas and Prescott (1974) as a mechanism to break this indeterminacy.

[4]There are, of course, several empirical papers studying gross worker flows in three-state frictional labor markets, but without modelling optimal intertemporal labor supply choices. These studies extend the matching function model (Mortensen and Pissarides, 1994), and employ a stock-flow accounting framework to model unemployment duration dependence, long-term unemployment, and non-participation. Recent examples, establishing the importance of workers' heterogeneity and the participation margin include Barnichon and Figura (2015), Elsby et al. (2015), and Kroft et al. (2016).

resulting equilibrium allocations imply labor market gross flows that are comparable to those obtained in the incomplete market economy. Thus, they conclude that uninsurable risk is not a necessary ingredient to obtain a satisfactory representation of labor market transitions.[5] In our paper, we show how the decentralized competitive equilibrium with complete markets can be obtained using either lotteries or sunspots, to produce constrained efficient allocations. At the same time, the resulting model is as successful at matching empirical gross worker flows as the incomplete markets model. In particular, using sunspots as the randomization mechanism generates heterogeneous employment histories across individuals that are conforming with realistic transitions across labor market states (comparable to what is achieved by Krusell et al., 2010, 2011, using idiosyncratic productivity shocks).

Finally, Krusell et al. (2017) augment the set-up developed in Krusell et al. (2010, 2011) with job-to-job transitions and aggregate shocks to labor market frictions, in order to study gross worker flows over the business cycle. We consider shocks to the job finding rate, and show how the model with indivisible labor supply, complete markets, and extrinsic randomization, can deliver either a countercyclical or a procyclical participation rate. Thus, the neoclassical growth model with search frictions can be reconciled with a mildly procyclical participation rate, that is supported by empirically realistic gross worket flows.

The remainder of the paper is organized as follows: Section 2 explains the environment; Section 3 establishes that an equilibrium with musical chairs and lotteries corresponds to a sunspot equilibrium; Section 4 presents the comparative steady state analysis; Section 5 concludes.

[5] In a model with indivisible labor supply choices but without frictional unemployment, Ljungqvist and Sargent (2008) obtain a similar result.

2 Model

2.1 Agents and markets

Time is discrete, indexed $t = 0, 1, \ldots$ The economy is populated by a continuum of infinitely-lived individuals, $i \in [0, 1]$. There is a single good, produced with capital and labor. Individuals buy consumption, c and invest in capital, k, depreciating at rate $\delta \in (0, 1)$, and face an indivisible participation choice: labor market participation imposes a utility loss, $\xi \geq 0$. Conditional on participation, individuals may be employed or unemployed. If employed, they incur an additional utility cost $-\ln\left(1 - \underline{h}\right) > 0$, as they sacrifice $\underline{h} \in (0, 1)$ units of their endowment of time (with $\underline{h}$ an exogenous parameter). Workers transition between three states: employment (e), unemployment (u), and non-participation (o). We denote labor market states by $\iota \in \mathcal{L} \equiv \{e, u, o\}$. Individuals have flow utility, $U(c) - \psi(\iota)$, with $U'(c) > 0$ and $U''(c) < 0$, and with $\psi(e) \equiv \xi - \ln\left(1 - \underline{h}\right)$, $\psi(u) \equiv \xi$ and $\psi(o) \equiv 0$.

Competitive firms have (identical) constant returns to scale technology which turns labor and capital into output, $F(k, n)$, that satisfy standard Inada conditions and (k, n) denote the demand for capital and labor. Firms pay wages w to hire workers, r to rent capital, and maximize profits, $F(k, n) - wn - rk$.

The economy consists of three islands, which we refer to as employment island, unemployment island and leisure island. Individuals that were unemployed (non-participants) in the previous period, start at the beginning of date t in the unemployment (leisure) island. If they decide not to participate, they relocate (remain) to the leisure island; and, if they decide to participate, they relocate to the employment island with probability f, or they remain (relocate) to the unemployment island with probability $1 - f$. New jobs become immediately productive.

Individuals previously employed, start date t in the employment island. An existing job is destroyed with probability λ, and upon destruction, previously employed individuals are allowed to search for another job and remain to the employment island with probability f. With probability $1 - \lambda$ the job is not destroyed and they continue with the existing employment relationship.

Labor frictions restrict access to the employment island; however, conditional on access, the labor market is competitive and wages reflect the marginal product of labor.[6] Goods markets are competitive, with capital moving freely across islands. There is no aggregate uncertainty, but frictional labor markets generate idiosyncratic risk.

2.2 Institutions

We consider two institutional trading arrangements. In the first, as in Andolfatto (1996), at the start of date t a game of musical chairs allocates individuals to different labor market states $\imath \in \mathcal{L}$. Subsequently, individuals buy lotteries over labor force participation and idiosyncratic shocks realise. Each period, insurance markets open before the realization of musical chairs and lotteries, with contracts traded at actuarial fair prices. At the end of each date, spots market open where individuals receive income, execute contracts, buy consumption and invest.

In the second market structure (the sunspot economy), we assume markets open only once, at date -1. Individuals trade contracts contingent on "sunspot" activity and idiosyncratic risk. Sunspots act as a coordination device much like the musical chairs and affect welfare because of indivisibilities in labor supply choices. This structure yields a competitive equilibrium with voluntary trade in contingent commodities, where sunspots coordinate actions among individuals. We label the first model "musical chairs" and the second "sunspots", and we

[6] The assumption of competitive markets in coexistence with search frictions has a long tradition and follows, for example, Lucas and Prescott (1974), Alvarez and Veracierto (1999) and Krusell et al. (2008, 2010, 2011). This approach is fruitful because, as we show in Proposition 2, it yields a constrained efficient equilibrium despite the search frictions. However, having competitive factor prices is not essential for the success of our model to match labor market transitions rates. In fact, in the steady state equilibrium, factor prices are constant and, thus, assuming non-competitive factor prices would not alter our analysis.

study each in turn.

2.2.1 Musical chairs

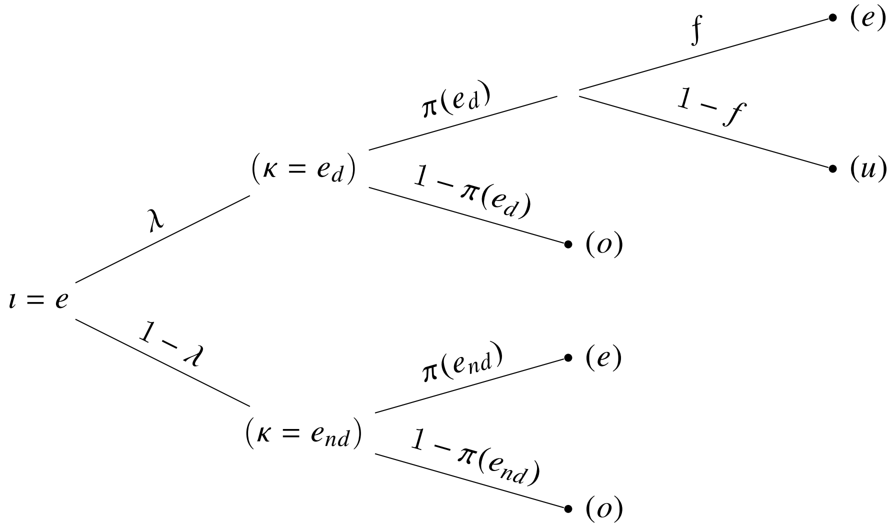
At the beginning of date t a game of musical chairs assigns individuals to a labor market state $\imath \in \mathcal{L} \in \{e, u, o\}$, with probability $\alpha_t(\imath)$. Figures 1a and 1b show the sequence of events conditional on the musical chairs randomisation. Specifically, individuals assigned to the employment island ($\imath = e$) observe the realisation of the idiosyncratic shock $\kappa \in \{e_d, e_{nd}\}$, where $\kappa = e_d$ denotes destruction (d) with probability λ, and $\kappa = e_{nd}$ denotes no destruction (nd) with $1-\lambda$; subsequently they buy lotteries over labor force participation, and conditional on the lottery outcome engage (or not) in search activity. Individuals assigned to the unemployment or leisure island ($\imath \in \{u, o\}$) buy lotteries over labor force participation and then engage (or not) in search activity. We denote by $\tilde{\imath} \in \{e_d, e_{nd}, u, o\}$ the consolidated set of states prior to the participation lottery stage, by $\jmath \in \mathcal{L} \in \{e, u, o\}$ the labor market state at the end of the period and by $\pi(\tilde{\imath})$ the lottery over labor force participation. The pair $(\tilde{\imath}, \jmath)$ denotes the labor market transitions of individuals during each period. Individuals discount the future with $\beta \in (0, 1)$.

The Bellman equation characterising each individual's decision is

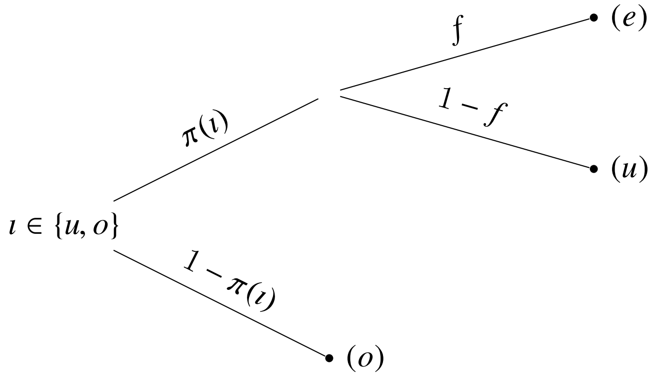
$$V_t\left(k_t, \bar{k}_t\right) = \max_{\{c,k,y,\pi\}} \left\{\alpha_t(e)\left[(1-\lambda_t)v_t\left(e_{nd}\right) + \lambda_t v_t\left(e_d\right)\right] + \alpha_t(u)v_t(u) + \alpha(o)v_t(o)\right\}$$

with

$$\begin{aligned} v_t(e_{nd}) = \pi_t(e_{nd})\left[U\left[c_t\left(e_{nd}, e\right)\right] - \psi(e) + \beta V_{t+1}\left(k_{t+1}\left(e_{nd}, e\right), \bar{k}_{t+1}\right)\right] + \\ (1-\pi_t(e_{nd}))\left[U\left[c_t\left(e_{nd}, o\right)\right] + \beta V_{t+1}\left(k_{t+1}\left(e_{nd}, o\right), \bar{k}_{t+1}\right)\right], \end{aligned}$$



(a) Sequence of events conditional on $\iota = e$



(b) Sequence of events conditional on $\iota \in \{u, o\}$

Figure 1: Sequence of events conditional on musical chairs' randomisation

and, for $\tilde{\iota} \in \{e_d, u, o\}$,

$$\begin{aligned} v_t(\tilde{\iota}) = \pi_t(\tilde{\iota}) f_t \left(U\left(c_t\left(\tilde{\iota}, e\right)\right) - \psi(e) + \beta V_{t+1}\left(k_{t+1}\left(\tilde{\iota}, e\right), \bar{k}_{t+1}\right)\right) + \\ \pi_t(\tilde{\iota})(1 - f_t) \left(U\left(c_t\left(\tilde{\iota}, u\right)\right) - \psi(u) + \beta V_{t+1}\left(k_{t+1}\left(\tilde{\iota}, u\right), \bar{k}_{t+1}\right)\right) + \\ (1 - \pi_t(\tilde{\iota})) \left(U\left(c_t\left(\tilde{\iota}, o\right)\right) + \beta V_{t+1}\left(k_{t+1}\left(\tilde{\iota}, o\right), \bar{k}_{t+1}\right)\right), \end{aligned}$$

subject to the budget constraint for each pair $(\tilde{\imath}, \jmath)$,

$$c_t\left(\tilde{\imath}, \jmath\right) + k_{t+1}\left(\tilde{\imath}, \jmath\right) + \sum_{\tilde{\imath}} \sum_{\jmath} q_t\left(\tilde{\imath}, \jmath\right) y_t\left(\tilde{\imath}, \jmath\right) = y_t\left(\tilde{\imath}, \jmath\right) + \left(r_t + 1 - \delta\right) k_t + w_t \underline{h} \mathbb{1}_{\jmath},$$

where $\mathbb{1}_{\jmath}$ is an indicator function that is equal to unity if $\jmath = e$ and zero otherwise. The relevant state space for individual optimisation consists of predetermined individual and aggregate capital stock, k and $\bar{k}$, respectively, and is independent of the previous period individual labor market state. At the end of date t, individuals buy consumption $c_t\left(\tilde{\imath}, \jmath\right)$, invest in capital stock $k_{t+1}\left(\tilde{\imath}, \jmath\right)$, execute contracts $y_t\left(\tilde{\imath}, \jmath\right)$ that are purchased at the beginning of date t (ex-ante) at price $q_t\left(\tilde{\imath}, \jmath\right)$, receive capital income and, if employed, labor income.

Actuarially fair insurance implies that marginal utilities of consumption $U_c\left[c\left(\tilde{\imath}, \jmath\right)\right]$ are equalised across all labor market states, which implies that $c\left(\tilde{\imath}, \jmath\right) = c$ for all pairs $(\tilde{\imath}, \jmath)$. In turn, it follows that the marginal return of one additional unit of captial $V_k\left(k\left(\tilde{\imath}, \jmath\right), \bar{k}\right)$ is equalised across labor market states, which implies $k\left(\tilde{\imath}, \jmath\right) = k$ for all pairs $(\tilde{\imath}, \jmath)$. The individual's decision is consolidated as follows:

$$\begin{gathered} V_t\left(k_t, \bar{k}_t\right) = \max \Big\{ U\left(c_t\right) - \alpha_t(e)(1-\lambda_t)\pi_t(e_{nd})\left(\xi - \ln\left(1-\underline{h}\right)\right) - \\ \left(\xi - f_t \ln\left(1-\underline{h}\right)\right)\left[\alpha_t(e)\lambda_t\pi_t(e_d) + \alpha_t(u)\pi_t(u) + \alpha(o)\pi_t(o)\right] + \beta V_{t+1}\left(k_{t+1}, \bar{k}_{t+1}\right) \Big\} \end{gathered} \tag{1}$$

subject to

$$\begin{gathered} c_t + k_{t+1} = \\ \left[\alpha_t(e)\pi_t(e_{nd})(1-\lambda_t) + \left(\alpha_t(e)\lambda_t\pi_t(e_d) + \alpha_t(u)\pi_t(u) + \alpha(o)\pi_t(o)\right) f_t\right] w_t \underline{h} + (r_t + 1 - \delta)k_t, \\ 0 \leq \pi_t(\tilde{\imath}) \leq 1. \end{gathered} \tag{2}$$

This represents the decision of a stand-in agent who chooses consumption, investment and lotteries over participation to maximise (1) subject to (2).

Wages and rental prices earn their respective marginal products. Insurance markets are segmented, in the sense that there exist separate markets for each contingency $(\tilde{\imath}, \jmath)$. Insurers in each market offer contracts $y(\tilde{\imath}, \jmath)$ at actuarially fair prices $q(\tilde{\imath}, \jmath)$, and free entry drives profits to zero.

Equilibrium is defined as follows:

Definition 1 *(Musical chairs) A full insurance equilibrium is a price system* $\{w, r, q\}$ *and probaility measures* $\alpha(\imath)$ *for* $\imath \in \mathcal{L}$*, a law of motion for aggregate capital* $\bar{k}$*, a collection of individual choices* $\{c, k, \pi, y\}$*, an individual value function* $V\left(k, \bar{k}\right)$ *and a collection of firm choices* $\{k, n\}$ *such that:*

1. *At given prices and* $\alpha(\imath)$*,* $\{c, k, \pi, y\}$ *and* $V\left(k, \bar{k}\right)$ *solve the individual's problem;*

2. *At given prices, all firms maximise profits;*

3. *Good's market clears,* $c + k_{+1} = f\,(k, n) + (1 - \delta)k$*;*

4. *Capital market clears,* $k = \bar{k}$*;*

5. *Labor market clears,*

$$n = \left[\alpha(e)\pi(e_{nd})(1 - \lambda) + \Big(\alpha(e)\lambda\pi(e_d) + \alpha(u)\pi(u) + \alpha(o)\pi(o)\Big)f\right]\underline{h};$$

6. $\alpha(\imath)$ *is equal to the previous period measure of individuals in labor market state* $\imath$*.*

The following remarks are in order. First, we show that any interior equilibrium satisfies $\pi(e_{nd}) = 1$ (corner solution) and $\pi(\tilde{\imath}) \in (0, 1)$, $\tilde{\imath} \in \{e_d, u, o\}$ (see Appendix A for details). Second, probabilities $\alpha(\imath)$ are determined by the measures of individuals in state $\imath \in \{e, u, o\}$ at the end of the previous period. Hence, although $\alpha(\imath)$ are taken as given by individuals, they are determined endogenously in equilibrium. Third, in Section 4 we offer a detailed characterisation of the equilibrium and discuss various comparative static exercises.

The hybrid model that we have analysed so far includes the musical chairs' framework of Andolfatto (1996) as a special case.

Lemma 1 *If $\xi = 0$, then our framework reduces to the musical chairs' model of Andolfatto (1996).*

Proof. Suppose $\xi = 0$. Set

$$N_t = \alpha_t(e)\pi_t(e_{nd})(1-\lambda_t) + \Big(\alpha_t(e)\lambda_t\pi_t(e_d) + \alpha_t(u)\pi_t(u) + \alpha(o)\pi_t(o)\Big)f_t,$$

with $N \equiv n/\underline{h}$ (see Section 4 for full details).

Then, (1)–(2), reduce to:

$$V_t\left(k_t,\bar{k}_t\right) = \max_{\{c,k\}}\left\{U\left(c_t\right) + N_t\ln\Big(1-\underline{h}\Big) + \beta V_{t+1}\left(k_{t+1},\bar{k}_{t+1}\right)\right\}, \tag{3}$$

subject to

$$c_t + k_{t+1} = w_t N_t \underline{h} + (r_t + 1 - \delta)k_t. \tag{4}$$

This corresponds to Andolfatto's model with N_t denoting the probability that an individual is allocated to employment and $1 - N_t$ the probability that is allocated to nonemployment. ■

This result requires that if the opportunity cost of participation is negligible, $\xi = 0$, then the randomisation devices prior and after the realisation of idiosyncratic shocks (see figures 1a nd 1b) reduce to the simple musical chair's randomisation in Andolfatto (1996).

2.2.2 Sunspots

In this Section we abstract from sequential trading and assume Arrow-Debreu (AD) markets, with trade occurring at date -1. Individuals trade contracts contingent on the publicly observed sunspot activity and idiosyncratic risk. Sunspot activity is constructed so that each period it induces a distribution of individuals across labor market states (islands) $\mathcal{L} = \{e, o, s\}$.

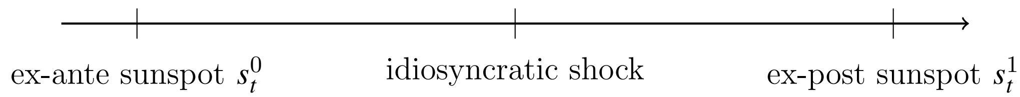


Figure 2: Sequence of events at date t

We employ the distinction between ex-ante and ex-post public randomisations (sunspots) within a given period that is discussed in Kehoe et al. (2002).[7] Figure 2 shows the sequence of events at date t. We denote ex-ante sunspot shocks with a superscript "0", and ex-post shocks with a superscript "1". At the beginning of date t individuals observe the ex-ante sunspot shock s_t^0, subsequently the idiosyncratic shock is realised and at the end of the period the ex-post sunspot shock s_t^1 is realised and transactions take place. For example, individuals induced by the ex-ante sunspot to start date t in the employment island, observe if the job is destroyed or not and the ex-post sunspot realisation allocates them to a labor market state at the end of the period; similarly, individuals induced by the ex-ante sunspot to start date t in the unemployment island, find employment with probability f and remain unemployed with probability $1 - f$, and at the end of the period observe the realisation of the ex-post sunspot and execute all their obligations.

The distinction between ex-ante and ex-post sunspots is important. It serves the following purpose. Ex-ante public randomisations replicate the distribution of musical chairs, overcome non-convexities arising from indivisibilities in labor supply, and influence the distribution of idiosyncratic risk (see below); while ex-post randomisation separate those individuals whose pre-existing jobs have been destroyed and need to be assigned into a labor market state, from those individuals whose jobs have survived (see Figure 1a). Hence, in the absence of idiosyncratic risk arising from job destruction, the ex-post public randomisation device is

[7] In their framework, only ex-post randomisations are important to overcome non-convexities arising from private information; and in fact, they show that the model with ex-ante and ex-post sunspots is equivalent in terms of allocations to the model with only ex-post sunspots. However, Cole (1989) showed that in a set-up with ex-ante sunspots and convex set of feasible allocations, the introduction of ex-post sunspots is still welfare improving because lotteries conditional on private information separate individuals with different risk profiles.

irrelevant.

Time and the resolution of uncertainty are described by an event-tree, a countable set. Denote the history of ex-ante and ex-post sunspot realisations up and until date t by $s^{0t} = \left[s_0^0, s_1^0, ..., s_t^0\right]$, $s^{1t} = \left[s_0^1, s_1^1, ..., s_t^1\right]$, the joint history by $s^t = \left[(s_0^0, s_0^1), (s_1^0, s_1^1), ..., (s_t^0, s_t^1)\right]$ and the history of idiosyncratic shocks up and until date t by $\phi^t = [\phi_0, \phi_1, ..., \phi_t]$. Let σ_t be the date-event consisting of ex-ante and ex-post sunspot realisations, s_t, and idiosyncratic shocks, ϕ_t, with history up to and including date t, $\sigma^t = [\sigma_1, \sigma_2, .., \sigma_t]$. We require the probability distribution of ex-post shocks to have a continuous density. We assume that s_t^1 is distributed uniformly on $[0, 1]$ and let $\mu_t^1\left(s^{0t}, \phi^t, s^{1t-1}\right)$ be the measure of date t ex-post sunspots states conditional on history $\left\{s^{0t}, \phi^t, s^{1t-1}\right\}$. The probability distributions of ex-ante and idiosyncratic shocks are obtained by appropriate construction as we demonstrate below. Let the unconditional probability of s^{0t} be $\mu_t^0\left(s^{0t}\right)$ and the probability of ϕ^t conditional on s^{0t} be $\gamma_t\left(\phi^t|s^{0t}\right)$. Let $\mu_t\left(s^{0t}, \phi^t\right) = \gamma_t\left(\phi^t|s^{0t}\right)\mu_t^0\left(s^{0t}\right)$. We assume that histories of ex-post shocks do not influence the distributions of ex-ante and idiosyncratic shocks.[8] Individuals trade contingent claims against future events σ^t at price $p_t\left(\sigma^t\right)$ and firms buy inputs and sell output against s^t at $p_t\left(s^t\right)$. Prices $p_t\left(s^t\right)$ are derived from $p_t\left(\sigma^t\right)$ by summing over ϕ^t.

The decision of an individual is

$$\max_{c,k} \sum_t \beta^t \sum_{\{s^{0t},\phi^t\}} \mu_t\left(s^{0t}, \phi^t\right) \int_{s^{1t}} \left[U\left(c_t\left(\sigma^t\right)\right) - \psi\left(\sigma^t\right)\right] ds^{1t}, \tag{5}$$

subject to

$$\begin{gathered}\sum_t \sum_{\{s^{0t},\phi^t\}} \int_{s^{1t}} p_t\left(\sigma^t\right)\left[c_t\left(\sigma^t\right) + k_{t+1}\left(\sigma^t\right)\right] ds^{1t} = \\ \sum_t \sum_{\{s^{0t},\phi^t\}} \int_{s^{1t}} p_t\left(\sigma^t\right)\left[\left(r_t\left(s^t\right) + 1 - \delta\right) k_t\left(\sigma^{t-1}\right) + \underline{h} w_t\left(s^t\right) \mathbb{1}\left(\sigma^t\right)\right] ds^{1t},\end{gathered} \tag{6}$$

where the indicator function $\mathbb{1}\left(\sigma^t\right)$ is equal to unity at date-events where individuals work

[8]This assumption follows Prescott and Townsend (1984), who assume that histories of lottery outcomes do not influence the distribution of types across the population.

and zero otherwise. We define multiple integrals $\int_{s^{1t}} \equiv \int_{s_0^1} \cdots \int_{s_t^1}$ up to and including date t and differentials $ds^{1t} \equiv ds_t^1 \cdots ds_0^1$ back to date zero.

Firms choose capital and labor to maximise profits:

$$\max_{k,n} \; \sum_t \sum_{s^{0t}} \int_{s^{1t}} p_t\left(s^t\right)\left[F\left(k_t\left(s^{t-1}\right), n_t\left(s^t\right)\right) - r_t\left(s^t\right) k_t\left(s^{t-1}\right) - w\left(s^t\right) n_t\left(s^t\right)\right] ds^{1t}. \tag{7}$$

Consider the following definition of a sunspot equilibrium.

Definition 2 *(Sunspots) A sunspot equilibrium is a price system* $\{w, r, p\}$*, a collection of individual choices* $\{c, k\}$*, and a collection of firm choices* $\{k, n\}$ *such that:*

1. *At given prices,* $\{c, k\}$ *solve the individual's problem* (5)-(6)*;*
2. *At given prices, firms maximise profits* (7)*;*
3. *Good's market clears,*

$$\int \left(c_t\left(\sigma^t\right) + k_{t+1}\left(\sigma^t\right) - (1-\delta) k_t\left(\sigma^{t-1}\right)\right) di = f\left(k_t\left(s^{t-1}\right), n_t\left(s^t\right)\right);$$

4. *Capital market clears,* $\int k_t\left(\sigma^t\right) di = k\left(s^t\right)$;
5. *Labor market clears,* $n_t\left(s^t\right) = \int \underline{h}\mathbb{1}\left(\sigma^t\right) di$.

3 Equivalence

The purpose of this Section is to demonstrate that the equilibrium allocations achieved by the sunspot economy and the musical chairs economy are equivalent. This equivalence result microfounds the hybrid model of musical chairs, idiosyncratic risk and lotteries over labor force participation. Subsequently, we demonstrate that the sunspot allocation is constrained Pareto optimal.

To demonstrate equivalence of equilibria between the two economies, we proceed in two steps. First, we establish in Proposition 1 that the same equilibrium allocations obtained with musical chairs and lotteries can be implemented as sunspot-equilibrium allocations and, thus, the lottery equilibrium corresponds to an equilibrium with sunspots. Specifically, in the proof of Proposition 1 we present a detailed construction of the sunspot probability distribution, so that the sunspot economy achieves the same equilibrium allocation as the target lottery equilibrium allocation. Second, using well-known results in the literature (see Garratt et al., 2002; Kehoe et al., 2002), we establish that the converse of Proposition 1 is also true if the sunspot state-space is sufficiently rich.

Proposition 1 *An equilibrium with musical chairs and lotteries corresponds to a sunspot equilibrium.*

Proof. The proof is constructive. Suppose an equilibrium with musical chairs exists. Then, we construct an equilibrium with sunspots supporting the same allocations as the musical chairs equilibrium.

Consider the stand-in agent's problem in the musical chairs economy, given by (1)–(2), implying the first-order conditions

$$U_c\left(c_t\right) = \beta R_{t+1} U_c\left(c_{t+1}\right), \tag{8}$$

$$\xi - f_t \ln\left(1 - \underline{h}\right) = f_t w_t \underline{h} U_c\left(c_t\right), \tag{9}$$

$$\xi - \ln\left(1 - \underline{h}\right) < w_t \underline{h} U_c\left(c_t\right), \tag{10}$$

where $R_{t+1} \equiv r_{t+1} + 1 - \delta$. Condition (8) is the Euler equation and (9), (10) are optimality conditions with respect to $\pi(\tilde{\imath}) \in (0,1)$, $\tilde{\imath} \in \{e_d, u, o\}$ and $\pi(e_{nd}) = 1$ (corner solution). Firm's optimality requires $r_t = F_k\left(k,n\right)$ and $w_t = F_n\left(k,n\right)$. We denote the equilibrium allocation under musical chairs with superscript "$*$".

Next, we construct a sunspot equilibrium where agent decisions are identical to those in the musical chairs equilibrium. To that end, we set the wage rate and the return on capital in the sunspot equilibrium to be equal to (R_t^*, w_t^*). Define AD prices as follows:

$$p_t\left(\sigma^t\right) \equiv \mu_t\left(s^{0t}, \phi^t\right) \times \left(\prod_{\tau=0}^{t} \left(R_\tau^*\right)^{-1}\right), \tag{11}$$

with $R_0 \equiv 1$. Define the investment portfolio x_{t+1} as follows:

$$x_{t+1} \equiv \sum_{\{s^{0t},\phi^t\}} \mu_t\left(s^{0t}, \phi^t\right) \int_{s^{1t}} k_{t+1}\left(\sigma^t\right) d^{s^{1t}}, \tag{12}$$

The investment portfolio x_{t+1}, and not its composition, is the relevant choice variable. In particular, individuals buy x_{t+1} at price $\prod_{\tau=0}^{t} \left(R_\tau^*\right)^{-1}$, and receive return $\left[\prod_{\tau=0}^{t+1} \left(R_\tau^*\right)^{-1}\right] R_{t+1}^* x_{t+1}$. Individual optimality with respect to x_{t+1} is satisfied at prices given by (11). Moreover, (11) implies that individual marginal utilities are equal across date-events, implying $c\left(\sigma^t\right) = c_t$, for all histories σ^t. Finally, under the given price system, the decisions of all the agents in the economy are well defined since $\lim_{t\to\infty} \sum_t \left(\prod_{\tau=0}^{t} \left(R_\tau^*\right)^{-1}\right)$ converges (equivalently, the musical chairs equilibrium is dynamically efficient).

Thus, problem (5)–(6) reduces to

$$\max \sum_t \beta^t \left(U\left(c_t\right) - \sum_{\{s^{0t},\phi^t\}} \mu_t\left(s^{0t}, \phi^t\right) \int_{s^{1t}} \psi\left(\sigma^t\right) ds^{1t}\right), \tag{13}$$

subject to

$$\sum_t \left(\prod_{\tau=0}^{t} \left(R_\tau^*\right)^{-1}\right)\left(c_t + x_{t+1}\right) = \sum_t \left(\prod_{\tau=0}^{t} \left(R_\tau^*\right)^{-1}\right)\left(R_t^* x_t + w_t^* \underline{h} \sum_{\{s^{0t},\phi^t\}} \mu_t\left(s^{0t}, \phi^t\right) \int_{s^{1t}} \mathbb{1}\left(\sigma^t\right) ds^{1t}\right). \tag{14}$$

Optimality with respect to consumption between two consecutive dates yields (8), so that $c_t = c_t^*$ is consistent with optimality. Moreover, we set $x_{t+1} = k_{t+1}^*$. To complete the argument we need to show that optimal allocations satisfy conditions (9)–(10) as well. From the consolidated problem (13)-(14) we can observe that keeping track the histories of ex-post realisations s^{1t-1} is not relevant anymore. To that end, we construct the conditional measure of ex-post states $\mu_t^1\left(s^{0t},\phi^t\right)$—dropping histories s^{1t-1}—and the probability measure $\mu_t^0\left(s^{0t}\right)$. Let, in turn, $S^1\left(s^{0t}\right)$ denote the set of individuals who, following history s^{0t}, are allocated to a pre-existing job that is destroyed with probability λ and survives with probability $1-\lambda$, as if starting from the employment island; $S^2\left(s^{0t}\right)$ the set of individuals who purchase lottery profile yielding employment with probability f, and unemployed with probability $1-f$, as if they started from the unemployment island; $S^3\left(s^{0t}\right)$ the set of individuals who purchase lottery profile yielding employment with probability f, and unemployed with probability $1-f$, as if they started from the leisure island; and, $S^4\left(s^{0t}\right)$ the set of individuals who choose not to participate upon observing s^{0t}.

Consider the following equilibrium conditions at history s^{0t}:

$$\begin{aligned}
\int_{i\in S^1\left(s^{0t}\right)} di &= \alpha_t^*\left(e\right), \\
\int_{i\in S^2\left(s^{0t}\right)} di &= \alpha_t^*\left(u\right)\pi_t^*\left(u\right), \\
\int_{i\in S^3\left(s^{0t}\right)} di &= \alpha_t^*\left(o\right)\pi_t^*\left(o\right),
\end{aligned} \tag{15}$$

and for each individual i

$$\begin{aligned}
\sum_{s^{0t}:\, i\in S^1\left(s^{0t}\right)} \mu_t^0\left(s^{0t}\right) &= \alpha_t^*\left(e\right), \\
\sum_{s^{0t}:\, i\in S^2\left(s^{0t}\right)} \mu_t^0\left(s^{0t}\right) &= \alpha_t^*\left(u\right)\pi_t^*\left(u\right), \\
\sum_{s^{0t}:\, i\in S^3\left(s^{0t}\right)} \mu_t^0\left(s^{0t}\right) &= \alpha_t^*\left(o\right)\pi_t^*\left(o\right),
\end{aligned} \tag{16}$$

where the pair (α^*,π^*) denotes the musical chairs' and participation probability measures evaluated at the musical chairs equilibrium. Conditions (15) are equilibrium conditions so that the measure of individuals at history node s^{0t} who face the prospect of job destruction or purchase each lottery profile after the sunspot realisation, is equal to the corresponding measure in the musical chairs equilibrium. Conditions (16) are consistency conditions so that the measures across history nodes where each individual faces the prospect of job destruction or purchases each lottery profile is equal to the measure of individuals at each history node s^{0t} who faces job destruction or purchase each lottery profile. Finally, construction of set $S^4\left(s^{0t}\right)$ follows residually.

Let, in turn, $Q^1\left(s_t^1|s^{0t},\phi^t\right)$ denote the fraction of individuals, among the measure of individuals who start from the employment island with a job that is destroyed following history $\{s^{0t},\phi^t\}$, who end up being employed at s_t^1; $Q^2\left(s_t^1|s^{0t},\phi^t\right)$ denote the fraction of individuals, among the measure of individuals who start from the employment island with a pre-existing job that is destroyed, who end up being unemployed; $Q^3\left(s_t^1|s^{0t},\phi^t\right)$ denote the fraction of individuals, among the measure of individuals who start from the employment island with a pre-existing job that is destroyed, who end up out of the labor force; $Q^4\left(s_t^1|s^{0t},\phi^t\right)$ the fraction of individuals, among the measure of individuals who start from the employment island with a job that is not destroyed, who end up employed; and $Q^5\left(s_t^1|s^{0t},\phi^t\right)$ the fraction of individuals, among the measure of individuals who start from the employment island with a job that is not destroyed, who end up out of the labor force.

Consider the following equilibrium conditions:

$$\begin{aligned}
&\int_{i\in Q^1\left(s_t^1|s^{0t},\phi^t\right)} di = \pi_t^*\left(e_d\right) f_t, &&\int_{i\in Q^4\left(s_t^1|s^{0t},\phi^t\right)} di = \pi_t^*\left(e_{nd}\right)\\
&\int_{i\in Q^2\left(s_t^1|s^{0t},\phi^t\right)} di = \pi_t^*\left(e_d\right)\left(1-f_t\right), &&\int_{i\in Q^5\left(s_t^1|s^{0t},\phi^t\right)} di = 1-\pi_t^*\left(e_{nd}\right)\\
&\int_{i\in Q^3\left(s_t^1|s^{0t},\phi^t\right)} di = 1-\pi_t^*\left(e_d\right),
\end{aligned} \tag{17}$$

and for each individual i

$$
\begin{aligned}
\mu_t^1\left(s^{0t},\phi^t\right) &= \pi_t^*\left(e_d\right)f_t, && \text{for each } i\in Q^1\left(s_t^1|s^{0t},\phi^t\right)\\
\mu_t^1\left(s^{0t},\phi^t\right) &= \pi_t^*\left(e_d\right)(1-f_t), && \text{for each } i\in Q^2\left(s_t^1|s^{0t},\phi^t\right)\\
\mu_t^1\left(s^{0t},\phi^t\right) &= 1-\pi_t^*\left(e_d\right), && \text{for each } i\in Q^3\left(s_t^1|s^{0t},\phi^t\right)\\
\mu_t^1\left(s^{0t},\phi^t\right) &= \pi_t^*\left(e_{nd}\right), && \text{for each } i\in Q^4\left(s_t^1|s^{0t},\phi^t\right)\\
\mu_t^1\left(s^{0t},\phi^t\right) &= 1-\pi_t^*\left(e_{nd}\right), && \text{for each } i\in Q^5\left(s_t^1|s^{0t},\phi^t\right)
\end{aligned}
\tag{18}
$$

where as before π^* denotes participation probability measures evaluated at the musical chairs' equilibrium. Conditions (17)–(18) are equilibrium and consistency conditions similar to (15)–(16). Finally, for individuals not belonging to the set $S^1\left(s^{0t}\right)$, the realisation of ex-post sunspots are irrelevant, so that the conditional measure of ex-post states is degenerate and equal to $\mu_t^1\left(s^{0t},\phi^t\right)=1$.

We require that idiosyncratic shocks and public signals are not independent events so that $\gamma_t\left(\phi^t|s^{0t}\right)$ depends on histories s^{0t}. In particular, we construct a dependence structure between shocks and signals consistent with summations over histories $\{s^{0t},\phi^t\}$ in (13)–(14) which yields the problem

$$
\begin{aligned}
&\max \textstyle\sum_t \beta^t\Big[U\left(c_t\right)-\alpha_t^*(e)(1-\lambda_t)\pi_t(e_{nd})\left(\xi-\ln\left(1-\underline{h}\right)\right)-\\
&\left(\xi-f_t\ln\left(1-\underline{h}\right)\right)\left[\alpha_t^*(e)\lambda_t\pi_t(e_d)+\alpha_t^*(u)\pi_t(u)+\alpha_t^*(o)\pi_t(o)\right]\Big]
\end{aligned}
\tag{19}
$$

subject to

$$
\begin{aligned}
&\textstyle\sum_t\left(\prod_t\left(R_t^*\right)^{-1}\right)\left(c_t+x_{t+1}\right)=\sum_t\left(\prod_t\left(R_t^*\right)^{-1}\right)R_t^*x_t+\\
&\textstyle\sum_t\left(\prod_t\left(R_t^*\right)^{-1}\right)\Big[\alpha_t^*(e)\pi_t(e_{nd})(1-\lambda_t)+\left(\alpha_t^*(e)\lambda_t\pi_t(e_d)+\alpha_t^*(u)\pi_t(u)+\alpha_t^*(o)\pi_t(o)\right)f_t\Big]w_t^*\underline{h}.
\end{aligned}
\tag{20}
$$

Optimality with respect to consumption and probability measures π satisfy (8)–(10). Thus, $c_t=c_t^*$, $x_{t+1}=k_{t+1}^*$, $\pi_t(e_d)=\pi_t^*(e_d)$, $\pi_t(e_{nd})=\pi_t^*(e_{nd})$, $\pi_t(u)=\pi_t^*(u)$, $\pi_t(o)=\pi_t^*(o)$ satisfy optimality. Feasibility at the given prices follows by multiplying (2) with $\prod_{\tau=0}^{t}\left(R_{\tau}^{*}\right)^{-1}$, and adding across time to obtain (20). Finally, this allocation is consistent with firm's optimality and market clearing conditions. ∎

The construction of the proof in Proposition 1 establishes that for any lottery-equilibrium, there is an associated sunspot-equilibrium. The proof is based on the property that sunspot prices are collinear with probabilities.[9] Moreover, we have assumed that the sunspot space is rich, allowing even for continuous ex-ante sunspot variables, so that the stochastic allocations induced by the coordination on the sunspot mimics the set of gambles available to an individual in the lottery economy.[10] Taken together, these two points imply that the converse of Proposition 1 is also true, completing our equivalence result. Below we elaborate on this, building on results in Garratt et al. (2002) and Kehoe et al. (2002).

In Proposition 1, we start with a target musical chairs equilibrium allocation, and show that it can be implemented as a sunspot equilibrium allocation, unique up to a relabelling of states, by using the construction (15)–(18). Conversely, a sunspot equilibrium allocation with prices collinear to probabilities as in expression (11) (which Garratt et al., 2002, call probability adjusted constant prices), corresponds to a musical chairs allocation with lotteries being pinned down by expressions (16) and (18). By construction, this candidate allocation is feasible in the musical chairs economy, yields the same factor prices, and provides the same utility level as the sunspot equilibrium allocation.[11] To complete the argument, we show that it is an equilibrium in the musical chairs economy. To that end, first, we show that any alternative lottery allocation that yields higher utility is not affordable; and second, that the candidate musical chairs allocation is affordable. The proof of the first part follows

[9] Garratt et al. (2002) call these prices constant probability adjusted prices. Furthermore, they show that in economies with complete markets all sunspot allocations can be supported by price functions that are collinear with probabilities if the sunspot variable is continuous.

[10] Garratt (1995) shows that for any lottery equilibrium there is an associated sunspot equilibrium, but the converse is not necessarily true. The equivalence fails when the sunspot variable is restricted. He provided an example where a sunspot equilibrium exists when trade is restricted to three equiprobable states, but the same allocation is not an equilibrium in the lottery economy.

[11] In the terminology of Kehoe et al. (2002) these two allocations are equivalent.

directly from the proof of Theorem 6.2 in Kehoe et al. (2002). A sketch of the argument is as follows. Suppose there exists an alternative lottery allocation that yields higher utility and is affordable. Then, from Proposition 1 we can use this alternative allocation to construct a sunspot allocation that is affordable at the given sunspot equilibrium prices yielding the same utility as the target allocation; hence, we arrive at our desired contradiction. Finally, the candidate allocation is affordable, since it induces the same factor prices as in the sunspot economy and, hence, satisfies budget constraints.

Proposition 1 has welfare implications. The sunspot allocation is Pareto efficient, given labor market frictions, if there is no alternative feasible allocation in which almost all households have no less utility and a positive measure of households have strictly more utility.

The following result applies.

Proposition 2 *The sunspot equilibrium allocation is Pareto efficient.*

Proof. Proposition 2 is a direct consequence of non-satiation of utility, and the first welfare theorem. To see this, consider (13)–(14) and rewrite it as an AD equilibrium under certainty so that the first welfare theorem applies. To this end, consider the following definitions:

$$\psi_t = \sum_{\{s^{0t},\phi^t\}} \mu_t\left(s^{0t},\phi^t\right)\int_{s^{1t}}\psi\left(\sigma^t\right)ds^{1t},\;\; h_t = \underline{h}\sum_{\{s^{0t},\phi^t\}} \mu_t\left(s^{0t},\phi^t\right)\int_{s^{1t}}\mathbb{1}\left(\sigma^t\right)ds^{1t},\;\; p_t = \prod_{\tau=0}^{t}\left(R^*_\tau\right)^{-1}. \tag{21}$$

Problem (13)–(14) modify as follows

$$\max \sum_t \beta^t [U(c_t) - \psi_t], \tag{22}$$

subject to

$$\sum_t p_t\left(c_t + x_{t+1}\right) = \sum_t p_t\left(R^*_t x_t + w^*_t h_t\right), \tag{23}$$

where ψ_t denotes the time-varying disutility cost at date t; h_t denote the time-varying endowment of time at date t; and, p_t denotes AD prices with $\sum_t p_t < \infty$. This is equivalent to the neoclassical growth model with time-varying endowments and preferences, so that the first welfare theorem applies. ■

4 Steady state analysis

In this Section we restrict attention to the steady state of the model and discuss comparative statics. We assume $U(c) = \ln(c)$ and $F(k, n) = k^\theta n^{1-\theta}$ with $0 < \theta < 1$. The system of equilibrium conditions consists of two blocks. The first block includes conditions (8)–(10) and market clearing conditions. The second block consists of motion equations for the aggregate labor market variables, as follows

$$n_t/\underline{h} \equiv N_t = (1 - u_t)\Pi_t, \tag{24}$$

$$N_t = \pi_t(e_{nd})(1 - \lambda_t)N_{t-1} + H_t f_t, \tag{25}$$

$$\Pi_t = \pi_t(e_{nd})(1 - \lambda_t) N_{t-1} + H_t, \tag{26}$$

$$H_t = \pi_t(u)U_{t-1} + \pi_t(o)O_{t-1} + \pi_t(e_d)\lambda_t N_{t-1}, \tag{27}$$

where N_t, U_t and O_t, denote measures of individuals, in turn, in the employment island, the unemployment island, and the leisure island (non-participants), at the end of date t; $\Pi_t \equiv U_t + N_t$, is the labor force measure, H_t denotes the measure of individuals searching for jobs, and $u_t \equiv U_t/\Pi_t$ is the unemployment rate.

The equilibrium is described by the following two systems of equations

$$\begin{cases} c_t^{-1} = \beta R_{t+1} c_{t+1}^{-1}, \\ \xi / f_t - \ln\left(1 - \underline{h}\right) = w_t \underline{h} c_t^{-1}, \\ c_t + k_{t+1} = k_t^{\theta} \left(\underline{h} N_t\right)^{1-\theta} + (1-\delta) k_t, \\ w_t = (1-\theta) \left(\frac{k_t}{\underline{h} N_t}\right)^{\theta}, \\ R_{t+1} = 1 - \delta + \theta \left(\frac{\underline{h} N_t}{k_t}\right)^{1-\theta}, \end{cases} \tag{28}$$

$$\begin{cases} N_t = (1 - u_t)\Pi_t, \\ N_t = (1 - \lambda_t) N_{t-1} + H_t f_t, \\ \Pi_t = (1 - \lambda_t)\, N_{t-1} + H_t, \\ H_t = \pi_t(u) U_{t-1} + \pi_t(o) O_{t-1} + \pi_t(e_d) \lambda_t N_{t-1}. \end{cases} \tag{29}$$

System (28) corresponds to the neoclassical growth model, with an endogenous labor market wedge in the second equation of the system, given by

$$\begin{aligned} w_t \underline{h} &= \text{labor wedge} \times \text{MRS} \\ &= \left[\frac{(\xi / f_t) - \ln\left(1 - \underline{h}\right)}{-\ln\left(1 - \underline{h}\right)}\right] \left[\frac{-\ln\left(1 - \underline{h}\right)}{1/c_t}\right], \end{aligned} \tag{30}$$

where $\text{MRS} = -\ln\left(1 - \underline{h}\right) c_t$, corresponds to the marginal rate of substitution between leisure and consumption in the absence of an opportunity cost of participation. The labor wedge is an outcome of search frictions, because the opportunity cost of participation is different from zero.[12]

[12]Our analysis follows much of the literature (for example, Chang and Kim, 2007; Shimer, 2013; Krusell et al., 2011, 2017) by only considering the extensive margin of adjustment in hours (hence, $\bar{h}$ is held constant). As we are considering steady state gross flows for a given labor market, relaxing this assumption would have no implications for the analysis of the competitive equilibrium that follows and, in particular, the results reported in Table 1. However, allowing for an intensive and an extensive margin is interesting if one wants to compare outcomes across economies with different sets of labor market policies. For example, different countries with different institutions (including unemployment insurance, employment protection and other

It follows from system (29) that the composition of H is indeterminate (see Ljungqvist and Sargent, 2008, for a related result). In the sunspot equilibrium, an equilibrium composition for H is selected through sunspots. Specifically, any restriction on parameters $[\pi(u), \pi(o), \pi(e_d)]$, maps into a sunspot equilibrium via conditions (16) and (18).

Next, we focus on the steady state of (28) and (29), and study how changes in search frictions affect the equilibrium level of employment, unemployment and participation. Moreover, using the same example, we examine the ability of the model to match gross worker flows, since an advantage of our model is that it identifies individual labor market transitions (in contrast to the Merz, 1995, large family model). Finally, we look at dynamics away from the steady state equilibrium by changing the job finding rate to mimic a typical recession and characterise the transition back to the steady state.

4.1 Search frictions and aggregate participation

The steady state of (28) and (29) is presented in Appendix B. In particular, the steady state labor market allocations are determined by the cost of participation, ξ, and parameters describing the labor market frictions, (λ, f), the Ins and Outs of unemployment. Following Krusell et al. (2010, 2011, 2017), we analyse how a reduction in the job-finding rate affects steady state labor market outcomes.

We establish the following Proposition:

Proposition 3 *A fall in the job finding rate f has the following impact on steady state labor*

tax and transfer policies) might have a different split of total hours between the intensive and extensive margins, and this would affect gross worker flows too. Fang and Rogerson (2009) offer a detailed treatment of how such policies may affect the split between employment and hours per worker across countries with different labor market institutions, in a canonical labor supply model with search frictions. If one extends our analysis beyond steady state comparisons, allowing for cyclical fluctuations in the amount of hours per worker is also interesting because it implies fluctuations in the opportunity cost of employment. In an influential paper, Chodorow-Reich and Karabarbounis (2016) explore this channel and show empirically that procyclical hours per worker contribute to making the opportunity cost of employment procyclical.

market outcomes:

1. *lowers aggregate employment,* N*;*
2. *raises the unemployment rate,* u*;*
3. *has an ambiguous effect on the labor force participation,* Π.

The proof of Proposition 3 is in Appendix B.

The reduction in the job finding rate lowers aggregate employment through the increase in the labor wedge, which from (30) implies that the MRS must fall (since the real wage is pinned down by technology and preferences, and is not affected by search frictions in steady state). Thus, consumption must fall, requiring a lower level of capital and employment.

The unemployment rate must increase since the steady state unemployment rate is determined only by the balancing between the inflow rate into unemployment and the outflow rate. As the inflow rate is constant, determined by the destruction rate λ, a reduction in the outflow rate, determined by f, must raise the unemployment rate in steady state.

The ambiguous effect on aggregate participation lead us to the following proposition.

Proposition 4 *There exists a threshold level of* ξ*, denoted* $\widehat{\xi}$*, such that at* $\xi > \widehat{\xi}$ *an increase in the finding rate,* f*, raises participation, and at* $\xi < \widehat{\xi}$ *an increase in* f *lowers participation. At* $\xi = \hat{\xi}$*, participation is acyclical. The threshold level is equal to*

$$\widehat{\xi} = -\left[\frac{\lambda \ln\left(1 - \underline{h}\right)}{1 - \lambda}\right]. \tag{31}$$

The proof of Proposition 4 is in the Appendix B.

A (permanent) change in the finding rate, f, affects the aggregate level of participation via three channels. The first channel is through returns to market work via the "effective" real wage rate, wf (the substitution effect). The second channel is through the opportunity cost of employment, the left hand side of the intratemporal condition in (28), so that increases in the finding rate increase the opportunity cost, which, in turn, discourages participation in the labor market. The net substitution effect, taking into account the opportunity cost channel, affects labor supply decisions via the labor wedge in expression (30). The third channel is the income effect from a permanent change in the finding rate, which affects the budget set of the stand-in agent through the effective real wage.

At $\xi = \widehat{\xi}$, the net substitution and income effects cancel out and aggregate participation is acyclical; while at $\xi > \widehat{\xi}$, the net substitution dominates and participation is procyclical, and at $\xi < \widehat{\xi}$, the income effect dominates and participation is countercyclical.

It follows from Proposition 4 that the model can deliver both a countercyclical or a procyclical participation rate, depending on the elasticity of the labor wedge to changes in f, controlled by the parameter ξ, the opportunity cost of participation. Thus, the neoclassical growth model with search frictions can be reconciled with a mildly procyclical participation rate. In turn, this result is particularly important given the tendency for models featuring intertemporal substitution in frictional labor markets to deliver excessively procyclical participation and, thus, procyclical unemployment (a problem stressed by Ravn, 2008; Shimer, 2013, for example).

4.2 Gross worker flows

Unlike the Merz (1995) large family set-up which only identifies net worker flows, our model yields equilibrium outcomes for gross worker flows. To illustrate this point, we present a simple quantitative exercise to evaluate the model's ability to explain the average gross flows in the data. In particular, despite its parsimony the model is able to account well for the transitions between unemployment and inactivity, which previous literature has shown to be challenging.

The model yields gross flows across the three states of employment, unemployment, and non-participation, resulting from individual optimal behaviour, given by

$$\begin{aligned}
\phi_{ee,t} &= (1-\lambda_t) + \pi_t(e_d)\lambda_t f_t, & \phi_{ue,t} &= \pi_t(u) f_t, & \phi_{oe,t} &= \pi_t(o) f_t, \\
\phi_{eu,t} &= \pi_t(e_d)\lambda_t\left(1-f_t\right), & \phi_{uu,t} &= \pi_t(u)\left(1-f_t\right), & \phi_{ou,t} &= \pi_t(o)\left(1-f_t\right), \\
\phi_{eo,t} &= \lambda_t(1-\pi_t(e_d)), & \phi_{uo,t} &= 1-\pi_t(u), & \phi_{oo,t} &= 1-\pi_t(o),
\end{aligned} \tag{32}$$

with $\phi_{ss',t}$ the transition rate from state s to state s', for $s, s' \in \{e, u, o\}$, in period t, implied by the labor market parameters, f and λ and the randomisation induced by the optimal choices for the lotteries over labor force participation. The latter may induce in equilibrium different participation probabilities chosen by the individuals starting in employment but who lose their jobs, those unemployment and those out of the labor force, in turn, $\pi_t(e_d)$, $\pi_t(u)$ and $\pi_t(o)$. We argue below that this feature is important for the success of the model to match the gross worker flows across the unemployment and non-participation states.

In the sequel we focus on steady state transition probabilities. For the US, we measure gross worker flows empirically from the longitudinal monthly Current Population Survey (CPS), as explained for example in Elsby et al. (2015) and Krusell et al. (2017). We test the model's ability to explain labor market transitions with a simple calibration experiment. We select values for the parameters determining labor market transitions, $[\lambda, f, \pi(o), \pi(e_d)]$, to minimise a distance criterion function of the deviations of the gross transitions from their empirical counterparts, given the equilibrium conditions (29), and for an employment rate set to $N = 65\%$.

Table 1 compares the gross flows implied by the calibrated example economy to their empirical counterparts (as reported in Krusell et al., 2017, based on the CPS longitudinal micro data), and reports the implied calibrated values for the vector vector of parameters. Despite the parsimonious set of parameters to match nine targets, the model does a relatively good job at matching gross flows, comparable to the results in Krusell et al. (2017), who develop a richer

Table 1: Gross worker flows (model and data)

	to s':		
$\phi_{s,s'}$	e	u	o
from s:			
e	0.977 (0.972)	0.017 (0.014)	0.006 (0.014)
u	0.229 (0.228)	0.637 (0.637)	0.134 (0.135)
o	0.010 (0.022)	0.027 (0.021)	0.963 (0.957)
		calibrated values	
λ		0.0290	
f		0.2645	
$\pi(o)$		0.0373	
$\pi(e_d)$		0.7848	
$\pi(u)$		0.8660	

Notes: In the first panel, values outside parenthesis are obtained from the model and the values in parenthesis are the empirical counterpart, obtained from Krusell et al. (2017), and used as targets. The lower panel reports the calibrated value for each parameter.

incomplete market model with heterogeneous agents. The model is particularly successful at matching the high transition rate from unemployment to inactivity, ϕ_{uo}, which the literature has found challenging.[13]

Key to the success of the model to match the transition from unemployment to inactivity is the indeterminacy in the composition of the stock of job searchers, H. This indeterminacy is resolved by the sunspot mechanism which yields different participation probabilities chosen by the individuals starting in employment but who lose their jobs, those unemployment and

[13] Garibaldi and Wasmer (2005), in a model with linear utility, and Krusell et al. (2011), in a model with concave utility and incomplete markets, both show that the transition rates from unemployment to inactivity are difficult to account for in three-state equilibrium models of the labor market, without additional heterogeneity across individuals to achieve the calibration target. Garibaldi and Wasmer (2005) experiment with permanent heterogeneity across workers, while Krusell et al. (2011) consider transitory productivity shocks to match the transition from unemployment to inactivity.

those out of the labor force, in turn, $\pi_t(e_d)$, $\pi_t(u)$ and $\pi_t(o)$. From equation (32) we see that $\phi_{uo} = 1 - \pi_t(u)$, the transition rate from unemployment to inactivity is entirely determined by $\pi_t(u)$. Thus, it is possible to construct a sunspot equilibrium from (15) and (16), to match successfully the ϕ_{uo} transition rate.

5 Conclusion

This paper shows that the same aggregation as in Andolfatto (1996) can be obtained without either lotteries or additional exogenous randomization (the game of musical chairs), when individual choices over contingent commodities are coordinated by sunspots. We show that this aggregation approach offers a tractable method to construct a general equilibrium model of gross worker flows. The upshot is that the economy with sunspots yields testable predictions about gross workers flows, which may be confronted with micro level data on labor market transitions.

Although lotteries are socially optimal in economies with indivisibilities, they have been repudiated by some as an employment allocation mechanism, on the grounds that such ideal device is not empirically plausible (Browning et al., 1999; Ljungqvist and Sargent, 2011). Previous work by Shell and Wright (1993), Garratt et al. (2002) and Kehoe et al. (2002) shows how to avoid the need for such implausible randomization mechanisms, by establishing the close connection between lottery economies and sunspot economies. We extend their approach to accommodate economies with labor market search frictions. The resulting model can obtain plausible individual employment histories, as illustrated by the empirically realistic gross worker flows in a calibrated example.

Turning to future work, the fact that an equilibrium with musical chairs can be decentralized with sunspots, opens the possibility to study adverse selection and moral hazard in labor markets with search frictions, using results for sunspot equilibra in incentive constrained economies (Kehoe et al., 2002).

Appendix

A Lottery equilibrium

Proposition 5 *An equilibrium with musical chairs and lotteries is characterised by* $\pi_t(\tilde{\imath}) \in (0,1)$*, for* $\tilde{\imath} \in \{e_d, u, o\}$*, and* $\pi_t(e_{nd}) = 1$*.*

Proof. Let us argue by contradiction. Suppose $\pi_t(e_{nd}) \in [0,1)$ and $\pi_t(\tilde{\imath}) \in [0,1]$. Then, $\pi_t(e_{nd}) \in [0,1)$ requires (10) to modify as follows:

$$\xi - \ln\left(1 - \underline{h}\right) \geq w_t \underline{h} U_c(c_t) \tag{A.1}$$

Multiplying both sides of (A.1) by f_t, yields

$$\xi - f_t \ln\left(1 - \underline{h}\right) > f_t \xi - f_t \ln\left(1 - \underline{h}\right) \geq f_t w_t \underline{h} U_c(c_t), \tag{A.2}$$

which in turn, requires $\pi_t(\tilde{\imath}) = 0$, for $\tilde{\imath} \in \{e_d, u, o\}$. Subsequently, the employment law of motion in (25) requires $H = 0$ and imposing the steady state restriction, it follows that $\pi(e_{nd}) = 1/(1-\lambda) > 1$, which is a contradiction. ■

B Steady state and comparative statics

In this Section we compute the steady state allocation and the comparative statics for the example economy presented in Section 4.

The steady state of the first block, system (28), after imposing steady state, reduces to

$$\frac{y}{k} = \left(\frac{1/\beta - 1 + \delta}{\theta}\right) \tag{B.1}$$

$$\frac{n}{k} = \left(\frac{1/\beta - 1 + \delta}{\theta}\right)^{1/(1-\theta)}, \tag{B.2}$$

$$\frac{c}{k} = \left(\frac{1/\beta - 1 + \delta}{\theta}\right) - \delta \tag{B.3}$$

$$n = \frac{f}{\xi - f \ln\left(1 - \underline{h}\right)} \frac{1 - \theta}{1 - \delta\,(k/y)}. \tag{B.4}$$

In turn, the steady state of the second block yields

$$H = \frac{\lambda}{\xi - f \ln\left(1 - \underline{h}\right)} \frac{1 - \theta}{1 - \delta\,(k/y)} \frac{1}{\underline{h}}, \tag{B.5}$$

$$\Pi = \left(\frac{\lambda}{f} + 1 - \lambda\right) \frac{n}{\underline{h}}, \tag{B.6}$$

$$O = 1 - \Pi, \tag{B.7}$$

$$U = \Pi - \frac{n}{\underline{h}}, \tag{B.8}$$

$$u = \frac{\lambda(1 - f)}{\lambda + f(1 - \lambda)}. \tag{B.9}$$

It follows from (B.4), (B.6), (B.9) that the elasticity of employment, unemployment rate and participation, respectively, with respect to f is equal to

$$\epsilon_{N,f} = \frac{\xi}{\xi - f \ln\left(1 - \underline{h}\right)} > 0, \tag{B.10}$$

$$\epsilon_{u,f} = -\frac{1}{1 - \lambda + \lambda/f} \frac{1}{1 - f} < 0, \tag{B.11}$$

$$\epsilon_{\Pi,f} = \frac{\xi}{\xi - f \ln\left(1 - \underline{h}\right)} - \frac{\lambda/f}{\lambda/f + 1 - \lambda}, \tag{B.12}$$

where $\epsilon_{X,Y} \equiv (dX/dY)\,(Y/X)$ denotes the elasticity of X with respect to Y. The result below follows directly from (B.12).

Corollary 1 *There exist* $\widehat{\xi}$ *such that* $\epsilon_{\Pi,f} = 0$. *For* $\xi > \widehat{\xi}$ *it follows that* $\epsilon_{\Pi,f} > 0$ *whereas for* $\xi < \widehat{\xi}$ *it follows that* $\epsilon_{\Pi,f} < 0$. *The threshold level is equal to*

$$\widehat{\xi} = -\left[\frac{\lambda \ln\left(1 - \underline{h}\right)}{1 - \lambda}\right]. \tag{B.13}$$

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