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The Earth's electromagnetic environment

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[1] It is found that the energy density of electromagnetic fields at the surface of the Earth follow a scaling law that extends over ~ 16 orders of magnitude from $\sim 10^{-9}$ Hz to $\sim 10^7$ Hz. The temporal variability of the field can be described with an $\sim 1/f^2$, or Brownian, noise power spectrum which reflects the superposition of numerous transient source processes. To the best of our knowledge, the spectral extent of this straightforward scaling law is unparalleled and outperforms any other scaling law in physics which describes a time dependent observable. The frequency dependence of the energy density can be approximated with the analytic description $u(f) = u_0(f_0/f)^2$ where $u_0 = 10^{-16} \text{ Jm}^{-3}\text{Hz}^{-1}$, $f_0 = 1$ Hz is a scaling constant, and f is the frequency of the electromagnetic field. The corresponding frequency dependence of the magnetic field is $B(f) = B_0(f_0/f)$ where $B_0 = 10^{-11} \text{ T/Hz}^{1/2}$. **Citation:** Füllekrug, M., and A. C. Fraser-Smith (2011), The Earth's electromagnetic environment, *Geophys. Res. Lett.*, 38, L21807, doi:10.1029/2011GL049572.

1. Introduction

[2] The Earth's atmospheric electromagnetic environment is composed of naturally occurring magnetic and electric fields which vary on all time scales. The overall frequency dependence of the Earth's atmospheric electromagnetic fields is not very well known and has been little studied. Electromagnetic field spectra have previously been approximated with scaling laws which use the frequency f with an exponent α to describe the frequency dependence with the functional form $f^{-\alpha}$. For example, background magnetic field spectra from $\sim 10^{-5}$ Hz to $\sim 10^5$ Hz measured at Arrival Heights in Antarctica [Fraser-Smith et al., 1992] suggest a description ranging from $\sim f^{-1}$ to $\sim f^{-1.5}$ [Lanzerotti et al., 1990]. Similarly, the electric field spectrum of an exemplary lightning discharge suggests a description ranging from $\sim f^{-1}$ to $\sim f^{-2}$ in the frequency range from $\sim 10^3$ Hz to $\sim 10^7$ Hz [Lanzerotti et al., 1989]. The energy density of these magnetic and electric field spectra has then a functional description ranging from $\sim f^{-2}$ to $\sim f^{-4}$ which clearly excludes self-organized criticality with a noise power proportional to $\sim f^{-1}$ [Bak et al., 1987] as a possible explanation for the energy density of the electromagnetic field. Yet, the large range of the observed exponents from $\alpha \approx -4$ up to $\alpha \approx -2$ and the relatively small frequency range of the assessment ~ 4 – 10 orders of magnitude inhibits a deeper understanding of any underlying scaling law. This contribution aims to reduce the current uncertainty by increasing the frequency range of the assessment and by

decreasing the possible range of exponents to infer a more significant analytic description of the average atmospheric electromagnetic field measured at the surface of the Earth.

2. Observations

[3] The observations are compiled from three contemporary reviews published in the scientific literature by senior authorities in their respective fields. The spectrum of the magnetic field $B(f)$ from $\sim 10^{-9}$ Hz to $\sim 10^3$ Hz is reported in units of $10^{-9} \text{ T/Hz}^{1/2}$ [Olsen, 2007] which is converted here to the corresponding electromagnetic energy density $u(f) = B^2(f)/\mu_0$ where $\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$ is the permeability of free space. The unit of the energy density is therefore given in $\text{Jm}^{-3}\text{Hz}^{-1}$, i.e., an energy per volume and per spectral bandwidth. The spectrum of the magnetic field from $\sim 10^{-5}$ Hz to $\sim 10^5$ Hz is reported in units of $10^{-15} \text{ T/Hz}^{1/2}$ [Lanzerotti et al., 1990] which is converted here to the corresponding electromagnetic energy density in the same way. Electric field measurements from $\sim 10^0$ Hz to $\sim 10^7$ Hz are reported in units of the noise figure $F_a(f)$ [Spaulding, 1995]. This noise figure measures the radio noise power (in units of W) from sources external to a lossless receiving antenna relative to the thermal, or Johnson, noise power available in the same bandwidth from a resistor at room temperature [Fraser-Smith, 2007]. The resulting unit of the noise figure is dB above KT_0b , where $k \approx 1.83 \times 10^{-23} \text{ J/K}$ is Boltzmann's constant, $T_0 = 288.15 \text{ K}$ is the reference room temperature, and b is the spectral bandwidth in Hz which is typically $\sim 10\%$ of its center frequency. The noise figure is converted here to the corresponding electromagnetic energy density in two steps. The first step is the calculation of the relative quantity

$$E_b(f) = F_a(f) + 20 \log_{10}(f/f_M) - 95.5 \quad (1)$$

where the frequency f of the electromagnetic field is referenced against the frequency $f_M = 10^6$ Hz and the constant 95.5 dB results from the aperture of a short grounded vertical monopole antenna used for the measurements [Fraser-Smith, 2007, 1995]. The physical significance of $E_b(f)$ is to represent the band-limited root-mean-square (rms) electric field strength $E_r(f)$ per spectral bandwidth b measured in dB relative to $E_0 = 10^{-6} \text{ Vm}^{-1}\text{Hz}^{-1/2}$ such that

$$E_b(f) = 10 \log_{10} \frac{E_r^2(f)}{bE_0^2}. \quad (2)$$

The second step is the calculation of the spectrum from the corresponding electric field strength

$$E_f = \frac{E_r(f)\sqrt{2}}{\sqrt{b}} = 10^{E_b(f)/20} E_0 \sqrt{2} \quad (3)$$

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which is converted here to the corresponding electromagnetic energy density by use of $u(f) = E^2(f)\epsilon_0$ where $\epsilon_0 \approx 8.85 \times 10^{-12} \text{ Fm}^{-1}$ is the permittivity of free space.

3. Results

[4] The resulting electromagnetic energy density from $\sim 10^{-9}$ Hz to $\sim 10^7$ Hz exhibits a constant decrease with increasing frequency which extends over the entire frequency range (Figure 1). It is interesting to note that the observed decrease applies simultaneously to the energy density derived from the magnetic field spectrum at lower frequencies and the energy density derived from the electric field spectrum at higher frequencies. The consistent decrease of the energy density also appears to be independent of the instrumentation used for the measurements which provides circumstantial evidence towards the significance of the published data.

[5] The electromagnetic energy density can be approximated with a scaling law of the form $u(f) = u_0(f_0/f)^2$ where $u_0 \approx 10^{-16} \text{ Jm}^{-3}\text{Hz}^{-1}$, $f_0 = 1$ Hz is a scaling constant, and f is the frequency of the electromagnetic field (Figure 1). The residuals of this approximation determine the uncertainty of u_0 in the scaling law. For example, if u_0 is increased by two orders of magnitude and decreased by two orders of magnitude, almost all the observed data are found within the bracketed range (Figure 1). This uncertainty allows for deviations from the overall scaling law in several parts of the energy density. These local deviations may indicate specific physical properties of the underlying source processes which contribute to the energy density. However, it is surprising that these deviations do not exhibit any significant persistent kink in the energy density over ~ 16 orders of magnitude. To the best of our knowledge, the spectral extent of this straightforward scaling law is unparalleled and out-

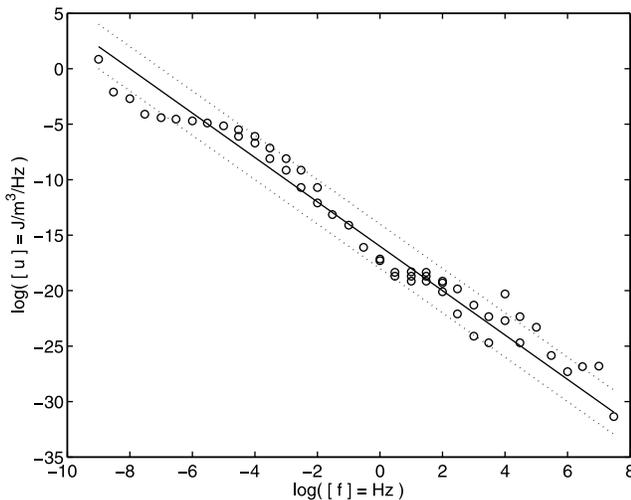


Figure 1. The energy density of electromagnetic fields measured at the surface of the Earth (circles) follows a scaling law which extends over ~ 16 orders of magnitude from $\sim 10^{-9}$ Hz to $\sim 10^7$ Hz (solid line). The energy density can be approximated by the scaling law $u = u_0(f_0/f)^2$ where $u_0 \approx 10^{-16} \text{ Jm}^{-3}\text{Hz}^{-1}$, $f_0 = 1$ Hz is a scaling constant, and f is the frequency of the electromagnetic field. The measured energy density exhibits deviations from the scaling law by $\sim \pm 2$ orders of magnitude across the entire frequency range (dotted lines).

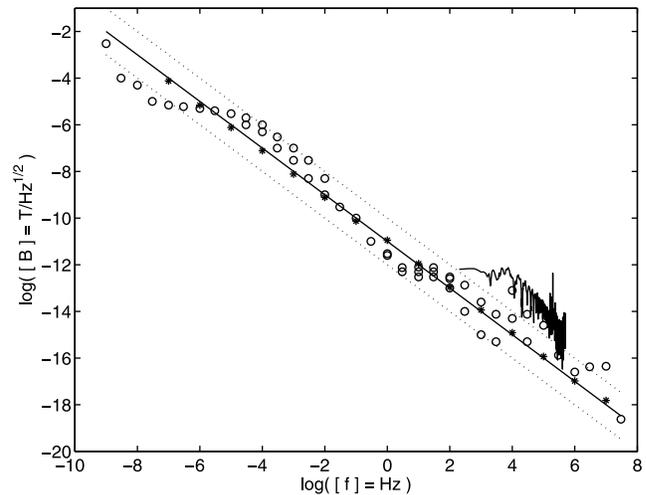


Figure 2. The magnetic field spectrum from $\sim 10^{-9}$ Hz to $\sim 10^7$ Hz (circles) can be approximated by the scaling law $B = B_0(f_0/f)$ where $B_0 \approx 10^{-11} \text{ T/Hz}^{1/2}$, $f_0 = 1$ Hz is a scaling constant, and f is the frequency of the magnetic field (solid line). The measured magnetic field exhibits deviations from the scaling law by $\sim \pm 1$ order of magnitude across the entire frequency range (dotted lines). The spectrum of an average lightning discharge, which is recorded with a radio receiver at a distance of ~ 510 km, exceeds the scaling law by ~ 2 orders of magnitude at frequencies from ~ 0.2 – 400 kHz. The spectrum of the lightning discharge is exceeded at 198 kHz by the narrow maximum from man made long wave radio transmissions, i.e., BBC radio 4 in the UK. The scaling law is simulated in the frequency range from 10^{-7} Hz to 10^7 Hz with a persistent normal distributed random noise process (stars) and exhibits an excellent agreement with the scaling law.

performs any other scaling law in physics which describes a time dependent observable.

[6] The electromagnetic energy density is converted to the corresponding magnetic field spectrum $B(f) = \sqrt{\mu_0 u(f)}$ to determine an equivalent scaling law for magnetic field measurements which are more commonly used in the Earth sciences community. The magnetic field spectrum can be approximated with a scaling law of the form $B(f) = B_0(f_0/f)$ where $B_0 \approx 10^{-11} \text{ T/Hz}^{1/2}$, $f_0 = 1$ Hz is a scaling constant, and f is the frequency of the electromagnetic field (Figure 2). The spatial and temporal variability of the atmospheric electromagnetic field results in deviations from this scaling law by $\sim \pm 1$ order of magnitude which determines the uncertainty of B_0 .

[7] It is interesting to note that particular source processes can exceed the average scaling law by several orders of magnitude. For example, the electric field strength of a lightning discharge is recorded with a wideband digital radio receiver [Fullekrug, 2010] at a distance of ~ 510 km. The spectrum of the measured electric field strength is converted here to a magnetic field spectrum by use of the relation $B(f) = \mu_0 E(f)/Z_0 = E(f)/c$ where $Z_0 = \sqrt{\mu_0/\epsilon_0} \approx 120\pi \Omega$ is the impedance of free space which is inferred from Faraday's law by use of the speed of light c and the dispersion relation for electromagnetic waves in vacuum. This spectrum of the lightning discharge exceeds the scaling law by ~ 2 orders of magnitude in the frequency range ~ 0.2 – 400 kHz (Figure 2).

The lightning spectrum is exceeded by a narrow maximum at 198 kHz which results from the long wave radio transmitter BBC 4 in the UK. This peak from man made radio transmissions exceeds the average scaling law by ~ 3 orders of magnitude.

4. Simulation

[8] The scaling laws for the energy density $\sim 1/f^2$ and the magnetic field spectrum $\sim 1/f$ indicate the presence of a $\sim 1/f^2$, or Brownian, noise power spectrum over a spectral range of ~ 16 orders of magnitude. A $1/f^2$ noise power spectrum can be simulated with an observable y which is generated by a persistent discrete random process r using an autoregressive process of the form

$$y_{i+1} = y_i + r_i \quad (4)$$

where the index i indicates a discrete time step with $y_0 = 0$ and r_i is the realization of a random number from a normal, or Gaussian, distribution with zero mean and standard deviation σ_r . A standard deviation of $\sigma_r = 6 \times 10^{-15}$ T is used here to simulate the scaling law of the magnetic field spectrum in the frequency range from 10^{-7} Hz to 10^7 Hz (Figure 2). The agreement between the scaling law and the simulation is excellent. Yet, whilst it is undoubtedly surprising to explain the scaling law over a frequency range of ~ 14 orders of magnitude with one single parameter σ_r , there are certainly many more physical processes which contribute to the atmospheric electromagnetic environment.

5. Discussion

[9] The energy density of the electromagnetic field can roughly be divided into two parts. The lower frequencies ~ 1 Hz are mainly associated with geomagnetic fields whilst the higher frequencies ~ 1 Hz are mainly associated with atmospheric electric fields. Geomagnetic field variations with frequencies $\sim 8 \times 10^{-9}$ Hz are mainly generated inside the Earth, whilst geomagnetic field variations from $\sim 10^{-8}$ – 10^{-3} Hz are generated by ionospheric current systems and superimposed geomagnetic storms [Olsen, 2007]. Rapid magnetic field variations are typically referred to as pulsations and occur irregularly in the frequency range from $\sim 10^{-3}$ – 10^0 Hz [Lanzerotti et al., 1990]. Atmospheric electric field fluctuations with frequencies in the range from $\sim 10^0$ – 3×10^7 Hz are mainly associated with lightning discharge processes, e.g., lightning continuing current, return strokes, and stepped leaders [Rakov and Uman, 2003]. Superimposed on these lightning discharge processes are irregularly occurring bursts of electromagnetic radiation from near-Earth space, e.g., hiss, chorus, and auroral kilometric radiation [Lanzerotti et al., 1990]. All these source processes are superimposed on each other and vary over large ranges of temporal and spatial scales. It is therefore surprising that the rich diversity of source processes produces, at least on average, a scaling law over a frequency range of ~ 16 orders of magnitude. In the search for the smallest possible denominator of all these source processes, two common characteristics may be relevant. The first characteristic is the transient nature of individual events. This transient nature is often characterized by a sudden rise and a slow decay which is, for example, apparent in geomagnetic storms and lightning discharges [Füllekrug, 2006]. The second characteristic

is the relationship between the recurrence time and the intensity of an event, e.g., strong events are usually more rare and weak events usually occur more often [e.g., *Chrissan and Fraser-Smith*, 2003]. Whether these recurring transient events are sufficient to explain the observed scaling law remains to be explored in future studies.

[10] It is interesting to note that the commonly used word ‘event’ results from a reductionist view on electromagnetic fields. In this view, an unusual excursion from the normally observed, or background, intensity of electromagnetic fields is the subject of study which involves theoretical modeling from first principles, e.g., Maxwell’s equations, and subsequent comparison with the measurements. In this context, the word ‘unusual’ normally means a large intensity measured against a background intensity, but it rarely means a small intensity measured against a background intensity. As a result, scientific efforts are often directed towards waiting for unusually intense events rather than studying the commonly occurring background intensity.

[11] In a more holistic view on electromagnetic fields, the scientific effort is directed towards the collection and integration of measurements across scientific subject boundaries to describe the background intensity. This approach remains a challenge to date which often requires an acute comparison of electromagnetic measurements with reference data during interdisciplinary research. For example, intense positive lightning discharges in the troposphere can cause transient luminous events above thunderclouds [Bocippio et al., 1995], which are referred to as sprites [Neubert et al., 2008; Füllekrug et al., 2006; Rakov and Uman, 2003; Sentman et al., 1995; Franz et al., 1990]. Spectacular sprites in the mesosphere can radiate electromagnetic waves [Pasko et al., 1998; Cummer et al., 1998] and occasionally cause pulsations in the ionosphere [Füllekrug et al., 1998; Fukunishi et al., 1997]. This chain of causal processes can span a frequency range from ~ 0.2 Hz up to $\sim 3 \times 10^7$ Hz and it may even extend down to $\sim 10^{-5}$ Hz if the time scale of thunderstorm generation and its possible impact on geomagnetic pulsations is considered [Fraser-Smith, 1993; Armstrong, 1987; Fraser-Smith and Roxburgh, 1969]. However, the critical question towards explaining the scaling law with such a complex chain of causal physical processes is whether or not the same processes contribute to the scaling law if they are not unusually intense. For example, intense positive lightning discharges represent only ~ 0.1 % of all lightning discharges [Füllekrug, 2006] and they can therefore not contribute significantly to the observed scaling law. But if all commonly occurring intra-cloud lightning discharges in the troposphere would generate some photons in the mesosphere, i.e., transient luminous events which are currently too weak to be detected [Pasko, 2010], and weak geomagnetic pulsations in the ionosphere, this exemplary chain of causal physical processes may be more important than currently thought.

[12] It remains surprising that the deviations from the scaling law only extend over some minor fraction of the spectrum and that these deviations do not persist over the remainder of the spectrum. As a result, no persistent kink seems to be present in the electromagnetic spectrum, whereas the cosmic ray flux [Nagano and Watson, 2000] and turbulence [Larsen and Kelley, 1982] in the Earth’s atmosphere exhibit significant kinks. This means that the superposition of numerous electromagnetic source processes varying on all temporal and spatial scales can not exceed some critical limitation. It is

therefore possible that the source processes are embedded in a self-organized structure of random states. It is speculated that this critical limitation may be needed to stabilize the Earth's atmospheric electromagnetic environment, similar to the limitation imposed by self-organized criticality in dynamic systems which is used to explain $1/f$ noise power spectra [Bak *et al.*, 1987].

[13] Another line of thought is to consider space time relationships of electromagnetic waves and their physical sources. For example, the dispersion relations for magnetohydrodynamic waves [Rytov, 2007; Lanzerotti, 1974] and Earth-ionosphere cavity resonances [Füllekrug, 2004; Kroll, 1971] have been used to support the standard model [Nakamura *et al.*, 2010] by placing an upper limit on the photon rest mass, based on an original assessment of the geomagnetic field [Fischbach *et al.*, 1994; Goldhaber and Nieto, 1971, 1968; Schrödinger, 1943]. Maxwell's equations are thought to be scale invariant, similar to other physical laws, e.g., Euler equations [Kelley *et al.*, 2011], such that the amplitudes of electromagnetic fields may scale according to the space time relationship of their physical sources. However, we are currently unaware of any corresponding theoretical conception which could explain the observed scaling law.

6. Application

[14] The observed scaling law has two important applications of scientific interest. The first application is to use the scaling law as a null hypothesis such that deviations from the scaling law of the Earth's electromagnetic field become of primary interest. The second application is to assist the design and construction of future generations of measurement equipment towards recording environmental electromagnetic fields over large frequency ranges. The $\sim 1/f$ dependence of the magnetic field spectrum suggests the pursuit of two possible strategies. The first strategy is to record differential electromagnetic fields and the second strategy is to build sensors with a response function which is proportional to frequency $\sim f$. The recording of differential fields corresponds to calculating the time derivative of the spectrum which results in a multiplication of the spectrum with the frequency and thereby compensates for the $\sim 1/f$ dependence of the scaling law. Sensors with a $\sim f$ response compensate for the $\sim 1/f$ dependence of the scaling law prior to the actual measurement of the electromagnetic field. In either case, the dynamic range of the recording equipment is of minor concern. As natural and man-made deviations from the intensity specified by the scaling law may span $\sim \pm 3$, i.e., a total of ~ 6 , orders of magnitude, the currently emerging digital technology with 24-bit dynamic range recording capability appears to be sufficient for extremely broadband recordings. Whether it will be possible to build sensors with a response function $\sim f$ over a frequency range of ~ 16 orders of magnitude remains to be seen. The key obstacle towards the development of future technology to record electromagnetic fields over a frequency range of ~ 16 orders of magnitude is the frequency dependent operation of electronic components, e.g., resistors, capacitors, transistors, and low-noise amplifiers.

7. Summary

[15] The frequency dependence of the Earth's atmospheric electromagnetic field is determined by compiling published

magnetic and electric field measurements over a frequency range of ~ 16 orders of magnitude. The resulting spectra exhibit a $\sim 1/f^2$, or Brownian, scaling law for the energy density and a $\sim 1/f$ dependence for the magnetic field spectrum. The scaling law may be explained with persistent random noise, recurring transient events, or the superposition of numerous transient source processes which contribute to the complexity of the Earth's atmospheric electromagnetic field. We favor the latter interpretation, even though the simulation of the scaling law with a persistent random noise model exhibits an excellent agreement. It is speculated that an as yet unknown mechanism remains to be discovered which imposes a critical limitation on naturally occurring electromagnetic fields and possibly stabilizes the Earth's atmospheric electromagnetic environment.

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References

- Armstrong, W. (1987), Lightning triggered from the Earth's magnetosphere as the source of synchronized whistlers, *Nature*, 327(6121), 405–408.
- Bak, P., C. Tang, and K. Wiesenfeld (1987), Self-organized criticality: An explanation of $1/f$ noise, *Phys. Rev. Lett.*, 59(11), 381–384.
- Boccippio, D., E. Williams, S. Heckman, W. Lyons, I. Baker, and R. Boldi (1995), Sprites, ELF transients, and positive ground strokes, *Science*, 269, 1088–1091.
- Chrissan, D. A., and A. C. Fraser-Smith (2003), A Clustering Poisson model for characterizing the interarrival times of sferics, *Radio Sci.*, 38(4), 1078, pp. 1–14, doi:10.1029/2002RS002693.
- Cummer, S. A., U. S. Inan, T. F. Bell, and C. P. Barrington-Leigh (1998), ELF radiation produced by electrical currents in sprites, *Geophys. Res. Lett.*, 25(8), 1281–1284.
- Fischbach, E., H. Kloor, R. Langel, A. Lui, and M. Peredo (1994), New geomagnetic limits on the photon mass and on long-range forces coexisting with electromagnetism, *Phys. Rev. Lett.*, 73(4), 514–517.
- Franz, R., R. Nemzek, and J. Winckler (1990), Television image of a large upward electrical discharge above a thunderstorm system, *Science*, 249, 48–51.
- Fraser-Smith, A. C. (1993), ULF magnetic fields generated by electrical storms and their significance to geomagnetic pulsation generation, *Geophys. Res. Lett.*, 20(6), 467–470.
- Fraser-Smith, A. (1995), Low-frequency radio noise, in *Handbook of Atmospheric Electrodynamics*, vol. I, edited by H. Volland, pp. 297–310, CRC Press, Boca Raton, Fla.
- Fraser-Smith, A. C. (2007), The effective antenna noise figure F_a for a vertical loop antenna and its application to extremely low frequency/very low frequency atmospheric noise, *Radio Sci.*, 42, RS4026, pp. 1–9, doi:10.1029/2005RS003387.
- Fraser-Smith, A., and K. Roxburgh (1969), Triggering of hydromagnetic “whistlers” by sferics, *Planet. Space Sci.*, 17, 1310–1312.
- Fraser-Smith, A., R. Helliwell, L. Lanzerotti, and T. Rosenberg (1992), Antarctic environmental concerns, *Science*, 256, 949–950.
- Fukunishi, H., Y. Takahashi, M. Sato, A. Shono, M. Fujito, and Y. Watanabe (1997), Ground-based observations of ULF transients excited by strong lightning discharges producing elves and sprites, *Geophys. Res. Lett.*, 24(23), 2973–2976.
- Füllekrug, M. (2004), Probing the speed of light with radio waves at extremely-low frequencies, *Phys. Rev. Lett.*, 93(4), 043901, pp. 1–3, doi:10.1103/PhysRevLett.93.043901.
- Füllekrug, M. (2006), Elementary model of sprite igniting electric fields, *Am. J. Phys.*, 74(9), 804–805, doi:10.1119/1.2206573.

- Füllekrug, M. (2010), Wideband digital low-frequency radio receiver, *Meas. Sci. Technol.*, 21, 015901, pp. 1–9, doi:10.1088/0957-0233/21/1/015901.
- Füllekrug, M., A. C. Fraser-Smith, and S. C. Reising (1998), Ultra-slow tails of sprite-associated lightning flashes, *Geophys. Res. Lett.*, 25(18), 3497–3500.
- Füllekrug, M., E. Mareev, and M. Rycroft (Eds.) (2006), *Sprites, Elves and Intense Lightning Discharges*, Springer, Dordrecht, Netherlands.
- Goldhaber, A., and M. Nieto (1968), New geomagnetic limit on the mass of the photon, *Phys. Rev. Lett.*, 21(8), 567–569.
- Goldhaber, A., and M. Nieto (1971), Terrestrial and extraterrestrial limits on the photon mass, *Rev. Mod. Phys.*, 43(3), 277–296.
- Kelley, M., E. Dao, C. Kuranz, and H. Stenbaek-Nielsen (2011), Similarity of Rayleigh-Taylor instability development on scales from 1 mm to one light year, *Int. J. Astron. Astrophys.*, in press.
- Kroll, N. (1971), Concentric spherical cavities and limits on the photon rest mass, *Phys. Rev. Lett.*, 27(6), 340–343.
- Lanzerotti, L. J. (1974), Magnetohydrodynamic waves in the magnetosphere and the photon rest mass, *Geophys. Res. Lett.*, 1(6), 229–230.
- Lanzerotti, L. J., D. J. Thomson, C. G. MacLennan, K. Rinnert, E. P. Krider, and M. A. Uman (1989), Power Spectra at radio frequency of lightning return stroke waveforms, *J. Geophys. Res.*, 94, 13,221–13,227.
- Lanzerotti, L. J., C. G. MacLennan, and A. C. Fraser-Smith (1990), Background magnetic spectra: $\sim 10^{-5}$ to $\sim 10^5$ Hz, *Geophys. Res. Lett.*, 17(10), 1593–1596.
- Larsen, M., and M. Kelley (1982), Turbulence spectra in the upper troposphere and lower stratosphere at periods between 2 hours and 40 days, *J. Atmos. Sci.*, 39, 1035–1041.
- Nagano, M., and A. Watson (2000), Observations and implications of the ultrahigh-energy cosmic rays, *Rev. Mod. Phys.*, 72(3), 689–732.
- Nakamura, K., et al. (2010), The review of particle physics, *J. Phys. G*, 37, 075021, pp. 1–1422, doi:10.1088/0954-3899/37/7A/075021.
- Neubert, T., et al. (2008), Recent results from studies of electric discharges in the mesosphere, *Surv. Geophys.*, 29(2), 71–137, doi:10.1007/s10712-008-9043-1.
- Olsen, N. (2007), Natural sources for electromagnetic induction studies, in *Encyclopedia of Geomagnetism and Paleomagnetism*, edited by D. Gubbins and E. Herrero-Bervera, pp. 696–700, Springer, Berlin.
- Pasko, V. P. (2010), Recent advances in theory of transient luminous events, *J. Geophys. Res.*, 115, A00E35, pp. 1–24, doi:10.1029/2009JA014860.
- Pasko, V. P., U. S. Inan, T. F. Bell, and S. C. Reising (1998), Mechanism of ELF radiation from sprites, *Geophys. Res. Lett.*, 25(18), 3493–3496.
- Rakov, V., and M. Uman (2003), *Lightning, Physics and Effects*, Cambridge Univ. Press, Cambridge, U. K.
- Ryutov, D. (2007), Using plasma physics to weigh the photon, *Plasma Phys. Controlled Fusion*, 49, 429–438, doi:10.1088/0741-3335/49/12B/S40.
- Schrödinger, E. (1943), The Earth's and the Sun's permanent magnetic fields in the unitary field theory, *Proc. R. Ir. Acad.*, 49(43), 135–148.
- Sentman, D. D., E. M. Wescott, D. L. Osborne, D. L. Hampton, and M. J. Heavner (1995), Preliminary results from the Sprites94 Aircraft Campaign: 1. Red sprites, *Geophys. Res. Lett.*, 22(10), 1205–1208.
- Spaulding, A. (1995), Atmospheric noise and its effects on telecommunication system performance, in *Handbook of Atmospheric Electrodynamics*, vol. I, edited by H. Volland, pp. 359–396, CRC Press, Boca Raton, Fla.

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