Optimizing two-dimensional vehicle loading and dispatching decisions in freight logistics

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Abstract

This paper introduces a multi-period, two-dimensional vehicle loading and dispatching problem, called Two-Dimensional Vehicle Loading and Dispatching Problem with Incompatibility Constraints (VLDP). The problem concerns preparing a single-origin single-destination transportation plan of loading required orders to vehicles at the origin and dispatching the vehicles to deliver the orders to the destination within their due dates. The decision maker uses their own fleet of vehicles, with each vehicle having a fixed transportation cost per trip, and may outsource additional vehicles at a higher cost. VLDP involves constraints regarding the due dates of the orders, pairwise incompatibility of orders packed in the same vehicle, incompatibility of orders and vehicles, as well as area and weight capacity of the vehicles. An order can be delivered earlier than its due date, incurring an earliness penalty due to storage requirements at the destination. The objective is to minimize the total vehicle usage and earliness penalty costs. A Mixed-Integer Linear Programming model is provided, as well as an Adaptive Large Neighbourhood Search (ALNS) algorithm. Results of computational experiments on instances derived from real-world data show the effectiveness of the ALNS algorithm.

Keywords: logistics, long-haul freight logistics, vehicle loading and dispatching, mixed integer programming model, adaptive large neighborhood search

1. Introduction

Transportation is one of the essential services that supports the sustainability of industrial operations as well as the livelihood of individuals. Many freight transporters provide shipping services with guaranteed delivery within a few days. Majority of freight transportation is performed using land vehicles. Factories, retail stores, gas stations, hospitals, banks, and many other commercial activities rely on shipments by vehicles to receive their supplies and to provide services and goods. Transportation accounts for a significant part of the final cost of products and represents an important component of the national expenditures of any country (Crainic & Laporte, 1997). Bureau of Transportation Statistics of the USA estimated that the total share of truck shipment over all land transportation modes was %63 in the United States in 2017 (Bureau of Transportation Statistics, 2019). Moreover, according to the same source, in 2015 the U.S. transportation system moved a

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daily average of about 49.3 million tons of freight valued at more than $52.5 billion, while tonnage was projected to increase at about 1.4 percent per annum between 2015 and 2045.

Logistics service providers have to offer highly reliable and high-quality services to satisfy their customers (Wieberneit, 2008). In particular, customers place great importance on total delivery time and service reliability. The competitive environment in the freight transportation industry forces the carriers to offer services at a lower price. Planning decisions for delivery networks have a direct impact on customer service and costs. Tactical planning of operations is comprised of a set of interrelated decisions that aim to ensure optimal allocation and utilization of resources to achieve the economic and customer service goals of the company. Tactical planning is particularly important for intercity freight carriers that make intensive use of consolidation operations (Crainic, 2000).

In this study, we focus on a long-haul transportation problem of a freight transportation company. We consider a single line of the provider between a given origin-destination pair. A set of customer orders are to be transported from the origin to the destination by a heterogeneous fleet of vehicles within a planning horizon and can be consolidated in the vehicles for more efficient use of vehicle capacity. We assume that the company has access to a spot market to outsource vehicles at a higher cost then their own fleet. Each order is indivisible, i.e. must be carried within a single vehicle, is prepacked to a known width and length, and orders cannot be stacked upon each other. The packing decisions within each vehicle should obey the weight capacity of the vehicle as well as non-overlapping constraints. Orders that arrive to their destination earlier than their due dates incur inventory storage costs. The service provider needs to decide when and how the orders should be loaded into the vehicles so that the transportation and inventory storage costs are minimized. We call this problem as the *Two-Dimensional Vehicle Loading and Dispatching Problem with Incompatibility Constraints* (VLDP)

VLDP combines decisions from the Two-Dimensional Bin Packing Problem (2DBPP) (Lodi et al. (2002), Wäscher et al. (2007)), as well as multi-period vehicle dispatching problems, while also deciding on the fleet mix in terms of outsourcing vehicles. Furthermore, operational constraints such as incompatibility of order-order and order-vehicle pairs are considered. To the best of our knowledge, only a few studies in the field of logistics simultaneously consider how to load vehicles and when to dispatch them over a planning horizon. Attanasio et al. (2007) study a transportation problem over a multi-day planning horizon, where order consolidation, vehicle dispatching as well as load packing into the vehicles are decisions to be made on a daily basis, as in our problem. The authors develop a two-stage matheuristic that first solves an integer programming model by relaxing the packing constraints and proceeds to find a feasible packing through a heuristic. An exact mathematical model is not provided. A real case study is solved and savings are reported and compared against the current practice.

Another related study is due to Kim & Kim (1999), where the authors studied the problem of dispatching vehicles to serve multiple retailers with a single commodity, one truckload at a time, to minimize the total cost of transportation and inventory holding. The authors modeled the single-retailer version of the problem as a shortest path problem. In subsequent work, Kim & Kim (2000) provided an updated model in which the vehicles could be dispatched multiple times within a period, and presented a Lagrangian relaxation algorithm to solve it. The VLDP differs from these two problems in terms of the two-dimensional packing requirement, and multi-commodity demand structure.

Arda et al. (2014) studied a multi-period vehicle loading problem with stochastic release dates, motivated by a real-life production-transportation problem, and stated that the main question of the company is whether to ship a given item today or wait to ship it together with another item which is expected to be released from production in the near future, with the objective of reducing transportation costs at the risk of incurring delay penalties. Although we study a deterministic problem, we consider a similar trade-off between reducing the transportation costs by more efficient packing and inventory holding costs for the orders that arrive early by controlling the timing of the shipments.

A considerable number of studies focus on solving variants of the VRP jointly with the 2DBPP. Iori et al. (2007) were the first to study the VRP with Two-Dimensional Loading Constraints (2L-CVRP), and provided a branch-and-cut algorithm as well as a constructive heuristic. The authors applied the constructive heuristic multiple times with different parameter values to find high quality solutions. Gendreau et al. (2008) presented a Tabu Search algorithm for the 2L-CVRP and provided computational results for five classes of instances with up to 255 items. To the best of our knowledge, Dominguez et al. (2014) and Dominguez et al. (2016) are the only studies on 2L-CVRP that allow for item rotations for packing. We refer the interested reader to the review by Pollaris et al. (2015) and a more recent paper by Alinaghian et al. (2017) and the references therein for an in-depth exposition to the studies on 2L-CVRP. We note that VLDP aims to serve a single origin-destination. Hence, vehicle routing decisions are not considered.

Our main contributions are a mathematical model and a heuristic solution algorithm that incorporate the joint scheduling, fleet mix, and packing decisions with incompatibility constraints. We develop a Mixed-Integer Linear Programming (MILP) formulation for the VLDP, as well as an Adaptive Large Neighbourhood Search (ALNS) algorithm, and conduct a computational study to observe and compare the performances of the exact model and the heuristic algorithm. We examine the effects of the cost parameters as well as the size of the problem on instances derived from the actual data of a logistics company.

The rest of this paper is organized as follows. The definition of the problem and the associated MILP model are given in Section 2. Our solution approach is explained in Section 4. In Section 5 we explain our method for instance generation and present test results for the MILP model as well as the ALNS algorithm. Section 6 provides concluding remarks and directions for future work.

2. Problem definition and formulation

The problem involves vehicle dispatching and the related packing decisions that occur within the planning horizon T, which consists of a set of consecutive days starting from day 1. The transportation time from the origin to the destination is a specified number of days, $p \geq 1$. The time to travel back is also equal to the same number of days. Hence, a vehicle departing on day t arrives at the destination after p days, comes back to the origin after $2p$ days, when it will be available for use again. Consequently, if a vehicle is loaded and dispatched on a specific day, it would not be available for the next $2p - 1$ days until it arrives back to the origin.

Each order is specified by a list of item types, together with the width, length, and weight of its package, earliest departure date, due date. The nature of the item types force the company to avoid packing incompatible items together, e.g. cleaning chemicals and food. Each order consists of compatible item types, and two orders are incompatible if they contain incompatible item types.

Similarly, certain orders cannot be carried by all the vehicles, e.g. orders containing dairy products require refrigerated vehicles.

The destination is assumed to have enough space to store the orders arriving earlier than their due dates. In case of an order arriving before its due date, the company incurs an earliness penalty due to the cost of owning or renting storage place at the destination. In what follows, we assume a fixed unit penalty cost per day specific to each order. Note that it may be economical for the company to transport the orders earlier than their due dates to use available capacity of the vehicles more effectively.

The problem is to find which items should be loaded into which vehicles and when the vehicles should depart so that the total cost of vehicle usage and earliness is minimized. The dispatching decisions are usually made for a week so this problem is at the tactical level, but it might be solved on a rolling horizon basis every day as new orders arrive. The notation in the formulation is given below.

Sets:

I	Set of orders
K	Set of vehicles
T	Planning horizon (days)
$P^{(I)}$	Set of incompatible order pairs, i.e., (i, j), that cannot be loaded in the same vehicle
$P^{(V)}$	Set of incompatible order-vehicle pairs, i.e., (i, k), where any pair $(i, k) \in P^{(V)}$ specifies that order i cannot be transported by vehicle k

Parameters:

p	One-way transportation time of a vehicle from the origin to the destination
$d_i^{(1)}$	Width of order $i \in I$
$d_i^{(2)}$	Length of order $i \in I$
a_i	Area requirement of order $i \in I$, i.e. $a_i = d_i^{(1)} \times d_i^{(2)}$
w_i	Weight of order $i \in I$
e_i	Earliest departure date of order $i \in I$
d_i	Due date of order $i \in I$
h_i	Earliness penalty cost per day for order $i \in I$
$\bar{d}_k^{(1)}$	Width of vehicle $k \in K$
$\bar{d}_k^{(2)}$	Length of vehicle $k \in K$
$\bar{a}_k$	Available area for vehicle $k \in K$, i.e. $\bar{a}_k = d_k^{(1)} \times d_k^{(2)}$
$\bar{w}_k$	Weight capacity of vehicle k, $k \in K$
f_{kt}	1, if vehicle k is available to be used at the beginning of day t based on the schedule of the vehicle in the previous planning horizon; 0, otherwise, $k \in K, t \in T$
c_k	Cost of using vehicle $k \in K$

Decision variables:

V_{kt}	Binary variable equal to 1, if vehicle k departs at the beginning of day t; 0, otherwise, $k \in K, t \in T$
A_{kt}	Binary variable equal to 1, if vehicle k is available for dispatching at the beginning of day t; 0, otherwise, $k \in K$, $t \in T$

B_{kt}	Binary variable equal to 1, if vehicle k is traveling on day t; 0, otherwise, $k \in K$, $t \in T$
D_{ikt}	Binary variable equal to 1, if order i departs in vehicle k at the beginning of day t; 0, otherwise, $i \in I, k \in K, t \in T$
X_{ikt}	X-coordinate of the left front corner of the position of the order i on vehicle k that departs at the beginning of day t, $i \in I$, $k \in K$, $t \in T$
Y_{ikt}	Y-coordinate of the left front corner of the position of the order i on vehicle k that departs at the beginning of day t, $i \in I$, $k \in K$, $t \in T$
Z_{ikt}	Binary variable equal to 1, if order i on vehicle k that departs at the beginning of day t is rotated 90° clockwise; 0, otherwise, $i \in I$, $k \in K$, $t \in T$
$Q^{(1)}_{ikt}$	Length covered by order i in X-coordinate of vehicle k that departs at the beginning of day t, $i \in I$, $k \in K$, $t \in T$
$Q^{(2)}_{ikt}$	Length covered by order i in Y-coordinate of vehicle k that departs at the beginning of day t, $i \in I$, $k \in K$, $t \in T$
L_{ijkt}	Binary variable equal to 1, if $X_{jkt} < X_{ikt} + Q^{(1)}_{ikt}$ for two orders i and j that are transported by vehicle k on day t, $i, j \in I, j \neq i$, $k \in K$, $t \in T$
R_{ijkt}	Binary variable equal to 1, if $Y_{jkt} < Y_{ikt} + Q^{(2)}_{ikt}$ for two orders i and j that are transported by vehicle k on day t, $i, j \in I, j \neq i$, $k \in K$, $t \in T$

We first provide a Mixed-Integer Nonlinear Programming (MINLP) formulation, which we then linearize.

$$\text{Minimize} \quad \sum_{k\in K}\sum_{t\in T}\left(c_k V_{kt} \quad + \quad \sum_{i\in I} h_i(d_i - t - p) D_{ikt}\right) \tag{1}$$

Subject to

$$\sum_{i\in I} w_i D_{ikt} \leq \bar{w}_k V_{kt}, \qquad \forall k \in K,\ t \in T \tag{2}$$

$$\sum_{i\in I} a_i D_{ikt} \leq \bar{a}_k V_{kt}, \qquad \forall k \in K,\ t \in T \tag{3}$$

$$Q^{(1)}_{ikt} = D_{ikt}\big(d^{(1)}_i(1 - Z_{ikt}) + d^{(2)}_i Z_{ikt}\big), \qquad \forall i \in I, k \in K,\ t \in T \tag{4}$$

$$Q^{(2)}_{ikt} = D_{ikt}\big(d^{(2)}_i(1 - Z_{ikt}) + d^{(1)}_i Z_{ikt}\big), \qquad \forall i \in I, k \in K,\ t \in T \tag{5}$$

$$X_{ikt} + Q^{(1)}_{ikt} \leq \bar{d}^{(1)}_k D_{ikt}, \qquad \forall i \in I, k \in K,\ t \in T \tag{6}$$

$$Y_{ikt} + Q^{(2)}_{ikt} \leq \bar{d}^{(2)}_k D_{ikt}, \qquad \forall i \in I, k \in K,\ t \in T \tag{7}$$

$$X_{ikt} + Q^{(1)}_{ikt} - X_{jkt} \leq \bar{d}^{(1)}_k (L_{ijkt} - D_{ikt} - D_{jkt} + 2), \qquad \forall i, j \in I : i \neq j, k \in K,\ t \in T \tag{8}$$

$$Y_{ikt} + Q^{(2)}_{ikt} - Y_{jkt} \leq \bar{d}^{(2)}_k (R_{ijkt} - D_{ikt} - D_{jkt} + 2), \qquad \forall i, j \in I : i \neq j, k \in K,\ t \in T \tag{9}$$

$$L_{ijkt} + L_{jikt} + R_{ijkt} + R_{jikt} \leq 3\ D_{ikt}, \qquad \forall i, j \in I : i \neq j, k \in K,\ t \in T \tag{10}$$

$$L_{ijkt} + L_{jikt} + R_{ijkt} + R_{jikt} \leq 3\ D_{jkt}, \qquad \forall i, j \in I : i \neq j, k \in K,\ t \in T \tag{11}$$

$$L_{ijkt} + L_{jikt} \leq 2, \qquad \forall i, j \in I : i \neq j, k \in K,\ t \in T \tag{12}$$

$$R_{ijkt} + R_{jikt} \leq 2, \qquad \forall i, j \in I : i \neq j, k \in K,\ t \in T \tag{13}$$

$$A_{kt} \leq f_{kt}, \qquad \forall k \in K,\ t \in T \tag{14}$$

$$V_{kt} \leq A_{kt}, \qquad \forall k \in K,\ t \in T \tag{15}$$

$$A_{kt} + B_{kt} \geq 1 + V_{kt}, \qquad \forall k \in K,\ t \in T \tag{16}$$

$$B_{k,t'} \geq V_{kt}, \qquad \forall k \in K,\ t \in \{1, ..., |T| - p + 1\},\ \min\{|T|, t + 2p - 1\} \geq t' \geq t \tag{17}$$

$$\sum_{t' \in \{t,...,t+2p-1\}} V_{k,t'} \leq 1, \qquad \forall k \in K,\ t \in \{1, ..., |T| - 2p + 1\} \tag{18}$$

$$D_{ikt} \leq V_{kt}, \qquad \forall i \in I,\ k \in K,\ t \in \{e_i, ..., d_i - p\} \tag{19}$$

$$\sum_{t \in \{e_i,...,d_i-p\}} \sum_{k \in K} D_{ikt} = 1, \qquad \forall i \in I \tag{20}$$

$$D_{ikt} + D_{jkt} \leq V_{kt}, \qquad \forall\, (i, j) \in P^{(I)},\ k \in K,\ t \in T \tag{21}$$

$$D_{ikt} = 0, \qquad \forall (i, k) \in P^{(V)}, t \in T \tag{22}$$

$$D_{ikt} = 0, \qquad \forall i \in I, k \in K, t \in T \setminus \{e_i, ..., d_i - p\} \tag{23}$$

$$V_{kt}, A_{kt}, B_{kt} \in \{0, 1\}\,, \qquad \forall k \in K,\ t \in T \tag{24}$$

$$D_{ikt}, Z_{ikt} \in \{0, 1\}\,, \qquad \forall i \in I,\ k \in K,\ t \in T \tag{25}$$

$$R_{ijkt}, L_{ijkt} \in \{0, 1\}\,, \qquad \forall i, j \in I, : i \neq j\ k \in K,\ t \in T \tag{26}$$

$$X_{ikt}, Y_{ikt}, Q^{(1)}_{ikt}, Q^{(2)}_{ikt} \geq 0, \qquad \forall i \in I,\ k \in K,\ t \in T. \tag{27}$$

The objective function (1) minimizes total cost (TC), comprised of the vehicle usage cost for owned and rented vehicles (VC) and the earliness penalty (EC). Constraints (2) and (3) correspond to the weight and area capacity limitations, respectively.

The lengths covered on X- and Y-coordinates of vehicle $k \in K$ by order $i \in I$ depend on whether the order is rotated 90° clockwise (i.e., $Z_{ikt} = 1$), or not (i.e., $Z_{ikt} = 0$) and are calculated through Constraints (4) and (5), respectively. Constraints (4) and (5) are non-linear as they include the product of two binary variables. These constraints can be linearized by introducing a new binary variable $U_{ikt} = D_{ikt} Z_{ikt}$ and including Constraints (28)-(33) instead of Constraints (4) and (5) as follows:

$$U_{ikt} \leq Z_{ikt}, \qquad \forall i \in I, k \in K,\ t \in T \tag{28}$$

$$U_{ikt} \leq D_{ikt}, \qquad \forall i \in I, k \in K,\ t \in T \tag{29}$$

$$U_{ikt} \geq D_{ikt} + Z_{ikt} - 1, \qquad \forall i \in I, k \in K,\ t \in T \tag{30}$$

$$Q^{(1)}_{ikt} = d^{(1)}_i D_{ikt} - U_{ikt}(d^{(1)}_i - d^{(2)}_i), \qquad \forall i \in I, k \in K,\ t \in T \tag{31}$$

$$Q^{(2)}_{ikt} = d^{(1)}_i D_{ikt} - U_{ikt}(d^{(2)}_i - d^{(1)}_i), \qquad \forall i \in I, k \in K,\ t \in T \tag{32}$$

$$U_{ikt} \in \{0, 1\}, \qquad \forall i \in I, k \in K,\ t \in T \tag{33}$$

Constraints (6) and (7) guarantee that the width or length of any vehicle is not exceeded.

Constraints (8) and (9) calculate the values of binary variables L_{ijkt} and R_{ijkt} for two different orders i and j that are transported by vehicle k on day t. Note that, if L_{ijkt} and L_{jikt} are both equal to one, then X-coordinate trajectories of orders i and j overlap. Similarly, if R_{ijkt} and R_{jikt} are both one, then Y-coordinate trajectories of orders i and j overlap. Constraints (10) and (11) guarantee that both X-coordinate and Y-coordinate trajectories, i.e., the placements, of any two orders in the same vehicle do not overlap. These constraints also ensure that the variables L_{ijkt} and R_{ijkt} are set to zero if either i or j is not transported by vehicle k on day t. Through Constraints (10) and (11), for any two orders i and j that are transported by the same shipment, either order i is placed to the left of the order j, or vice versa, for both width and depth dimensions, respectively.

Constraints (14) ensure the availability of vehicles based on their schedules in the previous planning horizon. Constraints (15) guarantee that an unavailable vehicle is not dispatched on a day. Constraints (16) define the relation between the availability, dispatching and traveling variables associated with each vehicle and planning horizon. If a vehicle is dispatched on a day, then it should be available and traveling on that day. If a vehicle is not traveling on a day, then it should be available for dispatching on that day. Finally, if a vehicle is traveling, it should be dispatched on that day or any former day. Constraints (17) and (18) control the travel duration and availability of the vehicles. If vehicle k departs at the beginning of day t it should be traveling and not available until it comes back to the origin after $2p$ days. Note that the vehicles departed at the beginning of day t will be back for use in the depot on day $t+2p$.

Constraints (19) ensure that order i can be loaded in vehicle k if the vehicle is used on day t and day t is in the time window of the order. Constraints (20) guarantee that order i is delivered within its time window.

Constraints (21) to (22) are compatibility constraints. Constraints (21) prevent the incompatible orders to be loaded in the same vehicle at the same time. Constraints (22) prevent the order-vehicle incompatibility. Constraints (23) prevent orders to be dispatched before their earliest departure dates or after their due dates.

Finally, Constraints (24) to (27) enforce integrality and non-negativity on the corresponding decision variables.

2.1. Strengthening the model

The model allows for using heterogeneous vehicles. However, for the special case of homogeneous vehicles, the following symmetry breaking constraints can be included to prevent usage of vehicle k at the beginning of day t, if any of the vehicles with indices from 1 to $k-1$ is not in-use, i.e. not traveling, on that day:

$$V_{k,t} \leq B_{k',t}, \quad \forall k = 2,\ldots,|K|,\ k' = 1,\ldots,k-1,\ t \in T \tag{34}$$

In addition, the weights and areas of the orders can be lifted according to the following procedure based on the lifting approach used in Erdoğan et al. (2010). For each item $i \in I$, the maximum weight of the shipment that obeys the weight capacity of a vehicle and that includes the item is found as $\bar{w}'$ by solving a knapsack problem. If $\bar{w}'$ is less than the smallest weight capacity of the vehicles in the fleet, i.e., $\bar{w}' < \min_{k\in K}\{\bar{w}_k\}$, then the weight of the item is increased by $\min_{k\in K}\{\bar{w}_k\} - \bar{w}'$. The areas of the orders (a_i), can be increased similarly without changing the dimensions of the orders.

2.2. Illustrative example

We now analyze the solution of instance with 11 orders to be transported and six vehicles, three of which are owned and the remaining will be rented in case of use. The planning horizon is one week, i.e. $T = \{1, ..., 7\}$, and the one-way transportation time of a vehicle is set to two days, i.e. $p = 2$. The data about the orders is provided in Table 1. All orders can be transported on any day meeting their due dates, i.e. $a_{it} = 1, \forall i \in I, t \in T$. All vehicles have the same dimensions and the same weight capacity $\bar{d}_k^{(1)} = 10, \bar{d}_k^{(2)} = 2$, and $\bar{w}_k = 2600,\ \forall k \in K$. The cost of using owned and outsourced vehicles are 1 and 2, respectively, i.e., $v_1 = v_2 = v_3 = 1$ and $v_4 = v_5 = v_6 = 2$. Both of the incompatibility sets are empty, i.e., $P(I) = P(V) = \emptyset$.

Table 1: Order data for the illustrative example

Order i	h_i	w_i	$d_i^{(1)}$	$d_i^{(2)}$	e_i	d_i
1	1	313.0	5	1	1	4
2	1	440.0	4	1	1	5
3	1	1414.0	10	1	1	5
4	1	2324.5	10	2	1	3
5	1	2125.5	10	1	1	7
6	1	310.0	5	1	1	5
7	1	405.0	10	1	1	7
8	1	1414.0	10	1	1	5
9	1	2400.0	10	2	1	3
10	1	1203.0	10	1	1	7
11	1	360.0	5	1	1	5

The linearized model provides an optimal solution for this small instance in 1218 CPU seconds (on our computational platform with details given in Section 5). The two dimensional placement of the orders inside the vehicles is illustrated in Fig 1. The shaded areas represent the empty spaces in the vehicles. The optimal solution makes use of all of the three owned vehicles and one additional vehicle is rented on the third day. Orders 3 and order 6 are both delivered one day earlier, leading to a total earliness cost of 2. The cost of using owned and rented vehicles are 5 and 3, respectively. Therefore, the total cost of the optimal solution is 10.

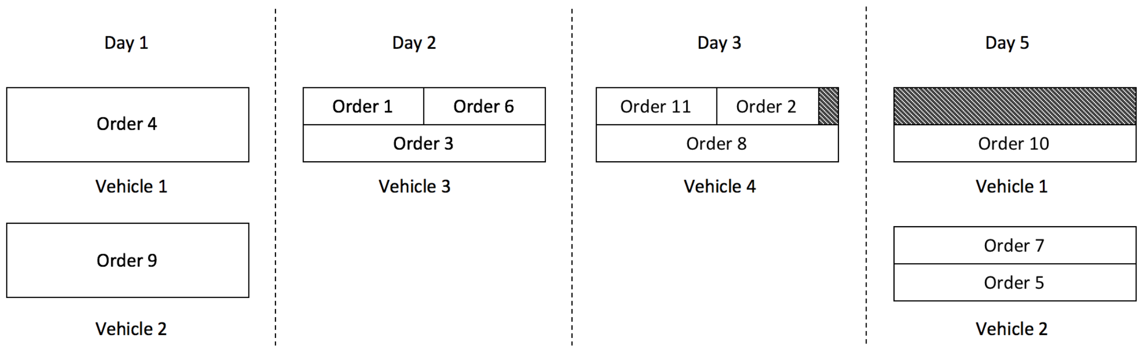


Figure 1: The placement of the orders inside the vehicles that are dispatched in different days for the optimal solution of the illustrative example

3. Extension - Joint Shipping

We next extend the model to handle the cases where multiple orders are preferred but are not obliged to be transported together. To that end, we add soft constraints in the model that penalize the split of the set of orders to be transported together to more than one vehicle and more than one day.

Let $P^{(R)}$ define the set of sets of orders that are preferred to be transported by the same shipment, where each element $I_r \in P^{(R)}$ contains a set of orders. We define four new decision variables: $N_r^{(s)}$ defines the number of distinct shipments of the orders in the set $I_r \in P^{(R)}$, $N_r^{(d)}$ defines the number of distinct shipment days of the orders in the set $I_r \in P^{(R)}$, $W_{rkt}^{(s)}$ is a binary variable indicating whether any order in set $I_r \in P^{(R)}$ is transported by a shipment that is performed by vehicle $k \in K$ on day $t \in T$, and $W_{rt}^{(d)}$ is a binary variable indicating whether any order in set $I_r \in P^{(R)}$ is transported on day $t \in T$.

We include Constraints (35)- (38) to determine the values of these new variables. We write $c_r^{(d)}$ and $c_r^{(s)}$ to denote the penalty associated with each additional distinct shipment day and each additional distinct shipment, respectively, for the orders of the set $I_r \in P^{(R)}$. Finally, we include the term $\sum_{I_r \in P^{(R)}} \left(c_r^{(d)}(N_r^{(d)}-1)+c_r^{(s)}(N_r^{(s)}-N_r^{(d)})\right)$ to the objective function, which penalizes the split of the set of orders to be transported together to more than one shipment and more than one day.

$$\sum_{i \in I_r} D_{ikt} \leq |I_r|\; W_{rkt}^{(s)}, \qquad \forall I_r \in P^{(R)},\; k \in K,\; t \in T \tag{35}$$

$$\sum_{(k,t)\in(K,T)} W_{rkt}^{(s)} \leq N_r^{(s)}, \qquad \forall I_r \in P^{(R)} \tag{36}$$

$$\sum_{i \in I_r}\sum_{k \in K} D_{ikt} \leq |I_r|\; W_{rt}^{(d)}, \quad \forall I_r \in P^{(R)},\; t \in T \tag{37}$$

$$\sum_{t=1}^{T} W_{rt}^{(d)} \leq N_r^{(d)}, \qquad \forall I_r \in P^{(R)} \tag{38}$$

4. Solution approach

We developed a constructive heuristic to obtain an initial solution and an ALNS heuristic to improve the initial solution. Detailed descriptions of the constructive heuristic and the ALNS algorithm are provided in Sections 4.1 and 4.2.

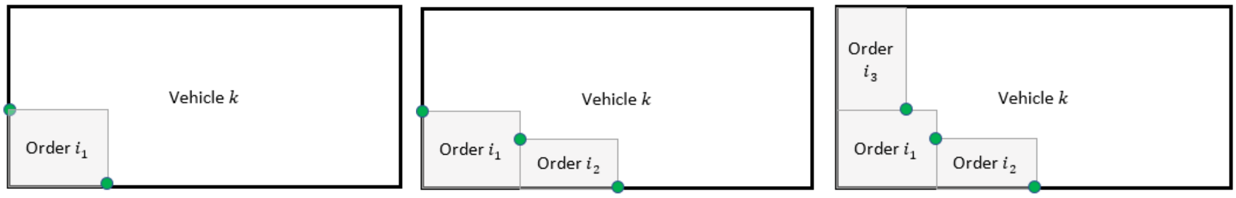
4.1. Constructive heuristic

The constructive heuristic (CH) starts with sorting the set of orders firstly in non-decreasing order of due dates (d_i), secondly in non-increasing order of weight (w_i), and thirdly in non-increasing order of area (a_i). That is, we aim to prioritize the orders that have closer due dates, greater weight, and larger area. Then, starting from the first order in the ordered list of orders, the algorithm evaluates the cost of two alternatives, transporting the order with an existing shipment or with a new shipment.

For the first alternative, existing shipments are evaluated based on their compatibility and the additional cost that would be incurred if the order is transported by that shipment, and incompatible shipments are disregarded. As the shipment cost has already been incurred for an existing shipment, the only additional cost that can be incurred is earliness cost.

In addition, for each compatible shipment, the placement alternatives are evaluated. The orders are placed sequentially and each order is located next to the already placed orders. Initially, the vehicle of the shipment is empty and the first order is placed at (0,0) coordinate of the vehicle that corresponds to the front left corner. After each placement, two new potential placement points become available for the next order in the list of orders. For instance, if order i_1 is the first order to be placed (without any rotation) then the set of potential placement points includes points $(d_{i_1}^{(1)}, 0)$ and $(0, d_{i_1}^{(2)})$ after its placement. Figure 2a illustrates the potential placement points after the placement of order i_1 by green circles. Figure 2b and 2c illustrate the potential placement points after the placement of the next two orders. If the coordinates of a potential placement point does not allow for any item to be placed, e.g. point $(\bar{d}_k^{(1)}, 0)$, then this point is discarded.

For the second order to be placed to the same shipment, these two points should be evaluated regarding placement feasibility, i.e. overlap with previously placed orders and the boundaries of the vehicle, with and without 90° rotation. A feasible potential point with the corresponding rotation information that leads to the smallest number of potential points is selected as the placement point for that order and that shipment. After calculating the additional cost and the number of potential points after the placement for each compatible existing shipment, the shipment with the smallest additional cost is selected as the first alternative. In case of a tie, the number of potential placement points is used as the tie-breaker.



(a) after placement of order i_1 **(b)** after placement of order i_2 **(c)** after placement of order i_3

Figure 2: Potential placement points (green circles) for a shipment performed by vehicle k

For the second alternative, the set of owned vehicles that are available on the latest possible departure date, the day before the latest possible departure day, up to the earliest possible departure day of the order are identified together with the associated additional earliness costs. Another option for the second alternative is an outsourced vehicle that departs on the latest possible departure date for the order with. The second alternative uses the available vehicle that incurs the smallest additional cost.

If there is no compatible existing shipment for an order, then the second alternative, i.e., new shipment with either an owned or an outsourced vehicle is selected. Otherwise, the alternative with the smaller cost is selected. Then, the order is placed in the potential placement point that is feasible and results in the smallest number of potential placement points after placement. The availability of the vehicle of the shipment during the two-trip is updated accordingly.

We note that the constructive heuristic can always find a feasible solution through outsourcing vehicles. The pseudocodes of the constructive algorithm and the placement of an order to a shipment are provided in Algorithms 1 and 2, respectively.

4.2. Adaptive Large Neighborhood Search (ALNS) algorithm

The initial solution obtained by the constructive heuristic presented in Section 4.1 is aimed to be improved through an ALNS algorithm. In every ALNS iteration, a destroy heuristic h^{destroy} and a repair heuristic h^{repair} are chosen based on their performance (computed as scores and weights) in previous iterations. Using the chosen pair, first, the orders are removed from shipments of the current solution s^{current} using the chosen destroy operator h^{destroy} and moved into the pool of removed orders. Then, using the selected repair operator h^{repair}, removed orders are reloaded into the shipments. If resulting solution s' meets the acceptance criteria, it replaces s^{current}. In case it is better than the best solution found so far, it replaces the best found solution s^{best}. Then, scores and weights of $(h^{\text{destroy}}, h^{\text{repair}})$ are updated and the search proceeds with the next iteration. This is repeated until the stopping criterion is met. The pseudocode of the ALNS algorithm is provided in Algorithm 3.

Algorithm 1 Initial Solution Construction

Input: I: set of orders, K: set of vehicles, T: number of time periods, p: one-way transportation time from origin to destination, all parameters for orders and vehicles

$I^{(o)} \leftarrow$ List of orders I sorted in first in non-decreasing order of d_i, second in non-increasing order of w_i, third in non-increasing order of a_i

$S \leftarrow \emptyset$ // List of shipments

for $t = 1, \ldots, T$ **do** // Initialize vehicles

  $K_t \leftarrow K$ // initially, all vehicles are available each day

end for

while $I^{(o)} \neq \emptyset$ **do**

  $i \leftarrow$ pop the first order from $I^{(o)}$

  $(\beta, \Delta c^{(\beta)}) \leftarrow$ **FindBestExistingShipmentAndCost**(S, i) // (shipment, additional cost) pair that result in the smallest additional total (earliness + split) cost that can transport i (in case of tie, the shipment that result with the smallest number of potential placement points is selected)

  $(k, \Delta c^{(k)}) \leftarrow$ **FindBestAvailableVehicleAndCost**$(K_{d_i - p}, i)$ // (vehicle, additional cost) pair that result in the smallest additional shipment cost that can transport i on day $d_i - p$, i.e., the latest possible day

  if $\Delta c^{(\beta)} \leq \Delta c^{(k)}$ **then** // use the existing shipment

    PackOrder(i, β)

  else// a new shipment

    $\beta^{(\text{new})} \leftarrow$ Create an empty shipment by vehicle k departing on day $d_i - p$

    PackOrder$(i, \beta^{(\text{new})})$

    $S \leftarrow S \cup \{\beta^{(\text{new})}\}$

    for each $t = d_i - p, \cdots, d_i + p - 1$ **do**

      $K_t \leftarrow K_t \setminus \{k\}$ // the vehicle will not be available during the two-way trip

    end for

  end if

end while

Output: S: Set of shipments

Algorithm 2 PackOrder(i, β)

Input: i: order, β: shipment, P_β: available location points of shipment β

$(p^{(\text{best})}, \text{rotation (yes or no)}) \leftarrow$ The best point $p(x, y) \in P_\beta$, rotation information pair that results in the smallest number of potential placement points after loading

Place order i at point $p^{(\text{best})}$ with or without rotation as specified

Update potential placement points of shipment β, P_β

Output: β: shipment with order i loaded

In the following subsections, the different destroy and repair heuristics and the acceptance criteria employed, and how a destroy–repair heuristic pair is chosen are described in further detail.

4.2.1. Destroy heuristics

Several different destroy heuristics have been employed in the ALNS literature. In the following, we briefly describe the ones employed in the proposed ALNS algorithm. Before a destroy heuristic is initiated, the degree of destruction, denoted by δ, has to be set. Depending on the destroy heuristic, the degree of destruction may specify the number of orders to be removed from the shipments or the number of shipments to be deleted. As in Pisinger & Ropke (2007), in every iteration it is chosen randomly between $\delta^{(\min)}$ and $\delta^{(\max)}$. Random shipment removal, type I and type II, worst

Algorithm 3 ALNS

Input: s: initial solution
$s^{\text{best}} \leftarrow s$
$s^{\text{current}} \leftarrow s$
$iter \leftarrow 1$
while stopping criteria is not met **do**
  if $iter \mod n^{(\text{adjust})} = 0$ **then** // once in $n^{(\text{adjust})}$ iterations
    Recompute weight $w(h)$ of each heuristic
  end if
  if $iter \mod n^{(\text{reloc})} = 0$ **then** // once in $n^{(\text{reloc})}$ iterations
    Perform shipment relocation for each shipment, separately
  end if
  if $iter \mod n^{(\text{ls})} = 0$ **then** // once in $n^{(\text{ls})}$ iterations
    $s^{\text{current}} \leftarrow LocalSearch(s^{\text{current}})$
  end if
  Choose a destroy heuristic h^{destroy} and a repair heuristic h^{repair} based on their weights
  $s' \leftarrow h^{\text{repair}}(h^{\text{destroy}}(s^{\text{current}}))$
  $u(h^{\text{repair}}) \leftarrow u(h^{\text{repair}}) + 1$
  $u(h^{\text{destroy}}) \leftarrow u(h^{\text{destroy}}) + 1$
  if $\text{Accept}(s', s^{\text{current}})$ **then**
    if $c(s') < c(s^{\text{best}})$ **then**
      $s^{\text{best}} \leftarrow s'$
      $s(h^{\text{destroy}}) \leftarrow s(h^{\text{destroy}}) + \alpha_1$
      $s(h^{\text{repair}}) \leftarrow s(h^{\text{repair}}) + \alpha_1$
    else if $c(s') < c(s^{\text{current}})$ **then**
      $s(h^{\text{destroy}}) \leftarrow s(h^{\text{destroy}}) + \alpha_2$
      $s(h^{\text{repair}}) \leftarrow s(h^{\text{repair}}) + \alpha_2$
    else
      $s(h^{\text{destroy}}) \leftarrow s(h^{\text{destroy}}) + \alpha_3$
      $s(h^{\text{repair}}) \leftarrow s(h^{\text{repair}}) + \alpha_3$
    end if
    $s^{\text{current}} \leftarrow s'$
  end if
  $iter \leftarrow iter + 1$
end while
Output: s^{best}: best found solution

shipment removal, random order removal, and related order removal are problem specific versions of the random, worst, and related removal heuristics proposed by Ropke & Pisinger (2006b) and Pisinger & Ropke (2007). Let $I^{(r)}$ denote the set of removed orders after the selected removal heuristic is applied.

Random shipment removal, type I (RSR-I): Randomly selects a shipment, removes its orders starting from the last order loaded and moves to the set of removed orders until δ percentage of the orders have been removed. If the selected shipment does not achieve δ, then another shipment is selected randomly and this is repeated until δ percentage of the orders have been removed.

Random shipment removal, type II (RSR-II): Randomly selects a shipment and removes all of its orders and moves to the set of removed orders. This process is repeated until δ percentage of the shipments have been removed.

Worst shipment removal (WSR): Calculates a score for each shipment as a difference of the normalized utilization and the normalized shipment cost. The normalized utilization is calculated based on the maximum of the weight and volume utilization. The shipment with smallest score is identified as the worst shipment. Then, all of the orders of the worst shipment are removed and moved to the set of removed orders. This process is repeated until δ percentage of the orders have been removed.

Random order removal (RaOR): Randomly selects an order and removes the orders of its shipment starting from the last order loaded until the selected order is removed. This is repeated until δ percentage of the orders have been removed.

Related order removal (ReOR): Randomly selects a set I_r, which specifies a set of orders preferred to be transported together, from the set $P^{(R)}$ and removes all orders in the set I_r from the corresponding shipments. When an order $i \in I_r$ is being removed from its shipment, the removal starts from the last order loaded to that shipment and continues until i is removed. This is repeated until δ percentage of the orders have been removed.

4.2.2. Repair heuristics

In order to repair a partially destroyed solution, several repair heuristics are utilized in the literature. All repair heuristics employed in the proposed ALNS try to reload as many removed orders as possible to any shipment. The random repair heuristic is included for diversification purposes. In all repair heuristics, first, time window feasibility and compatibility is checked.

Greedy insertion (GI): Applies the construction heuristic for the removed orders on the resulting partial solution.

Random insertion (RanI): Applies the construction heuristic for the removed orders on the resulting partial solution, with two modifications. Instead of finding the best existing shipment based on the additional cost criterion in line 8 of Algorithm 1, all compatible existing shipments are considered as a possible shipment. A shipment is selected among all compatible existing shipment alternatives and the new shipment alternative based on a roulette-wheel selection procedure that allocates each alternative an angle proportional to its additional cost in the roulette.

Related insertion (RelI): Applies the construction heuristic for the removed orders on the resulting partial solution, where related orders are inserted first.

Regret insertions (RegI): Regret heuristics are based on the greedy heuristic enhanced with a look-ahead feature (Potvin & Rousseau, 1993). Following Pisinger & Ropke (2007), the regret value for each removed order is calculated as follows. Let $\Delta c_k(i)$ denote the change in the total cost for loading order i at its best location in its k^{th}-cheapest shipment. We note that new shipment alternative is also considered in determining the k-cheapest shipment. In the basic version where $n = 2$, referred to as *ReI-2*, the order to be loaded next is selected such that the cost difference between loading it into its best shipment and its second best shipment is largest. In its generalized version, referred to as *ReI-$n^{(regret)}$*, where $n^{(\text{regret})}$ denotes the number of shipments to be considered, i.e. the cheapest until the $n^{(\text{regret})}$-cheapest one. Then, in each iteration, depending on the regret heuristic used, i.e., the value of $n^{(\text{regret})}$, the regret heuristic chooses the next order i^* to be shipped as follows:

$$i^* = \underset{i \in I^{(r)}}{\operatorname{argmax}} \left\{ \sum_{k=2}^{\min\{n^{(\text{regret})}, m_i\}} \left(\Delta c_k(i) - \Delta c_1(i) \right) \right\} \tag{39}$$

where m_i denotes the number of compatible shipments for order $i \in I^{(r)}$ including the new shipment alternative.

We note that the greedy heuristic and the regret heuristics are problem specific versions of the corresponding heuristics of Ropke & Pisinger (2006a,b); Pisinger & Ropke (2007). We also included a problem specific related insertion heuristic.

4.2.3. Periodic shipment relocation

Once in every $n^{(\text{reloc})}$ iterations, the following three-step relocation process is applied to each shipment separately. First, all the orders of the shipment are removed. Second, the orders are sorted in non-increasing order of their area, where orders having the same area are grouped according their width and length Third, *GI* is employed to load the orders into the same shipment in the order identified in the second step.

4.2.4. Local Search

Once in every $n^{(\text{ls})}$ iterations, a local search procedure is applied to the current solution. It includes two operations. In the first operation, all orders are traversed to check if replacing the order to another shipment improves the objective function. In the second operation, all possible order pairs are traversed to check if swapping their shipments results in a better solution.

4.2.5. Acceptance criteria

We employ the acceptance criteria of the simulated annealing framework of Kirkpatrick et al. (1983). That is, a solution s' is always accepted if it is better than s^{current}, i.e. if $c(s') < c(s^{\text{current}})$. In case s' is worse than s^{current}, s' replaces s^{current} with a probability of $e^{-(c(s')-c(s^{\text{current}}))/\mathcal{T}}$, where $\mathcal{T}$ denotes the current temperature. As in Ropke & Pisinger (2006b), $\mathcal{T}$ is initialized at the beginning of the search based on the total cost of the initial solution. The initial temperature is set to a value that ensures a solution with $(1+\omega)\ c(s)$ (that is, a solution having an objective function value of $(1+\omega)$ times the initial objective value) is accepted with probability 0.5. This enables controlling the level of diversification. At the end of each iteration *iter*, the temperature is updated as $\mathcal{T}_{iter} = \gamma\ \mathcal{T}_{iter-1}$, where γ is the cooling rate.

4.2.6. Weight adjustment for destroy and repair heuristics

As in Ropke & Pisinger (2006a,b), each destroy or repair heuristic h has a score $s(h)$ and a weight $w(h)$, and the weight of the heuristic is adjusted once in every $n^{(\text{adjust})}$ iterations. The score of each heuristic is initialized to zero in the beginning of each $n^{(\text{adjust})}$ iterations. Let $u(h)$ denote the number of times heuristic h is used in the last $n^{(\text{adjust})}$ iterations. After a heuristic h is applied in an iteration resulting in a solution s', $s(h)$ is increased by $\Delta s(c(s'), c(s^{\text{current}}), c(s^{\text{best}}))$, which is calculated as:

$$\Delta s(c(s'), c(s^{\text{current}}), c(s^{\text{best}})) = \begin{cases} \alpha_1, & \text{if } c(s') < c(s^{\text{best}}) \\ \alpha_2, & \text{if } c(s^{\text{best}}) \le c(s') < c(s^{\text{current}}) \\ \alpha_3, & \text{if } c(s') \ge c(s^{\text{current}}), \text{ but it is accepted} \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

To obtain a reasonable adjustment, we ensure the inequality $\alpha_1 > \alpha_2 > \alpha_3$ in our computational experiments. The weights are adjusted as:

$$w(h) = \begin{cases} (1-\rho)w(h) + \rho\, s(h)/u(h), & \text{if } u(h) > 0 \\ (1-\rho)w(h), & \text{if } u(h) = 0 \end{cases} \tag{41}$$

where the reaction factor $0 \leq \rho \leq$ controls the influence of historical and new information on the weights. The two end points $\rho = 0$ and $\rho = 1$ correspond to the cases where the weights remain constant at their initial level and only the recent success is considered, respectively.

Let $\mathcal{D}$ and $\mathcal{R}$ denote the sets of destroy and repair heuristics, respectively. The selection probability of a destroy heuristic h is $p(h) = w(h)/\sum_{h' \in \mathcal{D}} w(h')$ and the selection probability of a repair heuristic h is $p(h) = w(h)/\sum_{h' \in \mathcal{R}} w(h')$. The initial weights of the heuristics in D and R are $1/|D|$ and $1/|R|$, respectively.

4.2.7. Stopping criteria

The maximum number of iterations $n^{(\text{maxIter})}$ and the maximum number of non-improving iterations $n^{(\text{maxNonImpIter})}$ are both employed as the stopping criteria.

5. Computational Experiments

We evaluated the effectiveness of the proposed ALNS algorithm through computational experiments performed on randomly generated data sets that are based on data obtained from a freight logistics company. The method of generation and the resulting data sets are described in Section 5.1.

In this section, we first analyze the performance of the proposed framework compared to the mathematical model on small to medium size instances in Section 5.2. Second, we analyze the results of large-sized realistic instances in Section 5.3. Then, we analyze the effects of instance properties such as the diversity of the dimensions and areas of orders and the diversity of the due dates of the orders on the solutions in Sections 5.4 and 5.5, respectively. Finally, we analyze the effects of penalties associated with disjoint shipping of related orders in Section 5.6.

We solved the proposed mathematical model using CPLEX 20.1 with a run time limit of 12 h. We implemented the proposed ALNS using C#, and conducted computational experiments on a computer with a 2.3 GHz CPU and 64 GB of RAM. We performed 30 runs for the ALNS. The average results and the best found solution over the 30 runs are reported.

After an ad-hoc parameter tuning process performed on small-size instances, the parameters of the ALNS algorithm are set as provided in Table 2.

5.1. Data

Based on the data obtained from a freight transportation company that operates two lines, one of which has one-way transportation time of one day ($p = 1$) and the other with 2 ($p = 2$), we generated a set of benchmark data instances of three different sizes: small-sized (including 15 or 18 orders), medium-sized (20, 25, or 30 orders) and large-sized (40, 50, or 60 orders). The planning horizon is considered as seven days ($|T| = 7$). Order related parameters are generated as described in Table 3 based on an analysis of the real data. We randomly selected some pairs of orders as related orders preferred to be transported by the same shipment, where the ratio of the number of related pair of orders to the number of orders varies between 0 and 0.32 in all instances as in the real data. The penalty associated with each additional distinct shipment day and each additional distinct shipment for related orders are set to $c_r^{(d)} = 0.2$ and $c_r^{(s)} = 0.1$, respectively.

Table 2: Parameter values of the proposed ALNS

Parameter	Value
maximum no. of iterations, $n^{(\text{maxIter})}$	1000
maximum no. of non-improving iterations, $n^{(\text{maxNonImpIter})}$	300
minimum degree of destruction, $\delta^{(min)}$	0.5
maximum degree of destruction, $\delta^{(max)}$	1
shipment relocation period, in terms of no. of iterations, $n^{(\text{reloc})}$	15
local search period, in terms of no. of iterations, $n^{(\text{ls})}$	10
weight adjustment period, in terms of no. of iterations, $n^{(\text{adjust})}$	10
reaction factor in weight adjustment, ρ	0.6
score increase for the best-found improving solution, α_1	70
score increase for the current improving solution, α_2	50
score increase for the non-improving accepted solution, α_3	20
initial temperature determination parameter, ω	0.9
cooling rate, γ	0.99

Table 3: The values of order-related parameters in original instances (belonging to group G1)

Parameter	Value
width ($d_i^{(1)}$)	discrete, uniformly random in $[1, 5]$
length ($d_i^{(2)}$)	discrete, uniformly random in $[1, 20]$
weight (w_i)	discrete, uniformly random in $[100, 1000]$
due date (d_i)	discrete, uniformly random in $[p, \|T\|]$
earliest departure dates (e_i)	discrete, uniformly random in $[1, d_i - p + 1]$
earliness penalty costs per day (h_i)	discrete, uniformly random in $[1, 5]$

The vehicle fleet is assumed to be comprised of vehicles owned by the freight logistics company and rented vehicles, where the number of owned and rented vehicles are set to $\lceil |I|/\nu \rceil$ and and $\lceil |I|/\nu \rceil + 1$, respectively, i.e., $|K| = 2\lceil |I|/\nu \rceil + 1$. Based on an analysis of the real data, ν is set to 5. The unit cost of using an owned or a rented vehicle is \$50 or \$80, respectively. There are two types of owned or rented vehicles regarding their sizes and weight capacities: one type has a width of 5 units ($\bar{d}_k^{(1)} = 5$), a length of 25 units ($\bar{d}_k^{(2)} = 25$), and a weight capacity of 7500 units ($\bar{w}_k = 7500$), and the other has a width of 5 units ($\bar{d}_k^{(1)} = 5$), a length of 30 units ($\bar{d}_k^{(2)} = 30$), and a weight capacity of 9000 units ($\bar{w}_k = 9000$). The available day of an owned vehicle during the current planning horizon based on its delivery schedule in the previous planning horizon is determined as follows. Starting from the first day of the planning horizon, f_{kt} is set to 1 with probability 0.5. When $f_{kt'}$ is set to 1 for some $t' \in T$, then $f_{k,t'}$ is set to 0 for all $1 \leq t' < t$ and $f_{k,t'}$ is set to 1 for all $t < t' \leq |T|$.

The instances generated based on the above setting are considered as original instances and classified under group G1. Five additional groups of large-size data instances (Groups G2 to G6) are generated with controlled variety on sizes or due dates of orders in order to investigate the effect of the variety of sizes and the variety of due dates on the results. The average and standard deviation of the width, length, weight, and due date of orders of each group are provided in Table 4.

Table 4: Mean and standard deviation of the width, length, weight, and due date of orders of each group of large-size instances.

Instance Group	Characteristics*		width ($d_i^{(1)}$)		length ($d_i^{(2)}$)		weight (w_i)		due date (d_i)	
	DWV	DDV	μ	σ	μ	σ	μ	σ	μ	σ
G1	H	H	2.89	1.45	10.65	5.74	571.17	247.53	4.09	1.59
G2	L	H	3.04	0.82	10.46	2.85	546.26	151.58	4.87	1.58
G3	No	H	2.00	0.00	8.00	0.00	550.00	0.00	4.69	1.62
G4	M	H	3.03	0.80	10.86	4.77	538.66	227.76	4.84	1.53
G5	H	L	2.99	1.38	10.60	5.74	552.10	258.11	4.55	0.73
G6	H	M	2.94	1.38	10.50	5.67	541.68	257.93	4.66	1.46

* : DWV: Dimension and Weight Variety, DDV: Due Date Variety, H: High, M: Medium, L: Low.

All of the small- and medium-size instances belong to group G1 and there are 10 instances having the same number of orders in these sets. For large-size instances, there are 3 instances having the same number of orders that belong to the same group.

The name of each instance indicates five parts: "O", "K", "p", and "G" indicate the number of orders, number of owned and outsourced vehicles, one-way transportation time, and group of the instance, respectively. For example, the instance named O40K17p2G3_1 has 40 orders that satisfy properties of group G3 and 17 vehicles with one-way transportation time of one day and this is the first instance among others with the same properties.

The generated data is publicly available at Yücel et al. (2021).

5.2. Performance of the ALNS

In this section, we evaluate the performance of the proposed ALNS by comparison with the MILP formulation in Section 2 for small- and medium-size instances. The results for small- and medium-size instances are reported in Tables 5 and 6, respectively. In Tables 5 and 6, for each instance, the objective function values (i.e., TC) and the run times (in seconds) of the MIP and ALNS approaches are reported. In addition, the mean and standard deviation of the objective function values of the ALNS replications are reported under columns μ and σ, respectively. For all small-size instances, the MIP and ALNS find the optimal solution, where the run time of the ALNS is considerably shorter than that of the MIP. The MIP finds the optimal solution for only seven of the thirty medium-size instances (O20K9p2G1_1, O20K9p2G1_2, O20K9p2G1_3, O20K9p2G1_5, O20K9p2G1_9, O20K9p2G1_10, and O25K9p2G1_2) within the run time limit of 12 h. For medium-size instances, the percentage of improvement in the objective value provided by the ALNS compared to the MIP is reported under column *Diff (%)* in Table 6. As the results in the table shows, for all medium-size instances the ALNS provides an incumbent solution at least as good as that of the MIP. The incumbent solutions of the ALNS are on the average 4.57% better than those of the MIP in terms of total cost. We also note that for the medium-size instances that the MIP and ALNS cannot find the optimal solution, no improvement in TC is achieved through warmstarting the MIP with the incumbent solution of the ALNS for a run time of 1 h.

5.3. Analysis of the large-sized instances

In this section, we analyze the results of the constructive heuristic and ALNS for the original instances (G1). For each original instance, Table 7 reports the objective function values (i.e., TC) for the CH and ALNS. The run time of the CH is less than 1 second for all instances; therefore, the table

Table 5: Comparative results on small-size instances.

Instance	TC				Time(s)	
	MIP	ALNS	μ	σ	MIP	ALNS
O15K7p2G1_1	295.4	295.4	297.8	2.1	128	37
O15K7p2G1_2	175.2	175.2	176.7	2.5	125	109
O15K7p2G1_3	359.0	359.0	362.0	3.2	31	33
O15K7p2G1_4	287.0	287.0	287.8	2.5	53	34
O15K7p2G1_5	216.4	216.4	218.7	2.2	49	43
O15K7p2G1_6	236.2	236.2	238.3	1.9	59	44
O15K7p2G1_7	278.0	278.0	278.0	0.0	169	25
O15K7p2G1_8	214.2	214.2	216.0	1.4	285	34
O15K7p2G1_9	275.2	275.2	276.0	2.6	63	32
O15K7p2G1_10	223.2	223.2	226.0	3.8	53	41
O18K8p2G1_1	219.2	219.2	220.7	2.1	74	65
O18K8p2G1_2	314.0	314.0	315.9	2.4	4934	93
O18K8p2G1_3	325.3	325.3	264.8	1.8	355	39
O18K8p2G1_4	263.2	263.2	266.5	4.0	117	38
O18K8p2G1_5	274.2	274.2	276.0	2.1	92	52
O18K8p2G1_6	215.4	215.4	216.3	1.2	190	100
O18K8p2G1_7	210.0	210.0	214.7	3.7	101	52
O18K8p2G1_8	247.2	247.2	248.9	2.3	99	95
O18K8p2G1_9	262.4	262.4	267.2	3.5	104	224
O18K8p2G1_10	321.2	321.2	323.1	3.1	10928	222
				avg.	900.42	98.1

reports only the run times of the ALNS. In addition, the number of owned shipments, the number of total shipments, and the minimum, maximum and average area utilization of the shipments of the ALNS solutions are provided in the table. As the weight utilization of each shipment is less than or equal to the area utilization for each instance, the weight utilization values are not provided.

According to the results provided in Table 7, the ALNS results in 19.85 % on the average (9.32 % minimum and 33.84% maximum) improvement on the CH solution. The area utilization of the shipments of ALNS solutions ranges between 48 % and 100 %. The average area utilization of all shipments of all ALNS solutions is 83.89 %. In 10 of 15 original instances, owned vehicles are used for all shipments.

5.4. Analysis on the effect of diversity of the order dimensions and weights

Next, we analyze the effect of diversity of the order dimensions and weights on the ALNS solutions. As described in Section 5.1, considering that the original instance group has high diversity in dimensions and weights, the group G4, G2, and G3 correspond to the instances having medium, low, and no diversity, respectively. The detailed results for each instance in each group having order sizes of 40, 50, and 60 are provided in Appendix in Tables .11, .12, and .13, respectively. Table 8 reports the average results of each group in terms of TC (of the solutions of the CH and ALNS and the percentage improvement of the ALNS), and run time (in seconds), area utilization of shipments and number of shipments of the ALNS solutions. As in Section 5.3, the weight utilization values are not provided since the weight utilization of any shipment is not larger than the area utilization for any instance.

Table 6: Comparative results of medium-size instances.

Instance	TC				Diff (%)	Time(s)	
	MIP	ALNS	μ	σ		MIP	ALNS
O20K9p2G1_1	231.0	231.0	238.7	5.6	0.0	834	723
O20K9p2G1_2	311.2	311.2	316.2	8.1	0.0	23965	227
O20K9p2G1_3	355.4	355.4	362.1	7.3	0.0	381	247
O20K9p2G1_4	328.4	328.4	332.8	4.8	0.0	43200	275
O20K9p2G1_5	171.0	171.0	183.5	7.3	0.0	463	347
O20K9p2G1_6	259.1	259.1	259.3	0.4	0.0	43200	309
O20K9p2G1_7	419.4	419.4	419.5	0.4	0.0	43200	337
O20K9p2G1_8	451.4	451.4	453.2	2.0	0.0	43200	357
O20K9p2G1_9	213.0	213.0	219.0	6.1	0.0	1714	322
O20K9p2G1_10	304.0	304.0	309.5	3.6	0.0	342	252
O25K10p2G1_1	392.8	392.8	403.8	6.1	0.0	43200	740
O25K10p2G1_2	305.0	305.0	316.0	7.6	6.4	504	747
O25K10p2G1_3	411.4	367.5	377.5	9.7	10.7	43200	645
O25K10p2G1_4	386.4	368.3	376.1	11.6	4.7	43200	567
O25K10p2G1_5	404.5	404.5	418.7	11.7	0.0	43200	574
O25K10p2G1_6	405.4	390.8	394.2	3.2	3.6	43200	476
O25K10p2G1_7	315.3	315.3	320.7	3.3	0.0	43200	678
O25K10p2G1_8	392.0	392.0	394.0	3.3	0.0	43200	592
O25K10p2G1_9	352.2	325.2	339.5	11.1	7.7	43200	572
O25K10p2G1_10	574.2	572.4	574.2	2.8	0.3	43200	674
O30K13p2G1_1	562.3	493.3	499.1	8.1	12.3	43200	955
O30K13p2G1_2	319.4	273.5	286.1	7.3	14.4	43200	1103
O30K13p2G1_3	480.2	417.2	421.6	10.1	13.1	43200	871
O30K13p2G1_4	776.3	682.5	686.8	5.5	12.1	43200	6987
O30K13p2G1_5	438.4	359.4	364.8	5.0	18.0	43200	818
O30K13p2G1_6	640.6	564.9	574.5	6.4	11.8	43200	3630
O30K13p2G1_7	414	413.0	426.1	10.9	0.2	43200	859
O30K13p2G1_8	354.6	312.6	317.3	8.6	11.8	43200	1060
O30K13p2G1_9	574	537.0	542.2	5.64	6.4	43200	867
O30K13p2G1_10	736.9	663.9	666.0	11.8	9.9	43200	3698
				avg.	4.57	34060	1019

According to the results presented in Table 8, as the variety in dimension decreases the area utilization generally diminishes. To compare the unused area capacity over all shipments for varying DWV levels and order sizes, we introduce a new metric called Unused Area Capacity (UAC) to be calculated for each solution. The UAC of a solution is calculated as the product of average unused area capacity over all shipments (in percentage) and the average number of shipments. The Figure 3 presents a chart that shows the average UAC values of the solutions of each group of instances having different DWV levels and order sizes. As the chart demonstrates, the UAC is lower when DWV is high and higher when DWV is low to medium.

5.5. Analysis on the effects of diversity of the order due dates

Next, we analyze the effect of diversity of the order due dates on the ALNS solutions. As described in Section 5.1, considering that the original instance group has high diversity in dimensions and weights, the group G3 and G6 correspond to the instances having low and medium due date

Table 7: Results of large-sized instances.

Instance	TC			Time(s)	Area Utilization			#Owned /#Shipments
	CH	ALNS	Imp.(%)		min.	max.	avg.	
O40K17p1G1_1	852.4	701.4	21.53	1311	75.2	100.0	90.90	6/6
O40K17p2G1_2	905.2	756.3	19.69	1062	60.0	100.0	87.88	8/10
O40K17p2G1_3	643.7	520.7	23.62	1706	48.0	100.0	84.48	8/8
O40K17p2G1_4	727.3	543.4	33.84	1598	77.6	100.0	89.92	8/8
O40K17p2G1_5	843.3	704.2	19.75	1327	64.0	99.2	85.73	8/11
O50K21p1G1_1	927.5	787.5	17.78	1719	54.4	98.4	86.56	9/9
O50K21p2G1_2	859.6	759.8	13.14	1894	65.6	97.6	83.89	10/11
O50K21p2G1_3	924.8	733.8	26.03	2365	60.8	100.0	87.20	10/10
O50K21p1G1_4	739.4	674.4	9.64	2575	60.8	100.0	84.74	5/5
O50K21p1G1_5	806.4	669.3	20.48	2046	72.0	98.4	88.31	5/5
O60K25p1G1_1	995.7	910.8	9.32	2938	55.2	100.0	84.89	10/10
O60K25p2G1_2	1210.7	925.9	30.76	3290	60.8	100.0	87.34	12/13
O60K25p2G1_3	1182.7	1003.6	17.85	3122	70.4	100.0	86.93	12/15
O60K25p1G1_4	975.4	868.4	12.32	3042	60.8	100.0	84.38	8/8
O60K25p2G1_5	902.3	734.2	22.90	3616	80.0	100.0	90.69	12/12
avg.			19.85	2241	64.4	99.5	86.9	
min.			9.32	1062	48.0	97.6	83.9	
max.			33.84	3616	80.0	100.0	90.9	

Table 8: Average results of each group of instances having different level of variety in order dimensions and weights

DWV	Order Size	TC			Time (s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H (G1)	40	794.38	645.20	23.69	1401	64.96	99.84	87.78	8.6
M (G4)	40	854.78	689.18	23.94	1252	56.16	94.08	78.19	8.2
L (G2)	40	941.46	820.06	14.98	1154	48.80	91.52	73.95	9.6
No (G3)	40	377.40	355.36	6.19	2114	56.32	76.80	73.14	4.2
H (G1)	50	851.54	724.96	17.41	2120	62.72	98.88	86.14	8.0
M (G4)	50	1060.90	937.34	13.21	1805	49.44	96.32	76.50	11.4
L (G2)	50	1034.86	890.32	16.05	1880	56.16	96.96	78.81	11.2
No (G3)	50	478.26	454.16	5.31	3555	48.64	76.80	71.11	6.2
H (G1)	60	1053.36	888.58	18.63	3202	65.44	100.00	86.85	11.6
M (G4)	60	1308.38	1131.00	15.90	2583	55.20	97.12	79.50	14.6
L (G2)	60	1090.96	960.20	13.47	2947	63.04	95.84	80.79	10.6
No (G3)	60	535.30	521.68	2.61	4463	76.80	76.80	76.80	6.2

* : DWV: Dimension and Weight Variety, H: High, M: Medium, L: Low.

diversity, respectively. The detailed results for each instance in each group having order sizes of 40, 50, and 60 are provided in Appendix in Tables .14, .15, and .16, respectively. Table 9 reports the average results of each group in terms of TC (of the solutions of the CH and ALNS and the percentage improvement of the ALNS), and run time (in seconds), area utilization of shipments and number of shipments of the ALNS solutions. As in Section 5.3, the weight utilization values are not provided since the weight utilization of any shipment is not larger than the area utilization for any

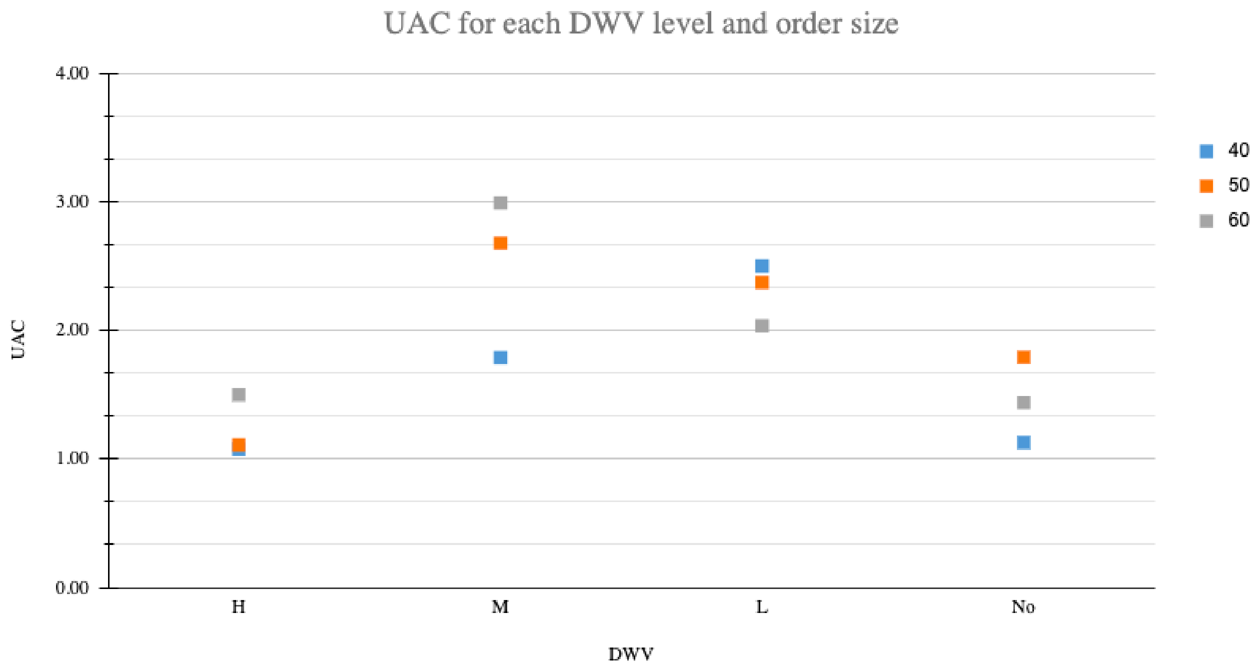


Figure 3: Unused area capacity for different DWV levels and order sizes

instance.

Table 9: Average results of each group of instances having different level of variety in order due dates

DDV	Order Size	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H (G1)	40	794.38	645.20	23.69	1401	64.96	99.84	87.78	8.6
M (G6)	40	766.00	629.70	21.79	1311	73.76	99.36	87.64	8.4
L (G5)	40	835.56	700.40	20.39	1393	60.16	99.52	84.08	10.8
H (G1)	50	851.54	724.96	17.41	2120	62.72	98.88	86.14	8.0
M (G6)	50	1093.98	938.74	16.87	2397	53.92	98.72	82.67	12.2
L (G5)	50	1095.64	875.00	25.17	1794	62.56	100.00	87.08	12.0
H (G1)	60	1053.36	888.58	18.63	3202	65.44	100.00	86.85	11.6
M (G6)	60	1073.86	868.32	23.65	3295	63.52	99.04	87.49	11.4
L (G5)	60	1053.52	857.10	22.90	3270	62.56	99.84	88.93	13.0

* : DDV: Due Date Variety, H: High, M: Medium, L: Low.

According to the results presented in Table 8, as the variety in due date decreases the area utilization does not change significantly. On the other hand, the number of shipments is largest when the due date variety is low. The Figure 4 presents a chart that shows the UAC values for each group of instances having different DDV levels and order sizes. As the chart demonstrates, the unused capacity is relatively stable for different DDV levels for large order sizes, however, varies more for different DDV levels for order sizes of 40 and 50.

5.6. Analysis on the effects of penalties of disjoint shipping of related orders

Next, we analyze the effect of penalties of disjoint shipping of the orders that are requested to be transported together on the ALNS solutions. The original instance group is used for this analysis.

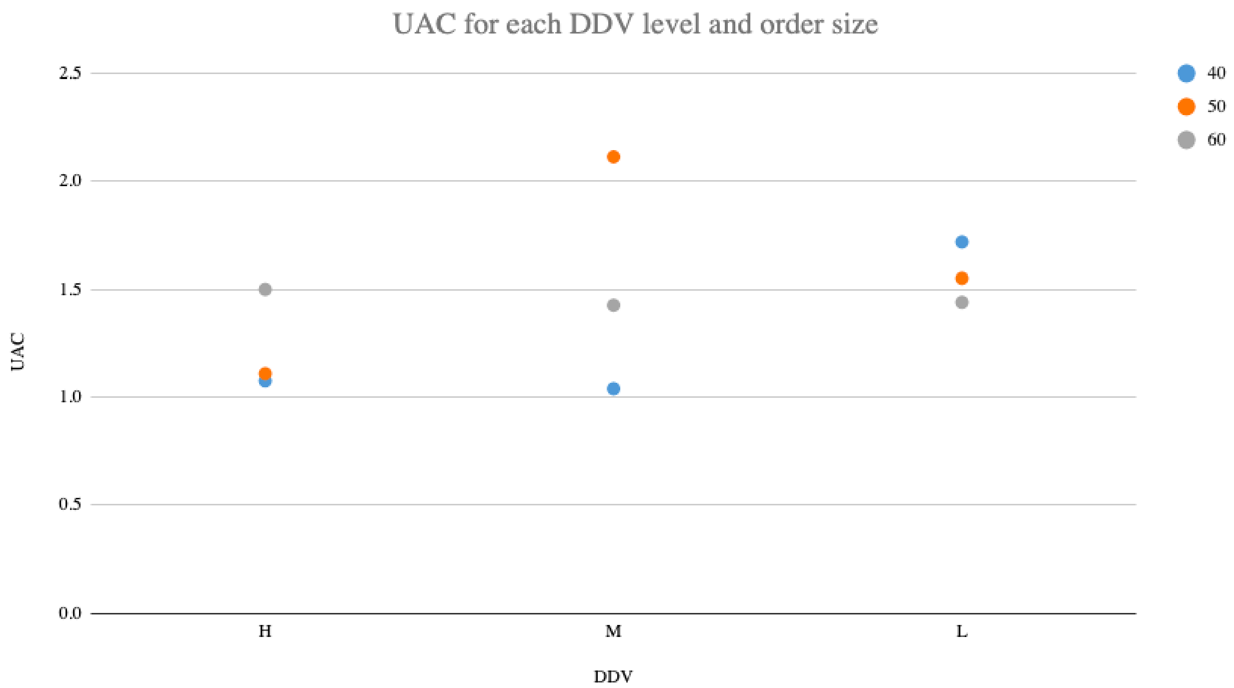


Figure 4: Unused area capacity for different DDV levels and order sizes

Table 10 reports the TC of the ALNS solution for the following pairs of penalties, ($c_r^{(s)}$,$c_r^{(d)}$):(0,0), (0.4,0.9), (1,2), (2,4), and (5,10), in addition to the original values, which are (0.1,0.2). In order to show the cases whose objective value differ just because of the difference between their penalty multipliers, for each pair of penalty, except $c_r^{(s)}$,$c_r^{(d)}$=(0,0), the sum of earliness and shipment costs are reported under the column ESC.

Table 10: Results of original instances having different levels of penalty costs ($c_r^{(s)}$ and $c_r^{(d)}$) related to joint shipments

($c_r^{(d)}$,$c_r^{(s)}$):	(0,0)	(0.1,0.2)		(0.4,0.9)		(1,2)		(2,4)		(5,10)	
Instance	TC	TC	ESC	TC	ESC	TC	ESC	TC	ESC	TC	ESC
O40K17p1G1_1	701	701.4	701	703.2	701	705	701	709	701	726	701
O40K17p2G1_2	756	756.3	756	757.2	756	759	756	762	756	771	756
O40K17p2G1_3	520	520.7	520	523,1	520	527	520	534	520	554	**524**
O40K17p2G1_4	543	543.4	543	544.6	543	547	543	551	543	563	551
O40K17p2G1_5	704	704.2	704	704.9	704	706	704	708	704	714	704
O50K21p1G1_1	770	770,6	770	772,7	770	776	770	782	770	800	770
O50K21p2G1_2	759	759.8	759	762.5	759	767	759	775	759	797	**767**
O50K21p2G1_3	733	733.8	733	736.6	733	741	733	749	733	773	733
O50K21p1G1_4	674	674.4	674	675.8	674	678	674	682	674	692	**682**
O50K21p1G1_5	669	669.3	669	671.2	669	672	669	675	669	681	**671**
O60K25p1G1_1	906	906.8	906	909.6	906	914	906	922	906	946	906
O60K25p2G1_2	911	911.7	911	914.1	911	918	911	925	911	946	911
O60K25p2G1_3	1001	1002.0	1001	1004.1	1001	1011	1001	1021	1001	1037	**1012**
O60K25p1G1_4	863	863.4	863	864.8	863	867	863	871	863	883	863
O60K25p2G1_5	734	734.2	734	734.9	734	736	734	738	734	744	734

According to the results in Table 10, the ALNS solution of a case is generally the same (in terms of the ESC and the joint shipment decisions) as the ALNS solution of the case having one step smaller penalty costs. Only the cases, of which the ESC is provided in bold, have a different ESC value and joint shipment decision compared to the case having one step smaller penalty costs. For these cases, it is better to pay larger earliness and shipment cost in order not to increase the joint shipment penalty. As these cases are observed for ($c_r^{(s)}$,$c_r^{(d)}$)=(5,10), it can be concluded that joint shipment penalties generally do not affect shipment decisions for lower levels of penalties.

6. Conclusions

In this paper, we have addressed the simultaneous vehicle loading and dispatching problem that arises in long-haul freight transportation. A set of orders need to be transported along a single origin-destination line such that the orders should reach their destination by their due dates and early arrivals cause a penalty cost. The problem involves multiple day planning of the shipments to be performed by the company owned fleet in addition to any needs from the spot market. The orders should be packed to the heterogeneous vehicles in a two-dimensional fashion and the shipment days of the vehicles should be determined. Constraints prevent packing incompatible orders into the same vehicle, while preventing packing of orders to incompatible vehicles and obeying area and weight capacity of the vehicles. The objective is to minimize the total vehicle usage and earliness penalty costs. We have provided a Mixed-Integer Linear Programming model, as well as an Adaptive Large Neighbourhood Search (ALNS) algorithm for the solution of this problem. Results of computational tests on instances derived from real-world data confirm the near-optimal performance of the ALNS algorithm.

Extension of the problem to address three-dimensional packing remains as a challenging avenue for future work. Stochastic arrival of new orders over the planning horizon may also be analyzed.

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Table .11: Results of the instances having 40 orders and different level of variety in order dimensions and weights

DWV	Instance	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H	O40K17p1G1_1	852.4	701.4	21.53	1311	75.2	100	90.89	6
H	O40K17p2G1_2	905.2	756.3	19.69	1062	60	100	87.88	10
H	O40K17p2G1_3	643.7	520.7	23.62	1706	48	100	84.48	8
H	O40K17p2G1_4	727.3	543.4	33.84	1598	77.6	100	89.92	8
H	O40K17p2G1_5	843.3	704.2	19.75	1327	64	99.2	85.73	11
	avg.	794.38	645.2	23.69	1401	64.96	99.84	87.78	8.6
M	O40K17p1G4_1	710.3	612.3	16.01	1175	60	97.6	78.47	5
M	O40K17p2G4_2	1076	799	34.67	1119	54.4	92.8	76.54	10
M	O40K17p2G4_3	870.1	760.2	14.46	1365	55.2	87.2	75.27	11
M	O40K17p2G4_4	844.3	615.2	37.24	1313	64.8	96.8	82.91	9
M	O40K17p1G4_5	773.2	659.2	17.29	1287	46.4	96	78.78	6
	avg.	854.78	689.18	23.93	1252	56.16	94.08	78.19	8.2
L	O40K17p1G2_1	647.4	561.4	15.32	1339	64	93.6	79.35	4
L	O40K17p2G2_2	1072.2	962.2	11.43	975	31.2	94.4	71.65	13
L	O40K17p2G2_3	999.1	882.2	13.25	1235	50.4	88	71.15	12
L	O40K17p1G2_4	928.3	855.2	8.55	1027	36	83.2	69.79	7
L	O40K17p2G2_5	1060.3	839.3	26.33	1197	62.4	98.4	77.83	12
	avg.	941.46	820.06	14.98	1154	48.8	91.52	73.95	9.6
No	O40K17p1G3_1	396.2	356.2	11.23	2134	51.2	76.8	73.14	3
No	O40K17p1G3_2	387.2	356.2	8.70	2251	51.2	76.8	73.14	3
No	O40K17p2G3_3	383.3	358.2	7.01	2299	64	76.8	73.14	6
No	O40K17p1G3_4	363.2	356.2	1.97	1825	51.2	76.8	73.14	3
No	O40K17p2G3_5	357.1	350	2.03	2059	64	76.8	73.14	6
	avg.	377.4	355.36	6.19	2114	56.32	76.8	73.14	4.2

Table .12: Results of the instances having 50 orders and different level of variety in order dimensions and weights

DWV	Instance	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H	O50K21p1G1_1	927.5	787.5	17.78	1719	54.4	98.4	86.56	9
H	O50K21p2G1_2	859.6	759.8	13.14	1894	65.6	97.6	83.89	11
H	O50K21p2G1_3	924.8	733.8	26.03	2365	60.8	100	87.2	10
H	O50K21p1G1_4	739.4	674.4	9.64	2575	60.8	100	84.74	5
H	O50K21p1G1_5	806.4	669.3	20.48	2046	72	98.4	88.31	5
	avg.	851.54	724.96	17.41	2120	62.72	98.88	86.14	8
M	O50K21p1G4_1	1137.4	1007.4	12.90	1792	54.4	94.4	72.48	9
M	O50K21p2G4_2	1147.2	1007.2	13.90	1655	40.8	98.4	75.34	14
M	O50K21p2G4_3	980.4	882.4	11.11	1983	48	98.4	79.41	14
M	O50K21p1G4_4	966.3	834.5	15.79	1755	60.8	96	81.25	7
M	O50K21p2G4_5	1073.2	955.2	12.35	1838	43.2	94.4	74.02	13
	avg.	1060.9	937.34	13.21	1805	49.44	96.32	76.50	11.4
L	O50K21p1G2_1	931.1	860.2	8.24	1712	57.6	95.2	75.67	7
L	O50K21p2G2_2	1122.3	962.3	16.63	1948	57.6	95.2	79.11	12
L	O50K21p2G2_3	926.3	857.3	8.05	1844	48	98.4	78	11
L	O50K21p2G2_4	1010.4	844.4	19.66	2132	57.6	97.6	80.69	12
L	O50K21p2G2_5	1184.2	927.4	27.69	1763	60	98.4	80.6	14
	avg.	1034.86	890.32	16.05	1880	56.16	96.96	78.81	11.2
No	O50K21p2G3_1	477.3	453.2	5.32	3468	51.2	76.8	71.11	8
No	O50K21p1G3_2	476.2	453.2	5.08	3236	51.2	76.8	71.11	4
No	O50K21p2G3_3	474.2	454	4.45	3859	51.2	76.8	71.11	8
No	O50K21p1G3_4	484.2	456.2	6.14	3056	51.2	76.8	71.11	4
No	O50K21p2G3_5	479.4	454.2	5.55	4157	38.4	76.8	71.11	7
	avg.	478.26	454.16	5.306	3555	48.64	76.8	71.11	6.2

Table .13: Results of the instances having 60 orders and different level of variety in order dimensions and weights

DWV	Instance	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H	O60K25p1G1_1	995.7	910.8	9.32	2938	55.2	100	84.89	10
H	O60K25p2G1_2	1210.7	925.9	30.76	3290	60.8	100	87.34	13
H	O60K25p2G1_3	1182.7	1003.6	17.85	3123	70.4	100	86.93	15
H	O60K25p1G1_4	975.4	868.4	12.32	3042	60.8	100	84.38	8
H	O60K25p2G1_5	902.3	734.2	22.90	3616	80	100	90.69	12
	avg.	1053.36	888.58	18.63	3202	65.44	100	86.85	11.6
M	O60K25p1G4_1	1230.6	1130.6	8.84	2836	51.2	100	74.22	10
M	O60K25p2G4_2	1309.4	1093.4	19.75	2106	51.2	100	80.55	15
M	O60K25p2G4_3	1444.3	1250.3	15.52	2649	65.6	88.8	78.06	17
M	O60K25p2G4_4	1322.3	1175.4	12.50	2769	67.2	98.4	82.12	17
M	O60K25p2G4_5	1235.3	1005.3	22.88	2553	40.8	98.4	82.58	14
	avg.	1308.38	1131	15.90	2583	55.2	97.12	79.50	14.6
L	O60K25p1G2_1	1028.4	965.5	6.51	2377	63.2	93.6	81.05	8
L	O60K25p2G2_2	1096.4	1002.5	9.37	3104	64.8	97.6	81.27	15
L	O60K25p2G2_3	1305.4	1046.4	24.75	3406	60	100	79.92	15
L	O60K25p1G2_4	1028.2	883.2	16.42	2871	69.6	93.6	82.45	8
L	O60K25p1G2_5	996.4	903.4	10.29	2978	57.6	94.4	79.24	7
	avg.	1090.96	960.2	13.47	2947	63.04	95.84	80.79	10.6
No	O60K25p1G3_1	535.4	521.4	2.68	4415	76.8	76.8	76.8	4
No	O60K25p1G3_2	551.4	526.4	4.75	4368	76.8	76.8	76.8	5
No	O60K25p2G3_3	531.2	524.2	1.34	4387	76.8	76.8	76.8	8
No	O60K25p1G3_4	522.3	519.2	0.60	4258	76.8	76.8	76.8	5
No	O60K25p2G3_5	536.2	517.2	3.67	4889	76.8	76.8	76.8	9
	avg.	535.3	521.68	2.61	44632	76.8	76.8	76.8	6.2

Table .14: Results of the instances having 40 orders and different level of variety in order due dates

DDV	Instance	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H	O40K17p1G1_1	852.4	701.4	21.53	1311	75.2	100	90.89	6
H	O40K17p2G1_2	905.2	756.3	19.69	1062	60.0	100	87.88	10
H	O40K17p2G1_3	643.7	520.7	23.62	1706	48.0	100	84.48	8
H	O40K17p2G1_4	727.3	543.4	33.84	1598	77.6	100	89.92	8
H	O40K17p2G1_5	843.3	704.2	19.75	1327	64.0	99.2	85.73	11
	avg.	794.38	645.2	23.69	1401	64.96	99.84	87.78	8.6
M	O40K17p1G6_1	605	539	12.24	1416	72	99.2	85.92	6
M	O40K17p2G6_2	820.2	664.2	23.49	1393	70.4	99.2	89.09	11
M	O40K17p2G6_3	918.2	738	24.42	1310	77.6	100	88.73	12
M	O40K17p1G6_4	772.2	677.1	14.05	1134	68.8	100	85.72	5
M	O40K17p2G6_5	714.4	530.2	34.74	1301	80	98.4	88.72	8
	avg.	766.	629.7	21.79	1311	73.76	99.36	87.64	8.4
L	O40K17p1G5_1	729.3	564.3	29.24	1302	60.8	100	86.98	8
L	O40K17p2G5_2	987	801	23.22	1603	48	100	81.6	13
L	O40K17p2G5_3	813.1	720.2	12.90	1385	66.4	100	85.4	12
L	O40K17p1G5_4	813.2	613.3	32.59	1211	61.6	100	83.6	8
L	O40K17p2G5_5	835.2	803.2	3.98	1464	64	97.6	82.83	13
	avg.	835.56	700.4	20.39	1393	60.16	99.52	84.08	10.8

Table .15: Results of the instances having 50 orders and different level of variety in order due dates

DDV	Instance	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H	O50K21p1G1_1	927.5	787.5	17.778	1719	54.4	98.4	86.56	9
H	O50K21p2G1_2	859.6	759.8	13.14	1894	65.6	97.6	83.89	11
H	O50K21p2G1_3	924.8	733.8	26.03	2365	60.8	100	87.2	10
H	O50K21p1G1_4	739.4	674.4	9.64	2575	60.8	100	84.74	5
H	O50K21p1G1_5	806.4	669.3	20.48	2046	72	98.4	88.31	5
	avg.	851.54	724.96	17.41	2120	62.72	98.88	86.14	8
M	O50K21p1G6_1	857.4	704.4	21.72	2102	44.8	98.4	84.06	6
M	O50K21p2G6_2	1170.2	1067.2	9.65	2330	57.6	100	80.89	17
M	O50K21p2G6_3	1404.2	1148.1	22.31	3709	54.4	97.6	79.6	18
M	O50K21p1G6_4	946.4	791.4	19.59	1778	56.8	100	84.85	7
M	O50K21p2G6_5	1091.7	982.6	11.10	2066	56	97.6	83.95	13
	avg.	1093.98	938.74	16.87	2397	53.92	98.72	82.67	12.2
L	O50K21p1G5_1	1223.3	928.3	31.78	1808	43.2	100	83.6	10
L	O50K21p2G5_2	1146.2	982.2	16.70	2141	72.8	100	87.3	16
L	O50K21p2G5_3	982.2	831	18.19	1769	60.8	100	88.63	14
L	O50K21p1G5_4	906.2	730.2	24.10	1719	76	100	90.11	10
L	O50K21p1G5_5	1220.3	903.3	35.09	1533	60	100	85.74	10
	avg.	1095.64	875.00	25.17	1794	62.56	100	87.08	12

Table .16: Results of the instances having 60 orders and different level of variety in order due dates

DDV	Instance	TC			Time(s)	Area Utilization			#Shipments
		CH	ALNS	Imp.(%)		min.	max.	avg.	
H	O60K25p1G1_1	995.7	910.8	9.32	2938	55.2	100	84.89	10
H	O60K25p2G1_2	1210.7	925.9	30.76	3290	60.8	100	87.34	13
H	O60K25p2G1_3	1182.7	1003.6	17.85	3123	70.4	100	86.93	15
H	O60K25p1G1_4	975.4	868.4	12.32	3042	60.8	100	84.38	8
H	O60K25p2G1_5	902.3	734.2	22.90	3616	80	100	90.69	12
	avg.	1053.36	888.58	18.63	3202	65.44	100	86.85	11.6
M	O60K25p1G6_1	993.5	851.3	16.70	3215	69.6	95.2	88.2	7
M	O60K25p2G6_2	1178.5	859.3	37.145	2947	60.8	100	89.07	15
M	O60K25p2G6_3	1266.5	1029.4	23.03	3403	64	100	88.38	17
M	O60K25p1G6_4	926.4	815.3	13.63	3595	43.2	100	80.3	6
M	O60K25p2G6_5	1004.4	786.3	27.74	3313	80	100	91.52	12
	avg.	1073.86	868.32	23.65	3295	63.52	99.04	87.49	11.4
L	O60K25p1G5_1	1254.4	893.4	40.41	2433	74.4	99.2	91.62	12
L	O60K25p2G5_2	1016.4	845.4	20.23	3328	72.8	100	91.2	15
L	O60K25p2G5_3	1096.2	952.2	15.12	3607	45.6	100	85.65	16
L	O60K25p1G5_4	958.3	830.2	15.43	3824	51.2	100	83.65	8
L	O60K25p2G5_5	942.3	764.3	23.29	3160	68.8	100	92.51	14
	avg.	1053.52	857.10	22.90	3270	62.56	99.84	88.93	13