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A Note on the Unemployment Volatility Puzzle*

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Abstract

This paper features an economy with frictional labour markets, Nash bargaining, cyclical opportunity cost of employment and demand-determined output. I show that the model generates big responses of unemployment to productivity movements. The key insight is that strongly countercyclical markups amplify the impact of aggregate shocks on the fundamental surplus, and as a consequence, on labour market outcomes. This answers the critique of [Chodorow-Reich and Karabarbounis \(2016\)](#), that the cyclical nature of the opportunity cost poses a challenge to the solution of the unemployment volatility puzzle.

Keywords: Demand-determined output, unemployment volatility, opportunity cost of employment, price markups, fundamental surplus

JEL classification: E24, E32, J24, J41, J63

1 Introduction

Understanding labor market fluctuations is an important issue in economics. [Shimer \(2005\)](#) argues that for common calibrations of the canonical equilibrium search model of unemployment, vacancies and unemployment are almost insensitive to productivity shocks, and the model cannot account for the

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key cyclical movements in labor market activity (the unemployment volatility puzzle). Following Shimer’s contribution, various modifications to the canonical model have been proposed to resolve the puzzle—notable contributions include [Hagedorn and Manovskii \(2008\)](#), [Hall and Milgrom \(2008\)](#), [Pissarides \(2009\)](#). However, [Chodorow-Reich and Karabarbounis \(2016\)](#) show that the opportunity cost of employment is procyclical in the US data, and that this cyclical nature poses a challenge to models relying on constant opportunity cost to solve the puzzle.

The contribution of this paper is to show that big responses of unemployment to productivity changes are possible even with Nash bargaining wage contracting and strongly procyclical opportunity cost. My approach incorporates into the canonical search model of unemployment two features: cyclical opportunity cost of employment and rationing of suppliers in the goods market, that is, demand-determined output. I show that the fundamental surplus ([Ljungqvist and Sargent, 2017](#)) is inversely related to price markups, and that strongly countercyclical markups amplify the impact of aggregate shocks on the fundamental surplus, and as a consequence, on labour market outcomes.

Models that feature real wage rigidities, e.g., [Hall \(2005\)](#), can attain big responses of unemployment to productivity even with cyclical opportunity cost. This is because wages do not reflect the cyclical nature of the worker’s outside option, and the opportunity cost has no impact on anything. Instead, [Chodorow-Reich and Karabarbounis \(2016\)](#) focus their critique on four models in which the opportunity cost affects market outcomes. These include [Mortensen and Pissarides \(1994\)](#) and [Hagedorn and Manovskii \(2008\)](#), featuring Nash bargaining, [Hall and Milgrom \(2008\)](#), featuring alternative-offer-bargaining, and environments pioneered in [Moen \(1997\)](#) with directed search and wage posting. In that respect, retaining the assumption of Nash bargaining, hardwired into the canonical model, allows me to address the critique and differentiates my paper from the literature on wage rigidities.

The economy features frictions in the labor market and a two-sector firm structure. Workers search for jobs; wholesale firms open costly vacancies to hire workers and produce intermediate output; and monopolistically competitive retail firms produce final goods with intermediate output, facing preset prices. The modelling of the retail sector follows the approach in [Korinek and Simsek \(2016\)](#). Wages are determined via Nash bargaining. The canonical model, augmented with cyclical opportunity cost, obtains when final good prices are flexible and monopolistic distortions are corrected.

Productivity changes impact wholesalers technology and induce changes in

the aggregate demand for final goods. Aggregate demand externalities spill-over to hiring decisions through changes in the relative price of intermediate output (real marginal cost of retailers). The relative price channel induces a substantial adjustment of hirings in response to productivity changes, despite the fact that the significant adjustment of wages, due to the cyclicity of the opportunity cost, absorbs part of the response.

To generate big responses of unemployment to movements in productivity, matching models require a high elasticity of market tightness with respect to productivity. The channel through which economic forces generate that high elasticity is the fundamental surplus (FS). It is an upper bound on what the “invisible hand” can allocate to vacancy creation. If productivity changes translate into large percentage changes in the FS, then a given percentage change in productivity translates into a much larger percentage change in resources used for vacancy creation and hence big responses of unemployment to movements in productivity.

In the current framework, the FS is equal to the relative price of intermediate output times productivity (value of an additional worker for wholesalers) less the opportunity cost of employment. Under the flexible prices benchmark, relative prices are constant and the response of the FS to productivity is muted because of the procyclicality of the opportunity cost. In contrast, under demand-determined output, relative prices are procyclical, amplifying the response of the FS to productivity changes, even with procyclical opportunity cost.

I compare the elasticity of market tightness to productivity changes between two economies (regimes) in steady state: the “benchmark” regime of flexible prices and the “demand-determined output” regime. The elasticity is decomposed in two terms: the first is determined by common calibrations and has limited influence on the elasticity, and the second is the elasticity of the FS to productivity. I conduct a calibration exercise where I evaluate the elasticity numerically for each regime.

The relative price of intermediate output can be associated to the markup of the price over marginal cost of retailers. The latter is equal to the inverse of the relative price of intermediate output, so that procyclicality of relative prices is equivalent to countercyclicality of the retailer’s price markup.

My paper is not the first to study the effects of countercyclical price markups on labour market outcomes. [Rotemberg \(2008\)](#) introduces large, imperfectly competitive firms into an otherwise standard search model of unemployment. Countercyclical markups have a big impact on employment by weakening the

worker’s bargaining power and making wages less procyclical. Changes in market power is a source of fluctuations and, importantly, the opportunity cost of employment is constant. I depart from these assumptions.

Another paper investigating the importance of price markups in explaining business cycle fluctuations is [Bils et al. \(2018\)](#). This paper decomposes the “labour wedge” into product market and labour market distortions. Using US data, they measure that price markup movements are at least as cyclical as wage markup movements. They conclude that countercyclical price markups should play a central role in explaining fluctuations alongside labour market frictions. The predictions of my model is consistent with their results. Specifically, the cyclical behaviour of the FS is controlled by two forces: countercyclical markups amplify the impact of shocks on the FS, while procyclical opportunity cost dampens that impact. In the absence of wage rigidities, the dampening force can be strong, so it is not immediate that an equilibrium model can generate big responses of the FS to productivity. My contribution is to build an equilibrium model consistent with such big responses.

Following Shimer’s contribution, the literature has advanced three main approaches: (1) specify a low value of forming a match, (2) introduce real wage rigidities, (3) incorporate labour market institutions.

[Ljungqvist and Sargent \(2017\)](#) review approaches (1)-(3), and identify a common channel through which economic forces that generate high elasticity of market tightness to productivity must operate. This channel, as we already noted, is the fundamental surplus. In the class of model they consider, productivity changes translate into large percentage changes in the fundamental surplus when the fundamental surplus is small.¹ Similarly, [Mortensen and Nagypal \(2007\)](#) combine several forces in a way that is consistent with the argument in [Ljungqvist and Sargent \(2017\)](#).

The first approach feature models where primitive parameters are calibrated to yield low FS. [Hagedorn and Manovskii \(2008\)](#) propose an alternative calibration of the canonical model that places a high value on the (constant) opportunity cost of employment.² [Hall and Milgrom \(2008\)](#) replace Nash bargaining with alternative-offer-bargaining. The opportunity cost of a match is equal to the value derived by [Hagedorn and Manovskii \(2008\)](#).

The second approach confronting the unemployment volatility puzzle features models with real wage rigidities. [Hall \(2005\)](#) introduces sticky wages into

¹More recently, [Ljungqvist and Sargent \(2021\)](#) show that the fundamental surplus is the key intermediating quantity in the matching models of [Christiano et al. \(2016, 2021\)](#).

²[Costain and Reiter \(2008\)](#) make a similar point.

the standard matching model in a way consistent with low FS; [Gertler and Trigari \(2009\)](#) adopt a staggered version of Nash bargaining with infrequent wage changes; [Blanchard and Galí \(2010\)](#) specify a real wage function that partially responds to productivity changes; and [Michaillat \(2012\)](#) specify a real wage function that responds as much to productivity changes as wages of new hires in the data.

The third approach enrich the structure of the matching models with a more detailed description of the labour market, including training costs, a welfare state and various labour market policy instruments. [Mortensen and Nagypal \(2007\)](#) and [Pissarides \(2009\)](#) show that the inclusion of training costs allows the model to match the cyclical movements in labor market activity; [Ljungqvist and Sargent \(2017\)](#) show that generous welfare states or layoff taxes amplify the impact of shocks by lowering the FS; and [Zanetti \(2011a\)](#) shows that the inclusion of labour instruments, like unemployment benefits, firing costs and income taxes, into the canonical search model of unemployment, amplify the impact of shocks on labour market variables.³

Finally, there exists a literature that incorporates labour market frictions into the New Keynesian monetary framework (see a review in [Blanchard and Galí, 2010](#)). The specification of monetary policy has nontrivial implications for the impact of shocks on aggregate outcomes. In the current paper, I abstract entirely from policy issues.

Section 2 presents the environment; Section 3 presents the main argument; Section 4 presents a numerical example; Section 5 offers various extensions; Section 6 concludes. Derivations are delegated into the online Appendix.

2 Environment

2.1 Agents and markets

Time is discrete and infinite ($t = 0, 1, \dots$). The economy is populated by a large household with a continuum of members of unit measure,⁴ and a two-sector firm structure with a continuum of wholesale firms, $j \in [0, 1]$, and a continuum of retail firms, $i \in [0, 1]$. Market participants trade differentiated consumption goods, $i \in [0, 1]$, an intermediate good, labor and money.

³[Thomas and Zanetti \(2009\)](#) and [Zanetti \(2011b\)](#) show that labour instruments have a substantial impact on aggregate fluctuations in a New Keynesian framework with frictional labour markets.

⁴I employ the large family assumption of [Merz \(1995\)](#), which assumes full risk sharing among family members.

Money is the numeraire. Household members search for jobs in frictional labor markets. The wholesale sector is competitive, with firms opening costly vacancies to hire workers in order to produce intermediate output; while the retail sector is monopolistically competitive, with firms buying intermediate output to transform them into differentiated (final) consumption goods.

Household's decision yields

$$\begin{aligned} \max_{\{c,m,n\}} \sum_{t=0}^{\infty} \beta^t & \left[c_t^{1-\zeta} / (1-\zeta) + (m_t/p_t)^{1-\delta} / (1-\delta) - \psi(n_t) \right], \\ \text{subject to} & \\ p_t c_t + m_t &= w_t n_t + m_{t-1} + \pi_t - p_t T_t, \\ n_t &= (1-s)n_{t-1} + u_t f(\theta_t), \end{aligned} \tag{1}$$

where $\beta \in (0, 1)$, $\zeta, \delta > 0$ are preference parameters. The household derives utility from consumption, c , end-of-period real money balances, m/p ,⁵ and experiences disutility by supplying labor, n , where $\psi(\cdot)$ is convex and differentiable. Its wealth consists of nominal profits, π , real lump-sum taxes, T , previous period money balances and labor income; at $t = 0$, the household is endowed with money balances $\bar{m} > 0$ and the growth rate of money is zero.⁶ The price (index) is denoted by p and the wage by w . Total employment is equal to pre-existing jobs, n_{t-1} , net of separations, with $s \in (0, 1)$ the separation rate, plus new matches $u \cdot f(\theta)$, with $u_t = 1 - (1-s)n_{t-1}$ beginning-of-period unemployment, and $f(\theta) \in (0, 1)$ the job-finding rate as function of market tightness θ . As in [Blanchard and Galí \(2010\)](#) and [Michaillat \(2012\)](#), I assume that new matches become immediately productive.

The consumption index is given by $c \equiv \left(\int_0^1 c(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$, with $\epsilon > 1$ (Dixit-Stiglitz aggregator), while the price index and the demand of each differentiated good, resulting from minimising consumption expenditure, are given by $p \equiv \left(\int_0^1 p(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ and $c(i) = (p(i)/p)^{-\epsilon} c$, respectively, with $p(i)$ denoting the price of each differentiated good.

The necessary first-order conditions of (1) reduce to

$$\frac{c_t^{-\zeta}}{p_t} = \frac{(m_t/p_t)^{-\delta}}{p_t} + \beta \frac{c_{t+1}^{-\zeta}}{p_{t+1}}, \tag{2}$$

⁵Money in the utility function serves only as a convenient “short-cut” to construct a stationary equilibrium, where labour market frictions and aggregate demand externalities influence aggregate outcomes (see Section 2.3 for details).

⁶I extend the analysis to include constant, nonzero, growth and show that all results follow intact (see Section 5 for details); thus, this assumption is without loss of generality.

$$S_t^H = \frac{w_t}{p_t} - z_t + \beta \Delta_{t+1,t} (1-s)(1-f(\theta_{t+1})) S_{t+1}^H, \quad (3)$$

and the transversality condition is

$$\lim_{t \rightarrow \infty} \beta^t c_t^{-\zeta} (m_t/p_t) = 0.$$

Here, $\Delta_{t+1,t} \equiv (c_{t+1}/c_t)^{-\zeta}$ and z denotes the opportunity cost of employment in terms of consumption, $z_t = c_t^\zeta \psi'(n_t)$. Condition (2) states that along an optimal path, the utility cost of holding \$1.00 at t must be equal to the utility gain from holding that dollar for a period and converting it back into consumption at $t + 1$. Condition (3) states that along an optimal path, the marginal value to the household of an additional worker at t in terms of consumption, S_t^H , is equal to the flow value plus the present discounted marginal value at $t + 1$. The flow value consists of the flow gain from wages and a flow loss associated with moving an individual from unemployment to employment.

All wholesalers are identical and produce the intermediate good according to the technology $y^W = an$, where a is productivity. They post vacancies to hire new employees and lose employees at the rate s . The cost of each additional vacancy is equal to $p\gamma$, with $\gamma > 0$ denoting recruitment costs. Since job postings are homogenous to final goods, each wholesaler j buys final goods $v(j, i)$ from each i retail firm so as to minimise expenditure, subject to $v(j) \equiv \left(\int_0^1 v(j, i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$, with $v(j)$ denoting the number of vacancies posted by wholesaler j . The demand by wholesalers for final goods produced by retail firm i is $v(j, i) = (p(i)/p)^{-\epsilon} v(j)$. Vacancies are filled at the rate $q(\theta) \in (0, 1)$, which denotes the vacancy-filling rate as function of market tightness, so that new hirings of each wholesaler j are $h(j) = q(\theta)v(j)$. In the sequel, I will work with hirings rather than vacancies. The number of workers available for production in wholesale firm j are $n_t(j) = (1-s)n_{t-1}(j) + h_t(j)$. At the aggregate level, workers available for production at t equal $n_t = (1-s)n_{t-1} + h_t$, with $n_t = \int_0^1 n_t(j) dj$ and $h = \int_0^1 h(j) dj$.

Wholesale firms sell their output to retailers in a competitive market at the price p^W . The decision of wholesaler j yields

$$\begin{aligned} \max_{\{y^W, n, h\}} \sum_{\kappa} \beta^{\kappa} \Delta_{t+\kappa,t} \left[\frac{p_{t+\kappa}^W}{p_{t+\kappa}} y_{t+\kappa}^W(j) - \frac{w_{t+\kappa}}{p_{t+\kappa}} n_{t+\kappa}(j) - \frac{\gamma}{q(\theta_{t+\kappa})} h_{t+\kappa}(j) \right], \\ \text{subject to} \\ n_{t+\kappa}(j) = (1-s)n_{t+\kappa-1}(j) + h_{t+\kappa}(j), \quad \kappa = 0, 1, \dots \\ y_{t+\kappa}^W(j) = a_{t+\kappa} n_{t+\kappa}(j), \quad \kappa = 0, 1, \dots \end{aligned} \quad (4)$$

The necessary and sufficient first-order condition reduces to

$$\frac{\gamma}{q(\theta_t)} = \frac{p_t^W}{p_t} a_t - \frac{w_t}{p_t} + \beta(1-s)\Delta_{t+1,t} \frac{\gamma}{q(\theta_{t+1})}. \quad (5)$$

Condition (5) states that along an optimal path, the marginal cost of hiring an extra employee is equal to the flow value plus the present discounted value of the marginal cost at $t+1$. The flow value consists of the marginal benefit of an additional worker, $(p^W/p)a$, and the flow loss associated with the wage cost of an additional worker. I assume free entry in the vacancy creation process so that the marginal value to the wholesale firm of an additional employee, S^W , is equal to the marginal cost, that is, $S_t^W = \gamma/q(\theta_t)$. Any current worker can be immediately replaced with someone who is unemployed by paying the hiring cost.

Retail firms are monopolistically competitive and transform one-to-one the intermediate good into differentiated (final) goods, $y_t(i) = x_t(i)$, where $x(i)$ is the quantity of the intermediate good demanded by retailer i . I distinguish between two regimes with respect to retailer decisions. In the first regime, retail firms can set prices freely (flexible prices) and this regime will serve as a benchmark; while in the second regime, I assume that retailers have preset nominal prices that are equal to each other and that never change, $p_t(i) = p$ for all t . This implies that the final good price is also constant, $p_t = p$ for all t .

When retailers set prices freely, their decisions are

$$\begin{aligned} \max_{\{p(i), y(i)\}} & \left(\frac{p_t(i)}{p_t} y_t(i) - \frac{p_t^W}{p_t} y_t(i) (1 - \tau(i)) \right), \\ & \text{subject to} \\ & y_t(i) \leq \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon} \tilde{y}_t. \end{aligned} \quad (6)$$

Each retailer is constrained by an isoelastic demand for its good i , $y(i) = (p(i)/p)^{-\epsilon} \tilde{y}$, where \tilde{y} denotes the aggregate demand for final goods. Here, $\tau(i)$ capture linear subsidies to retailer i , which are financed by lump-sum taxes $T_t = (p_t^W/p_t) \int_0^1 \tau(i) y_t(i) di$, levied on households. I introduce subsidies in order to correct the distortions that arise from monopolistic competition. This serves the purpose that the benchmark regime corresponds to the canonical search model of unemployment, augmented with cyclical opportunity cost. I set $\tau(i) = 1/\epsilon$ for all $i \in [0, 1]$, so that the optimality conditions of (6) yields $p(i)/p = p^W/p$ (zero price markups).

Set against the previous benchmark, the second regime requires that retailers have preset nominal prices $p(i) = p$. Their decisions can be written as

$$\begin{aligned} \max_{\{y(i)\}} & \left(\frac{p_t(i)}{p_t} y_t(i) - \frac{p_t^W}{p_t} y_t(i) (1 - \tau(i)) \right), \\ & \text{subject to} \\ & y_t(i) \leq \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon} \tilde{y}_t, \end{aligned} \tag{7}$$

where $p(i)/p$ is the retailer's fixed relative price, which is equal to unity by symmetry. The retailer chooses how much to produce subject to the constraint that its output cannot exceed the demand of its goods. I assume that subsidies are set as in the benchmark regime, that is, $\tau(i) = 1/\epsilon$ for all $i \in [0, 1]$.⁷ In the equilibria that I analyse, the retailers always accommodate aggregate demand, $y_t(i) = \tilde{y}_t$, since its marginal cost is strictly below its price (positive markups). As a result, output in this regime is ultimately determined by aggregate demand of final goods.

Matches are produced by a constant returns to scale matching technology, $h(u, v)$, where u are job searchers and $v = \int_0^1 v(j) dj$ are aggregate vacancies. The matching function is differentiable and strictly increasing in both arguments and satisfy $h(u, v) \leq \min(u, v)$. Conditions in the labour market are summarised by the labour market tightness $\theta \equiv v/u$. An unemployed household member finds a job with probability $f(\theta) \equiv h(u, v)/u$; and a vacancy is filled with probability $q(\theta) \equiv h(u, v)/v$. A useful property is $f(\theta) = \theta q(\theta)$.

Wages are determined via period-by-period Nash bargaining, where $\omega \in (0, 1)$ is the worker's bargaining power and the Nash solution requires

$$(1 - \omega)S_t^H = \omega S_t^W. \tag{8}$$

2.2 Steady state equilibria

A benchmark equilibrium is a path of prices, $\{p_t(i), p_t, p_t^W, w_t\}$, allocations, $\{c_t, c_t(i), m_t, n_t, y_t^W(j), h_t(j), v_t(j, i), v_t(j), n_t(j), y_t(i), \theta_t, u_t, \tilde{y}_t\}$, such that (i) households minimise consumption expenditure and solve (1), (ii) wholesalers minimise vacancies expenditure and solve (4), (iii) retailers solve (6), (iv) Nash solution is satisfied, and (v) all markets clear. An equilibrium under the demand-determined regime is similarly defined but final goods prices are preset for all t , which implies that retailers solve (7) instead of (6). I

⁷In online Appendix A.2 I show that demand-determined equilibria exist even if $\tau(i) = 0$ for all $i \in [0, 1]$.

restrict attention to symmetric non-stochastic steady state equilibria, thus all labels i, j are dropped. (See online Appendix A for detail derivations of the equilibrium under each regime.)

Consider the benchmark regime. Combining the household's and wholesalers' optimality, expressions (3) and (5) respectively, the Nash solution, (8), the free entry condition, $S^W = \gamma/q(\theta)$, and imposing steady state, yields

$$\frac{p^W}{p}a - z = \frac{\gamma}{q(\theta)} \left(\frac{\omega}{1-\omega} \beta(1-s)f(\theta) + \frac{1-\beta(1-s)}{1-\omega} \right). \quad (9)$$

Expression (9) is the key equilibrium relation to determine market tightness θ under the benchmark regime. Combining symmetry with retailer's optimality yields $p^W/p = 1$. The LHS denotes the FS, $a - z$, which is equal to productivity less the worker's opportunity cost of employment in terms of consumption. The opportunity cost is a function of aggregate household demand c and aggregate employment n , recall that $z = c^\zeta \cdot \psi'(n)$. To complete the characterisation under the benchmark equilibrium we express z as function of tightness, substitute it into (9) and solve for equilibrium tightness.

Equilibrium in the labour market requires that inflows to unemployment $s \cdot n$ equals outflows from unemployment $[1 - (1-s)n] \cdot f(\theta)$ and market tightness θ is related to employment by the Beveridge curve

$$n(\theta) = \frac{f(\theta)}{s + (1-s)f(\theta)}. \quad (10)$$

If firms post more vacancies, market tightness increases, which raises the job-finding rate $f(\theta)$ and increases employment n . In turn, aggregate hirings are equal to outflows from unemployment (equal to inflows in steady state), that is, $h(\theta) = s \cdot n(\theta)$.

Intermediate good's market clearing equates demand, x , with supply, y^W , and combining with producer technologies, yields $an = y^W = x = y$. The aggregate demand for final goods consists of two components: household's demand, which, after imposing steady state and $m = \bar{m}$ in (2), becomes a function of real balances,

$$c(\bar{m}/p) = (1-\beta)^{\frac{1}{\zeta}} \cdot \left(\frac{\bar{m}}{p} \right)^{\frac{\delta}{\zeta}},$$

and wholesalers demand for final goods, $(\gamma/q(\theta)) h$. Final goods market equilibrium requires $\tilde{y} \equiv c + (\gamma/q(\theta)) h = y$, which, combined with intermediate good's equilibrium, yields

$$c(\bar{m}/p) = an(\theta) - \frac{\gamma}{q(\theta)} h(\theta). \quad (11)$$

The RHS denotes output net of hiring costs and the LHS household consumption demand.

Taken together, (10) and (11) allow me to express the opportunity cost as function of productivity and tightness, $z(a, \theta) = (c(a, \theta))^\zeta \cdot \psi'(n(\theta))$, and substituting it into (9), I solve for equilibrium market tightness. The price level p adjust to satisfy (11) residually. Observe that household demand at equilibrium affects the FS through its impact on the opportunity cost z ; however, aggregate demand externalities are muted since relative prices are acyclical $p^W/p = 1$. This is because demand c accommodates any supply expressed by sellers through adjustments of the price level, placing no additional restriction on the equilibrium set, which imply that market tightness is determined independently of factors affecting household demand $c(\bar{m}/p)$.

It is useful for the subsequent analysis to provide a graphical interpretation of (11). Figure 1 plots net output (equal to household consumption in equilibrium) as a function of tightness. Assumptions in the primitives (see Appendix A.1) ensure that it starts from the origin, reaches a maximum at $\theta = \theta^*$ ($\partial c/\partial \theta = 0$), and cuts the x -axis at $\theta = \hat{\theta}$. Equilibrium under the benchmark regime does not place a priori any restrictions as to whether equilibrium tightness should be below, equal or above θ^* ; however, equilibrium under the demand-determined regime does. This is what we turn to next.

I introduce aggregate demand externalities by assuming that prices p are preset. Expressions (9), (10) and (11) are still relevant to describe equilibrium, but the logic of the previous argument does not apply. At preset prices, household demand is pinned down by real balances which, focusing on (11), implies that equilibrium market tightness cannot be determined independently from demand conditions. In other words, (11) imposes additional non-trivial restrictions at the equilibrium set. Simultaneously, relative prices p^W/p are a function of market tightness in order for (9) to be satisfied. The closed loop of interactions is initiated with the impact of consumption on tightness, which spillover to hiring decisions via its impact on relative prices which, in turn, affect aggregate demand via hiring costs and so on.

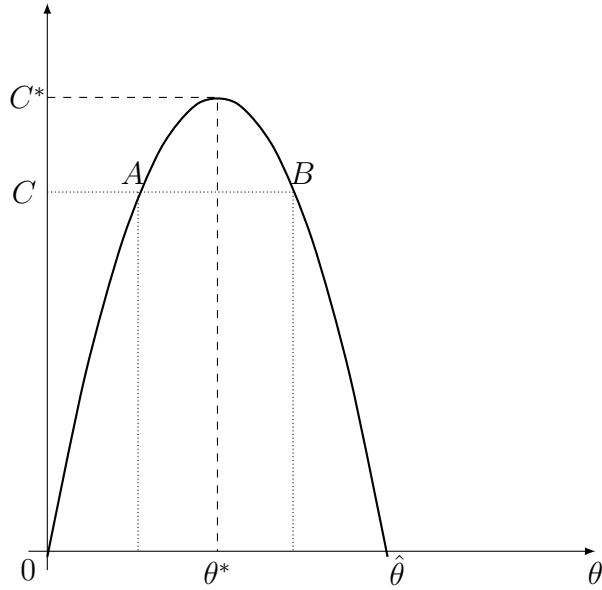


Figure 1: Expression (11)

Cyclicity of relative prices translates to cyclicity of price markups. Denote the retailer's price markup by \mathcal{M} . It is defined as the inverse of the real marginal cost, $\mathcal{M} \equiv 1 / [(1 - \epsilon^{-1})(p^W/p)]$. In turn, the fundamental surplus is equal to $\text{FS} \equiv [(1 - \epsilon^{-1})\mathcal{M}]^{-1} a - z$. Cyclicity in markups have a direct impact on the fundamental surplus.

Construction of a demand-determined equilibrium is explained with the help of Figure 1. Suppose an equilibrium under the benchmark case is somewhere in the graph, say at point B. Then, I fix the (final good) price close to the equilibrium price under the benchmark case. It follows that one of the solutions in (11) is in the neighbourhood of point B. Simultaneously, relative prices adjust to satisfy (9) and existence require price markups to be positive, $\mathcal{M} > 1$. A demand-determined equilibrium in the neighbourhood of the benchmark implies that frictions due to aggregate demand externalities are small (or even negligible). This is an appealing property given that demand externality spillovers persist in the long-run and the analysis has abstracted from policy.

However, at given preset prices, satisfaction of (11) is consistent with two equilibria (see Figure 1). All demand-determined equilibria to the left of the peak ($0 < \theta < \theta^*$) imply that improvements in productivity, a , generate a

contraction in the labour market⁸ and depress the opportunity cost, z (countercyclicality). While equilibria to the right ($\theta^* < \theta < \hat{\theta}$) are consistent with procyclical opportunity cost. (This is shown in Section 3 where I analyse comparative statics.) To that end, I restrict the analysis to demand-determined equilibria in the neighbourhood of the benchmark (small frictions) and to the right of the peak in Figure 1.

2.3 Discussion of the assumptions

I discuss three assumptions of the model: money in the utility function, separation between final and intermediate good production and preset prices.

The presence of money in the utility function is a convenient short-cut to obtain an interesting concept of aggregate demand in a model without savings; otherwise households would spend mechanically all their income on the produced good (Say's law). To see this, let me abstract from money in the utility. Then, consumption adjust to satisfy budget balance and (3) is the only relevant optimality condition. In turn, this implies that (11) does not impose any restrictions at equilibrium since c adjusts mechanically to satisfy any supply expressed by producers. We uncover the property of the benchmark equilibrium that market tightness is determined independently from demand. To avoid this problem, households must have the choice between consumption, labor and something else. The assumption of money in the utility function has been employed, among many others, by Blanchard and Kiyotaki (1987) and more recently by Michailat and Saez (2015), in order to avoid Say's Law.

The separation between retailers with monopoly power and perfectly competitive intermediate good producers is done to avoid interactions between price setting and wage bargaining at the firm level. This assumption has been employed by Ravenna and Walsh (2008) and Blanchard and Galí (2010), among others.

The modelling of retailer decisions, namely, (6) and (7), is borrowed from Korinek and Simsek (2016).⁹ The assumption of preset prices is made for analytical tractability, which allows a clean characterisation of comparative statics. To check the robustness of the results, I develop a version of the

⁸The same property is true in some New Keynesian models (Galí, 1999).

⁹Other models with fix prices and aggregate demand externalities include Barro and Grossman (1971), Benassy (1975) and Malinvaud (1977); more recently, Kokonas (2016) provided conditions for uniqueness of fix price equilibria in an economy with monetary and fiscal policy.

model in which prices are partially sticky (see Section 5 for details). Specifically, a fraction of retailers reoptimize their price level every period. These firms solve problem (6). The remaining fraction have preset prices in the sense that they do not reoptimize their price level every period. These firms solve problem (7). Also, I extend the previous version to include constant inflation and price indexation to long run inflation. After reviewing the microeconomic evidence on price adjustments (see Section 5 for related literature), I perform various quantitative exercises to show the robustness of my results.

3 Elasticities

This section characterises the elasticity of market tightness with respect to productivity changes, denoted by $\varepsilon_{\theta,a}$. In steady state the unemployment rate is $u = 1 - (1 - s) \cdot n(\theta)$, where the elasticity of unemployment to productivity changes is given by $\varepsilon_{u,a} = -(1 - u) \cdot \varepsilon_{f,\theta} \cdot \varepsilon_{\theta,a}$. High values of $\varepsilon_{\theta,a}$ produce big responses of unemployment to productivity changes. I compute the elasticity of market tightness to productivity changes for each regime in steady state, namely, the benchmark regime of flexible prices and the demand-determined output regime.

Proposition 1. (Elasticities)

(i) (Benchmark)

$$\varepsilon_{\theta,a} = \frac{\omega f(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}}{\omega f(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}\varepsilon_{q,\theta}} \cdot \frac{a - z\varepsilon_{z,a}}{a - z};$$

(ii) (Demand-determined output)

$$\varepsilon_{\theta,a} = \frac{\omega f(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}}{\omega f(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}\varepsilon_{q,\theta}} \cdot \left\{ \frac{(p^W/p)a - z\varepsilon_{z,a}}{(p^W/p)a - z} + \frac{(p^W/p)a}{(p^W/p)a - z} \varepsilon_{p^W/p,a} \right\}.$$

In the remainder of this section, I demonstrate the proof of Proposition 1.

Let me start with the benchmark regime. Rearranging (9), using the property $f(\theta) = \theta q(\theta)$ and ensuring that $p^W/p = 1$, yields

$$\frac{(1 - \omega)}{\gamma} (a - z(a, \theta)) = \omega\theta\beta(1 - s) + \frac{1}{q(\theta)} (1 - \beta(1 - s)); \quad (12)$$

recall that $z(a, \theta) = (c(a, \theta))^\zeta \cdot \psi'(n(\theta))$ after substituting (10) and (11) into z . Equation (12) defines equilibrium tightness as function of productivity. Differentiating it with respect to productivity, and rearranging, yields

$$\varepsilon_{\theta,a} = \frac{\omega\theta q(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}}{\underbrace{\omega\theta q(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}\varepsilon_{q,\theta}}_{\equiv \Upsilon}} \cdot \varepsilon_{FS,a}, \quad (13)$$

where the first term is defined as Υ for simplicity, and the second term is the elasticity of the fundamental surplus to productivity, $\varepsilon_{FS,a}$, given by

$$\varepsilon_{FS,a} = \frac{a - z\varepsilon_{z,a}}{a - z}. \quad (14)$$

Furthermore, the elasticity of the vacancy filling rate to tightness is given by $\varepsilon_{q,\theta} \equiv -\theta q'/q > 0$ and the elasticity of the opportunity cost to productivity is given by

$$\varepsilon_{z,a} = \zeta \cdot \{\varepsilon_{c,a} + \varepsilon_{c,\theta} \cdot \varepsilon_{\theta,a}\} + \varepsilon_{\psi',n} \cdot \varepsilon_{n,\theta} \cdot \varepsilon_{\theta,a}, \quad (15)$$

where $\varepsilon_{\psi',n} > 0$ denotes the inverse of the Frisch elasticity, $\varepsilon_{n,\theta} > 0$ derives from the Beveridge curve, $\varepsilon_{c,a} > 0$ is the direct impact of productivity on net output and $\varepsilon_{c,\theta}$ denotes its slope, which is positive (negative) if $\theta < \theta^*$ ($\theta > \theta^*$) (see Figure 1). The cyclicity of the opportunity cost is due to the cyclicity of the marginal utility of consumption and of the disutility cost of employment.

The term Υ has limited influence on the elasticity ($\Upsilon \approx 1.15$ in our calibration). If $\varepsilon_{z,a} = 0$, which obtains with linear utility ($\zeta = 0$) and infinite elastic labor supply ($\varepsilon_{\psi',n} = 0$), (13) and (14) coincide with the formulas of the canonical model. In that case, low values of $a - z$ generate large values of the elasticity (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017). Instead, augmenting the canonical model with procyclical opportunity cost ($\varepsilon_{z,a} \approx 1$ in our calibration) weakens substantially the impact of $\varepsilon_{FS,a}$ on the elasticity and the amplification mechanism. This is the Chodorow-Reich and Karabarbounis critique.

Next, let me proceed to the demand-determined regime. Equilibrium requires

$$\frac{(1-\omega)}{\gamma} \left(\frac{p^W}{p} a - z(\theta; \bar{m}/p) \right) = \omega\theta\beta(1-s) + \frac{1}{q(\theta)} (1 - \beta(1-s)), \quad (16)$$

$$c(\bar{m}/p) = an(\theta) - \frac{\gamma}{q(\theta)} h(\theta), \quad (17)$$

with $z(\theta; \bar{m}/p) = (c(\bar{m}/p))^\zeta \cdot \psi'(n(\theta))$ since consumption is pinned down by real balances at given preset prices. System of equations (16)-(17) define equilibrium tightness and relative prices as function of productivity. Differentiating (16) with respect to productivity yields

$$\varepsilon_{\theta,a} = \frac{\omega\theta q(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)}}{\underbrace{\omega\theta q(\theta) + \frac{1-\beta(1-s)}{\beta(1-s)} \varepsilon_{q,\theta}}_{\equiv \Upsilon}} \cdot \varepsilon_{FS,a}, \quad (18)$$

with

$$\varepsilon_{FS,a} = \frac{(p^W/p)a - z\varepsilon_{z,a}}{(p^W/p)a - z} + \frac{(p^W/p)a}{(p^W/p)a - z} \varepsilon_{p^W/p,a} \quad (19)$$

and

$$\varepsilon_{z,a} = \varepsilon_{\psi',n} \cdot \varepsilon_{n,\theta} \cdot \varepsilon_{\theta,a}.$$

The elasticity of the fundamental surplus to productivity is affected by two opposing forces. Procyclical relative prices ($\varepsilon_{p^W/p,a} > 0$) (or countercyclical markups) amplify the impact of productivity shocks on the FS by generating high values in (19), while procyclical opportunity cost ($\varepsilon_{z,a} > 0$) dampens that impact. The latter elasticity is determined only by cyclical fluctuations in the disutility cost of employment, given that household consumption is pinned down by real money balances.

Observe that (18) and (19) link the elasticity of market tightness to the elasticity of relative prices. To complete the characterisation, I differentiate (17) with respect to productivity to determine the elasticity of tightness and then, I substitute it back to (18) to determine the elasticity of relative prices.

Substituting the Beveridge curve and $h(\theta) = s \cdot n(\theta)$ into (17), yields

$$c(\bar{m}/p) = \frac{f(\theta)}{s + (1-s)f(\theta)} \left(a - s \frac{\gamma}{q(\theta)} \right);$$

differentiating it with respect to productivity, and subsequently, rearranging using the Beveridge curve once more and the property $f(\theta) = \theta q(\theta)$, yields

$$0 = n(\theta) + \frac{\partial \theta}{\partial a} \cdot \left\{ [a - (1-s) \cdot c(\bar{m}/p)] \cdot f'(\theta) - s\gamma \right\} \cdot \frac{n(\theta)}{\theta q(\theta)}, \quad (20)$$

where the two terms in the RHS characterise the direct and indirect impact of productivity on net output respectively. Rearranging yields

$$\varepsilon_{\theta,a} = -aq(\theta) \cdot \left\{ [a - (1-s) \cdot c(\bar{m}/p)] \cdot f'(\theta) - s\gamma \right\}^{-1}. \quad (21)$$

The first term is bounded above by a and has limited influence on the elasticity. The second term is the inverse of the slope of net output, and becomes unboundedly large as the slope goes to zero in the neighbourhood of θ^* (Figure 1). If net output is increasing in tightness ($\theta < \theta^*$), then the second term is positive and productivity increases induce a contraction in the labour market, and the opportunity cost is countercyclical ($\varepsilon_{z,a} < 0$); I do not focus on these type of equilibria. Instead, I restrict the analysis to equilibria in a neighbourhood of θ^* , where net output is decreasing in tightness ($\theta > \theta^*$), elasticity $\varepsilon_{\theta,a}$ attains large values and the opportunity cost is procyclical.

Large values of $\varepsilon_{\theta,a}$ are equivalent to strongly procyclical relative prices (strongly countercyclical markups). Solving (18) and (19) with respect to $\varepsilon_{p^W/p,a}$ yields

$$\varepsilon_{p^W/p,a} = \frac{(p^W/p)a - z}{(p^W/p)a} \left[\Upsilon \cdot \varepsilon_{\theta,a} + \frac{z}{(p^W/p)a - z} \cdot \varepsilon_{z,a} - \frac{(p^W/p)a}{(p^W/p)a - z} \right].$$

It follows that large values of $\varepsilon_{\theta,a}$ (and of $\varepsilon_{z,a}$) induce strongly procyclical relative prices.

An intuitive explanation goes as follows. A small fall in productivity induces a large fall in hiring costs and aggregate demand for final goods \tilde{y} . This translates into large falls in the intermediate inputs demand, which require large falls (increases) in relative prices (price markups) to clear the intermediate

good's market. Aggregate demand externalities spill-over to hiring decisions via lower relative prices. In turn, the marginal value of an additional worker falls more than the initial fall in productivity. Wages fall, reflecting both the lower opportunity cost and the lower value of an additional worker for the firm, but cannot absorb the fall in the marginal value of an additional worker. Thus, hirings adjust substantially. Compare with the mechanism under the benchmark case. Namely, reductions in the marginal value of an additional worker are absorbed almost entirely by wages, so that amplification of shocks is weak.

Remark 1. (Opportunity cost): a property of the model with preset prices is that the marginal utility of consumption is constant and does not affect the opportunity cost. This is relaxed in the model with partial stickiness, where marginal utility fluctuates with productivity. Furthermore, the analysis has abstracted from fluctuations in the intensive margin (hours of work), which actually, plays an important role in the Chodorow-Reich and Karabarbounis analysis of the opportunity cost. To that end, I extend the model to include fluctuations in the hours of work, and show that all results apply intact (see Section 5).

4 Numerical example

I compute a numerical example to illustrate the mechanism discussed in the previous section. As a benchmark against which to evaluate the results, I use the estimate of Chodorow-Reich and Karabarbounis (2016) for the elasticity of unemployment to productivity in the data, $\varepsilon_{u,a} \approx -9.5$.

I assume a Cobb-Douglas matching function $h(u, v) = \kappa u^\alpha v^{1-\alpha}$, with $\kappa > 0$, $\alpha \in (0, 1)$, and $\psi(n) = \chi n^{1+\xi}/(1+\xi)$, with $\xi > 0$ the inverse Frisch elasticity of labor supply and $\chi > 0$.

The exogenous parameters of the model are

$$\Xi = \left\{ \xi, \alpha, \omega, s, \zeta, \delta, \bar{m}, \beta, \gamma, \chi, \kappa, a, \epsilon \right\}.$$

I set $\delta = \zeta = 1$ (log-utility in consumption and real balances) and normalise $\bar{m} = a = 1$. Furthermore, following Chodorow-Reich and Karabarbounis (2016), I set externally $\beta = 0.99$, $s = 0.045$, $\alpha = 0.6$;¹⁰ and using common

¹⁰Mortensen and Nagypal (2007) use the same value for the elasticity of the matching function to vacancies.

calibrations in the business cycle literature, I set $\xi = 1$ and $\epsilon = 6$. The remaining parameters $\{\kappa, \omega, \gamma, \chi\}$ are used to hit four target.

Consider the benchmark equilibrium. Following the (monthly) estimates in Chodorow-Reich and Karabarbounis, I hit the following three targets: $\tilde{f} = 0.704$, $\tilde{q} = 0.71$ and $\tilde{z} = 0.74$,¹¹ which imply $\tilde{\theta} = 0.9915$ and $\tilde{u} = 0.062$. Consistent with my analysis, I restrict the equilibrium to the right of the peak in Figure 1. This is achieved by targeting a negative value for the slope of equilibrium net output. A convenient value to target is $\varepsilon_{c,\theta} = -0.013$, which places the benchmark equilibrium close to maximum net output.

The numerical value of parameters used to target the above aggregates are $\kappa = 0.7064$, $\omega = 0.16$, $\chi = 0.82$ and $\gamma = 0.95$.¹² The numerical value of the endogenous variables are $\tilde{c} = 0.9223$, hiring costs $:= \gamma\tilde{h}/\tilde{q} = 0.059$, $\tilde{p} = 0.010842$ and $n(\tilde{\theta}) = 0.981431$.

The elasticity of tightness to productivity is decomposed in two terms: Υ multiplied by the elasticity of FS to productivity. It follows that $\Upsilon = 1.15$, $\varepsilon_{z,a} = 1.07$, which imply $\varepsilon_{\theta,a} = 0.9175$. The elasticity of unemployment to productivity is $\varepsilon_{u,a} = -(1 - \tilde{u})(1 - \alpha) \cdot \varepsilon_{\theta,a} = -0.345$. The upshot is that the elasticity of unemployment to productivity under the benchmark case is almost 30 times below its empirical counterpart.

Notice that the elasticity of the opportunity cost (15) is a function of $\varepsilon_{c,\theta}$. Performing robustness checks, it turns out that small values of $\varepsilon_{c,\theta}$ have a negligible impact on the amplification mechanism under the benchmark case.

Next, consider the demand-determined equilibrium. The parameters in Ξ are equal to the values assigned above. The equilibrium is indexed by (preset) prices p , and existence requires positive markups, $\mathcal{M} > 1$. The only parameter to calibrate here is p . Assigning a numerical value should satisfy two goals. First, the price is calibrated close to \tilde{p} , so that allocations and relative prices under the demand-determined regime are close to the benchmark equilibrium values (small frictions arising from demand externalities). Second, the calibrated parameter should be consistent with price markups in the 10–20 percent range.¹³ Attaining both goals is made possible because of the assumption that monopoly subsidies are set as in the benchmark regime, so that \mathcal{M} is a function of relative prices and ϵ . I set the price at $p = 0.0108425$

¹¹Mortensen and Pissarides (1994) and Hall and Milgrom (2008) use an estimate for z close to our target, namely, $z = 0.71$.

¹²My estimate for γ is close to the estimate of 0.986 in Hall (2005); and the estimate of ω is close to the value of 0.1 used in Jäger et al. (2020).

¹³See, e.g., Rotemberg and Woodford (1993) and Basu and Fernald (1997).

($\approx \tilde{p}$) consistent with $\mathcal{M} = 1.19854$ (slightly below 20% markups).

The elasticity of tightness to productivity is computed from (21) and is equal to $\varepsilon_{\theta,a} = 79$.¹⁴ The opportunity cost is strongly procyclical with the relevant elasticity equal to $\varepsilon_{z,a} = 1.975$. Combining these results, we obtain $\varepsilon_{p^w/p,a} = 24$. Finally, the elasticity of unemployment to productivity is $\varepsilon_{u,a} = -29$, which is three times higher, in absolute terms, than its empirical counterpart. Evidently, aggregate demand spillovers via relative prices produce a strong amplification mechanism.

It is important to note that the elasticity of tightness computed from (21) is inversely related to the slope of net output. Low values of the latter induce very strong amplification, while higher values moderate the responses of the model, at the expense of higher matching costs as percentage of GDP. The targeted value of $\varepsilon_{c,\theta} = -0.013$ is a reasonable compromise.

5 Extensions

5.1 Intensive margin

I extend the model to include variation in hours per worker (intensive margin). The disutility cost of the large family is $\chi n_t (e_t^{1+\phi}/(1+\phi))$, where $\chi, \phi > 0$, n denotes variations along the extensive margin and e denotes variations of hours per worker. Linearity with respect to n implies that variations along the extensive margin do not affect the opportunity cost. This assumption allows me to isolate the impact of variation along the intensive margin on the opportunity cost. I continue assuming Nash bargaining contracting, but now workers and firms bargain over wages and hours. (All derivations and proofs are in online Appendix B.1.)

Proposition 2. (Elasticities)

(i) (Benchmark)

$$\varepsilon_{z,\bar{a}} = 1, \quad \varepsilon_{\theta,\bar{a}} = \Upsilon;$$

(ii) (Demand-determined output)

$$\varepsilon_{z,\bar{a}} = 1 + \varepsilon_{p^w/p,\bar{a}}, \quad \varepsilon_{\theta,\bar{a}} = \Upsilon \cdot \{1 + \varepsilon_{p^w/p,\bar{a}}\},$$

with $\bar{a} \equiv a \cdot e$.

¹⁴Interestingly enough, [Mortensen and Nagypal \(2007\)](#) report a similar magnitude for the search model with exogenously fixed real wages.

The analysis is similar to Section 3. Specifically, the term Υ has a limited influence on the elasticity and as a result, the amplification mechanism under the benchmark case is weak. In contrast, strongly procyclical relative prices (countercyclical markups) amplify the impact of aggregate shocks on the fundamental surplus, and as a consequence, on labour market outcomes.

5.2 Money growth

I extend the model to include constant, nonzero, growth rate of money. Let $m_t = \mu \cdot m_{t-1}$, where $\mu > 0$ is the gross growth rate and $m_{-1} = \bar{m} > 0$ is the initial condition. The government budget is given by $m_t - m_{t-1} = H_t$, where H are nominal transfers that accrue to the household budget constraint. Stationarity requires $m_t/p_t = m_{t-1}/p_{t-1} = \dots = m_0/p_0 = (\mu \cdot m_{-1})/p_0$, starting from an initial condition, say m_{-1} ; constant inflation (or deflation) $\Pi_t = p_t/p_{t-1} = \mu$; real transfers pinned down by real balances $H_t/p_t = (m_t/p_t)(\mu - 1)/\mu$; and relative prices and real variables are constant. Note that nominal preset prices p_t are indexed by calendar time; however, what is important for the analysis is that real variables and relative prices are constant in the steady state.

The consumption demand is determined from (2) and after imposing stationarity, yields

$$c = \left(1 - \frac{\beta}{\mu}\right)^{\frac{1}{\zeta}} \cdot \left(\mu \cdot \frac{m_{-1}}{p_0}\right)^{\frac{\delta}{\zeta}}.$$

The analysis in the baseline model assumed $\mu = 1$. The growth rate affects the equilibrium only through its impact on c . However, the analysis in Sections 2.2 and 3 did not depend on the specific functional form of household's consumption demand; and in the calibration exercise, the initial condition was normalised to unity without loss of generality. All the theoretical results follow intact as long as $\mu > \beta$; and given a suitable numerical value for μ , a re-normalisation of initial conditions attains exactly the same calibration.

5.3 Partially sticky prices

The baseline model features an extreme form of nominal price stickiness. I develop a version of the model in which prices are partially flexible and conduct various quantitative exercises. (Derivations of formulas and details pertaining to the quantitative exercise can be found in online Appendix B.2.)

A fraction v of firms have flexible prices, in the sense that they reoptimize their price level every period. These firms solve (6), which yields

$$\frac{p^{flex}}{p} = \frac{p^W}{p}, \quad (22)$$

with $\tau(i) = 1/\epsilon$ for all $i \in [0, 1]$. The remaining $1-v$ firms face sticky (preset) prices p^{sticky} as in the baseline model. Given preset prices, these firms solve problem (7). The price markup is $\mathcal{M} = (p^{sticky}/p) / [(1 - \epsilon^{-1})(p^W/p)]$.

Given the Dixit-Stiglitz technology, the aggregate price index is

$$p^{1-\epsilon} = v (p^{flex})^{1-\epsilon} + (1-v) (p^{sticky})^{1-\epsilon}. \quad (23)$$

Equations (9) and (10) are still relevant for the determination of equilibrium; and aggregation in the goods market imply that (11) reduces to

$$an(\theta) = \left(c(\bar{m}/p) + \frac{\gamma}{q(\theta)} h(\theta) \right) \cdot \Psi, \quad (24)$$

where household consumption depends on real balances as before, with p determined by (23), and Ψ denotes the degree of price dispersion, which is defined as

$$\Psi \equiv v \left(\frac{p^{flex}}{p} \right)^{-\epsilon} + (1-v) \left(\frac{p^{sticky}}{p} \right)^{-\epsilon}.$$

The equilibrium reduces to four unknowns, $\{p^{sticky}/p, p^{flex}/p, p^W/p, \theta\}$, in four equations, (9) and (22)-(24). Existence of a demand-determined equilibrium with partial stickiness requires positive markups, $\mathcal{M} > 1$.

The key object in the analysis is the fundamental surplus. It is equal to

$$\text{FS} \equiv \frac{p^W}{p} \cdot a - z = \underbrace{\frac{p^{sticky}}{p} \cdot \frac{1}{(1 - \epsilon^{-1})\mathcal{M}}}_{= p^W/p} \cdot a - z,$$

where the second equality follows from the definition of \mathcal{M} . As in the baseline analysis, suppose z and p^W/p are procyclical. Combining (22), (23) and the procyclicality of p^W/p , it follows that p^{sticky}/p and \mathcal{M} are countercyclical.

The impact of countercyclical markups on the cyclicity of the FS is now muted by the opposing countercyclicality of p^{sticky}/p . The key parameter that controls the relative cyclicity of p^{sticky}/p and \mathcal{M} is the frequency of price changes, proxied by the fraction ν of firms that reoptimize prices every period. If $\nu = 0$, then $\Psi = 1$ and $p^{sticky}/p = 1$, which suggests that the model with partial stickiness collapses to the baseline model with preset prices.

I extend the analysis to include constant inflation, $\Pi = p_t/p_{t-1} = \mu > 1$. The behaviour of firms with flexible prices is unchanged. The remaining $1 - \nu$ of firms passively index their prices to the long run inflation of the economy. Specifically, they set

$$\begin{aligned} p_t^{sticky} &= \lambda_t \cdot p_{t-1}^{sticky} \cdot \Pi \text{ for } t = 1, 2, \dots \text{ with} \\ p_0^{sticky} &\text{ predetermined,} \end{aligned}$$

where $\{\lambda_t\}_{t=1}^{\infty}$ denote predetermined indexation parameters. As before, these firms solve (7). Characterisation of equilibrium relative prices and tightness follows from the previous argument; however, one additional step is required, namely, the sequence $\{\lambda_t\}_{t=1}^{\infty}$ adjust appropriately so as to keep the inflation rate constant at $\mu > 1$. In turn, the indexation parameter at each date is a function of a and ν . If $\nu = 0$ and $\lambda_t = 1$, then the partial stickiness model collapses to the model with preset prices and constant inflation (Section 5.2), as long as p_0^{sticky} is common between the models.

To map the model to the data, I use the version with constant inflation. In turn, the relevant variables to link with the data are the frequency of price increases, ν ,¹⁵ and the inflation rate, $\mu - 1$. The empirical study of Nakamura and Steinsson (2008) on price adjustments in the United States shows that the frequency of price increases covaries strongly with inflation, whereas the frequency of price decreases does not;¹⁶ Klenow and Kryvtsov (2008) report a similar result. Nakamura and Steinsson (2013) review the literature and discuss evidence that firms adjust prices more frequently in periods of high inflation. This evidence points to upward price stickiness at the micro level.

¹⁵The passive rule of the remaining $1 - \nu$ of firms adjust to productivity changes without affecting real allocations and relative prices.

¹⁶The same results have been documented in the Euro areas as well.

Table 1: Elasticities

source	v	$\varepsilon_{\theta,a}$	$\varepsilon_{u,a}$	$\varepsilon_{z,a}$
non-stochastic menu cost	2%	78	-29	1.95
Nakamura and Steinsson (2008)	9%	76	-28.5	1.9
Nakamura and Steinsson (2008)	12%	75	-28	1.87
Bils and Klenow (2004)	21%	71.5	-26.9	1.79
Klenow and Kryvtsov (2008)	27.3%	69	-25.9	1.73

One way to capture the upward stickiness is with the rule: $v = \mu - 1$. The latter is consistent with the key prediction of any menu cost model, namely, that the fraction of firms that reprice in a given time interval is an increasing function of the inflation rate (see, among others, Sheshinski and Weiss, 1977; Caplin and Spulber, 1987; Golosov and Lucas, 2007; Nakamura and Steinsson, 2008). The average annual inflation rate in the US between 1990-2020 is 2%, and setting $v = 2\%$ ¹⁷ yields $\{\varepsilon_{u,a} = -29, \varepsilon_{\theta,a} = 78, \varepsilon_{z,a} = 1.95\}$, (almost) identical to the magnitude generated by the fix-price model (see Section 4).

An alternative calibration is to set v equal to the (monthly) estimate of the frequency of price changes (which includes price decreases) as reported by various papers in the literature, namely, Nakamura and Steinsson (2008), Klenow and Kryvtsov (2008) and Bils and Klenow (2004). Specifically, I set $v = 9\%$ (or $v = 12\%$),¹⁸ the lower (upper) estimate in Nakamura and Steinsson (2008); or $v = 21\%$ ¹⁹ as reported by Bils and Klenow (2004); or $v = 27.3\%$ ²⁰ as reported by Klenow and Kryvtsov (2008). (The results of the quantitative exercise are reported in Table 1.) Paralleling the discussion in Section 4, the partially sticky price model with strongly procyclical opportunity cost generates strong amplification relative to the flexible price (benchmark) model with strongly procyclical opportunity cost. The latter produces an estimate of $\varepsilon_{u,a}$ almost 30 times below its empirical counterpart ($\varepsilon_{u,a} \approx -9.5$), which is exactly the Chodorow-Reich and Karabarbounis critique; while the partially sticky price model produces strong amplification - $\varepsilon_{u,a}$ is 2.7 times above its empirical counterpart - and, despite its simplicity,

¹⁷Golosov and Lucas (2007) show that a calibrated version of a non-stochastic menu cost model with constant inflation to US data yields a repricing frequency approximately 2%.

¹⁸Implied (median) duration of price rigidity between 8 and 11 months.

¹⁹Implied (median) duration of 4.3 months.

²⁰Implied (median) duration of 3.7 months.

fits the data better.²¹

6 Conclusions

This note shows that big responses of unemployment to productivity changes are possible in an economy with procyclical opportunity cost, flexible wages and demand-determined output. The magnitude of the elasticity of market tightness is controlled by the elasticity of price markups to productivity changes. The upshot is that aggregate demand externalities spill-over to hiring decisions via the real marginal cost of retailers, amplifying the effects of productivity changes on hiring decisions. This mechanism highlights the important role played by aggregate demand and the strong cyclical property of markups in explaining the cyclical properties of labor market aggregates.

²¹As discussed in Section 4, $\varepsilon_{u,a}$ can be estimated closer to its empirical counterpart at the expense of higher matching costs as percentage of GDP.

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