# Crack detection in dielectric objects using electrical capacitance tomography imaging

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**Abstract-** Dielectric materials are becoming widely used because of their versatility and low-cost. Examples include carbon fibre, glass reinforced plastic, polymers and ceramics. These materials are increasingly used for aircraft structures and other important areas, such as transportation in gas pipelines, where materials require regular inspection. There is a need for a new technology for rapid inspection of dielectric materials. This paper focuses on one of the most challenging material inspection problems using an electrical capacitance tomography (ECT) system. The results show that volume cracks can be detected using ECT data and a state-of-the-art shape reconstruction algorithm. The reconstruction of cracks is difficult using conventional image based approaches.

Keyword: Crack detection in dielectric materials, Electrical capacitance tomography, permittivity mapping, inverse problem, level set method

## 1. Introduction

Electrical Capacitance Tomography (ECT) is a technique which uses external capacitance measurements to obtain the dielectric permittivity distribution within an object. The uses of this emerging technology are numerous. Particular applications are in the monitoring of industrial processes, such as finding out the concentration of one fluid in another such as water for oil in oil wells [6], monitoring the flow of fluids within pipes, or even the distribution of solids within fluids. ECT systems have also been shown to work particularly well when imaging combustion, due to the free electrons and ions influencing the permittivity. This paper proposes the use of ECT for non-destructive evaluation (NDE) of dielectric materials [8]. A very challenging ECT application in NDE is the detection of cracks. This paper presents a preliminary simulated study of the potential application of ECT identifying cracks in dielectric materials.

## 2. ECT System

ECT is a new and emerging tomographic imaging technique. The main advantage of this technique is, that it is non-invasive and non-destructive. However it is also reasonably inexpensive and fast. Figure 1 shows a complete system consisting of the sensor, a data acquisition system and a computer.

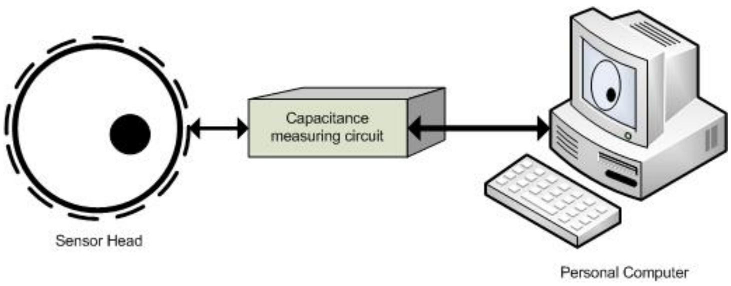



*Figure 1: An ECT system*

An ECT sensor basically consists of a non-conducting pipe, on which an array of electrodes is externally mounted, with an earthed screen around it. This earthed screen protects the data readings from being affected by external electrical field changes. The earthed screen may have further radial earthed screens in an attempt to improve the quality of the image by preventing flux density between adjacent electrodes. The design and effect of these radial screens are discussed in [1]. However, if there is a metal vessel, then the electrodes are mounted internally, and the vessel itself acts as the earthed electrical screen [5].

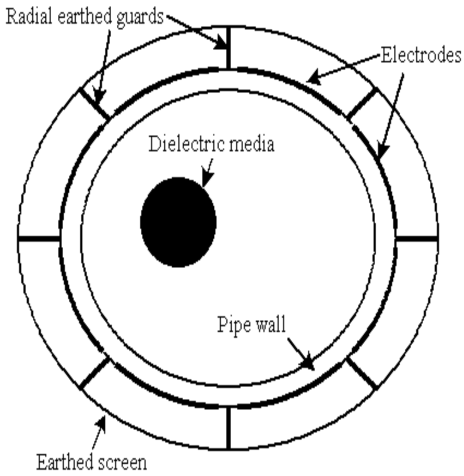



*Figure 2: A diagram of an ECT sensor with eight internally-mounted electrodes*

To obtain an image of the region under investigation forward and inverse problems need to be solved. The first stage involves solving the forward problem, which is: given an estimate for permittivity distribution ε and the electric potential at the boundary, find the electrical response $\underline{u}$, if

$$\nabla \cdot \left(\underline{\varepsilon}_0 \quad \nabla \underline{u}\right) \;=\; 0 \text{ on } \Omega \qquad (1)$$

$$\underline{u} \;=\; b \text{ on } \partial\Omega$$

Ω is the entire region including shielding area, and b is the electric potential. The inverse problem can be solved using the results of the forward problem. The inverse problem aims to find a new approximation for the permittivity distribution ε, if

$$J(\varepsilon o)\underline{x}=\underline{y} \qquad (2)$$

where $\varepsilon_0$ is the permittivity estimate used in the forward problem, J is the sensitivity matrix which is the Jacobian of the capacitance with respect to pixels evaluated at $\varepsilon_0$, $\underline{x} = \varepsilon - \varepsilon o$ is the difference between the permittivity distribution solution and the previous estimate of the permittivity distribution, and $\underline{y} = C(\varepsilon)-C(\varepsilon o)$ where C is the capacitance, calculated using $\underline{u}$ and the equation

$$C(\varepsilon) = \frac{1}{Area \quad of \quad electrode} \int \varepsilon \frac{\partial u}{\partial n} \qquad (3)$$

The new ε can then be fed back into the forward problem and the cycle can be repeated to improve the permittivity approximation [4].

**3. Level set method**

The level set method is a popular technique in image processing. It enables the user to easily follow shapes under a change in topology, and thus track the evolution of an interface. It is possible to perform numerical computations involving surface and curves on a Cartesian grid, without needing to parameterise the objects[2], [3],[5]. This is an Eulerian approach. Figure 3 shows how a level set representation can be used to account for shape evolution.

The main advantage of the method is that it is much easier to work with the level set function of a shape than the shape directly, since one can then avoid having to find new models when the shape deforms.

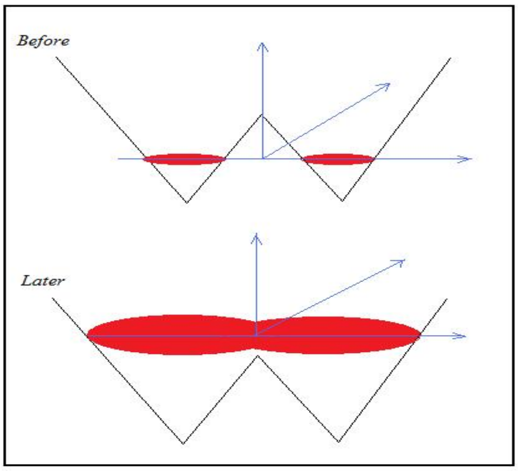



*Figure 3: The level set method naturally handling changes in topology through evolution*

The computation time can be reduced by taking what is known as the narrow band approach, which involves working only in a specific region neighbouring the zero level set, corresponding to the front [2].

The choice of narrow band width is very important. Sethian in[2] states that experience has shown that an appropriate band width is 6 points either side of the front. If used effectively this method can be ten times faster than the full matrix approach on a 160 by 160 grid[2].

This paper examines the use of the narrow band level-set method for detecting cracks. The algorithm is starting by choosing a level set function as an initial guess for the interface between the two materials and from this an initial estimate for the permittivity $\varepsilon$ distribution can be obtained:

$$\underline{\varepsilon} = \begin{cases} \varepsilon_i & for \quad interior \\ \varepsilon_e & for \quad exterior \end{cases} \tag{4}$$

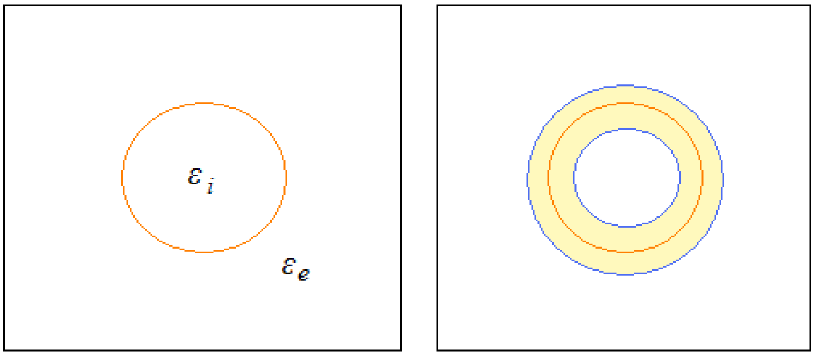



*Figure 4: Initial interface and narrowband*

The permittivity at each point $r$ can be described in terms of the level-set function depending on the position of the point $r$ with respect to the boundary $\partial D$ of the inclusion D as

and

$$\partial D = \{r : \Phi(r) = 0\}\,. \tag{5}$$

Here $\varepsilon_i$ and $\varepsilon_e$ are the permittivity of the inclusion and the background respectively. In crack monitoring the background permittivity is that of the dielectric structure and inclusion (here crack) has dielectric permittivity of air. The describing level-set function is a function form $R^3 \rightarrow R$ for three-dimensional case, and its value is zero on the boundary, it has a negative sign inside and a positive sign outside of the boundary.

The inverse boundary value problem is to find the boundary $\partial D$ (which in turn describes a conductivity distribution) that minimizes the mismatch between the measured and fitted voltage data. The mismatch function, $\Psi(\partial D)$, is defined as

$$\Psi(\partial D) = \left\| C_m - C(\partial D(\varepsilon)) \right\|_2^2 + G(\partial D(\varepsilon)) \tag{6}$$

where $C_m$ are the measured capacitances, and $C(\partial D(\varepsilon))$ are the capacitances calculated from the conductivity distribution, $\varepsilon$, derived from the corresponding boundary $\partial D$ and $G$ .is a regularisation term applied to the interface. In this paper the regularisation matrix is the identity matrix. The derivative of the voltage to a change in interface was derived in [5] and has been used in this study. Regularised Gauss-Newton update formula[4], [5] has been used to reconstruct the interface. The regularisation matrix is the identity matrix. Small values of the mismatch between measured and simulated capacitances indicate good locations for the boundary where as large values indicate poor estimation of the boundary. In particular, the boundary which makes this as small as possible is used, along with the inclusion and background permittivity, to give the level-set reconstruction. In practice this minimum cannot be found in a closed form and so a numerical procedure is required. The method can then be summarised as follows:

1. Solve the forward problem over the whole domain as usual using $\underline{\varepsilon}_0$ in order to obtain an estimate for the potential $\underline{u}$.
2. Calculate the derivative of this potential with respect to the normal to the curve. This will be used in solving the inverse problem to obtain a new approximation for the permittivity, $\underline{\varepsilon}$.
3. Solve the inverse problem over the narrow band. From this the new level set function can be obtained. The Jacobian in the inverse problem is the change in capacitance with respect to the motion of the interface, evaluated at the previous permittivity estimate. It directs the evolution of the interface towards the actual solution. The difference between the measured capacitance C and $f(\underline{\varepsilon}_0)$ gives the distance the solution should move.
4. If the residual $||C - f(\varepsilon_0)||_2$ is satisfactorily small then stop, otherwise continue on to step 1.
5. Create a band for the new interface by again selecting neighbouring points.
6. Return to step 1 where the new $\underline{\varepsilon}$ is fed back into the forward model as $\underline{\varepsilon}_0$, and

repeat the process.

**4. Results & discussions**

Figure 5 shows the target image of a crack in a two phase material, which we will attempt to reconstruct this using the method described above. Figure 5 also includes a comparison of the reconstruction achieved using this narrow band level set method and the reconstruction achieved by using the standard Gauss-Newton approach with Tikhonov regularisation. Firstly it is clear that the level set method produces a much improved reconstruction. There is some notable discontinuity in this reconstruction, but this may be improved using a finer mesh, as demonstrated later. Figure 6 shows the evolution of our level set function from an initial poor guess of a circle in the centre of the grid to a good reconstruction of the crack.

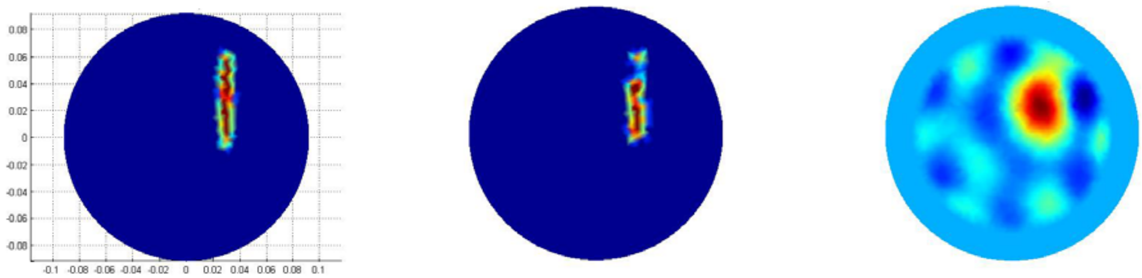
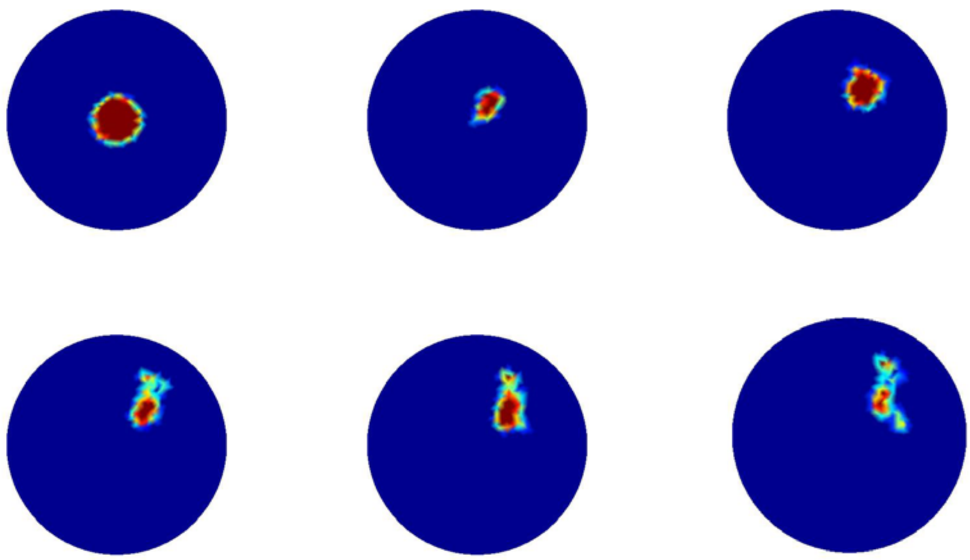



*Figure 5: True image, image reconstructed via narrow band level-set approach, image reconstructed via Gauss-Newton approach.*



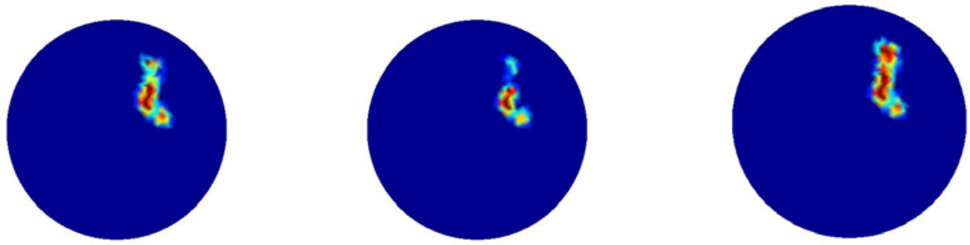


*Figure 6: Evolution of image reconstruction for crack problem*

The image can be further improved by using a higher density mesh. Figure 7 shows a similar target crack to test proposed approach and the improved image reconstruction using a denser mesh. This finer mesh has removed many of the inaccuracies such as discontinuities and extra pixels that were present in the narrow band level set reconstruction when using the more sparse mesh.

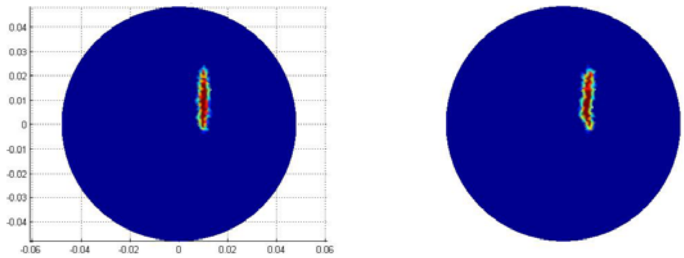



*Figure 7: True image and reconstructed image using narrow band level set approach and a finer mesh*

The narrow band level set approach also produces reasonable reconstructions for more complicated shapes such as the one seen in Figure 8. When compared with the reconstruction via the Gauss Newton approach, the improvement is dramatic.

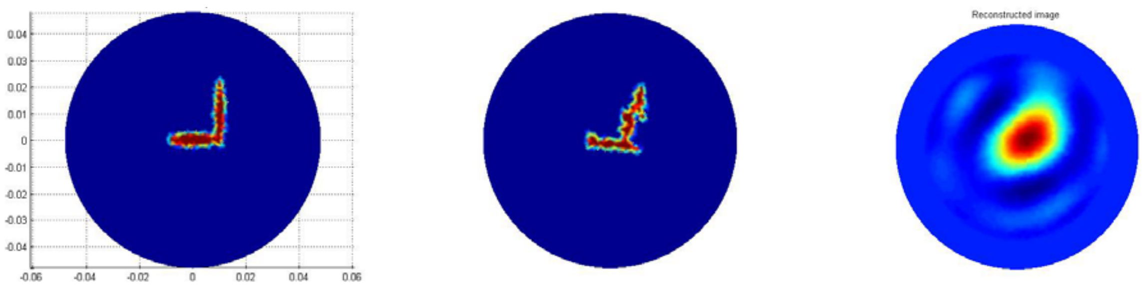



*Figure 8: True image with increased complexity, reconstructed image using narrow band level set approach and a finer mesh, reconstruction used Gauss-Newton pixel based method.*

Figure 9 shows that the method is capable of handling thin cracks. Figure 10 shows the convergence of the level set algorithm where the L2 norm differences between measured and estimated capacitance data has been presented. Figure 11 shows that it is possible to reconstruct multiple cracks using this method. However it is quite clear that the addition of separate cracks does reduce the quality of the reconstruction, but the reconstruction is still a lot better than that of the pixel based method.

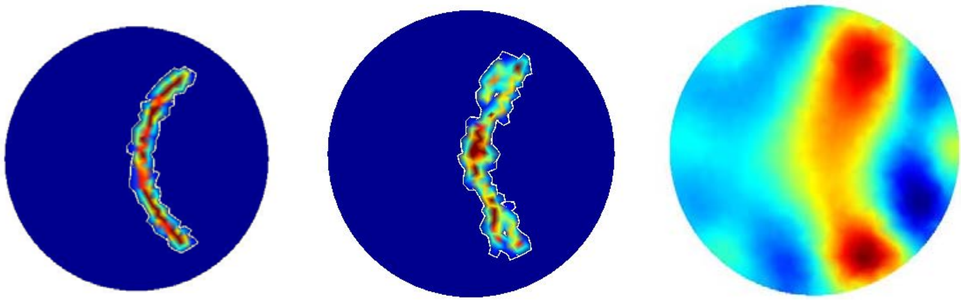


*Figure 9: Complex slim crack, true image and reconstruction via narrow band level set and traditional image reconstruction*

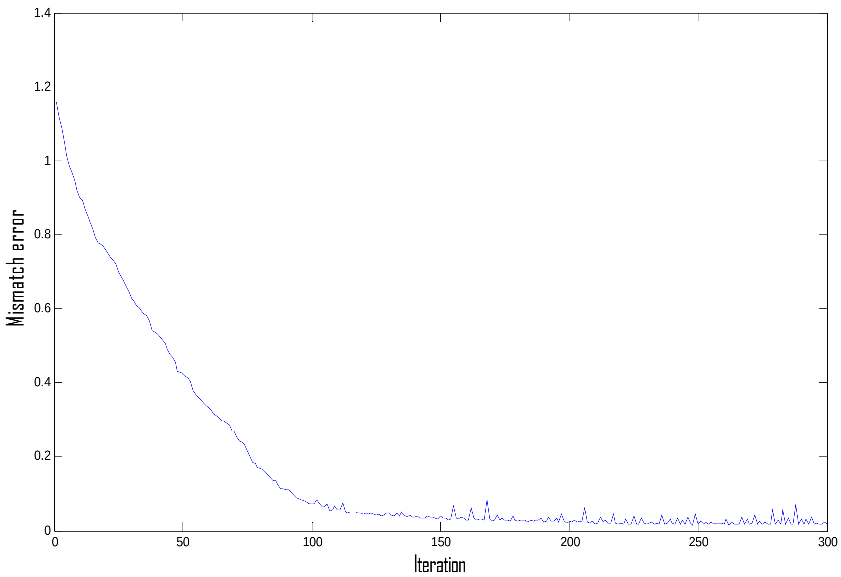



*Figure 10: The convergence plot showing the norm of error between measured capacitance and simulated one in each iteration of the level set algorithm*

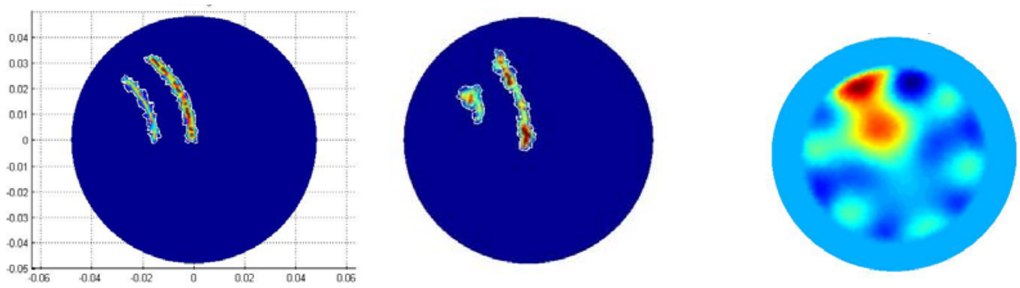



*Figure 11: True double crack image, image reconstruction using narrow band level set approach, image reconstruction using traditional pixel based approach*

## 5. Conclusion

These results suggest there is potential for this narrow band level-set approach to be used in the field of crack detection. Difficulties may arise for narrower cracks; however the results suggest that some improvement can be gained by creating a finer mesh. The cracks detected here are still volume cracks and we will further investigate the reconstruction of smaller cracks using specific methods developed for crack reconstruction (Alvarez et al 2006). We will further investigate the results of this work using experimental data.

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