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The Optimal Neglect of Inflation: An Alternative Interpretation of UK Monetary Policy during the “Great Moderation”

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Abstract
This paper argues that UK monetary policymakers did not respond to the inflation rate during most of the “Great Moderation” that ran from the early 1990s to the mid-2000s. We derive a generalisation of the New Keynesian Phillips curve in which inflation is a nonlinear function of the output gap and show that the optimal response of the policy rule to inflation depends on the slope of the Phillips curve; if this is flat, manipulation of aggregate demand through monetary policy does not affect inflation and so policymakers cannot affect inflation. We estimate the monetary policy rules implied by a variety of alternative Phillips curves; our preferred model is based on a Phillips curve that is flat when output is close to equilibrium. We find that policy rates do not respond to inflation when the output gap is small, a situation that characterised most of the “great moderation” period.

JEL: C51, C52, E52, E58
Keywords: monetary policy, Phillips curve, non-linearity

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1) Introduction

This paper argues that the behaviour of monetary policymakers is more subtle and complex than implied by the simple monetary policy rules that are widely used in the literature. The response of policy rates to changes in inflation depends on the slope of the Phillips curve relationship that relates inflation to the level of output. If, as many have argued, the Phillips curve is nonlinear, then the response of policy rates to inflation is not constant. More radically, if the Phillips curve is flat, inflation is unresponsive to output. In such regions, policy rates cannot affect inflation and it is therefore optimal for policymakers not to respond to inflation. Policymakers exhibit the optimal neglect of inflation.

Our evidence from UK data suggests that policy rates are unresponsive to inflation when the output gap is small but increasingly responsive as output moves away from equilibrium. As a result, we argue, policymakers have not responded to inflation when output was within 0.25% of equilibrium, while the Taylor Principle that real policy rates should move in the same direction as inflation is only satisfied when the output gap is above 1%. As the output gap widens, the effect of policy rates on inflation increases and so the response of policymakers is increasingly vigorous. This implies a rather different account of UK monetary policy over the past fifteen years, as output was close to equilibrium for most of this period. We identify a response of policy rates to inflation in 1993-4 and in 2000-1 only. Policymakers therefore have neglected the inflation rate for most of the inflation-targeting period. They have instead mainly responded to the output gap.
The paper is structured as follows. In section 2) we derive the optimal monetary policy rule when the Phillips curve is non-linear. The standard model of the New Keynesian Phillips curve, as described in Gali et al (1999) derives a linear relationship between inflation and proportional movements in real marginal cost around steady-state. A constant inter-temporal elasticity of elasticity labour supply function is then assumed, resulting in a proportional relationship between real marginal cost and the output gap. This then gives the familiar linear relationship between inflation and the output gap. We generalise this analysis by dropping the assumption of a constant inter-temporal elasticity labour supply. Doing so breaks the proportionality between movements in the output gap and real marginal cost and results in a nonlinear relationship between inflation and the output gap. To derive a monetary policy rule, we assume policymakers choose the nominal interest in order to minimise a quadratic loss function taking into account the macroeconomic structure defined by the aggregate demand and Phillips curves. This results in an optimal monetary policy rule that resembles the familiar Taylor rule, but where the response of policy rates to inflation is a function of the output gap; we argue that this interaction is a distinctive characteristic of the optimal policy rule with a non-linear Phillips curve.

In section 3), we consider the impact on monetary policy rules of the various Phillips curves that have been proposed in the literature. Most studies use a convex or concave Phillips curve; since these are flat when there is a slump or a boom respectively, they imply that the response of policy rates to inflation will be highly cyclical (cf Dolado et al, 2004, and Kesriyeli et al 2006). We argue that these effects may well be difficult to detect in our sample
period, which covers the “Great Moderation” that ran from the early 1990s to the middle 2000s, when booms and slumps were largely avoided. We also consider an alternative form of the Phillips Curve due to Solow (1969) in which inflation is highly sensitive to output in booms or slumps but unresponsive at other times. This implies that the optimal response of policy rates to inflation would be small in periods of stability but large when output is more volatile, which would suggest a low response of policy rates to inflation during the great moderation. We close this section by proposing a functional form for the Phillips curve that encompasses these cases and deriving the implied optimal policy rule.

In section 4) we outline our empirical methodology, explaining how we estimate policy rules for the alternative Phillips curve described above and how we confront the lack of identification of key parameters that bedevils work in this area. We present our estimates in section 5). We find that the data imply a policy rule consistent with a “Solow-type” Philips curve that is flat when output is close to equilibrium but which becomes steep as output moves away from equilibrium. Section 6) discusses the implications of these estimates for UK monetary policy in recent years. As discussed above, we suggest a different interpretation of recent policy decisions in which policymakers have largely neglected inflation. Section 7) summarises and concludes.
2) The optimal monetary policy rule when the Phillips curve is nonlinear

Our model is based closely on Gali (2008). The representative household has the utility function

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \]

Where \( C_t \) is a consumption bundle defined as \( C_t = \left( \int_0^1 \int_{\epsilon}^{1-\epsilon} C_t(i) \, di \right)^{\frac{1}{\epsilon}} \), \( N \) is hours of work and \( \beta \) is the discount factor. Maximising \( C_t \) subject to the budget constraints

\[ \int_0^1 P_t(i) C_t(i) \, di + Q_t B_t = B_{t-1} + W_t N_t, \]

where \( P_t(i) \) is the price of good \( i \), \( B_t \) are bond purchases, \( Q_t \) is the price of bonds and \( W_t \) is the wage rate, yields the demand functions

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{\frac{1}{\epsilon}} C_t. \]

where \( P_t = \left( \int_0^1 C_t(i)^{1-\epsilon} \, di \right)^{\frac{1}{\epsilon}} \) is the aggregate price index. With these, the budget constraint can be re-written as \( P_t C_t + Q_t B_t = B_{t-1} + W_t N_t \). The optimal choice of consumption and hours of work to maximise (1) subject to this constraint satisfies

\[ \frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t}. \]
There is a continuum of firms who have the production function

\[ Y_i(i) = A_i N_i(i) \]

who each face the demand curve in (3). Here we depart from Gali (2008) in assuming constant returns, for simplicity. Following Calvo (1983), each firm can adjust price with fixed probability \( 1 - \theta \). As is well-known (see, eg, Gali and Gertler, 1999), this model implies the following log-linearised Phillips curve relationship

\[ \pi_t = \beta E_t \pi_{t+1} + \left(1 - \theta \right) \left(1 - \beta \theta \right) \dot{mc} \]

where \( \dot{mc} \) is the proportional deviation of aggregate real marginal cost from its steady-state value.

To derive a Phillips curve relationship between inflation and the output gap, we initially follow Gali (2008) in assuming

\[ U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \]

In this case (3) simplifies to \( w_t - p_t = \sigma c_t + \phi n_t \), where a lower case variable denotes the natural logarithm of the corresponding upper case variable. Since the log of real marginal cost equals the log real wage less the log marginal product of labour and since consumption equals output, the log of
real marginal cost can be expressed as $mc_i = \sigma y_i + \phi n_i - a_i$. Using the argument in Gali (2008), p48), the proportional deviation of aggregate real marginal cost from its steady-state value can then be expressed as

\begin{equation}
\hat{mc}_i = (\sigma + \phi)(y_i - y^{ss}_i)
\end{equation}

Where a superscript “ss” denotes steady-state values. Substituting (7) into (5) we obtain the linear Phillips curve relationship

\begin{equation}
\hat{\pi}_i = \beta E \hat{\pi}_{i+1} + \frac{(1-\theta)(1-\beta\theta)(\sigma + \phi)}{\theta} \hat{y}_i
\end{equation}

where $\hat{y} = y_i - y^{ss}_i$, is the output gap (cf Gali, 2008, eqn 3.21).

It is worth noting that this derivation of a linear Phillips curve only holds in the special case of the constant intertemporal elasticity of labour supply assumed in the utility function in (6). This ensures that variations in labour supply are proportional to variations in the real wage and hence that variations in output are proportional to variations in real marginal cost, allowing us to make the transition from the linear “marginal cost” Phillips curve in (5) to the linear “output gap” Phillips curve in (8). Aside from this special case, variations in output are not proportional to variations to marginal cost, implying that the $\hat{mc}_i$ term in (5) is in general a nonlinear function of the output gap. As a result, the “marginal cost” Phillips curve in (5) will be linear but the “output gap” Phillips curve in (8) will in general be nonlinear.

The remainder of the section formalises this point. Dropping the assumption of a constant inter-temporal elasticity we generalise (6) to be
which implies that (3) becomes $w_t - p_t = \sigma c_t + \log \phi'(N_t)$ and log real marginal cost is $mc_t = \sigma y_t + \log \phi'(N_t) - a_t$. Using the argument on Gali (2008), p46, we can express the proportional deviation of aggregate real marginal cost from steady-state

(10) $\hat{mc}_t = \sigma (y_t - y^{ss}_t) + \log \phi'(N_t) - \log \phi'(N^{ss}_t) = \sigma \hat{y}_t + \log \phi'(N_t) - \log \phi'(N^{ss}_t)$

Here $\hat{mc}$ is not proportional to the output gap (other than in the special case of a constant elasticity labour supply curve, in which case (10) simplifies to (7)).

Since we are not assuming a simplifying special case, we have to use approximations in order to derive a more general relationship between $\hat{mc}$ and the output gap. Denoting $\log \phi'(N_t)$ by $g(N_t)$, noting $g(N_t) = g(Y_t / A_t)$, assuming a steady-state value of TFP of unity (following Blanchard and Gali, 2008) and using the approximation $g(Y_t / A_t) \approx g(Y_t) - g'(Y_t)Y_t(A_t - 1)/A_t^2$ gives

(11) $g(N_t) - g(N^{ss}_t) \approx g(Y_t) - g(Y^{ss}_t) - \frac{A_t - 1}{A_t^2} a_t (g'(Y_t)Y_t - g'(Y^{ss}_t)Y^{ss}_t)$

Since the second term on the RHS is likely to be close to zero for all but large deviations of TFP from steady-state, we can simplify this as to be $g(N_t) - g(N^{ss}_t) \approx g(Y_t) - g(Y^{ss}_t)$. We can further approximate this as
\[(12) \quad g(Y_i) - g(Y_{i-1}) = g(e^\nu) - g(e^{\nu_i}) = g'(e^{\nu_i})(y_i - y^*_i) + \frac{1}{2} g''(e^{\nu_i})(y_i - y^*_i)^2 = h(\hat{y}_i)\]

Substituting (12) into (10) gives

\[(13) \quad \hat{m}_c = \sigma \hat{y}_i + h(\hat{y}_i) = f(\hat{y}_i)\]

In (13) \(\hat{m}_c\) is a nonlinear function of the output gap, reflecting the fact that variations in labour supply are not proportional to deviations in the real wage and hence in real marginal cost. Substituting this into (5) gives

\[(14) \quad \pi_t = \beta E_{t+1} \pi_{t+1} + (1 - \theta)(1 - \theta) f(\hat{y}_i)\]

This is a generalisation of the well-known linear “output gap” Phillips curve in (8) in which the relationship between inflation and the output gap is nonlinear (unless the inter-temporal elasticity of labour supply is constant, in which case (14) simplifies to (8)). Both linear and non-linear forms of the Phillips curve share the “divine coincidence” that stabilising inflation is achieved by stabilising the output gap, a property highlighted by Blanchard and Gali (2007).

As Gali (2008), pp48-9 discusses, the first-order condition for consumption from the model above implies that aggregate demand is given by the “dynamic IS curve”
\begin{equation}
\hat{y}_t = -\rho(i_t - E_t \pi_{t+1} - \rho) + E_t \hat{y}_{t+1} + e_t^d
\end{equation}

where \( i = -\log Q \) is the nominal policy rate and \( \rho = -\log \beta \).

We assume that policymakers choose the policy rate in order to minimise a loss function defined over inflation and the output gap and taking into account the macroeconomic structure defined by the aggregate demand relationship in (7) and the nonlinear Phillips curve relationship in (6). We assume the per-period loss function is

\begin{equation}
L_t = \frac{1}{2} \left( \pi_t - \pi^* \right)^2 + \frac{\lambda}{2} \hat{y}_t^2 + \frac{\mu}{2} (i_t - i^*)^2
\end{equation}

where \( \pi^* \) is the inflation target or desired inflation rate, \( i^* \) is the equilibrium or desired policy rate and \( \lambda \) and \( \mu \) are positive coefficients that capture the relative weights on output and policy rates in the loss function. This is a conventional quadratic loss function. The first term expresses the loss from inflation as a quadratic function of the deviation of inflation from target, the second term expresses the loss from output as being quadratic in the output gap (both assumptions are standard, Clarida et al, 1999). The final term in the loss function ensures that the policy rate equals the equilibrium value, given by \( i^* = \pi^* + \pi \), where \( \pi \) is the equilibrium real policy rate, when inflation equals the target and the output gap is zero.

\footnote{There may be other ways of obtaining this result, for example, by using a model that uses union-firm bargaining or efficiency wages in place of labour market clearing (Blanchard and Gali, 2007, 2010).}
Assuming that policymakers choose the nominal policy rate at the beginning of period $t$ on the basis of information available at the end of period $(t-1)$, their optimisation problem is

$$ (17) \quad \min_{\hat{i}_t} \mathbb{E}_{t-1} \sum_{j=0}^{\infty} \beta^j L_{t+j} $$

Following the existing literature by solving this optimisation problem under discretion, the first-order condition is

$$ (18) \quad E_{t-1} \left[ \frac{\partial L_t}{\partial \pi_t} \frac{d\pi_t}{dt} \frac{d\hat{y}_t}{dt} + \lambda \frac{\partial L_t}{\partial \hat{y}_t} \frac{d\hat{y}_t}{dt} + \mu \frac{\partial L_t}{\partial \frac{\partial L_t}{\partial \pi_t}} \right] = 0 $$

Using the loss function and aggregate demand and Phillips curve relationships, the optimal monetary policy rule is

$$ (19) \quad \hat{i}_t = i^* + \frac{\partial \lambda}{\mu} E_{t-1} \hat{y}_t + \frac{\partial \gamma}{\mu} E_{t-3} f'(\hat{y}_t) (\pi_t - \pi^*) $$

where $\hat{i}$ is the optimal policy rate and $\gamma = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. This is the optimal monetary policy rule when the Phillips curve is nonlinear.

The key feature of this policy rule is that the response of policy rates to inflation depends on the slope of the Phillips curve, given by $f'(\hat{y}_t)$. The policy rule is a simple generalisation of the familiar Taylor rule, which is
obtained if the Phillips curve is linear, so \( f'(\hat{y}_t) \) is a constant. If the Phillips curve is flat, so \( f'(\hat{y}_t) = 0 \), then the policy rate does not respond to inflation; therefore this policy rule exhibits the optimal neglect of inflation. Of course, “optimal neglect” does not imply that policymakers are indifferent to inflation since the policy rule is driven by the desire to stabilise inflation. It simply implies that the inflation gap in effect drops out of the optimal policy rule when the Phillips curve is flat, implying that the best way for policymakers to stabilise inflation is to stabilise the output gap. More generally, the response of monetary policy to deviations of inflation from the target is stronger when the Phillips curve is steeper, implying that a given change in the policy rate has a stronger impact on inflation. We also note that the interaction between inflation and the output gap in (19) is a distinctive feature of policy rules when the Phillips curve is nonlinear. In the literature, nonlinearity is often introduced by assuming non-quadratic terms in the loss function but assuming that the Phillips curve is linear (e.g., Nobay and Peel, 2003, and Surico, 2007). To see the implications of this, we can express the loss function as

\[
L_i = L_x (\pi_t - \pi^*) + \lambda L_y (\hat{y}_t) + \frac{\mu}{2} (i_t - i^*)^2
\]

where \( L_x \) and \( L_y \) capture the impact of inflation and the output gap respectively on the loss function. If the slope of the Phillips curve is constant (given by \( \phi \)), then in this alternative model the optimal policy rule becomes
\[ i_t = i^* + \frac{\rho_\lambda}{\mu} E_{t-1} L_\pi'(\hat{y}_t) + \frac{\rho_y \phi}{\mu} E_{t-1} L_y'(\pi_t - \pi^*) \]

where \( L_\pi' \) and \( L_y' \) are the first derivatives of their respective functions. This loss function is nonlinear if \( L_\pi \) and \( L_y \) are not quadratic. However, it does not exhibit the interaction between inflation and the output gap that is in (19). That interaction term is therefore characteristic of a nonlinear Phillips curve\(^2\).

3) The effect of specific nonlinear Phillips curves

Having derived the general optimal monetary policy rule, we consider the implications of nonlinear Phillips curves that have been proposed in the literature. The most popular of these is a convex Phillips curve, first proposed by Turner (1995) and Laxton et al. (1995). Most existing models of monetary policy with a nonlinear Phillips curve consider this case (e.g., Dolado et al. 2004, and Kesriyeli et al. 2006). The convex Phillips curve is depicted in figure 1a), where it is apparent that inflation becomes highly sensitive to the output gap as output rises above equilibrium but that inflation and output become increasingly disconnected as output falls below equilibrium. In terms of the model of section 2), this implies that the elasticity of labour supply is decreasing in employment, so increases in output require only small increases in the real wage when employment is low but imply sharply increasing real wages as employment rise above its steady-state level. The implications of this for monetary policy are apparent from (11). The response of the policy

\(^2\) Another consequence of a nonlinear Phillips curve is that policy rates may fall when inflation increases (cf. Boinet and Martin, 2008)
rate to the inflation gap \( \frac{\partial i_t}{\partial (\pi_t - \pi^*_t)} \) depends on the slope of the Phillips curve. When output is well below equilibrium, the Phillips curve is horizontal and so the policy rate does not respond to the inflation gap. As output increases, the Phillips curve becomes steeper and so the policy rate becomes more responsive to the inflation gap. As output rises well above equilibrium, the Phillips curve is steep and the policy rate is highly sensitive to the inflation gap. In short, it is optimal for policymakers to respond strongly to the inflation gap in booms but to neglect this in slumps. This is summarised in figure 1b) where the optimal response of the policy rate to the inflation gap is depicted as a function of the output gap.

An alternative form for the Phillips curve has been proposed by Stiglitz (1997) and Eisner (1997) who argue that the Phillips curve is concave. As depicted in figure 1a) this is the polar opposite of the previous case, since inflation is now highly sensitive to the output gap when output is well below equilibrium and disconnected when output is well above equilibrium. This implies that the labour supply elasticity is an increasing function of employment. The optimal response of policymakers is therefore for the policy rate rates to be highly sensitive to the inflation gap in slumps but to neglect inflation in booms. This response is shown in figure 1b).

These Phillips curves arguably have little relevance to monetary policy in recent years. In both cases, the Phillips curve is approximately linear when output is close to equilibrium. Since fluctuations in output have been small during the recent period of the “great moderation”, any variations in the slope of the Phillips curve have been small and so a constant response of the policy rate to the inflation gap has not been too far from optimal.
However an alternative form of the Phillips curve implies that the stability of recent decades has increased the relevance of optimal neglect. Solow (1969) proposes an alternative form for the Phillips curve\(^3\) that is concave when the output gap is negative and convex when the output gap is positive (Dupasquier and Ricketts, 1998 consider a similar relationship while Filardo, 1998, considers related piece-wise linear Philips curve that is flat when the output gap is small). This type of Philip curve is depicted in figure 2a). The Phillips curve is flat when output is in a zone close to equilibrium and has an increasing slope as output moves away from equilibrium. This type of Phillips Curve is implied by a labour supply function that is highly elastic when employment is close to its steady-state value but increasingly inelastic as employment moves away from steady-state in either direction (Solow uses an argument based on capacity utilisation in his derivation; eg Solow, 1969, pp 10-12).

The implied optimal response of the policy rate to the inflation gap \(\frac{di_t}{d(\pi - \pi^*)}\) is depicted in figure 2b). When the output gap is small, the Phillips curve is horizontal. This implies optimal neglect; the policy rate does not respond to the inflation gap as policymakers recognise that they cannot affect inflation through monetary policy. As output moves further from equilibrium, in either direction, the slope of the Phillips curve increases and so the policy rate become more responsive to the inflation gap. This type of Phillips curve implies that optimal neglect was been common during the period of recent period of unusual output stability. Figure 2a) depicts two

\(^3\) We thank Adrian Winnett for suggesting this reference
alternative Phillips curves of this type. The first is symmetric (specifically, it has rotational symmetry around the vertical axis). This implies the same marginal response of the policy rate to the inflation gap when output is, for example, 2% below or 2% above equilibrium. The other is asymmetric, allowing for different marginal responses depending on whether output is above or below equilibrium.

We close this section by proposing a flexible functional form for the Phillips curve that can capture these alternative cases, given by

\[
(22) \quad f(y_t) = \phi y_t^I e^{\omega y_t} - 1
\]

where \( \phi \) and \( \omega \) are parameters (\( \phi > 0 \) but \( \omega \) is unrestricted) and \( I_\phi \) is a positive-valued integer. If \( I_\phi = 1 \) the Phillips curve is \( f(y_t) = \phi e^{\omega y_t} - 1 \), a functional form proposed by Nobay and Peel (2003). The slope of the Phillips curve is \( \phi e^{\omega y_t} \), which implies that the monetary policy rule is

\[
(23) \quad \hat{i}_t = i^* + \frac{\rho}{\mu} E_{t-1} y_t + \frac{\rho \phi}{\mu} E_{t-1} e^{\omega y_t} (\pi_t - \pi^*)
\]

The Phillips curve in (23) is convex if \( \omega > 0 \), in which case the marginal response of the policy rate to the inflation gap in (21) is also convex. The Phillips curve is concave if \( \omega < 0 \) in which case the response to inflation is concave. A linear Phillips curve is obtained when \( I_\phi = 1 \) and \( \omega \to 0 \). In that
case the slope simplifies to \( f'(y_t) = \phi \); substituting this into the policy rule in (21) we obtain the familiar linear Taylor rule

\[
i_t = i^* + \frac{\rho \lambda}{\mu} E_{t-1}y_t + \frac{\rho \gamma \phi}{\mu} E_{t-1}(\pi_t - \pi^*)
\]

If \( I_\phi = 2 \), the Phillips curve is \( f(y_t) = \phi y_t \frac{e^{\omega y_t^2} - 1}{\omega} \). This is the symmetric relationship depicted in figure 2a). The optimal policy rule is

\[
i_t = i^* + \frac{\rho \lambda}{\mu} E_{t-1}y_t + \frac{\rho \gamma \phi}{\mu} E_{t-1}\left(\frac{e^{\omega y_t^2} - 1}{\omega} + 2y_t^3 e^{\omega y_t^2}\right)(\pi_t - \pi^*)
\]

If \( I_\phi = 3 \) the Phillips curve is \( f(y_t) = \phi y_t^2 \frac{e^{\omega y_t^2} - 1}{\omega} \), which is the asymmetric relationship depicted in figure 2a). The optimal policy rule in this case is

\[
i_t = i^* + \frac{\rho \lambda}{\mu} E_{t-1}y_t + \frac{\rho \gamma \phi}{\mu} E_{t-1}y_t\left(2 \frac{e^{\omega y_t^2} - 1}{\omega} + 3y_t^3 e^{\omega y_t^2}\right)(\pi_t - \pi^*)
\]

The marginal responses of the policy rate to the inflation gap in these cases correspond to those shown in figure 2b). In both cases, the policy rate is unresponsive to the inflation gap when output is close to equilibrium, rising as the output gap widens. The policy rules in (25) and (26) are consistent with the symmetric and asymmetric responses respectively. In the latter case, the
direction of the asymmetry is determined by the parameter $\omega$; if this is positive, there is a stronger response when output is above equilibrium.

4) Empirical Methodology

In this section we discuss how to estimate the parameters of the non-linear optimal monetary policy rule in (21) when the Phillips curve is given by the class of non-linear functions in (22). Using (22), the policy rule can be written as

$$\hat{i}_t = i^* + \frac{\rho\phi}{\mu} E_{t-1} y_t + \frac{\rho y^2}{\mu} E_{t-1} y_t^2 (\frac{e^{\omega y}}{\omega} - 1 + I_\phi y_t e^{\omega y}) \left( \pi_t - \pi^* \right)$$

This is an extended Taylor rule, where the response of the policy rate to the inflation gap is successively

$$\frac{\rho\phi}{\mu} E_{t-1} e^{\omega y}, \quad \frac{\rho y^2}{\mu} E_{t-1} (\frac{e^{\omega y^2}}{\omega} + 2 y_t^2 e^{\omega y})$$

or

$$\frac{\rho\phi}{\mu} E_{t-1} y_t (\frac{2 e^{\omega y^2}}{\omega} + 3 y_t^2 e^{\omega y})$$

when $I_\phi$ equals 1, 2 or 3.

Our empirical strategy will be to estimate the policy rule in (27) for different values of $I_\phi$ and assess which model provides the best empirical account of policymakers’ actions. However it is apparent that many of the parameters in the policy rule are not identified, in particular, we cannot identify the parameter $\omega$. In order to identify this, we take a first order approximation of (18) around the point where $\omega = 0$. In doing so, we follow the approach of studies of non-quadratic preferences, that, in effect, approximate the nonlinear
policy rule around the linear Taylor rule. Using the approximations

\[ e^{\alpha y_t} \approx 1 + \alpha y_t, \quad \text{and} \quad \frac{e^{\alpha y_t} - 1}{\alpha} \approx y_t^2 + \frac{\omega}{2} y_t^2 \]

and substituting into (19), we obtain

\[
\hat{i}_t = \hat{i}^* + \omega_t E_{t-1} y_t + \omega_1 \left( 2 \phi_t - 1 \right) y_t^{2h_t} \pi_t - \pi^* + \frac{\omega}{2} \left( 3 \phi_t - 1 \right) y_t^{3h_t} \pi_t - \pi^* + \varepsilon_t
\]

where \( \omega_t = \frac{\rho^T}{\mu} \) and \( \omega_t = \frac{\rho^T \phi}{\mu} \).

We make two further refinements before estimation. First, we use rational expectations, giving

\[
\hat{i}_t = \hat{i}^* + \omega_t y_t + \omega_2 \left( 2 \phi_t - 1 \right) y_t^{2h_t} \pi_t - \pi^* + \frac{\omega}{2} \left( 3 \phi_t - 1 \right) y_t^{3h_t} \pi_t - \pi^* + \varepsilon_t
\]

where \( \varepsilon_t \) captures the expectational errors induced by replacing the expected values of variables with actual values. Second, we recognise that policy rates adjust slowly towards their optimal values (perhaps for reasons described in Woodford, 2003), so \( \hat{i}_t = \kappa(L)i_{t-1} + (1 - \kappa(L))i_t \), where \( \kappa(L) = \kappa_0 + \kappa_1 L + \kappa_2 L^2 + \ldots \) is a polynomial in the lag operator \( L \) and \( \kappa_0, \kappa_1, \kappa_2, \ldots \), etc are real-valued parameters. Using this in (29) we obtain our empirical model as
This is our empirical policy rule, which we estimate using GMM, exploiting the orthogonality condition in (28)

The following section presents estimates of this non-linear monetary policy rule. We do not attempt joint estimation of the policy rule with the non-linear Phillips curve. The recent controversy on specification and in particular on the measurement of key variables in the literature on Phillips curves (cf Gali, Gertler, Lopez-Salido, 2001, 2005, Sbordone, 2002, 2005, Rudd and Whelan, 2005 and Linde, 2005) means this is beyond the scope of this paper. We discuss this issue further in section 6).

5) Econometric Estimates

In this section we present estimates of the optimal monetary policy rule in (30). We use quarterly data for the UK for 1992Q4-2005Q1, covering both the “great moderation” period and the period of inflation targeting that began in late 1992. The policy rate is the 3-month treasury bill rate; this is widely used in the literature because, as Nelson (2003) discusses, it is closely correlated with the various policy instruments used in recent decades. We measure inflation as the annual change in the retail price index, following Kesriyeli et al (2006). We model the output gap as the difference between real GDP and a Hodrick-Prescott trend of real GDP; this also follows Kesriyeli et al (2006). Other authors (eg Nelson, 2003) model the output gap using a quadratic trend rather than the Hodrick-Prescott filter; Mihailov (2005) concludes that this
makes little difference to estimates of Taylor rules in a UK context. We find
that inflation and the output gap are stationary but that the order of integration
of the policy rate is more ambiguous; we assume that all variables are
stationary (see Clarida et al, 1998 and Dolado et al, 2004 for a discussion of
similar issues; also see Christensen and Neilsen, 2003, for a discussion of the
difficulties encountered when estimating policy rules as a cointegrating
relationship).

We begin with estimates of the simple Taylor rule, to provide
comparability with the rest of the literature and to act as a baseline for
estimates of our nonlinear models. We use two lags of the policy rate. We
find Column (i) of table 1) presents estimates of the Taylor rule in (16). We
estimate that $\omega_1 = 1.17$ and $\omega_2 = 2.17$, indicating a much stronger weight on
inflation than on the output gap. These estimates are consistent with other
recent estimates (Martin and Milas, 2004, Mihailov, 2005). The Taylor rule
assumes $I_\phi = 1$ and $\omega \to 0$. The estimates in column (ii) of table 1) also
assume $I_\phi = 1$ but allow $\omega$ to be estimated from the data. By doing so we
impose that the response of the policy rate to the inflation gap is an
asymmetric function of output but allow the data to determine whether this
asymmetric response is convex or concave. This model is not successful.
Although the estimates of $\omega_1$ and $\omega_2$ are similar to those in column (i), the
estimate of parameter $\omega$ is not significant and the equation standard error is
higher than in column (i). This is perhaps not surprising. As we noted above,
this model assumes a Phillips curve that exhibits nonlinearity in either booms
or slums but which is approximately linear at other times. This type of model
is unlikely to be successful in our sample period, which covers the “great moderation”.

We next consider estimates of models based on a Philips curve that is flat when output is close to equilibrium. Column (iii) of table 1) presents estimates of the model obtained when $I_\phi = 2$. This model requires that the response of policy rates to the inflation gap is a symmetric function of output in which the response is smallest when the output gap is small, but allows the data to determine how strongly policy rates respond to inflation when output is further from equilibrium. This model is also unsuccessful, having a higher standard error than the Taylor rule. Column (iv) of table 1) presents estimates of the model obtained when $I_\phi = 3$. This model is successful, with a standard error markedly lower than in other models the Taylor rule. We estimate $\omega = 0.20$; this implies that the Phillips curve is steeper, and thus that the response to inflation is stronger, when the output gap is positive.

The estimates in table 1) assume the policy rate responds to the contemporaneous values of inflation and output. Since models exist in which policy rates respond instead to the expected inflation gap in the next period, we also estimated two further models. The first is identical to (22) but where policy rates respond to the expected inflation rate one period ahead rather than to the contemporaneous inflation rate. The second is identical to (22) but where policy rates respond to both the expected inflation rate and output gap one period ahead. Estimates of these alternative models are presented in tables 2a) and 2b) respectively. In each case, the estimates are similar to estimates of the comparable model in table 1) and the best fit is again obtained when $I_\phi = 3$. In each case, however, the estimates in table 1) have
a lower standard error. Our preferred model is therefore that in column (iv) of table 1).4

6) Implications for UK monetary policy

Our estimates suggest that monetary policymakers in the UK have followed a more subtle policy rule than that implied by the Taylor rule. The conclusion that the best model is obtained when $I_\phi = 3$ implies that the response of policy rates to the inflation gap depends on output, being larger when output is further from equilibrium; the finding that $\omega = 0.2$ implies an asymmetric response that is stronger when the output gap is positive. Figure 5) depicts the estimated response of policy rates to the inflation gap as a function of the output gap; this is obtained by calculating the value of

$$\frac{\rho_y}{\mu} f'(y)$$

for different values of the output gap, using the parameter estimates in column (iv) of table 1). There is no response to inflation when output is between -0.25% and 0.2% of equilibrium. The Taylor principle that policy rates should not accommodate inflation is only satisfied when the output gap is above 0.75% or below -0.85%. The response to inflation increases sharply thereafter.

The implications of this for the policy rate are shown in figure 6). This depicts the deviation of policy rates from equilibrium implied by the Taylor

---

4 The estimates in tables 1)-2) use a common instrument set. To check the sensitivity of these estimates to the instruments, we found, using different instrument sets for each model, that estimates of models with $I_\phi = 1$ and $I_\phi = 2$ could be improved but that estimates for $I_\phi = 3$ remained superior. It appears, therefore, that our estimates are robust to this.
rule, calculated as \( \frac{\rho_y \phi}{\mu} (\pi, -\pi^*) \) using the estimates in column (i) of table 1).

The Taylor rule calls for constant adjustment of the policy rate but the larger adjustments include increases in 1992-4, 1995-9 and in 2003-4 and decreases in 1999-2003. Figure 6) also depicts the deviation of policy rates from equilibrium implied by the estimates of our preferred zone-asymmetric model in column (iv) of table 1), calculated as \( \frac{\rho_y}{\mu} f'(y) (\pi, -\pi^*) \). This model implies much less frequent adjustment of policy rates. There was no response to inflation for most of this period. Policy rates deviated from equilibrium in two periods. First, rates were substantially above equilibrium from early 1993 to late 1994, when output was up to 2% below equilibrium and inflation was up to 3.5%. Second, rates were up to 2% below equilibrium in 2000-1 when output was up to 1% above equilibrium and inflation was some way below the target. However we note that although the deviation of inflation from the target in 1997-9 and in 2002-3 was at least as large as that in 2001, there was no response to inflation in these periods. This is because the output gap was smaller in 1997-9 and 2002-3 than in 2000-1. Our estimates imply that policymakers did not respond to these larger deviations of inflation from target because they recognised that they were unable to affect inflation while the economy was in a flat region of the Phillips curve and so inflation was unresponsive to aggregate demand.

This evidence suggests that policymakers have been rather passive in their response to inflation for the past fifteen years. In contrast to the constant adjustments to the policy rate that are implied by the Taylor rule, our preferred estimates suggest there was only an appreciable response to inflation when
deviations of inflation from the target coincided with output gaps of 1% or more. This does not, however, imply that policymakers were inactive. Rather, they attempted to keep inflation close to the target by keeping output as close as possible to equilibrium.

7) Conclusions
This paper has argued that the optimal response of the policy rate to the inflation gap is a function of the slope of the Phillips curve. This implies that the response to the inflation gap is a function of the output gap if the Phillips curve is non-linear. Non-linear Phillips curves in the literature have regions that are flat; our analysis implies that policy rates do not respond to the inflation gap in these regions. When estimated on UK data for the inflation targeting period that began in 1992, our model suggests that policymakers did not respond to the inflation gap because the economy was in a region where the Phillips curve was flat for most of this period.

Although we would argue that this analysis is interesting and plausible and produces new insights into monetary policymaking in this period, there is an obvious extension to the model. We have not attempted joint estimation of non-linear monetary policy rule and a non-linear Phillips curve in this paper, arguing that the lack of consensus on specification and estimation in the literature on Phillips curves makes this beyond the scope of this paper. We intend to return to this issue in future work.
Table 1

GMM Estimates of Nonlinear Monetary Policy Rules 1992Q4-2005Q1

\[ i_t = \kappa_1 i_{t-1} + \kappa_2 i_{t-2} + (1-\kappa_1-\kappa_2)(i^* + \omega_1 y_t + \omega_2 \{2I_{la} - 1\} y_t^{3I_{la} - 2}(\pi_t - \pi^*) + \frac{\omega_3}{2}(3I_{la} - 1)y_t^{3I_{la} - 2}(\pi_t - \pi^*)} + \varepsilon_t) \]

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<tr>
<td></td>
<td>Taylor rule</td>
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<tr>
<td></td>
<td>( \phi \rightarrow 0 )</td>
<td>( I_{la} = 1 )</td>
<td>( I_{la} = 2 )</td>
<td>( I_{la} = 3 )</td>
</tr>
<tr>
<td>( i^* )</td>
<td>5.372 (0.138)</td>
<td>5.261 (0.196)</td>
<td>5.189 (0.191)</td>
<td>5.393 (0.260)</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>1.165 (0.349)</td>
<td>1.045 (0.278)</td>
<td>2.281 (0.567)</td>
<td>1.388 (0.434)</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>2.169 (0.588)</td>
<td>2.091 (0.563)</td>
<td>2.320 (0.643)</td>
<td>0.342 (0.117)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>-0.288 (0.226)</td>
<td>-0.314 (0.023)</td>
<td>0.199 (0.023)</td>
<td></td>
</tr>
<tr>
<td>( \kappa_2 )</td>
<td>1.306 (0.087)</td>
<td>1.314 (0.089)</td>
<td>1.206 (0.086)</td>
<td>1.276 (0.072)</td>
</tr>
<tr>
<td>( \kappa_1 )</td>
<td>-0.522 (0.069)</td>
<td>-0.533 (0.057)</td>
<td>-0.401 (0.079)</td>
<td>-0.416 (0.080)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.901</td>
<td>0.895</td>
<td>0.894</td>
<td>0.909</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.347</td>
<td>0.362</td>
<td>0.363</td>
<td>0.336</td>
</tr>
<tr>
<td>J statistic</td>
<td>0.69</td>
<td>0.61</td>
<td>0.74</td>
<td>0.56</td>
</tr>
<tr>
<td>Het</td>
<td>0.46</td>
<td>0.08</td>
<td>0.40</td>
<td>0.58</td>
</tr>
<tr>
<td>norm</td>
<td>0.86</td>
<td>0.30</td>
<td>0.86</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Notes:
1) \( r \) is the 3-month treasury bill rate, \( \pi \) is the annual change in the retail price index and \( y \) is the proportional deviation of real GDP from a Hodrick-Prescott trend. We assume \( \pi^* = 2.5\% \)
2) Standard errors are reported in parentheses
3) In all columns, the instrument set comprises a constant and two lags of the policy rate and \((\pi - \pi^*)\) and four lags of \( y \) and \( y^*(\pi - \pi^*) \).
4) The J-Statistic reports the p-value of Hansen's test for the over-identifying restrictions
5) Het reports the p-value for the White test for the null of no heteroskedasticity and norm reports the p-value of the Jarque-Bera test of the null of normality of the residuals
Table 2
GMM Estimates of Nonlinear Monetary Policy Rules 1992Q4-2005Q1

\[ i_t = \kappa_1 i_{t-1} + \kappa_2 i_{t-2} + (1 - \kappa_1 - \kappa_2) (i^* + \omega y_{t-1} + \omega_2 (2I_\phi - 1) y_{t-1}^{2I_\phi - 2} (\pi_{t-1} - \pi^*) + \frac{\phi}{2} (3I_\phi - 1) y_{t-1}^{3I_\phi - 2} (\pi_{t-1} - \pi^*) + \epsilon_t) \]

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</tr>
<tr>
<td></td>
<td>( (\omega \rightarrow 0 ) )</td>
<td>( I_\phi = 1 )</td>
<td>( I_\phi = 2 )</td>
<td>( I_\phi = 3 )</td>
</tr>
<tr>
<td>( i^* )</td>
<td>5.824 (0.136)</td>
<td>5.697 (0.185)</td>
<td>5.449 (0.363)</td>
<td>5.399 (0.211)</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>0.847 (0.289)</td>
<td>0.826 (0.278)</td>
<td>2.066 (0.704)</td>
<td>2.530 (0.742)</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>3.322 (0.708)</td>
<td>2.860 (0.563)</td>
<td>2.066 (0.704)</td>
<td>1.104 (0.266)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>-0.187 (0.218)</td>
<td>-0.330 (0.023)</td>
<td>0.177 (0.004)</td>
<td></td>
</tr>
<tr>
<td>( \kappa_2 )</td>
<td>1.101 (0.090)</td>
<td>1.133 (0.094)</td>
<td>1.185 (0.074)</td>
<td>1.225 (0.075)</td>
</tr>
<tr>
<td>( \kappa_1 )</td>
<td>-0.364 (0.049)</td>
<td>-0.394 (0.060)</td>
<td>-0.351 (0.054)</td>
<td>-0.373 (0.059)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.846</td>
<td>0.858</td>
<td>0.825</td>
<td>0.897</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.436</td>
<td>0.425</td>
<td>0.471</td>
<td>0.362</td>
</tr>
<tr>
<td>J statistic</td>
<td>0.70</td>
<td>0.59</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
<td>het</td>
<td>0.02</td>
<td>0.01</td>
<td>0.07</td>
<td>0.41</td>
</tr>
<tr>
<td>norm</td>
<td>0.62</td>
<td>0.30</td>
<td>0.80</td>
<td>0.90</td>
</tr>
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</table>

\[ i_t = \kappa_1 i_{t-1} + \kappa_2 i_{t-2} + (1 - \kappa_1 - \kappa_2) (i^* + \omega y_{t-1} + \omega_2 (2I_\phi - 1) y_{t-1}^{2I_\phi - 2} (\pi_{t-1} - \pi^*) + \frac{\phi}{2} (3I_\phi - 1) y_{t-1}^{3I_\phi - 2} (\pi_{t-1} - \pi^*) + \epsilon_t) \]

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<td>( I_\phi = 1 )</td>
<td>( I_\phi = 2 )</td>
<td>( I_\phi = 3 )</td>
</tr>
<tr>
<td>( i^* )</td>
<td>5.125 (0.236)</td>
<td>5.300 (0.263)</td>
<td>5.398 (0.299)</td>
<td>5.501 (0.336)</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>2.519 (0.715)</td>
<td>0.602 (0.667)</td>
<td>4.680 (2.104)</td>
<td>4.529 (1.503)</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>3.319 (0.841)</td>
<td>3.449 (0.981)</td>
<td>6.663 (2.710)</td>
<td>1.870 (0.593)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>-0.297 (0.236)</td>
<td>-0.365 (0.022)</td>
<td>0.221 (0.008)</td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{|c|c|c|c|c|}
\hline
\kappa_2 & 1.003 (0.117) & 1.068 (0.129) & 1.190 (0.111) & 1.255 (0.095) \\
\kappa_1 & -0.294 (0.106) & -0.346 (0.121) & -0.355 (0.112) & -0.402 (0.096) \\
R^2 & 0.854 & 0.864 & 0.857 & 0.872 \\
s.e. & 0.426 & 0.415 & 0.426 & 0.403 \\
J \text{ statistic} & 0.95 & 0.72 & 0.81 & 0.76 \\
Het & 0.12 & 0.01 & 0.21 & 0.10 \\
Norm & 0.22 & 0.63 & 0.75 & 0.77 \\
\hline
\end{array}
\]

Notes: see Table 1)
Figure 1a) “Linear Phillips curve” depicts the Phillips curve when $\phi = 1$ and $\omega \rightarrow 0$.

“Convex Phillips curve” depicts the Phillips curve when $\phi = 1$ and $\omega > 0$.

“Concave Phillips curve” depicts the Phillips curve when $\phi = 1$ and $\omega < 0$.

Figure 1b) “Linear response to inflation” in the case where $\phi = 1$ and $\omega \rightarrow 0$.

“Convex response to inflation” in the case where $\phi = 1$ and $\omega > 0$.

“Concave response to inflation” in the case where $\phi = 1$ and $\omega < 0$.
note: “zone symmetric Phillips curve” depicts the Phillips curve when $I_\phi = 2$;

“zone asymmetric Phillips curve” depicts the Phillips curve when $I_\phi = 3$

note: this figure depicts the response of policy rates to inflation $\frac{di}{d\pi}$, implied by eqn (11). “Zone symmetric response to inflation” depicts this in the case when $I_\phi = 2$

“Zone asymmetric response to inflation” depicts this in the case when $I_\phi = 3$

this figure depicts the response of policy rates to inflation $\frac{di}{d\pi}$, implied by eqn (11) using the parameters from column (iv) of table 1)
Figure 3b)

notes:

“Nonlinear response” depicts the deviation of policy rates from equilibrium implied by the estimates of our preferred model in column (iv) of table 1), calculated as $\frac{\rho y}{\mu} \cdot f'(y)(\pi_t - \pi^*)$; “Linear response” depicts the deviation of policy rates from equilibrium implied by the Taylor rule, calculated as $\frac{\rho y \phi}{\mu} (\pi_t - \pi^*)$ using the estimates in column (i) of table 1).
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