COMPRESSING AND PROPAGATING
SOLITONS IN HOLLOW CORE
PHOTONIC CRYSTAL FIBRE

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A thesis submitted for the degree of Doctor of Philosophy
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August 2010

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Acknowledgements

Firstly, I would like to thank my supervisors, Jonathan and William for their time and patience and for being brave enough to take a chance on me.

Tori, thank you for putting up with my strops at having to work weekends, nights and every spare hour.

Pete and Charles, thank you for reading through this thesis; removing semicolons and improving its readability.

Jim, ay cor bait avin yam around: yam bostin ar kid.

I would like to thank anyone I played football with throughout my Ph.D. sorry if my tackling left you wondering if I knew the name of the game.

Fred, thanks for putting up with my ginger jokes and for allegedly emptying the bin that time.

I would like to thank my parents for always being there.

To anyone I have overlooked, I hope your lives were better for knowing me, knowing you bettered mine.

Geoff, there are three, that’s got to be a pint?
Abstract/Overview

The development of photonic crystal fibre from conventional optical fibre follows a trend in the development of materials, to create composites and structured materials on smaller and smaller scales. In fact the great success of photonic crystal fibre is largely due to the ability to structure it on scales comparable to the wavelength of light. It is this micron size structure that allows the creation of an (out of plane) optical bandgap in silica and allows hollow core fibre to guide light in an air core freeing the guided mode from the properties of bulk silica.

This thesis focuses on the propagation and compression of high peak power optical solitons in hollow core fibre. As the Kerr nonlinear response of air is approximately a thousand times less than that of silica, the air core of hollow core fibre can support much higher peak powers than conventional optical fibre without the manifestation of nonlinear effects, making it ideal for the delivery of high peak power laser pulses. Coupled with this, hollow core fibre has a large region of anomalous dispersion in its transmission window allowing optical pulses to be transmitted as temporal solitons freeing them from the effects of dispersion. The author started his Ph.D. in 2006, three years after the first demonstration of soliton propagation in hollow core fibre and as the first demonstrations of soliton compression in hollow core fibre were being undertaken. Work by the author to build upon these early demonstrations is presented in this thesis in the following manner:

Chapters 1, 2 & 3 are theory chapters. Chapter 1 explains the background waveguide theory and theory of nonlinear optics that is used throughout the thesis. Chapter 2 details the properties of photonic crystal fibres focusing on hollow core fibre. Chapter 3 details recent papers relevant to the propagation and compression of solitons in hollow core fibre.

Chapters 4, 5 & 6 are experimental chapters reporting work undertaken by the author. Chapter 4 focuses on modifying the nonlinearity of hollow core fibre and measuring the dispersion of hollow core fibre accurately. Chapter 5 focuses on the compression of chirped and unchirped picosecond pulses in dispersion decreasing hollow core fibre tapers. Chapter 6 reports the compression in hollow core fibre of femtosecond pulses centred at 540nm wavelength through soliton effect compression.
1 Introduction

1.0 Chapter overview

This chapter details the background theory necessary to understand the following chapters.

It has the following structure:

- Section 1.1 details the design of conventional step index fibre, spatial modes and attenuation.
- Section 1.2 details dispersion in a waveguide with no nonlinear response.
- Section 1.3 details nonlinear effects in a dispersionless waveguide.
- Section 1.4 details solitonic effects in a dispersive waveguide with a nonlinear response.

1.1 Background

Section 1.1 outlines basic theory about conventional step index fibres, it is not meant to be comprehensive and is designed to give an overview of the background theory necessary to understand the following chapters. For a comprehensive review the reader is referred to (Snyder and Love, 1983).

1.1.1 Step Index Fibre

Optical fibre in its most common from consists of a glass cladding with a doped glass core that has a slightly higher refractive index. The fibre is then jacketed in a polymer to provide protection (Fig. 1.1). As this polymer jacket has a higher refractive index than the cladding it also serves to remove light that might otherwise be guided in the cladding.

![Fig. 1.1. A schematic illustration of the cross section of a conventional step index optical fibre, showing the refractive index profile of the fibre.](image-url)

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Guidance of light in these fibres can be thought of in terms of a simplistic ray picture (Snyder and Love, 1983, pp3), whereby rays are guided by total internal reflection, such that the angle of incidence \( \theta_{inc} \) of a guided ray on the core cladding interface must satisfy the following equality.

\[
\frac{n_{clad}}{n_{core}} < \sin \theta_{inc}
\]  

(1.1)

A more accurate picture involves considering the wavevector of the propagating light. It is well known that the wavevector \( k \) of light propagating in an infinite bulk medium, is set by the refractive index of the medium \( n \) and the freespace wavevector \( k_0 \) such that \( n = |k_0|/|k| \). In a waveguide this wavevector (which is a vector quantity) is commonly split into two scalar components; the transverse wavevector \( k_T \) and the longitudinal wavevector \( \beta \) (which is referred to as the propagation constant). For a wave to be confined within the core, the propagation constant of the light (which is continuous across the core/cladding boundary) must be larger than the maximum wavevector supported by the cladding material.

\[
n_{clad} < \frac{\beta}{k_0} \leq n_{core}
\]  

(1.2)

1.1.2 Modes

Thinking of light as rays is useful to describe the guidance of light in macroscopic media, but is of little use for describing many of the effects seen when light propagates in optical fibres; as light is confined to structures comparable in size to its wavelength and therefore manifests many wave-like properties.

When Maxwell’s equations are used to perform a full analysis of how light propagates in optical fibre, a discrete set of transverse electric field profiles which propagate without (confinement) loss are found (Snyder and Love, 1983, pp209); these solutions are called bound modes (and will henceforth be referred to as modes). Modes have a transverse field profile that does not vary in magnitude with propagation, only the phase of the field varies in a manner determined by the propagation constant of the mode (section 1.2.1).

To further this explanation the modes of a one dimensional slab waveguide made of silica surrounded by air are shown in Fig. 1.2. A fundamental mode is always guided; this is a Gaussian like field profile of constant phase. As the index contrast or the size of the waveguide increases additional higher order modes become supported, and already supported modes become more localised in the silica. Waveguides that only support the fundamental mode are referred to as single moded, waveguides that support additional higher order modes are referred to as multi moded.

The modes of a slab waveguide can be thought of as analogous to the bound wavefunctions of a square potential well in quantum mechanics, whereby larger or wider wells supports more bound wavefunctions.
In a conventional step index optical fibre higher order modes begin to be supported when the normalized frequency $V$ is greater than 2.405, where:

$$V = \frac{2\pi r_{\text{core}}}{\lambda} \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2}$$  \hspace{1cm} (1.3)

A two dimensional waveguide such as a step index optical fibre can support light polarised in two transverse directions. If the index contrast between the core and cladding is small the allowed two dimensional field profiles can be thought to comprise two orthogonal linearly polarised modes (Gloge, 1971). Single moded fibres therefore are not truly single moded as they support two separate orthogonally polarised modes. As physically created optical fibres do not exhibit exact rotational symmetry due to slight imperfections which arise during fabrication, these two orthogonally polarised modes have different modal properties. This variation of modal properties with polarisation is referred to as birefringence.

It should be noted that in Fig. 1.2, no frequency dependence of the refractive index was simulated and that a frequency dependence of the propagation constant exists. This effect is referred to as waveguide dispersion. As the size of the core increases or the wavelength of light decreases the guided mode becomes more confined to the core of the fibre; the evanescent field decays in the cladding more quickly. The frequency dependence of the propagation constant (dispersion) is the subject of section 1.2.
1.1.3 Attenuation

As the amount of light lost in a length $L$ of a fibre takes the form of a fraction of the input light, decibels are used to specify an attenuation constant $\alpha_{dB}$.

$$\alpha_{dB} = \frac{-10}{L} \log \left( \frac{P_{out}}{P_{in}} \right)$$  \hspace{1cm} (1.4)

A typical value for commercially available fibre can be as low as 0.2dB/km at 1550nm. This value varies by several orders of magnitude across the guided wavelength range. Fig. 1.3 shows a typical attenuation spectrum for a step index fibre in which three sources of loss are identified, the attenuation reaches a minimum value at 1550nm in a wavelength range which is referred to as the telecommunications window.

**Losses occur from three main sources:**

- Losses for Rayleigh scattering increase as the wavelength of light approaches that of the size of density fluctuation frozen in the glass during the manufacturing process. Loss from Rayleigh scattering scales as:

  $$\alpha_{Rayleigh} = \frac{C_R}{\lambda^4}$$  \hspace{1cm} (1.5)

  where $C_R$ typically takes a value between 0.7-0.9dB/(km.$\mu$m$^2$) (Agrawal, 2007, pp6) in silica fibres.

- Silica has vibrational resonances in the infra red and losses increase rapidly beyond 1.6$\mu$m.

- Across the transmission window loss is affected by impurities present in the glass, the most significant being absorption from the OH group. The group’s fundamental vibrational resonance is centred at 2.73$\mu$m, it however has harmonics at 1.38$\mu$m, 0.95$\mu$m and 0.72$\mu$m as well as a peak at 1.23$\mu$m. Typically the number of OH ions in high grade optical silica is less than one part per hundred million. This is achieved by displacing them during the manufacturing process with a more reactive ion such as Cl which has resonances in the UV.

![Graph showing Attenuation vs Wavelength](image_url)

**Figure 1.3.** The typical attenuation of a commercial step index fibre, the contributions to the attenuation from Rayleigh scattering and from vibrational resonances in the infra red are also shown.
1.2 Dispersion

Optical pulses require electric fields of differing frequencies to combine to create a region of intense field. As refractive index is in general frequency dependent it can be thought that these frequencies travel at different speeds, leading to the pulse broadening. This effect is referred to as dispersion and is the subject of section 1.2.

1.2.1 Pulse Propagation

In a bulk medium the phase of a monochromatic wave propagating solely in the x-direction \( E(x, t) = E_0 \exp i(\beta x - \omega t) \) varies with a wavevector defined by the refractive index of the medium:

\[
\beta(\omega) = n(\omega) k_0 = n(\omega) \frac{\omega}{c} = \frac{1}{v_p} \omega
\]  

(1.6)

where \( k_0 \) is the freespace wavevector of light, \( n(\omega) \) is the frequency dependent refractive index of the material and \( v_p \) is the phase velocity of the material: the velocity at which a point of constant phase propagates through the material. In a waveguide where waveguide dispersion exists as well as material dispersion this refractive index is replaced with an effective refractive index \( n_{\text{eff}} \) of the longitudinal propagating electric field.

Mathematically the frequency variation of the propagation constant is expressed in terms of its derivatives using a Taylor expansion around the central frequency \( (\omega_0) \) of the pulse such that

\[
\beta(\omega) = \beta(\omega_0) + \frac{\beta_1(\omega_0 - \omega)}{1!} + \frac{\beta_2(\omega_0 - \omega)^2}{2!} + \frac{\beta_3(\omega_0 - \omega)^3}{3!} + \cdots
\]  

(1.7)

where \( \beta_m = \frac{d^m \beta}{d\omega^m}|_{\omega_0} \)

(1.8)

How the different coefficients of the Taylor expansion \( (\beta_1, \beta_2, \beta_3 \ldots) \) affect pulse propagation are explained separately:

The first derivative: \( \beta_1 \)

When multiple waves of different frequencies combine to form a pulse, the envelope of the pulse propagates with a group velocity \( (v_g) \) defined as:

\[
\frac{1}{v_g} = \frac{d}{d\omega} = \beta_1 = \frac{d(n(\omega) \frac{\omega}{c})}{d\omega} = \frac{1}{c} \left( n(\omega) + \omega \frac{dn(\omega)}{d\omega} \right) = \frac{1}{v_p} + \omega \frac{d}{d\omega} \left( \frac{1}{v_p} \right)
\]  

(1.9)
The second derivative $\beta_2$

The frequency dependence of group velocity causes the different frequency components within a pulse to travel at different speeds. This process causes time bandwidth limited pulses (section 1.2.1) to broaden (disperse) as they propagate. The rate of this broadening is often expressed in terms of the variation of $\beta_1$ with wavelength; so called group velocity dispersion (GVD) which is often abbreviated to $D$.

$$D = \frac{d\beta_1}{d\lambda} = \frac{-2\pi c}{\lambda^2} \beta_2 = \frac{-2}{\lambda^2} \left( \frac{2}{d\omega} \frac{dn}{d\omega} + \omega \frac{d^2n}{d\omega^2} \right)$$  \hspace{1cm} (1.10)

If group velocity increases as the wavelength increases, then $D$ is negative and the dispersion is said to be normal. If group velocity decreases as wavelength decreases the dispersion is said to be anomalous. The wavelength where $D = 0$ is referred to as the zero dispersion wavelength (ZDW).

Higher order derivatives $\beta_3, \beta_4, \beta_5 ...$

$\beta_3, \beta_4, \beta_5 ...$ are referred to as higher order dispersion terms; the contribution of $\beta_3$ is referred to as third order dispersion, the contribution of $\beta_4$ fourth order dispersion and so on. These terms become important close to the ZDW or when the bandwidth of a propagating pulse becomes large (Agrawal, 2007, pp62). Third order dispersion is one of the main perturbations that affect soliton propagation in HC-PCF (sections 1.4.5 & 3.1).

1.2.2 Sellmeier Equation

The refractive index of a material is linked to the resonances of bound electrons in the medium and as such is frequency dependent. Far from these resonances the refractive index of bulk silica is well approximated by the following Sellmeier equation (Agrawal, 2007, pp6), where $\lambda_j$ represents the wavelength of a resonance and $a_j$ its strength.

$$n = 1 + \sum_{j=1}^{3} \frac{a_j\lambda_j^2}{\lambda^2 - \lambda_j^2}$$  \hspace{1cm} (1.11)

$\lambda_1 = 0.0684043 \mu m \ a_1 = 0.6961663$
$\lambda_2 = 0.1162414 \mu m \ a_2 = 0.4079426$
$\lambda_3 = 9.896161 \mu m \ a_3 = 0.8974794$

Using this relation and the expressions in section 1.2.1 the refractive index, group velocity and dispersion of silica is shown in Fig. 1.5.
1.2.3 Dispersive Wave Equation

The effect of dispersion on a pulse in a waveguide can be modeled using the following equation (Agrawal, 2007, pp62). It uses a retarded reference frame such that \( T = t - z/v_g \), \( A(z, t) \) is the envelope of the electric field along the waveguide and is related to the electric field of the pulse through \( E(z, t) = A(z, t)F(x, y)\exp(\text{i}\omega t) \) where the transverse electric field is assumed to be invariant with length as the pulse is assumed to exist in a guided mode

\[
\frac{i}{2} \frac{\partial A}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{i\beta_3}{3} \frac{\partial^3 A}{\partial T^3} + \ldots
\]

(1.12)

This equation can be readily solved in frequency space; solutions have the form

\[
\mathcal{A}(z, \omega) = \mathcal{A}(0, \omega) \exp \left( \frac{i\beta_2 \omega^2 z}{2!} \right) \exp \left( -\frac{i\beta_3 \omega^3 z}{3!} \right) \ldots
\]

(1.13)
If the input pulse takes the form of a Gaussian:

\[ A(z, 0) = \exp \left(- \frac{T^2}{2T_0} \right) \]  \hspace{1cm} (1.14)

and higher order derivatives of the propagation constant are zero \((\beta_3, \beta_4, \beta_5 \ldots = 0)\) it can be readily shown that the envelope will evolve in the following manner

\[ A(z, T) = \frac{T_0}{\sqrt{T_0^2 - i\beta_2 z}} \exp \left( \frac{-T^2}{2(T_0^2 - i\beta_2 z)} \right) \]  \hspace{1cm} (1.15)

Where the 1/e temporal width of the pulse broadens in the following manner

\[ \frac{T_{1/e}(z)}{T_0} = \left[ 1 + \left( \frac{i\beta_2 z}{T_0^2} \right)^2 \right]^{1/2} \]  \hspace{1cm} (1.16)

The length over which dispersive effects become important is often defined as \(L_D = T_0^2 / |\beta_2|\), and referred to as the dispersion length. An example of a pulse that has been broadened over 5 dispersion lengths is shown in Fig. 1.7, where the pulse envelope \((A(z, T))\) has been multiplied by a monochromatic carrier frequency \((\exp(\text{i} \omega t))\) to represent the electric field of the pulse. It is clear from Fig 1.7 and equation (1.15) that not only the envelope of the pulse has been affected by dispersion there is a frequency sweep visible across the pulse. This variation in the rate of change of the phase across the pulse is called chirp, (section 1.2.4).

---

**Fig 1.7.** The left pulse is a (time bandwidth limited) Gaussian pulse with a monochromatic carrier wave, the right pulse represents the propagation of the left pulse through a material with normal dispersion. A chirp is visible on the right pulse; the phase oscillates slower at the back of the pulse than at the front. The pulses are arbitrary shifted in time. The spectra of both pulses are identical. This figure was calculated using equation (1.15) where, \(T_0 = 1\), \(\beta_2 = 1\) & \(z = (0 \text{ and } 5)\) A monochromatic sinusoidal carrier frequency of \(\omega = 5\) was added to the envelope.
1.2.4 Time Bandwidth Limited Pulses and Chirp

To characterise a pulse not only the different frequencies that comprise the pulse must be known, the (relative) phase of these frequencies also must be known. The shortest possible pulse duration occurs when the phase of all the frequencies is constant at the centre of the pulse. In this case the pulse can be represented by a (real) envelope and single monochromatic carrier frequency.

If the spectrum of a Gaussian shaped envelope with single carrier frequency is calculated:

\[ E(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right) \exp(i \omega_0 t) \quad \hat{E}(\omega) = \sqrt{2\pi}\sigma_0 \exp\left(-\frac{\omega^2}{2}\right) \] (1.17)

a quantity called the time bandwidth product can be calculated as:

\[ f_{FWHM}t_{FWHM} = \frac{2\ln(1/2)}{\pi} \approx 0.44 \] (1.18)

where \( f_{FWHM} \) is the full width half maximum (FWHM) of the spectrum and \( t_{FWHM} \) is the FWHM of the electric field. This product sets the minimum pulse duration that a Gaussian shaped spectrum can generate. Different pulse shapes have different time bandwidth products, notably for solitonic applications (section 1.4) a sech pulse has a time bandwidth product of 0.315. Pulses that have the lowest allowable time bandwidth product (and hence a monochromatic carrier frequency) are referred to as time bandwidth limited.

Non time bandwidth limited pulses are referred to as chirped, the phase of the frequency components that creates them is non constant at the centre of the pulse. As a linear phase shift between different frequency components translates a pulse in time, chirped pulses occur when the relative phase of frequency components varies in a quadratic or higher order manner. Chirp arises when time bandwidth limited pulses propagate in a medium where \( \beta_2 \) is non zero. A quadratic phase shift arises when \( \beta_2 \) is non zero, a cubic phase shift when \( \beta_3 \) is non zero and so on, (equation 1.7).

A quadratic phase shift with frequency creates a linear change in the carrier frequency across the pulse which is referred to as linear chirp. Pulses that are broadened through normal dispersion are referred to as positively chirped; with the low frequency components being at the front of the pulse. Pulses broadened through anomalous dispersion are referred to as negatively chirped; with the high frequency components being at the front of the pulse.

---

**Key points of dispersion**

- Dispersion alters the phase of spectral components.
- This usually broadens the temporal envelope of the pulse and introduces a frequency sweep to the carrier wave.
- No new spectral components are created.
1.3 Nonlinear Optics

In linear optics the propagation constant of a wave (travelling in an isotropic medium) is modelled only to be function of frequency. However when intense fields are applied it becomes necessary to treat the propagation constant also as a function of applied field strength. The addition of this variable can change the frequency of a propagating light wave, something that is not possible in linear optics.

1.3.1 Introduction

In linear optics it is assumed that the induced polarisation of a dielectric by an applied electric field is linearly proportional the applied field.

\[ \vec{P} = \varepsilon_0 \chi^{(1)} \vec{E} \]  

(1.19)

Physically, this assumes that valence band electrons are moved a distance from the nucleus of the atom that is linearly proportional to the magnitude of the applied field, and hence the potential well in which the electrons are bound has an assumed shape. This assumption is valid when “weak” fields are applied, where electrons are displaced “small” distances, however it is not valid for “intense” fields where the electrons are displaced “large” distances. For these intense fields the induced polarisation is expanded as a power series of the applied field(s) where the induced polarisation varies nonlinearly with the applied field(s).

\[ \vec{P} = \varepsilon_0 \left( \chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \vec{E} + \chi^{(3)} \vec{E} \vec{E} \vec{E} + \cdots \right) \]  

(1.20)

1.3.2 Kerr Nonlinearity

In Kerr nonlinearity three additional assumptions are made.

- Firstly because the applied electric field(s) and induced polarisation are vector quantities even terms of the susceptibility tensor \( \chi^{(2n)} \) are assumed to be zero for materials with inversion symmetry; such as silica. These terms imply that there is a preferred direction in the material whereby the induced polarisation can be in the opposite direction to the applied field(s).
- Secondly it is assumed that terms higher than \( \chi^{(3)} \) are assumed to be negligible.
- Thirdly it is assumed that only one field is applied, such that the polarisation and applied electric field can be treated as scalar rather than vector quantities and only one component of the susceptibility tensor \( \chi^{(3)}_{xxxx} \) contributes to the induced polarisation.

The induced polarisation can therefore be written in the following form:

\[ P = \varepsilon_0 \left( \chi^{(1)} E + \chi^{(3)}_{xxxx} E^3 \right) \]  

(1.21)

When this expression is substituted into a definition of electric displacement field (Snyder and Love, 1983, pp398),

\[ \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 n^2 \vec{E} \]  

(1.22)

a nonlinear refractive index can be defined as (Agrawal, 2007, pp33)(Appendix 1):

\[ n(\omega, |E|^2) = n(\omega) + n_2 |E|^2 \quad \text{where} \quad n_2 = \frac{3}{8n} Re \left( \chi^{(3)}_{xxxx} \right) \]  

(1.23)

A typical value for the nonlinear index of silica is \( n_2 = 3 \times 10^{-20} \text{ m}^2 / \text{W} \), (Agrawal, 2007, pp17).
1.3.3 Self Phase Modulation: An Introduction

One of the most frequently observed nonlinear effects responsible for spectral broadening is self phase modulation (SPM). In linear optics the wavevector of a wave in an amorphous bulk medium is defined as \( k = k_0 n(\omega) \) and therefore is constant for a monochromatic wave irrelevant of circumstance. In a Kerr nonlinear medium this is no longer true as the refractive index is intensity dependent (equation 1.23) and hence the wavevector is intensity dependent.

\[
k = k_0 n(\omega, |E|^2) = \frac{-2\pi c}{\lambda}(n + n_2 |E|^2)
\]  

(1.24)

This variation in the wavevector means that different sections of what was a monochromatic wave no longer have a constant phase relation and oscillate with different frequencies.

\[
\omega = \frac{d\phi}{dt} = \frac{dk_x}{dt} = -\frac{2\pi c}{\lambda} \left( n + n_2 \frac{d|E|^2}{dt} \right) x \quad \delta\omega = -\frac{2\pi c}{\lambda} \left( n_2 \frac{d|E|^2}{dt} \right) x
\]  

(1.25)

In silica where \( n_2 \) is positive the frequency components at the front of the pulse (where \( d|E|^2/dt > 0 \)) are down shifted in frequency (\( \delta\omega < 0 \)) and frequency components at the tail of the pulse (where \( d|E|^2/dt < 0 \)) are up shifted (\( \delta\omega > 0 \)). This frequency shift chirps the pulse, section 1.2.4. The key point to note from equation (1.25) is that the effects of SPM are strongest for temporally short, high peak power pulses.

1.3.4 Self Phase Modulation: The Nonlinear Phase Shift Equation

The nonlinear phase shift equation is often used to model the effect of self phase modulation on a pulse in a waveguide.

\[
\frac{\partial A}{\partial z} = i\gamma |A|^2 A \quad \text{where} \quad \gamma = \frac{n_2(\omega_0)\omega_0}{c A_{eff}} \quad \text{and} \quad A_{eff} = \frac{\iint_{-\infty}^{\infty} F(x,y)^2 dx dy}{\iint_{-\infty}^{\infty} F(x,y)^4 dx dy}
\]  

(1.26)

Note: In HC-PCF \( \gamma \) can be thought to arise for two separate contributions: the overlap of the mode with the silica structure and the overlap of the mode with air. Thus \( \gamma \) is often replaced with two values: one arising from the air \( \gamma_a \) and one arising from the silica \( \gamma_s \) such that \( \gamma = \gamma_a + \gamma_s \). The effective areas used to calculate \( \gamma_s \) and \( \gamma_a \) are calculated by modifying the expression in equation (1.26) so that the integral in the denominator is only the silica or air regions respectively (the numerator remaining unchanged as it is the square of the total power of the mode), for example the effective area of the silica regions is:

\[
A_{eff(silica)} = \frac{\iint_{\text{silica}} |F(x,y)|^2 dx dy}{\iint_{\text{all regions}} |F(x,y)|^4 dx dy}
\]
1.3.5 **Self Phase Modulation: An Example**

If an initial input pulse is assumed to be Gaussian in shape and propagates in a medium in which only self phase modulation occurs we can write:

\[
A(0, T) = \exp\left(-\frac{T^2}{2T_0^2}\right) \\
A(z, T) = \exp\left(-\frac{T^2}{2T_0^2}\right) \exp\left(i\gamma \exp\left(-\frac{T^2}{2T_0^2}\right)^2 z\right)
\]

(1.28)  
(1.29)

By numerically taking the Fourier transform of this pulse one can easily compute its spectrum, Fig. 1.8. The reason for the oscillatory nature of the spectra is there are multiple contributions to the Fourier integral for a given frequency that occur at different times. Dependent on the exact frequency and spacing in time they either combine constructively or destructively.

![Graphs showing self phase modulation](image)

**Fig. 1.8.** The characteristics of the phase shift induced by self phase modulation. Three different phase shifts are shown \(\phi_{\text{max}} = 0, 1.5\pi, 3.5\pi\); where \(\phi_{\text{max}} = \gamma z P_0\) and is the maximum phase shift applied to the pulse. The electric fields distributions were generated from equation (1.29) by setting \(2T_0^2 = 1\) and adding a monochromatic sinusoidal carrier frequency of \(\omega = 20\). All quantities are normalized.

**Key points of SPM**
- **Through SPM the wavevector becomes intensity dependent.**
- **In pulses this creates new frequencies of light.**
- **The temporal envelope of the pulse remains unchanged.**
1.3.6 Raman Gain

As well as SPM another form of nonlinearity, Raman gain is frequently observed in HC-PCF. The Raman effect is an inelastic process, which occurs when a pump wave of energy $\hbar \omega_p$ is scattered by a molecule, to a lower energy state $\hbar \omega_s$ exciting a vibration/rotational state of the molecule $\hbar (\omega_p - \omega_s) = \hbar \Omega_R$. The lower energy scattered light is known as stokes radiation.

\[
\frac{dI_s}{dz} = g_r I_p I_s
\]

(1.30)

where $g_r (\Omega_R)$ is the Raman gain coefficient, for silica it is plotted in Fig 1.10.

Silica is an amorphous solid with a continuum of vibrational/rotational states stretching from $\Omega_R=0\text{THz}$ to $40\text{THz}$. At a pump wavelength of 1$\mu$m the Raman gain coefficient reaches its maximum value of $g_r (\Omega_R)=1\times10^{-6}$ for $\Omega_R=13.2\text{THz}$.

Fig 1.10. Adapted from Stolen et al. (1989). An example of the Raman gain curve of silica as a function of pump Stokes wave detuning.
1.4 Solitons

Sections 1.2 and 1.3 focused on the effects of dispersion and Kerr nonlinearity acting independently. In section 1.4 these effects will be treated jointly and a description of soliton propagation will be outlined.

1.4.1 Introduction

It was shown in sections 1.2 and 1.3 that when either dispersion or Kerr nonlinearity act independently on a pulse, either the temporal or spectral shape of the pulse changes. However when dispersion and nonlinearity act jointly, it is possible for a pulse to propagate in a waveguide with neither a change of its temporal or spectral shape. Such pulses are called fundamental (temporal) solitons and arise from a balance between anomalous dispersion and self phase modulation. This balance can be thought to occur in the following manner; self phase modulation down shifts frequency components at the front of the pulse, frequency up shifting the tail of the pulse. Anomalous dispersion redistributes these frequency components; shifting the high frequency components to the front of the pulse and the low frequency components to the tail.

This balance between self phase modulation and anomalous dispersion allows the pulse to maintain a constant temporal and spectral shape and occurs for pulses with the following temporal envelope.

\[ A(T) = \sqrt{\frac{|\beta_2|}{T_0^2}} \text{sech} \left( \frac{T}{T_0} \right) \]  \hspace{1cm} (1.31)

In telecommunications the broadening of pulses through dispersion limits the maximum achievable bit rate. Therefore the ability of solitons to maintain a constant temporal duration when propagating in the presence of dispersion has been explored in the telecommunication community for over the last 30 years. However solitons are yet to be used in this application, primarily for two reasons; the presence of additional undesirable nonlinear effects acting on the solitons due to the high peak powers necessary and the impressive performance of the alternative option of transmitting pulses linearly in ultralow dispersion fibres (Haus and Wong, 1996). Solitons are ideal though for the transmission of femtosecond pulses through meter long lengths of hollow-core photonic crystal fibre, as will be shown in the subsequent chapters.

1.4.2 The Nonlinear Schrödinger Equation

The equation which governs the propagation of pulses in a medium with both (second order) dispersion and Kerr nonlinearity is the nonlinear Schrödinger equation (NLSE).

\[ i \frac{\partial A}{\partial z} + \beta_2 \frac{\partial^2 A}{2! \partial T^2} = -\gamma |A|^2 A \]  \hspace{1cm} (1.32)

Though this equation can be solved using the inverse scattering method to prove the existence of solitons (Agrawal, 2007, pp41), numerical modelling is required for the vast majority of physical situations. The methodology most commonly used is the split step Fourier method, in which the NLSE is treated as two separate equations which were introduced in sections 1.2.3 & 1.34

The dispersive wave equation

\[ \frac{\partial A}{\partial z} = -i \beta_2 \frac{\partial^2 A}{2! \partial T^2} \]  \hspace{1cm} (1.33)

The nonlinear phase shift equation

\[ \frac{\partial A}{\partial z} = i\gamma |A|^2 A \]  \hspace{1cm} (1.34)
These equations are solved separately with small steps being taken iteratively between the nonlinear phase shift equation and the dispersive wave equation Fig 1.11. If the step size is small relative to the nonlinear and dispersion lengths, then the final solution accurately approximates the exact solution of the NLSE. This technique requires multiple Fourier transforms into the frequency domain and inverse Fourier transforms back into the time domain to be performed and therefore its computational speed is highly reliant on the speed of finite Fourier transform algorithms.

Fig 1.11. A schematic of the split step Fourier method being applied to solve the nonlinear Schrödinger equation.

The NLSE is frequently expanded to include terms such as higher order dispersion and Raman gain.

\[
\frac{\partial A}{\partial z} = - \left( \frac{\beta_2}{2!} \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{3!} \frac{\partial^3 A}{\partial T^3} + \ldots \right) + \left( i \gamma |A|^2 A - i \gamma T_r A \frac{\partial |A|^2}{\partial T} \right) \tag{1.35}
\]

The methodology of the split step Fourier method for solving this expanded equation or other modified NLSE equations is the same as for the NLSE. Small iterative steps are taken with terms being applied either in the frequency domain or time domain.
1.4.3 Higher Order Solitons

There is a second class of solitons that can exist in materials with anomalous dispersion and Kerr nonlinearity: higher order solitons. Unlike fundamental solitons, higher order solitons do not maintain a constant temporal shape and spectrum as they propagate. The temporal shape and spectrum undergoes a periodic evolution, recovering their initial form after a distance called the soliton period ($z_0$).

$$z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|}$$  \hspace{1cm} (1.36)

Higher order solitons are characterised by a positive integer called the soliton number $N = \sqrt{L_D/L_N}$, and at some point in their periodic evolution have a temporal envelope of the following form. (This form will henceforth be referred to as the initial temporal profile as it is close to the profile of pulses from solid state, transform limited, laser systems).

$$A(T) = N \sqrt{\frac{|\beta_2|}{T_0 Y}} \text{sech} \left( \frac{T}{T_0} \right)$$ \hspace{1cm} (1.37)

where $E_{\text{pulse}} \approx \int_{-\infty}^{+\infty} A(T)^2 dT = 2N^2 \frac{|\beta_2|}{T_0 Y} = \frac{1.76 N^2 \lambda^3 D A_{\text{eff}}}{2 \pi c T_{\text{PWHM}} n_2}$

Higher order solitons undergo a periodic evolution because the effects of SPM and dispersion are not balanced. From their initial time bandwidth limited sech shape, self phase modulation prevails over dispersion generating additional bandwidth. Temporally the chirp created by SPM is partially compensated by dispersion to form a chirped pulse that contains a compressed portion. For an N=2 soliton, dispersion then becomes more dominate and broadens the pulse back to its initial temporal shape; the pulse recovering its initial spectral shape though SPM. This behaviour is more complicated for N>2 solitons, as the initial pulses splits into (N-1) pulses. The temporal and spectral evolutions of the first two higher order solitons (N=2&3) are plotted in Fig. 1.12.

![Figure 1.12](image)

Fig 1.12. The spectral and temporal evolution of a fundamental soliton and the first two higher order solitons (N=1&2), propagating in a material with solely second order dispersion and Kerr nonlinearity. The colour scale is individually normalized for each plot and shows spectral intensity and intensity. The results were produced from equation (1.32) by setting $\beta_2=1$, $\gamma=1$ and $T_0=1$. The soliton period is $z_0 = \pi/2$. All quantities are normalized.

26
Through perturbation theory it can be shown (Agrawal, 2007, pp.138) that fundamental solitons are robust phenomena and are readily created. For instance if an N=1.2 pulse is initiated in a waveguide it will narrow temporally evolving asymptotically into a fundamental soliton as it propagates (Agrawal, 2007, pp.138). Fundamental solitons can be formed from pulses of a sech shape where N>0.5 and also are readily formed from chirped pulses or pulses of a different temporal shape. In the influence of perturbative effects such as loss and Raman gain; fundamental solitons evolve their temporal and spectral shapes to retain that of an N=1 fundamental soliton. A rule-of-thumb often used is that a perturbed fundamental soliton will evolve to regain an N=1 shape as long as the effect of the perturbation is small over the soliton period. In contrast to the stability of fundamental solitons, higher order solitons do not regain their shape when perturbed and propagations over multiple soliton periods are rarely seen outside theory, (Friberg and Delong, 1992).

1.4.4 Intrapulse Raman Scattering

Intrapulse Raman scattering is one of the most important higher order effects that should be considered when propagating solitons in optical fibres. Its effects become appreciable for picosecond or shorter solitons (Agrawal, 2007, pp.163) causing them to continuously down shift in frequency as they propagate through a phenomenon referred to as the soliton self frequency shift (Mitschke and Mollenauer, 1986). Physically this shift can be understood in terms of stimulated Raman scattering. Solitons which have a duration of approximately one picosecond or less have sufficient spectral bandwidth such that Raman gain can amplify the low frequency components of the pulse with the high frequency components acting as the pump.

In a waveguide with negligible loss and higher order dispersion, it can be shown (Agrawal, 2007, pp.163) that if the soliton retains its initial pulse length then the self frequency shift grows linearly with distance as:

\[ \Omega_p(z) = \frac{8\gamma_P |\beta_2|}{15\gamma_3^2} z \]  

(1.38)

Though experimentally the rate of this shift decreases with length as Raman gain is an inelastic process and therefore the energy of the soliton decreases as the propagation length increases. This decrease in energy is accompanied by the temporal duration of the soliton increasing as it evolves to regain a N=1 shape. An example evolution of a soliton self frequency shifting is shown in Fig.1.13.

Fig 1.13. The spectral evolution of an N=2 soliton in a medium with Raman gain. Higher order solitons do not retain a periodic evolution in the presence of perturbative effects. The fundamental soliton that is formed from the initial N=2 soliton undergoes a continuous self frequency shift to lower frequencies as it propagates. The rate of this shift decreases with propagation length as the temporal width and energy of the soliton decreases. This figure was generated by solving equation 1.35, using SSPROP (Murphy et al. 2007) setting \( \beta_2 \sim 1, \beta_3, \beta_4, \beta_5 \ldots = 0, \gamma = 1, T_0 = 1 \) and T_R=0.1. All quantities are normalized.
1.4.5 Higher Order Dispersion

Introducing higher order dispersion terms can complicate the observed nonlinear dynamics considerably. For simplicity solely the effect of third order dispersion (TOD) will be considered here. The primary effect of positive TOD is to delay the soliton in time (Agrawal, 2007, pp138), negative TOD advancing the soliton in time (relative to the retarded reference frame).

The temporal and spectral evolution of an N=2 soliton in a material with TOD is shown in Fig. 1.14. Higher order solitons do not retain their periodic nature under the influence of perturbations: the initial N=2 soliton evolving into a fundamental soliton. Observing Fig. 1.14.(a),(b)&(c) the radiation at \(\omega=12\) is dispersive radiation in the normally dispersive regime which is created at \(z=1\) when the spectral bandwidth of soliton overlaps into this regime, at \(z=10\) this dispersive radiation is located at \(t=8.5\) (which is not shown). At \(z=10\) the soliton is centred \(t=19\), \(\omega=1.5\). Also visible is some dispersive radiation centred at \(t=3.8\), \(\omega=0.5\) which beats with the soliton to create the fringe pattern visible at \(\omega=0.5\) in Figs. 1.14.(a,d). The fringes have a period of \(\omega=0.275\) which corresponds to the inverse of the separation in time between the dispersive radiation at \(t=3.8\) and the soliton at \(t=19\).
1.4.6 Supercontinuum Generation

Supercontinuum generation is the creation of a broad spectrum of light from a spectrally narrow source, using one or more nonlinear processes. Supercontinuum generation was first demonstrated in strongly non linear bulk mediums (Shimizu 1967, Alfano and Shapiro 1970). However in the past decade silica PCFs (section 2.1.3) have become the medium of choice to generate broad supercontinua instead of other “more” nonlinear mediums, because:

- Silica has a wide transmission window in the visible/IR wavelength range.
- The small waveguiding core allows a long interaction length with a small “spot size”.
- The dispersion of silica PCF can be altered from that of bulk silica allowing pulses to remain temporally narrow over a long length.

Supercontinuum generation in PCF can take place in many different regimes: CW, nanosecond, picosecond or femtosecond pumps sources can be launched into fibres where they are affected by anomalous, normal or anomalous and normal dispersion. Furthermore different techniques can be used to aid the broadening such as dual pumping, intermodal mixing and fibre tapering. This flexibility explains the wealth of papers that have been published on the subject however explaining these effects is outside the scope of this thesis; the reader is referred to Dudley et al. (2006).
2 Photonic Crystal Fibres

2.1.1 Introduction

Conventional optical fibres are solid structures that rely on dopants to provide the core/cladding index contrast required for total internal reflection to occur (section 1.1.1). Photonic crystal fibres (PCFs) (also known as microstructured fibres) do not rely on dopants for guidance. The cross sections of PCFs can consist of a regular array of air holes that run along the length of the fibre, which can provide a core/cladding index contrast (Fig. 2.1.c&d), (Kaiser and Astle 1974).

However PCFs are capable of much more than simply guiding light without dopants. The stack and draw method that is commonly used to fabricate them (section 2.3.2), allows the fabrication of silica and doped silica structures with incredible flexibility. Moreover it is possible to structure PCFs on scales comparable to the wavelength of light allowing significant control of fibre properties such as the number of supported modes, the group velocity dispersion (D) and the nonlinearity (γ) (Birks et al. (1997), Knight et al. (2000) and Cregan et al. (1999) respectively). Arguably one of the most impressive effects that can be achieved in PCF is the low loss guidance of light in the air core of a hollow core photonic crystal fibre (HC-PCF) (Fig. 2.1.a) this will be focus of section 2.2.

The fabrication of PCF and conventional optical fibres will be covered in section 2.3.

Fig. 2.1. SEMs of commonly used PCFs. (a) 7-cell hollow core fibre. (b) All solid band gap fibre. (c) Strand in air highly nonlinear fibre. (d) Endlessly single mode fibre. In (a),(c)&(d) black is air, grey is silica. In (b) the white dots are germanium doped silica situated in a (grey) silica background.
2.1.2 Quantifying PCF Structures

When PCFs are quantified three parameters are often used, Fig. 2.2:

- The pitch ($\Lambda$): the distance between adjacent air holes (or inclusions), this is equivalent to the lattice constant in crystalline solids.
- The air hole (or inclusion) size ($d$): approximated as a circle.
- The air filling fraction of the cladding: which is proportional to $(d/\Lambda)^2$.

Though these parameters are useful to quickly quantify the cladding of a PCF, air holes (or inclusions) are rarely well approximated by circles and additional parameters are used in numerically modelled claddings (Amezcua-Correa et al. 2006).

![Diagram of PCF parameters](image_url)

Fig. 2.2. Adapted from Amezcua-Correa et al. (2006). A schematic illustration of how the claddings of PCFs are described.

2.1.3 Forms of PCF

This section gives an overview of four forms PCF other than HC-PCF (which is covered in section 2.2).

1. Strand in air fibre
2. Endlessly single mode fibre
3. All solid band gap fibre
4. Kagome

The flexible nature of the stack and draw process (section 2.3.2) used to manufacture PCFs means that there is a continuum of different designs that can be created and therefore the four designs presented here should not be considered an exhaustive list.

1. Strand in air fibre

Strand in air fibres which are also known as highly nonlinear (HNL) fibres consist of a silica core suspended by a web of thin silica strands and are frequently used to observe nonlinear effects. The high index contrast created between the silica core and air cladding strongly confine light to a core that can be less than 1µm in diameter; the guided mode of which experiences a large nonlinearity $\gamma$ (equation 1.26). Furthermore by tailoring the pitch and air filling fraction of the cladding, significant control over the dispersion of the fibre is possible allowing a high figure of merit for nonlinear interactions to be obtained. (Dudley et al. 2006).
2. Endlessly single mode fibre

Endlessly single mode (ESM) fibre is designed to only guide the fundamental mode invariant of wavelength. The structure of the fibre is the same as that of the strand in air fibre but has an air filling fraction d/A <0.4 (Kuhlmeier et al. 2002). The single mode nature of the ESM fibre is due to dispersive nature of the cladding.

As the wavelength decreases, light in the silica regions of the cladding decays quickly in the air regions and the effective index of the cladding tends to that of silica. This creates a low index contrast between the core and cladding keeping the fibre single moded (equation 1.3), however also making it susceptible to bend loss (Birks et al. 1997).

As the wavelength increases, light in the silica regions of the cladding decays slower in the air regions and the effective index of the cladding decreases. However the fibre remains single moded because the ratio of core size to the guided wavelength decreases (equation 1.3). Single mode transmission at long wavelengths (like short wavelength transmission) is limited by bend loss, as the guided mode becomes less confined in the core.

3. All solid band gap fibre

The cladding of all solid band gap fibre consists of a low index (silica) background with high index (germanium doped) inclusions. The index of these inclusions is typically ~2% higher than that of silica; therefore the effective index of the cladding is higher than that of the core (Birks et al. 2006).

Like HC-PCF, light is guided in all solid band gap fibre because the cladding forms a photonic bandgap (section 2.2.2). The states of which are well explained by considering solely the modes of the high index inclusions (Litchinitser et al. 2003).

4. Kagome fibre

Kagome fibres are highly multimode, allowing the guidance of light in an air core with a typical loss of 1dB/m. The cladding is larger than that of HC-PCFs (a typical pitch of ~12μm, compared to ~4μm) and does not support a bandgap (section 2.2.2) (Couny et al. 2006).

The low loss of the core guided modes is due to the small overlap integral between the core and cladding modes. Cladding modes are highly confined in the silica struts and exhibit a fast transverse phase oscillation. Core modes are confined to the air exhibiting a slow transverse phase oscillation (Couny et al. 2008).

Furthermore this inhibited coupling between core and cladding modes is supported by a low density of states guided in the cladding (Pearce et al. 2007).
2.2 Hollow Core Fibre

2.2.1 Introduction

Hollow core photonic crystal fibre (Fig. 2.3) was first fabricated by Cregan et al. 1999. Its cladding is structured on a sub-micron scale; consisting of glassy apexes joined by thin struts. This cladding structure forms a photonic bandgap that allows light to be guided in an air core. The dispersion and nonlinearity of the guided modes can be radically different from the modes of conventional fibres that are guided in silica cores via total internal reflection. Though HC-PCFs are typically quasi multi-moded supporting lossy higher order modes (Petrovich et al. 2008), the following discussion will focus only on the fundamental mode.

![Fig. 2.3. A 7-cell HC-PCF; the core of the fibre was formed by omitting the 7 central capillaries from the cladding during the stacking process.](image)

Dispersion

The dispersion of the fundamental mode of all-solid conventional fibres is dominated by the dispersion of bulk silica. In solid-core air-clad PCF this is can be heavily modified by waveguide dispersion, however material dispersion still plays a significant role (Knight et al. 2000).

In contrast material dispersion contributes negligibly to the dispersion of HC-PCF, as the dispersion of air is low and typically only ~1% of the power of the core mode is guided in silica. Instead the dispersion is dominated by waveguide dispersion, leading to anomalous dispersion across a large region of the bandgap (Fig. 2.4). One of the key features of HC-PCF is that because the dispersion is set by microstructure of the core and photonic cladding, this allows the dispersion curve of a fibre to be shifted in frequency by scaling the fibre’s microstructured cladding/outer diameter.
Arguably the property of HC-PCF that most surpasses what is achievable in solid core fibres is the reduced nonlinearity $\gamma$. The nonlinearity $\gamma$ of the air guided mode of a HC-PCF can be several orders of magnitude less than a silica guided mode. This is because the nonlinear index of air ($n_2=3.0\times10^{-9}\text{m}^2/\text{W}$) is three orders of magnitude less than that of silica ($n_2=2.9\times10^{-14}\text{m}^2/\text{W}$).

The nonlinear response of a HC-PCF does not solely come from air. Part of the guided mode (typically \sim 1%) overlaps with the glass; this can contribute significantly to the nonlinear response. The precise interplay of how the silica and air contribute to the nonlinear response can strongly depend on nanometre scale differences in the fibre’s structure (Hensley et al. 2007).

### 2.2.2 Guidance Mechanism

It is well known that when a wave is incident upon a boundary between materials, the component of the wavevector tangential to the boundary is conserved (Snyder and Love, 1983, pp241). In an optical fibre this component is the longitudinal wavevector (which is also known as the propagation constant $\beta$). Light of a given frequency can be confined in the core of an optical fibre if its longitudinal wavevector is not supported by the cladding.

In conventional step index fibre total internal reflection can occur because the material that forms the core has a higher refractive index than the material that forms the cladding (and as such it supports larger wavevectors). In HC-PCF the reverse is true; the index of the core is lower than that of the cladding. Guidance in an air core is possible because the microstructured cladding (unlike solid silica) does not support a continuum of frequencies and longitudinal wavevectors; there are bands of frequency/longitudinal wavevector that are not supported. This guidance mechanism is called bandgap guidance, the structure of the supported frequencies and longitudinal wavevectors is called the bandstructure.

The main difference to note between conventional step index fibres and HC-PCF is that light can always be guided in (the fundamental mode of) a step index fibre, for all frequencies. In HC-PCF a guided mode of the air core can only exist for frequencies at which a photonic bandgap exist. This frequency range is typically \sim 20\% of the frequency at the central of the bandgap (Amezcua-Correa et al. 2008).
2.2.3 How Can A Bandgap Exist?

To explain how a band gap occurs in a HC-PCF it is useful to consider the allowed longitudinal wavevectors of the following three scenarios, (see Fig. 2.5).

1. In an **infinite silica medium** the wavevector of light is \( \mathbf{k} = n_{\text{silica}} \mathbf{k_0} \), this wavevector can point in any direction and hence the longitudinal wavevector can take a continuum of values such that \( \beta \leq n_{\text{silica}} k_0 \).

2. In a **1D step index waveguide** made of silica surrounded by air; boundary conditions mean that light can only be supported in the silica for a discrete set of modes and therefore a discrete set of longitudinal wavevectors (section 1.1.2). If the air surround is considered infinite then the wavevector of light in the air can point in any direction and hence can take a continuum of values such that \( \beta \leq k_0 \).

3. The allowed longitudinal wavevectors of an **array of 1D step index waveguides**, not only depends on the modes supported by individual waveguides but on the modal overlap between adjacent waveguides. This coupling of light between waveguides alters the modes of the array from that of a single waveguide and hence the bandstructure is also changed. The low index regions can no longer be considered infinite and light guided in these regions is also restricted to a discrete set of modes. As such light guided in neither the high or low index regions can take a continuum of values; it is this that allows a bandgap to exist.

It should be noted that, although it is useful to consider the modes of individual waveguides, when coupling of light between waveguides occurs, the supermodes of the entire system should be considered (Huang et al. 1994).

![Fig. 2.5. A schematic representation of the different bandstructures of three structures.](image-url)
2.2.4 Density of States Plots

Calculating the bandstructure of a HC-PCF requires numerical modelling. Often the core is ignored and the cladding is modelled as an infinite lattice; solving Maxwell's equations using a plane wave expansion method (Pottage et al. 2003). The resulting bandstructure of these calculations is displayed in a density of states (DOS) plot (Fig. 2.6), which shows the calculated number of modes per unit area (δωδβ). β and ω are displayed in the dimensionless units; usually effective index n_{eff} = β/k_0 and normalised frequency k_0Λ.

![Density of States Plot](image)

Fig. 2.6. Results adapted from Couzy et al. (2007). An example DOS plot of a HC-PCF's cladding; regions with a high DOS are shown in white, regions with zero DOS in black. The shaded red region is where the bandgap extends below the airline; a guided mode supported in the air core can exist in this region. Three computed plots of cladding modes are also shown.

The shape of the bandgap can be thought to arise from three types of modes; the short frequency edge of the bandgap being defined by the modes of the glass apexes, the high frequency edge by the modes of the struts and the depth of the bandgap by the modes of the air holes, Fig. 2.6. Maximising the core guided bandwidth of HC-PCFs is a complex issue (Couzy et al. 2007 and Light et al. 2009). As a rule of thumb, increasing the air filling fraction of a HC-PCF (thinning the struts) increases the core guided bandwidth (Benabid, 2006), as the strut guided modes become more airy making their bandstructure flatter to the airline.
2.2.5 HC-PCF Loss Mechanisms

The minimum attenuation of \( \sim 0.15 \text{ dB/km} \) for conventional all silica fibre is determined by the fundamental limits of scattering and absorption in ultra-high purity silica glass (section 1.1.3). For a typical 7-cell HC-PCF \( \sim 99\% \) of power of the fundamental mode is guided in air (sections 3.12 & 4.2), this vastly reduces these loss mechanisms allowing the possibility of a lower minimum attenuation.

However there are three other loss mechanisms that can exist in HC-PCF:

1) Light can tunnel through the photonic cladding to the solid outer cladding.
2) The surface roughness of the microstructure can cause light to be scattered out of the fundamental mode into cladding modes.
3) Dependent on how the cladding is terminated around the core, surface modes can be guided at this termination. Light guided in the fundamental mode can be readily scattered into these surface modes.

These loss mechanisms will be explained individually.

1. Tunnelling

Tunnelling or confinement loss occurs due to the exponential tail of the guided mode extending beyond the microstructured cladding. Increasing the size of the microstructured cladding (both by increasing the number of unit cells and the size of the unit cells) can reduce the extent of this tunnelling. In the current generation of HC-PCFs the minimum attenuation obtainable is negligibly affected by this mechanism (West et al. 2004).

Although this loss can occur in standard conventional fibres, it is easier to minimise as the cladding is solid and can be made arbitrarily large.

2. Surface roughness

When HC-PCFs are made, vibrational waves exist on the surface of the molten capillaries and as the fibre is cooled these waves are frozen in. These waves are thermodynamically inevitable and set the ultimate limit for the attenuation of HC-PCF (Roberts et al. 2005).

The presence of these frozen-in vibrational waves causes loss by scattering light from the fundamental core mode to modes of the photonic cladding. The extent of this scattering (and hence the attenuation) scales proportional to \( \lambda^{-3} \); in contrast to Rayleigh scattering which scales proportional to \( \lambda^{-4} \).

This difference in scaling and the low overlap of the guided mode with silica means that the minimum attenuation wavelength of HC-PCF is different to that of conventional all solid fibre. In conventional all solid fibre Rayleigh scattering and IR absorption combine to yield a minimum attenuation wavelength of \( \sim 1550\text{nm} \) (section 1.1.3). In HC-PCF surface scattering and (reduced) IR absorption yield a minimum attenuation wavelength of \( \sim 1900\text{nm} \), Fig. 2.7.
3. Surface modes

Surface modes are modes that exist in the cladding glass that surrounds the core defect. They are undesirable (for most applications), being within the bandgap created by the photonic cladding and hence limiting the low loss bandwidth, Fig. 2.8. The absolute minimum attenuation is also increased as light can be scattered to surface modes more readily then being scattered to cladding modes (West et al. 2004). Surface modes can be thought to arise from the imperfect termination of the photonic cladding at the core defect. Although surface modes are unavoidable for realistic HC-PCF designs their effects can be shifted to the edges of the bandgap if care is taken in designing the core surround (Amezcua-Correa et al. 2008 and 2006).

It should be noted that because surface modes have a high overlap with the cladding, perturbations such as surface roughness and variations in the cladding structure can easily scatter light from them into cladding modes and therefore they are lossy. Another consequence of surface modes being strongly peaked in the glass is that they dispersive. They anti-cross with the air guided core mode increasing its dispersion (Engeness et al. 2003). A numerically modelled bandstructure of an air mode anti-crossing with a surface mode is shown in Fig. 2.9.

Fig. 2.7. Taken from Roberts et al. 2005. Modelled results showing the minimum attenuation of HC-PCF as a function of design wavelength. Two 19-cell HC-PCF designs are displayed where 99.5% and 99.8% of the power in the guided mode overlaps with air. It should be noted that the minimum attenuation wavelengths of these two fibres are different because the relative contributions of IR absorption and surface scattering are different.

Fig. 2.8. Results taken from Smith et al. 2003 and Roberts et al. 2005a. Attenuation curves showing surface modes in 7-cell and a 19-cell HC-PCFs: (a)&(b) respectively. It can be seen that the larger core wall of the 19-cell fibre supports more surface modes.
Fig. 2.9 Adapted from Amezcua-Correa et al 2006. The modelled bandstructure of a core mode (blue) anti-crossing with a surface mode (red), the edges of the band gap are shown in green. The hybridisation of the surface mode with the core guided mode is shown at 4 points on the bandstructure labelled (i-iv).

2.3 Optical Fibre Fabrication

2.3.1 Fabrication of Conventional Fibre

The fabrication of the low loss conventional all solid optical fibres occurs in two stages.

In the first stage, a vapour deposition method (Agrawal, 2007, pp4) is used to create a cylindrical rod made of ultra pure SiO₂ doped with impurities in the region that will form the core of the fibre. The dopants increase the refractive index of the silica and their application is controlled such the relative cross section of the rod is the same as that of the desired fibre. This rod is referred to as preform; typically they are of the order of a meter long and centimetres in width, however can be much larger.

In the second stage, the preform is stretched in a furnace to create fibre. This is achieved by slowly feeding the preform into a furnace whilst drawing it out of the furnace at a much faster speed. During this process the relative core cladding dimensions and the index profile are preserved.
2.3.2 Fabrication of PCFs: The Stack and Draw Method

Photonic crystal fibre is usually fabricated using a similar methodology to that described for conventional fibre. The fibre’s structure is built on the macroscopic scale and then drawn down to a microscopic scale. This method of PCF fabrication is referred to as the stack and draw method and is outlined in the following bullet points and Fig. 2.10.

- Factory bought glass tubes are drawn to thin hollow tubes (capillaries).
- The capillaries are arranged in a hexagonal stack.
- The corners of the stack are removed and replaced with solid rods so that the stack approximates a circle and fits tightly in a jacketing tube.
- The jacketed stack is drawn down to canes.
- The canes are re-jacketed and drawn down to fibre.

Fig. 2.10. A schematic illustration of the steps undertaken to produce photonic crystal fibres using the stack and draw method. The sizes indicated are typical values.
2.3.3 Drawing Tower

The following schematic (Fig. 2.11) is a description of the fibre drawing tower at The University of Bath. The outer diameter of glass tubes/canes/preforms are reduced by feeding them into a furnace at a slower rate than they are pulled out at.

**Pressure controllers:** Pressures are applied to different sections of the cane to counteract the effects of surface tension and to allow holes to be inflated/deflated. (Section 2.3.4.)

**Preform feeder:** Holds PCF preforms, silica rods and tubes feeding them slowly into the furnace at controlled speeds of typically ~5 mm/min.

**Furnace:** Heats glass to ~1900°C by passing electricity through a graphite element.

**Laser micrometer:** Measures the diameter of glass exiting the furnace.

**Cane Puller:** Pulls preforms, rods and capillaries out of the furnace at a controlled speed. Typically ~2 m/min. (Not used for drawing fibre)

**Coating cup:** Adds a protective polymer to the outside of fibre.

**UV lamp:** Cures the protective polymer coating.

**Capstan:** Draws fibre from the furnace. Typical speed ~50 m/min.

**Drum winder:** Stores drawn fibre. Fully drawing a 1m long, 2cm diameter preform would yield 40km of 100μm fibre.

Fig. 2.11. A pictorial description of the fibre drawing tower at The University of Bath.
2.2.4 Pressurisation

It is important to control the cross sectional profiles of PCFs as they are drawn from the mesoscopic scale created in the stacking process to the microscopic scale of the finished fibre. When canes and preforms are molten in the furnace, surface tension acts to collapse the air holes. This effect can be partially negated by drawing the fibre at as low temperature as possible, however a pressure is usually also applied inside the holes during the drawing process. Pressurisation not only allows surface tension to be counteracted and the size of the holes maintained, but allows holes to be inflated or deflated in a controlled manner.

When fabricating HC-PCF three different pressures are typically applied, Fig. 2.12:

- The first is applied to the core defect using a thin glass capillary (~200μm) that is inserted a short distance (~15 cm) into the cane.
- The second pressure is applied to end of the cane and allows the pitch/air filling fraction of the cladding to be increased or decreased.
- The third is a vacuum line that is applied to the end face of the jacketing tube to remove any air gap between the cane and the jacketing tube that exists.

Fig. 2.12. A schematic of how pressures are applied when fabricating 7-cell HC-PCF. The end face of the capillary insert, cane and jacketing tube are compartmentalised; pressures being applied to them individually. Typical applied pressures are 10kPa and 18kPa in the core and cladding respectively.
3  Literature Review: Solitons in HC-PCF

3.0  Chapter Overview

This chapter summarises previous research into the use of HC-PCF for the propagation and compression of femtosecond solitons as well as the creation of femtosecond solitons from chirped picosecond amplified fibre laser pulses. Although some of this work was carried out at Bath, the author did not contribute to it.

The structure of the chapter is as follows:

- Section 3.1 details the propagation of solitons in HC-PCF summarising two papers: Ouzounov et al. (2003) and Luan et al. (2004).
- Section 3.2 details the compression of solitons in HC-PCF, it is split into two subsections; adiabatic soliton compression and soliton effect compression. Three papers are summarised in this section: Ouzounov et al. (2005), Gérôme et al. (2007) and Travers et al. (2007).
- Section 3.3 details the creation of femtosecond solitons from chirped picosecond amplified fibre laser pulses using HC-PCF. One paper is summarised in this section; Gérôme et al. (2008).
- Section 3.4 details the relevance of the work reported in sections 3.1, 3.2 & 3.3, to work undertaken by the author (which is reported in chapters 4, 5 & 6).

3.1  Propagation of Solitons in HC-PCF

3.1  Introduction

The first demonstration of the propagation of femtosecond optical pulses in HC-PCF was by Ouzounov et al. (2003). They demonstrated that due to the low nonlinearity $\gamma$ of the fibre’s air core it is possible to propagate pulses with megawatt energies through meter lengths of fibre. These pulse energies are orders of magnitude larger than can be propagated in conventional solid core fibre; in fact if these megawatt pulses where launched into even a few centimetres of conventional fibre they would be distorted by nonlinear effects such as self phase modulation and Raman gain (Agrawal, 2007, pp79). To prevent the pulses broadening during propagation because of dispersion, Ouzounov et al. took advantage of the large region of anomalous dispersion in the guided spectral bandwidth of HC-PCF to propagate solitons. The output pulse duration after propagation through the HC-PCF was considerably less than that expected for the case of linear propagation.

This section focuses on two papers that describe the propagation of solitons in HC-PCF, firstly the aforementioned Ouzounov et al. (2003) paper, and then a similar paper by Luan et al. (2004) is reported. In this paper the nonlinearity of the guided mode is calculated; the modal overlap with the silica cladding and air core being presented.
3.1.1 Propagation of Solitons in HC-PCF

Ouzounov et al. (2003) were the first to demonstrate soliton pulse propagation in HC-PCF. They launched 110fs pulses centred at 1470nm into a 3m length of HC-PCF with a zero dispersion wavelength of 1425nm. The dispersion at 1470nm was measured to be 15.9ps/(nm.km) corresponding to an input dispersion length of 24cm. The input 110fs pulses would therefore broaden to approximately 1400fs under purely linear propagation. As they increased the input power towards that of the soliton threshold energy they saw the output pulse duration decrease and the time bandwidth product of the pulses tend to the Fourier limit (Fig. 3.1). The minimum pulse duration observed (although longer than the input pulse) was much less than that expected for linear propagation, coupled with the aforementioned low time bandwidth product this is characteristic of the behaviour of optical solitons.

![Graphs](image)

Fig. 3.1. Results from Ouzounov et al. (2003) summarising the propagation of femtosecond pulses through 3m of HC-PCF. (a) shows the change in output pulse autocorrelation width as a function of the coupled pulse energy, red points represent experimental data, the blue line is from modelling that was undertaken. (b) shows the measured time bandwidth product of the output pulses as a function of pulse energy.

Both Fig. 3.1a&b have two distinct regions; for energies less than approximately 500nJ the measured pulse duration and time bandwidth product of the output pulse increases, for energies above 500nJ both these parameters plateau. 500nJ corresponds to the fundamental soliton threshold energy, for lower energies dispersion dominates over SPM, temporally broadening the pulse. At the threshold energy and at higher energies the output pulse duration is always larger than that of the input pulse because the perturbative effects of TOD (section 1.4.5) and Raman gain (section 1.4.3) are stronger for large spectral bandwidths; these effects limit the output pulse duration to 320fs. Raman and TOD are the two main perturbative effects that limit the performance of solitons in HC-PCF. Spectrally the effect of Raman gain is apparent through the observed self frequency shift in (Fig. 3.2) which increases in magnitude with the input pulse energy.
Fig. 3.2. Results from Ouzounov et al. (2003) showing the observed soliton self frequency shift of pulses in a HC-PCF. (a) shows example input and output spectra, visible in the output spectrum is a soliton centred at 1530nm as well as dispersive radiation at shorter wavelengths created because of perturbative effects acting on the soliton. (b) shows the wavelength shift of the soliton as a function of input pulse energy, the red points are experimentally recorded data, the blue curve is from modelling.

Ouzounov et al. stated that the nonlinearity of their HC-PCF came solely from the air core with a negligible contribution coming from the glass cladding. As such it should be possible to prevent this self frequency shift by filling the HC-PCF with a non Raman active gas rather than air. To this end, they filled 1.7m of the same HC-PCF with non Raman active xenon to atmospheric pressure, launching 7.5fs pulses centred at 1510nm. Xenon is a non Raman active gas because it is monatomic; Raman vibrational and rotational states relate to molecular degrees of freedom. It is likely that pulses centred at 1510nm rather than at 1470nm as previously reported were used to increase the ratio of $\beta_2: \beta_3$, limiting the effects of TOD. For an output pulse energy of 470nJ they observed little deviation in the output pulse’s temporal or spectral shape from that of the input pulse (Fig. 3.3). The absence of a self frequency shift supporting their claim that (for this fibre) only a small contribution to the Raman gain comes from the guided mode’s overlap with silica, with almost all the light being guided in the xenon core.

Fig. 3.3 Results for Ouzounov et al. (2003), showing the ability of a 1.7m long xenon filled HC-PCF to transport high peak power pulses without a large degradation of the pulses. (a) shows the measured autocorrelation profiles of the input and output pulses, (b) shows the corresponding spectra for these pulses.
After this first demonstration of soliton propagation by Ouzounov et al. (2003), Luan et al. (2004) transmitted femtosecond solitons centred at 800nm through 5m of HC-PCF. As the experiment was at a shorter wavelength than that reported by Ouzounov et al. the core area of the fibre is smaller, increasing its nonlinear response $\gamma$ and decreasing the observed fundamental soliton threshold energy from 500nJ to 60nJ. Observing Fig. 3.4a this threshold energy is apparent; for energies less than 60nJ the 140fs input pulses undergo significant temporal broadening. For energies greater than 60nJ the pulse duration remains almost constant at 300fs, close to the plateau at 320fs observed by Ouzounov et al. (2003).

Fig. 3.4. Results from Luan et al. (2004) showing the propagation of femtosecond solitons through 5m of HC-PCF. (a) shows the measured autocorrelation width as a function of output pulse energy. (b) shows the spectral changes that occurred for different output pulse energies. As the soliton shifts to longer wavelengths a tail of dispersive radiation is left behind, this dispersive radiation was observed by Luan et al. as a pedestal in the autocorrelations they recorded.

Luan et al. performed an analysis of the guided modes of a (modelled) HC-PCF similar to that used experimentally using a plane wave expansion method, Fig. 3.5. (Pottage et al. 2003). Approximately 99% of the energy of the fundamental core guided mode was found to propagate in the air with only 1% propagating in the glass, with the nonlinear contributions to $\gamma$ from the air and glass being found to be comparable for this modelled fibre. This is in contrast to Ouzounov et al. (2003) who showed that the overlap of the guided mode with the cladding of their fibre was almost negligible. Luan et al. point out that the core area of their modelled fibre is lower than that of the experimentally fabricated fibre and as such the overlap of the guided mode with the silica may have been greater in the modelled fibre. They also note that varying the air filling fraction of the modelled fibre from 92% to 87.9% doubled the contribution of the silica cladding to the nonlinearity coefficient $\gamma$ and therefore small variations in the design of HC-PCFs can have a large impact on the properties of the guided mode.

Fig. 3.5. Results from Luan et al. (2004) showing an intensity plot of the guided mode of a modelled HC-PCF, shown in (a) & (b) respectively. The air filling fraction of the cladding was 92%. The highest intensity of the fundamental core guided mode occurring in the glass is 16% of the peak intensity of the mode occurring at the centre of the HC-PCF.
3.1.2 Summary: Propagation of Solitons in HC-PCF

Ouzounov et al. and Luan et al. showed that it is possible to transmit high peak power femtosecond solitons at different wavelengths through several meters of HC-PCF without a significant degradation of the input pulses, something that would be impossible in a single mode silica fibre. These modest lengths of a few meters are short relative to the distances spanned by modern fibre based telecommunication networks however they are sufficient to transport high peak power pulses the small distances which are required for laser machining and a range of \textit{in vivo} experiments.

The results observed in both these experiments differed from those of solitons propagating in a medium with solely Kerr nonlinearity and second order dispersion, primary because of TOD and Raman gain, which are more perturbative the greater the spectral bandwidth of the soliton. The effect of TOD can be minimised by carefully choosing the wavelength at which light is launched into a given HC-PCF and by creating HC-PCFs with as broad as possible transmission windows and therefore as small as possible values of TOD. Ouzounov et al. showed that it is possible to limit the effect of Raman gain by filling the fibre with a non Raman active gas.

During the course of the author’s Ph.D. Gorbach and Skryabin (2008) showed that the Raman response of silica core and air core fibres differ greatly. They state that solitonic pulses travelling in air core fibres with durations less than or close to 100fs suffer from strong energy losses to non-solitonic radiation through Raman gain; limiting the minimum pulse duration that light can be transmitted through air core fibres. For silica core fibres the maximum Raman gain occurs at a detuning frequency that is approximately five times greater than for air (Fig. 3.6). As such comparable losses in silica to non solitonic radiation occur when the pulse duration is much shorter than for an air core fibre. Furthermore the Raman gain of silica is relatively flat as a function of detuning for small bandwidths, whereas the Raman gain of air increases rapidly as a function of detuning.

![Fig. 3.6. Modelled results adapted from Gorbach and Skryabin (2008), showing the difference in the Raman gain curves of silica and air in black and grey respectively as a function of detuning frequency. The curves are arbitrary and independently normalised.](image-url)
3.2 Compression of Solitons in HC-PCF

3.2 Introduction

Following the publication of the two papers outlined in section 3.1 detailing the propagation of solitons in HC-PCF came two papers demonstrating the temporal compression of optical solitons in HC-PCF. Ouzounov et al. (2005) utilised soliton effect compression and Gérôme et al. (2007) an adiabatic decrease in dispersion along a fibre’s length. Both these compression techniques have been demonstrated before in conventional fibres (Mollenauer et al. 1980 and Chernikov 1992), however these papers demonstrated that they were viable techniques in HC-PCF.


3.2.1 Soliton Effect Compression

Soliton effect compression occurs when a higher order soliton is initiated in a waveguide. As explained in section 1.4.3 higher order solitons undergo a periodic evolution, temporally narrowing at the beginning of each soliton period. By an appropriate choice of fibre length, higher order solitons can be made to exit at the point where they are temporally narrowest. If higher order effects (such as Raman gain and TOD) are ignored then the performance of soliton effect compression depends solely on the input soliton number. This behaviour was studied by Mollenauer et al. (1983) and is shown in Fig. 3.7. One of the main disadvantages of this form of compression is that unlike adiabatic soliton compression (section 3.2.2) only a proportion, $Q_C$, of the input energy forms a compressed pulse, the remaining energy forming a pedestal.

![Graphs showing soliton effect compression](image)

Fig. 3.7. (a) Results adapted from Mollenauer et al. 1983 summarising the performance of idealised soliton effect compression, shown is the optimal compression length as a fraction of the soliton period $z_{opt}/z_0$, the maximum compression ratio $F_C$ and the fraction $Q_C$ of the input power in the compressed pulse. Shown in (b) is a selected area of this diagram for soliton numbers $N<5$, calculated by the author.
Ouzounov et al. (2005) were the first to demonstrate soliton effect compression in a HC-PCF. Using pulses centred at 1.500nm they compressed a 31.5nJ 120fs input pulse to 50fs in a 24cm long length of HC-PCF. This fibre was filled with xenon to a pressure of 9atm, to remove the Raman response of the air core. An above atmospheric pressure of xenon was used for two reasons; firstly the increased nonlinear response of the pressurized core reduced the contribution of the glass cladding to the nonlinear coefficient of the fundamental mode $\gamma$ and hence reduced the glass cladding’s contribution to the Raman gain. Secondly it increased the nonlinear coefficient $\gamma$ of the fundamental mode allowing higher order solitons to be generated using lower input powers.

Using the numbers provided by Ouzounov et al. the input soliton number can be calculated as $N=3.2$ and therefore (referring to Fig. 3.7) in the absence of higher order effects the minimum pulse duration would occur at $z_{opt}/z_0=0.2$ (which can be calculated to be 8cm), with a compression ratio of $F_C=10$ and a quality factor of $Q_C=0.65$. As the Raman gain of the xenon filled HC-PCF was negligible Ouzounov et al. state that the achieved compression ratio of 2.4 was restricted by the finite width of the HC-PCF’s bandgap, through higher order dispersion and the creation of frequencies of light that were not guided within the bandgap. They state that by using longer input pulses, larger compression ratios should be achievable as the spectral bandwidth of these pulses would be smaller in relation to the width of the HC-PCF’s bandgap. The presence of a pedestal is visible in Fig. 3.8, however the proportion of the energy it contains (an experimental value of $Q_C$) was not quantified by Ouzounov et al.

![Fig. 3.8. Results from Ouzounov et al. (2005) showing soliton effect compression in 24cm of xenon filled HC-PCF. (a) shows autocorrelations of the input pulse and the compressed output pulse. (b) shows the corresponding spectra.](image)
3.2.2 Adiabatic Soliton Compression

Fundamental solitons are robust phenomena; they can adapt to maintain a solitonic profile in the presence of loss, changes of dispersion or changes in nonlinearity, as long as these changes occur slowly with respect to the soliton period. This robustness can be used to compress pulses, for instance if the dispersion of a waveguide is decreased along its length then SPM will prevail over dispersion increasing the spectral bandwidth of the soliton until dispersion and SPM become rebalanced; this balance occurring for a fundamental soliton with a shorter temporal duration. Unlike soliton effect compression (section 3.2.1), in theory this process can be done adiabatically: such that the compressed output soliton has the same energy as the input soliton. An adiabatic change in the dispersion/nonlinearity of a fibre along its length was first proposed by Tajima (1987) as a technique to maintain the temporal duration of a soliton in the presence of loss, Kuchl (1988) then proposed that it could be used to compress solitons. Gérôme et al. (2007) were the first to demonstrate this type of compression in a HC-PCF, using a fibre in which the dispersion felt by the soliton decreases along the length of the fibre.

During a HC-PCF draw they increased the capstan speed in a controlled manner, whilst keeping the rate at which the preform was fed into the furnace constant, to created a fibre with an outer diameter that decreased linearly along its length Fig. 3.9a, (see Fig. 2.11. for a schematic of the fibre drawing tower used). They then turned this fibre around such that the outer diameter increased along the length of the fibre such that along the length of the fibre: the size of the microstructured cladding increasing, the bandgap shifted to longer wavelengths and the dispersion at a fixed wavelength within the bandgap shifted to less anomalous values. From this fabricated profile a small 8m tapered section was removed, in this fibre the dispersion at the pump wavelength of 800nm decreased from approximately 80ps/(nm.km) at the input of the fibre to 0ps/(nm.km) at the output, Fig. 3.9.b.

![Diagram showing the characteristics of a HC-PCF taper.](image)

Fig. 3.9. Results from Gérôme et al. (2007) showing the characteristics of a HC-PCF taper. (a) shows the selection of the taper used from a longer manufactured taper, the data shown is from a laser micrometer that was active during the fibre draw. (b) shows the measured dispersion of the input and output of the 8m taper as well as a transmission measurement through the taper using a broadband source.

Time bandwidth limited 195fs input pulses were launched into the 8m taper and the output spectrum and autocorrelation width were measured as a function of the output pulse energy, Fig. 3.10. A minimum output pulse duration of 90fs was observed for an output pulse energy of 69nJ. For comparative purposes the performance of an 8m length of untapered fibre with the same dispersion as the input of the taper was also studied, Fig. 3.10. Spectrally, pulses from the tapered fibre had broader bandwidths than from the linear fibre and as such were transform limited to shorter temporal durations. Temporally, solitons from the taper were approximately 3 times shorter than from the uniform fibre and were approximately half the duration of the input pulses. As stated in section 3.1.2, the minimum pulse duration obtained is likely to have been limited by the Raman gain of air (Gorbach and Skryabin 2008), and by TOD.

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Fig. 3.10. Results taken from Gérôme et al. (2007) showing the performance of the tapered and untapered HC-PCFs. (a) shows the spectral evolution of pulses in the tapered fibre as a function of input power. (b) shows the spectral evolution of pulses in an untapered fibre. (c) shows the temporal evolution of both the tapered and untapered fibres as a function of output pulse energy.

In a separate experiment Travers et al. (2007) showed that it is possible to achieve large compression ratios in dispersion decreasing tapers by exciting higher order solitons at the input of the taper rather than fundamental solitons. Working in highly nonlinear PCF (which has a silica core), the dispersion of their 50m taper (at the pump wavelength of 1064nm) decreased from 40ps/(nm.km) to 10ps/(nm.km) and therefore cannot explain the compression ratio of 15 times that was observed, from 830fs to 55fs (Fig. 3.11). The compression was partly due to soliton effect compression and partly due to the dispersion decreasing taper. The input pulse for which the minimum pulse duration was observed had a soliton number of N=2. The oscillatory nature of the output pulse duration as a function of input pulse energy in Fig. 3.11.a can be explained in terms soliton effect compression whereby the optimal compression length is a function of the input pulse energy. All the presented autocorrelations were without pedestals; in other words the compression was adiabatic, even though the taper was only an estimated 3 (input) soliton periods (of ≈15m) long and the compression was aided by soliton effect compression.
Fig. 3.11. Results taken from Travers et al. (2007) showing the compression of femtosecond pulses in a 50m tapered highly nonlinear PCF. (a) shows the measured output pulse duration as a function of input pulse energy and input soliton number. (b) shows the corresponding spectra.

The minimum pulse duration achieved by Travers et al. (2007) was less that achieved by Gérôme et al. (2007) because the perturbative effects of Raman gain and TOD are reduced in silica core fibres; TOD is reduced because the fibre no longer relies on a bandgap for guidance and Raman gain is reduced because the maximum Raman gain of silica occurs for a frequency detuning that is approximately five times greater than for air (Gorbach and Skryabin 2009). It should be noted that the compression of higher order solitons in dispersion decreasing tapers was explored theoretically by Pelusi and Liu (1997).

3.2.3 Summary: Soliton Compression

The main advantage of adiabatic soliton compression over soliton effect compression is that in theory all of the energy of input pulse can be contained within the compressed output pulse. The main disadvantage is that the fibre must be several soliton periods long for the compression to be adiabatic. This means that the propagating pulses experience the effects of Raman gain, TOD and fibre attenuation over longer lengths than in soliton effect compression and as such the minimum obtainable pulse duration is limited to larger values. It has been shown experimentally by Travers et al. (2007) that impressive compression ratios can be achieved by launching higher order solitons into tapered dispersion decreasing fibres, to create pedestal free pulses in short tapers.
3.3 Pulse De-Chirping in HC-PCF

3.3 Introduction

Recently, amplified fibre lasers systems have become viable alternatives to solid state systems for
applications that require pulses energies of several multi-hundred nanojoules and pico/femtosecond
pulse durations. Unlike solid state systems, fibre lasers are inexpensive, have small footprints and
require little or no maintenance: to name but a few advantages. However the main problem with
generating high peak power pulses in amplifiers is that of nonlinearity in the gain medium. Solid state
laser systems have short, high gain amplifiers which minimize the effects of nonlinearity, however the
interaction length in fibre amplifiers is much larger and hence nonlinear effects have a much greater
influence on the output, yielding pulses that are several times the transform limit. To overcome this
problem large mode area fibres are used (which are limited in area if single mode operation is desired),
or alternatively a chirped amplification scheme is employed.

Chirped amplification schemes work via chirping/dispersing oscillator pulses before they enter the
amplifier fibre (Strickland and Mourou 1985), reducing the peak power of the pulses and hence
reducing the nonlinear effects observed. After a pulse is amplified it is then recompressed to a high
peak power pulse. This recompression can take place in freespace dispersion compensation elements
such as gratings. However the low nonlinearity $\gamma$ of HC-PCF means that it can be used as a dispersion
compensation element, to make all fibre systems. Gérôme et al. (2008) and de Matos and Taylor (2005)
showed that under certain conditions it was not necessary to prechirp the input pulse to the amplifier
fibre, as it is possible to compress the additional bandwidth generated by the occurrence of nonlinear
effects in the amplifier fibre using a HC-PCF. A full numerical model of this type of compression was
reported by Lægsgaard and Roberts (2008 & 2009b). Section 3.3.1 will focus on the work of Gérôme et
al. (2008).

3.3.1 Pulse De-Chirping in HC-PCF

The amplified fibre laser system used by Gérôme et al. (2008), produced 320nJ positively chirped 5.5ps
pulses centred at 1064nm with a spectral bandwidth of 12nm (corresponding to a time bandwidth
product of 17.5). The positive chirp on the pulses occurs because of the normal dispersion of the
amplifier fibre and the occurrence of SPM within it. These pulses were launched into 8m of HC-PCF
with a dispersion of 93ps/nm/km at 1064nm where 77% of the coupled power was compressed to 520fs
(time bandwidth limited) solitons, Fig. 3.12.a&b. Gérôme et al. state that attenuating the laser to a low
power (such that the propagation can be thought of as linear) the chirp of the laser pulses was
compensated after $\approx$5m of HC-PCF and then the pulse broadened to approximately 3ps in the
remaining 3m of fibre.

A temporal and spectral evolution of the 320nJ laser pulses through the HC-PCF is shown in Fig.
3.12.c&d. After approximately $\approx$4.5m of what can be modelled as linear propagation the pulse becomes
sufficiently compressed that nonlinearity redistributes the spectral components into one solitonic peak
at the long wavelength side of the spectrum. By 6.5m a clear 520fs soliton is formed, that can be seen to
self frequency shift to longer wavelengths. This observed self frequency shift of the soliton away from
the dispersive radiation could have been used to separate the soliton from the dispersive radiation via
the use of a long-pass filter to create pulses without pedestals, however Gérôme et al. used a different
method. They frequency doubled the soliton in a temperature tuned 5mm thick lithium triborate crystal
with a 2nm phase matching bandwidth to creating near time bandwidth limited 500fs pulses at $\approx$540nm.

The dispersion at the input of the taper was chosen to be high (93ps/(nm.km)) for two reasons; firstly if
a lower dispersion had been chosen then a longer piece of fibre would be necessary and fibre loss may
have become an issue. Secondly this point on the dispersion curve represents the maximum ratio
between dispersion and dispersion slope and therefore the chirp of the laser pulses is likely to be most
effectively compensated at this point.
3.3.2 Summary: Pulse De-Chirping in HC-PCF

The use of HC-PCF to convert chirped and spectrally complex laser pulses to transform limited solitons is a promising area of research as it represents an uncomplicated and inexpensive manner of improving the performance of amplified fibre laser systems to close to that of solid state systems. It should be highlighted that passing the laser pulses through HC-PCF not only temporally compressed them (as a grating compressor could do), the nonlinear response of the fibre converts the spectrally complex pulses into spectrally clean pulses.

Fig. 3.12. Results for Gérôme et al. (2008) showing the compression of chirped picosecond amplified laser pulses to femtosecond solitons in a HC-PCF. (a) shows the measured output autocorrelation width as a function of output pulse energy, (b) shows input and output spectra for an output pulse energy of 220µJ, (c) & (d) show temporal and spectral evolutions of the 220µJ output pulse as a function of propagation length; this data was collected via a cut back method and is presented on a linear intensity scale with linear interpolation.
3.4 Relevance to Chapters 4, 5 & 6

Building upon the work reported in this chapter, the author undertook the following work:

In section 4.1, a 3-cell HC-PCF is described, this fibre was created to span the gap in nonlinearity $\gamma$ that exists between 7-cell HC-PCFs and solid core fibres. Soliton propagation is demonstrated in this 3-cell HC-PCF.

In section 4.2, building upon the work of Luan et al. (2004) work undertaken by a college (R. Amezcua-Correa) is presented, were the soliton threshold energies of a 7-cell HC-PCF modelled with different air filling fractions are analysed.

In section 4.3, a technique is described which enables the dispersion of a HC-PCFs to be known accurately, such that in future soliton propagation experiments the influence of TOD can be minimised.

In section 5.1, building upon the work of Gérôme et al. (2007) a tapered HC-PCF is described that compresses picosecond pulses to femtosecond durations. The input soliton number in this taper was $N\approx2$ and therefore similar to the regime explored by Travers et al. (2007).

In section 5.2, building upon the work of Gérôme et al. (2008) & (2007) a HC-PCF is described that has first a linear section then a tapered section. The linear section creates a soliton from a chirped amplified fibre laser pulse in a manner similar to Gérôme et al. (2008), the tapered section then adiabatically compresses the soliton in a manner similar to Gérôme et al. (2007).

In section 6, building upon from the work of Gérôme et al. (2008) and Ouzounov et al. (2005), pulses from a chirped amplified fibre laser are compressed in a HC-PCF, then frequency doubled in a non critically phase matched LBO crystal before being launched into a 539nm guiding HC-PCF where they undergo soliton effect compression.
4 Development of HC-PCF for soliton propagation applications

4.0 Chapter Overview

This chapter details work carried out into understanding and improving HC-PCF for soliton propagation applications.

The structure of the chapter is as follows:

- Section 4.1 details the fabrication of a 3-cell HC-PCF which has a higher nonlinearity $\gamma$ than its 7-cell counterpart. Solitons were generated in the fibre and its performance was compared to a 7-cell fibre through both modelling and experiment.
- Section 4.2 details through modelling how variations in the pitch/air filling fraction of a 7-cell HC-PCF affect its dispersion and nonlinearity $\gamma$.
- Section 4.3 details a measurement of the dispersion of a 7-cell HC-PCF using an improved measurement technique. The accuracy of this measurement allowed the ratio of dispersion to dispersion slope to be known with certainty and without making any additional assumptions.

The author took the leading role in all the work presented in this chapter with one exception. The numerical modelling of the dispersion and nonlinearity of fibres shown in Figs. 4.5, 4.6, 4.10 & 4.16d was performed by a colleague (R. Amezcua-Correa).

4.1 3-cell HC-PCF

Section 4.1 reports the fabrication of a 3-cell HC-PCF. This is a HC-PCF where the core of the fibre consists of a defect of 3 missing cells rather than the usual 7 cells. A 3-cell fibre was reported by Petrovich et al. (2008) whilst the author was undertaking the work reported here.

Amplified fibre lasers, compared to their solid state counterparts are cheap, have small footprints and require little or no maintenance. The pulse energies output by these systems are of the order of the soliton threshold energy in 7-cell HC-PCF (in fact they are limited to this energy in systems that use 7-cell HC-PCF to recompress the chirped amplified pulses to solitons, section 3.3). Therefore soliton experiments in 7-cell HC-PCF that use amplified fibre lasers as a pump source are often limited by the insufficient pulse energy of the laser. The fabrication of a 3-cell HC-PCF was designed to solve this problem by creating a HC-PCF where the soliton threshold energy is lower than standard 7-cell fibre. It would also inhabit the gap in nonlinearity $\gamma$ (of several orders of magnitude!) that exists between conventional solid core fibre and 7-cell HC-PCF.

A 3-cell HC-PCF should have a significantly higher nonlinearity than its 7-cell counterpart for the following reasons. Firstly the nonlinearity $\gamma$ of HC-PCF does not come solely from the air core. Part of the guided mode (typically ~1%) overlaps with the glass. This contributes significantly to the Kerr nonlinearity because the nonlinear refractive index ($n_2$) of silica is three orders of magnitude greater than that of air. Reducing the core area will increase the overlap of the light with the silica cladding and hence increase the nonlinearity $\gamma$. Secondly reducing the core area confines the guided mode to a smaller area and hence the intensity of the guided light is greater.
4.1.1 Fabrication

Although the 3-cell HC-PCFs presented in this section were fabricated using a standard stack and draw method (section 2.2.1) it required this method to be adapted; steps had to be taken that are not necessary for the fabrication of 7-cell HC-PCF.

These steps were needed when the fabricated canes Fig. 4.1a were drawn to fibre as two problems occurred:

- Typically when 7-cell HC-PCF canes are drawn to fibre, a single capillary insert is used to control the pressure in the core defect. The core defect of a 7-cell HC-PCF cane approximates a circle and therefore a single (circular) capillary inserted into the core defect can fill much of its cross sectional area, preventing pressure bleed into the defect from the cladding. The core defect of 3-cell HC-PCF canes do not approximate a circle, and therefore when these canes were drawn to fibre it was not possible to control the pressure in the core using a single capillary insert as pressure bleed into the core from the cladding occurred. To overcome this problem three capillaries were inserted into the core as the (combined) cross section of the three capillaries approximated the cross section of the defect and therefore limited the pressure bleed from the cladding into the core that occurred.

- At first it was not possible to maintain three of the cladding holes around the core when the canes were drawn to fibre, Fig. 4.1b. These “small” cladding holes caused many surface crossings (section 2.2.5) to be visible in transmission measurements and the fibres to be of little use. This problem was overcome by pressuring these three holes independently with 3 capillary inserts, whilst pressuring the rest of the cladding as normal.

A schematic of how pressures were applied during the fabrication of 3-cell HC-PCF is shown in Fig. 4.2, and a schematic of how pressures are typically applied during the fabrication 7-cell HC-PCF is shown in Fig. 2.21.

![Image](image_url)

**Fig. 4.1.** (a) An image of the end face of the canes used to draw 3-cell HC-PCF. (b) An SEM of an early attempt at drawing 3-cell HC-PCF, it can be seen that 3 of the cladding holes around the core have nearly collapsed; these holes caused surface crossings to be evident in transmission measurements. Furthermore the relative size of the core is enlarged from the cane because of pressure bleeding from the cladding into the core.
Fig. 4.2. A schematic of how pressures were applied during the fibre drawing process to successfully fabricate 3-cell HC-PCF with a qualitatively periodic cladding around the core, and therefore suppressed surface crossings. Four separate pressures were applied; the first controlled the size of the core using three capillaries inserted into the core. The second controlled the size of three of the holes surrounding the core using three capillaries inserts, one inserted into each of the holes. The third controlled the size of the rest of the holes in the cladding and was applied to the end face of the cane. The fourth was a vacuum line applied in between the cane and jacket to ensure they fused together with no air pockets.

4.1.2 3-cell HC-PCF Properties

A set of 3-cell HC-PCFs were drawn using the aforementioned methodology, these fibers had virtually the same cladding structure, but the structure of their cores and the cells surrounding their cores differed. From this set of two 3-cell HC-PCFs were selected because their core geometries maximised the transmitted bandwidth of the fundamental mode. The first of these fibres had a large 11µm diameter core which significantly perturbed the surrounding cladding and the second had an 8µm diameter core which perturbed the surrounding cladding far less. SEMs of these fibres are shown in Fig. 4.3 as well as a state-of-the-art 7-cell HC-PCF (Amezcua-Correa et al. 2008) which is included for comparison, it has a core diameter of 17µm.

The attenuation and dispersion of the 3-cell fibres was measured and is shown in Fig. 4.4. The minimum attenuation of both of these fibres is less than that reported by Petrovich et al. (2008) (approximately 200dB/km) although the bandwidth of both fibres is less. For both 3-cell HC-PCFs the dispersion and dispersion slope was found to be high across much of the bandgap with the larger 11µm core fibre having a relatively flat region at the centre of the bandgap.
Fig. 4.3. Scanning electron micrographs (shown on the same scale) of the two 3-cell HC-PCFs and a state-of-the-art 7-cell HC-PCF. (a) (b) and (c) respectively. The core diameter sizes are 8µm, 11µm and 17µm respectively.
Fig. 4.4. In (a), the measured attenuation and dispersion of the 8μm core 3-cell HC-PCF. (b) The 11μm core 3-cell HC-PCF. (c) The state-of-the-art 7-cell HC-PCF. The zero dispersion wavelengths are 1490nm, 1465μm and 1502μm respectively.

It is difficult to draw any quantitative conclusions comparing the 3-cell and 7-cell fibres in Fig. 4.4 because the pitches and air filling fractions of the fibres differ. It was not possible to draw 3-cell fibres with a matching, higher air filling fraction because when the cane was inflated, the cladding structure became irregular and therefore the fibre was very lossy. Therefore to help compare the 3-cell and 7-cell HC-PCF designs, a 3-cell HC-PCF with the same core geometry as the 8μm fibre (Fig. 4.5) and a 7-cell HC-PCF were modelled. The respective core sizes were 11μm and 15μm the pitch and air-filling fraction was the same for both fibres, and was 5μm and d/A=0.98 respectively. This pitch and air-filling fraction was chosen so that the bandgap of the fibres was centred at 1550μm.
Comparing the modelled 3-cell and 7-cell HC-PCFs, (Fig. 4.6, Table 1.1), although 10nm from the ZDW the nonlinear response (γ) of the 3-cell fibre is 7 times greater, the dispersion of the 3-cell HC-PCF is also greater by a factor of 4. Hence the soliton energy at this wavelength is only decreased by a factor of approximately 2 between these two designs (equation 1.31). These ratios will not hold true for all 3-cell fibres and 7-cell fibres as the properties of HC-PCF differ significantly with changes in pitch and air-filling fraction as shown in section 4.2.

<table>
<thead>
<tr>
<th></th>
<th>7-cell</th>
<th>3-cell</th>
<th>Ratio (Approximate)</th>
</tr>
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<tbody>
<tr>
<td>Dispersion (10nm from ZDW)</td>
<td>25ps/(nm.km)</td>
<td>7ps/(nm.km)</td>
<td>4:1</td>
</tr>
<tr>
<td>Nonlinear coefficient (γ)</td>
<td>0.0016 (W km)$^{-1}$</td>
<td>0.011 (W km)$^{-1}$</td>
<td>1:7</td>
</tr>
<tr>
<td>Contribution to γ from air</td>
<td>0.0013 (W km)$^{-1}$</td>
<td>0.0025 (W km)$^{-1}$</td>
<td>1:2</td>
</tr>
<tr>
<td>Contribution to γ from glass</td>
<td>0.00032 (W km)$^{-1}$</td>
<td>0.0084 (W km)$^{-1}$</td>
<td>1:26</td>
</tr>
<tr>
<td>Ratio of air/glass contributions</td>
<td>4:1</td>
<td>1:3.5</td>
<td>1:13</td>
</tr>
</tbody>
</table>

Table 1.1. The nonlinear coefficient (γ) and dispersion of the modelled 3-cell and 7-cell fibres 10nm from their respective ZDWs.
4.1.3 Observing Solitons in 3-cell HC-PCF

After fabricating the two 3-cell HC-PCFs detailed in section 4.1.3, experiments were performed to investigate the viability of using these fibres to propagate low energy femtosecond solitons. The laser system used was a femtosecond optical parametric amplifier (OPA) pumped by a regeneratively amplified Ti-Sapphire laser (Coherent: OPA9800, Rega9000 and Mira9000). The amplifier delivered 200fs pulses centred at 802nm and with energies up to 5µJ at a repetition rate of 125kHz (having an average power of 625mW). The OPA generated 90fs pulses which could be tuned across the entire bandgap of both 3-cell fibres. The maximum pulse energy obtainable at 1500nm was 27.5nJ (34mW).

Spectrally, no nonlinear response was observed when launching this energy at any wavelength into the 7-cell fibre. For both 3-cell fibres spectral splitting at their respective ZDWs was observed, this splitting occurs because for frequencies close to the ZDW Kerr nonlinearity dominates over dispersion, these frequencies are shifted away from the ZDW where they are no longer dominated by Kerr nonlinearity. For both 3-cell fibres the position of this splitting agreed with the values from the low coherence interferometer dispersion measurement. Fig. 4.7 shows this agreement for the 8µm 3-cell fibre putting the ZDW at 1490nm.

![Graph](image)

Fig. 4.7. In grey the spectral splitting observed through 5m of the 8µm fibre for 80nJ(10mW) input pulses (energy prior to the coupling optics) shown in dashed black. Both curves are individually normalized. In black, part of the measured dispersion taken from Fig. 4.4a.

Launching pulses centred at 1492nm into a 5m long length of the 8µm core fibre, a self frequency shift was observed with increasing pulse energy, Fig. 4.8. Launching 27.5nJ (34mW) pulses at the fibre input face (the coupled energy was less due to the estimated 30% coupling efficiency), an output energy of 80nJ (10mW) was measured and a 433fs pulse was observed on an autocorrelator based upon the nonlinear response of a LED (Reid et al. 1997). This is much shorter than the multi-picosecond pulse expected for linear propagation given that the dispersion is approximately 100ps/(nm.km) at 1500nm, and that an output spectral width of 5nm was seen after 5m. The input pulse spectrum was of a similar shape to that in Fig. 4.7. Though spectral splitting was observed through the 11µm 3-cell HC-PCF, femtosecond pulses were not observed and as such no propagation characterisations are presented here.
In an effort to reproduce the results in Fig. 4.8 modelling was done using a split step Fourier method, (equation 1 from Gorbach and Skryabin 2008), Fig. 4.9. Fibre losses were not included because of the low loss of the 5m length of fibre (approximately 0.5dB). Higher-order dispersion terms were taken from a polynomial fit to the data in Fig. 4.4a. Observing Fig. 4.6a the relative nonlinear contributions of air and glass were taken to be in a 1:3 ratio, γ was assumed to be 0.0138(W.km)^{-1}. This value of γ was chosen to be slightly higher than the modelled value so that excellent agreement between the experimental and modelled results was found. The computed output pulse duration was 330fs for the input of a 90fs, 80nJ (10mW) pulse.

Fig. 4.9. The modelled response of 5m of the 8μm 3-cell fibre as a function of output energy for a 90fs sech^2 shaped input pulse centred at 1492nm. (a) Shows the output spectra as a function of output energy, (b,c)&(d) Shows XFROG spectrograms (Treinho et al. 1997) of the output pulse for output energies of 80, 60 & 38nJ respectively.
In these modelled results the short wavelength radiation in the normal dispersive regime is created from the spectral overlap of the input pulse with the normally dispersive regime. For an output energy of 80nJ (10mW) there is a single compressed pulse in the time domain (Fig. 4.9b), this pulse contains of 67% of the energy and is centred at 1499nm. This femtosecond pulse is maintained if the simulation is extended to larger propagation lengths; for a simulation length of 30m the output pulse has a duration of 550fs. Loss is not present in this simulation and therefore it is not an accurate description of what would occur experimentally over such a length. The oscillatory structure that is present in Fig. 4.9a is due to the beating of the soliton with dispersive radiation which separated in time from the soliton but not in frequency (as outlined in section 1.4.3). Observing Fig. 4.9.a,b&c the time delay between the dispersive radiation and the soliton directly relates to the frequency of the oscillations, the frequency overlap setting the wavelength of the oscillations.

4.1.4 Discussion

In evaluating the 3-cell design over the 7-cell design for the propagation of low energy solitons, it can be concluded that the potential benefit associated with the increase in nonlinearity is effectively negated by increased dispersion. The magnitude of the group-velocity dispersion across the bandgap is large because of the dispersion slope. The fabricated 7-cell had a dispersion slope at the centre of the bandgap of 0.3fs/(nm.m) compared to over 2fs/(nm.m) for the 8μm 3-cell. The modelled 7-cell and 3-cell fibres have dispersion slopes of 0.2fs/(nm.m) and 1.6fs/(nm.m) respectively.

Although modelling shows that the 3-cell HC-PCFs inherently have higher dispersion slopes than 7-cell HC-PCFs, the dispersion of the manufactured 3-cell fibres could have been decreased if the pitch/air filling fraction of the cladding was increased (section 2.2.4). Observing the reported 3-cell HC-PCFs in Fig. 4.3, the cladding holes at either side of the off-centred cores are different sizes, this made it hard to increase the air fraction without distorting the fibre structure further. Placing the guiding core of the 3-cell HC-PCF in the centre of the cladding structure rather than off-centre would have made the stacking process more difficult; however it might have enabled the fabrication of 3-cell fibres with higher air filling fractions and therefore reduced dispersion slopes. How variations in air filling fraction and pitch affect the dispersion and nonlinearity of 7-cell HC-PCFs is explored in section 4.2. Qualitatively, placing a 7-cell core in the centre of a cladding is readily achievable as the 6-fold symmetry of the core leads to cladding designs that typically exhibit 6-fold symmetry and can be jacketed with a minimal use of packing rods. The 3-cell core only exhibits 3-fold symmetry and possible cladding designs require more packing to achieve a tight fit in the jacking tube.

Though it has been shown that 3-cell HC-PCF is not a suitable candidate for low energy soliton propagation a gap in nonlinearity of several orders of magnitude still exist between solid core and 7-cell HC-PCF. Further work into fibre designs that span this gap would be useful. There are two obvious routes to achieving this. The nonlinearity γ of solid core fibre can be decreased, most obviously by increasing the core diameter; though steps have to be taken to keep the fibre single modeled (Birks et al. 1997), (equation 1.3). The second approach is to increase the nonlinearity of HC-PCF as attempted in this work. One option that has been shown to work for increasing the nonlinearity of HC-PCF whilst maintaining low dispersion is to fill the fibre with nonlinear gases or high pressure air (Laegsgaard and Roberts 2009a). This method also allows the Raman response to be tailored; for instance a non Raman active gas (such as xenon) could be used to reduce the Raman response (Ouzounov et al. 2003) or a highly Raman active gas (such as SF6) could be used to increase the Raman response. In a portable system this however requires the end faces of the fibre to be sealed leading to potential coupling and nonlinearity induced problems.
4.2 Engineering the Nonlinearity of 7-cell HC-PCF

(This work was undertaken by R. Amezcua-Correa)

As shown in section 4.1 the creation of a HC-PCF with a reduced soliton threshold energy requires understanding of how both the dispersion and nonlinearity vary with any change in fibre structure. To better this understanding three 7-cell HC-PCFs were modelled, with pitches and air filling fractions chosen in such a way that the high frequency edge of the bandgap was fixed. The dispersion and the nonlinearity as a function of wavelength of these fibres was then calculated, Fig. 4.10.

![Graphs showing the relationship between wavelength, dispersion, and nonlinearity in 7-cell HC-PCFs.](image)

Fig. 4.10. The modelled dispersion (a) and nonlinearity (b) of 3 different 7-cell fibres, designed such that the high frequency edge of their bandgaps occur at the same wavelength. (c) Images of the cores of the three fibre designs, the pitches were 5.5, 4.5 and 3.5μm.

Increasing the air filling fraction and pitch in such a way, increases the spectral width of the bandgap and increases the core size; reducing waveguide dispersion. This also reduces the guided mode’s overlap with the silica: reducing the nonlinear response γ of the guided mode.

Changing the pitch from 5.5μm to 3.5μm (whilst keeping the high-frequency edge of the bandgap fixed) can increase the nonlinearity by as much as 4 times within the bandgap. However, this change in pitch also increases the dispersion by changing the dispersion slope. For example, 10nm away from the zero dispersion wavelength the dispersion is increased by a factor of 4. Hence, at this wavelength there would be no change in the soliton energy and a different approach is necessary if a fibre that supports lower energy solitons is to be found.
4.3 Accurate Dispersion Measurements

Sections 4.1 & 4.2 showed the importance of accurately knowing the dispersion of HC-PCFs for soliton propagation applications. This section details an extension to the technique previously used at Bath for measuring the dispersion of HC-PCFs. The improved accuracy of this technique allowed for the first time the ratio of dispersion to dispersion slope of a 7-cell HC-PCF to be presented.

The method which the CPPM group at Bath previously used to measure the dispersion of HC-PCFs relied on a low-coherence interferometer with the test fibre in one arm (Appendix 2). This was used to measure the wavelength dependence of the group index. By differentiating this data the group velocity dispersion was derived, and in principle, a second numerical derivative provides the dispersion slope. However in practice this is not possible. When a second derivative of any previously published data (Bouwmans et al. 2003) is taken even apparently smooth dispersion data is in fact shown to be too noisy to be helpful. Therefore it is not possible to deduce the dispersion slope from these seemingly accurate plots, without making additional assumptions. This problem is exacerbated with the current generation of HC-PCFs which have reduced dispersion compared to those previously studied (Amezcua-Correa et al. 2006 & Amezcua-Correa et al. 2008). The change in their group index with wavelength is smaller and the signal-to-noise ratio of the measurement decreases. Using longer lengths of test fibre would increase the absolute value of group delay, thus increasing the accuracy of the measurement. However this would require a long free-space delay arm which becomes impractical for long lengths of test fibre.

In these past measurements, the wavelength-tunable low-coherence source for the interferometer consisted of a filtered supercontinuum from a low repetition rate (kHz), sub-nanosecond, microchip laser at 1064 nm pumping a nonlinear fibre. If a high repetition rate mode-locked laser is used as a pump source instead, it is possible to interfere supercontinuum pulses which have been delayed by travelling though a long length of test fibre, with the supercontinuum pulses generated by the next pulse from the oscillator. This allows the use of an almost unlimitedly long test fibre without the use of a similarly long delay arm as the length of the test fibre can be increased by integer multiples of the pump laser’s cavity length.

4.3.1. Supercontinuum Coherence

The supercontinuum to be used in this measurement has to have a high level of shot-to-shot coherence. Supercontinua generated in highly nonlinear PCFs have widely varying noise characteristics depending on the characteristics of the nonlinear fibre and the pump source. Commonly, broadband supercontinua rely on soliton propagation, over some part of the bandwidth, for spectral broadening. If modulation instability is present in the system, it amplifies low level incoherent noise present in the system such that when solitons are subsequently formed (incorporating this noise) they have differing amplitudes and durations from shot to shot of the laser. These solitons self frequency shift at differing rates causing shot to shot variation in the generated spectrum and temporal shot to shot variations through chromatic dispersion, rendering the source useless for the measurement described. To overcome this shorter pulses are often used as the pump source of the supercontinuum, so that self phase modulation plays a greater role in the spectral broadening and the effects of modulation instability are reduced (Dudley and Coen 2002).

The setup in Fig. 4.11.a. was used to investigate the wavelength dependent (shot-to-shot) coherence of different supercontinua generated from a Ti-Sapphire oscillator (Coherent Mira 900) that emitted 200fs pulses at 75.3MHz, with a total power of 550mW (7.3µJ). As this laser system was fixed the only flexible element in the generation of different supercontinua was the length and type of the nonlinear fibre use. In the setup in Fig 4.11.a the arms of the interferometer were unbalanced by a length corresponding to that of the laser cavity, to interfere adjacent continuum pulses in a manner similar to Lu and Knox (2004), establishing over what wavelength range it was possible to obtain fringes.
The pulses from the Ti:Sapphire oscillator (Coherent Mira 900) were passed through a band-pass filter and isolator to minimize feedback. The generated supercontinuum were attenuated before entering the endlessly single mode (ESM) fibre which fed the interferometer so that the all subsequent propagation was linear, wavelength selection was provided by a 3nm band-pass monochrometer. After testing various nonlinear fibres and lengths a long wavelength limit of $\sim$1080nm was found; no fibre tested produced a continuum that was coherent beyond this wavelength.

![Diagram of experimental setup](image)

Fig. 4.11. Two separate interferometric setups (a & b), outlined with dashed lines. They use a common pump source. (a) Shows an interferometer where the arms are unbalanced by a length corresponding to the cavity length in order to test the pulse to pulse coherence of a generated supercontinuum as a function of wavelength. (b) Shows the interferometric setup used to measure the dispersion of a test fibre, where the test fibre is of a length comparable to the length of the laser cavity.

Nonlinear fibres with “large” cores have a zero dispersion wavelengths at long wavelengths far from the (800nm wavelength) pump and the effect of modulation instability is lessened in the supercontinuum generation process (Lu and Knox 2004), making any continuum generated more coherent. However the increased (normal) dispersion combined with the reduced nonlinear response $\gamma$ of the “large” core means that the generated continuum is not broad (Dudley et al. 2006). A similar effect is seen for different lengths of highly nonlinear fibre, whereby longer lengths generate a broader spectrum but are not as coherent. An example of this shown in Fig. 4.12, a 4cm long piece of 1.5$\mu$m core diameter fibre produces coherent light to a long wavelength limit of 1080nm, whereas a 30cm length of fibre generates a broader continuum that is only coherent to 850nm.
4.3.2. Dispersion Measurements

To measure the dispersion of a test fibre, setup Fig. 4.11b was used, with endlessly single mode fibre being used to deliver the signal to the detector in order to eliminate the influence of higher order modes. A lock-in amplifier allowed the fringes to be measured without the intensity of the light in both arms being balanced, and a thin film polarizer allowed one guided polarization to be selected at a time.

HC-PCFs are of particular interest at three wavelengths: 800, 1064 and 1550nm which match the available laser sources. However it was only possible to complete a full measurement on a fibre designed for 800nm using this technique because of the lack of coherence at longer wavelengths. As well as measuring the relative group velocity as a function of wavelength (GVD), it was also possible to deduce the test fibre’s absolute group index, using the following method. Interference fringes were first observed interfering adjacent pulses (with a 4m length of fibre in the test fibre arm). Then this length of fibre was cut very accurately such that the arms of the interferometer became rebalanced. If the length of the removed fibre and the repetition rate of the laser are known accurately then the group index can be deduced by calculating the ratio of the removed fibre length to the length of the laser cavity.

The measured group index and group-velocity dispersion curves are shown in Fig. 4.13. These highly accurate measurements allow a curve of the ratio of dispersion to dispersion slope to be plotted. Accurate knowledge of this ratio is essential as higher order dispersion has a detrimental effect on many of the techniques that use the anomalous dispersion of HC-PCFs such as soliton propagation, pulse compression and modelocked lasers, (Ouzounov et al. 2003, Legsgaard and Roberts 2008 & Lim and Wise 2004 respectively). The group index is close to unity and of a magnitude slightly less than that given by Bouwmans et al. (2003). This is not unexpected as the pitch is larger (2.2µm compared to 1.94µm), the cladding air-filling fraction is higher, and the glass wall that surrounds the core is thinner in the fibre used in the current measurements (Amezcua-Correa et al. 2006 & Amezcua-Correa et al. 2008). The main source of inaccuracy in these measurements is the uncertainty in interpreting the centre of the recorded interference fringes. If this is done with an accuracy of ± 0.005mm and points are taken at a spacing in wavelength of 10nm then it follows that the accuracy of the dispersion points is ± 1.6ps/(nm.km), for a 4.14m fibre length, Fig. 4.14.
Fig. 4.13. The properties of a HC-PCF guiding at 800nm. (a) The group index of the two guided polarizations of the fundamental mode, (b) Dispersion curves calculated for Fig. 4.13a; points show the two-point difference of the recorded group index points, the line is the derivative of a fitted 6th order polynomial, (c) The ratio of dispersion to dispersion slope; the points correspond to the second differential between adjacent points in Fig. 4.13a, the curve is a second derivative of the polynomial fit, (d) The attenuation of the fibre.

Fig. 4.14. (a) Example experimental results of an interference fringe packet, shown in grey. To compute the centre of the packet the fringes were removed using a numerical Fourier filter and a Gaussian curve was fitted to the data, shown in black. The peak of the Gaussian curve is at 5.936nm. The dotted black lines are plotted at 5.936± 0.005mm, (b) Shows a portion of the fringe packet shown in (a) visible are individual fringes.
An accurate measurement of the dispersion of a fibre guiding at 1064nm was also made using a similar 4 m length of test fibre (Fig. 4.16). Due to the lack of coherence of the supercontinuum source in this wavelength range it was not possible to interfere a pulse with next repetition of the laser. Therefore a long free space delay arm was used. A microchip laser was used as the supercontinuum pump source for simplicity.

For a comparison the dispersion of a simulated HC-PCF (Fig. 4.16 d) with a similar structure to the fibre in Figs. 4.16.a,b&c was modelled. The pitch was 3.3µm, the strut thickness 100nm and the size of the glass regions where the cladding struts meet was 310nm. The curve of the ratio of dispersion to dispersion slope is qualitatively similar to that of the measured fibre. The differences between the two fibres are due to the difficulties in measuring and then replicating an actual fibre’s structure when modelling, as slight (µm scale) changes can have large effects on the fibre’s properties. Using a scanning electron microscope (SEM) the measured parameters of the 800nm and 1064nm fibres, were found to be as follows, the pitches were 2.2µm and 3.4µm, the strut thicknesses were 129nm and 152nm and the size of the glass regions where the cladding struts meet were 317nm and 394nm respectively. SEM’s of these fibres are shown in Fig. 4.15.

![Fig. 4.15. Scanning electron micrographs of the 800nm and 1064nm guiding HC-PCF’s shown in (a) and (b) respectively. The core structure of the 800 nm guiding fibre exhibits a large asymmetry; this was introduced unintentionally during the fibre drawing process.](image-url)
The ratio of dispersion to dispersion slope of the 1064nm fibre is much larger than that of the 800 nm fibre, with values of approximately 70nm and 30nm respectively. Frequently dispersion is expressed in terms of $\beta$, Comparing the two fibres the maximum values of $\beta/\beta_c$ are approximately -0.1 and -0.08fs$^2$ for the 1064nm and 800nm fibres respectively. A typical value for a grating pair compressor is -0.7fs$^2$ (at 800nm), (Backus et al. 1998). Attenuation is also of interest in dispersion compensation experiments in Fig. 4.17 the dispersion per 3dB loss length is shown for both the 800nm and 1064nm HC-PCF. The magnitude of the dispersion per 3dB loss for both fibres can be seen to rise towards both edges of the bandgap. It should be noted that a polarisation dependent loss measurement was not undertaken and the attenuation of the two polarisation axes may differ.
4.3.3 Discussion

A technique for precisely measuring the dispersion of low dispersion fibres was outlined and applied to a low dispersion HC-PCF designed to operate at 800nm. The flexibility of this technique was limited by the low shot to shot coherence of supercontinuum which it was possible to generate from the Ti-Sapphire oscillator used as a pump source. The simplest means of improving this coherence would have been to use a laser system that output pulses of shorter durations (Dudley et al. 2006).

A check of the viability of this technique to use non adjacent pulses from the laser and therefore longer delays was not performed. Though it follows that these pulses will have a degree of coherence, as adjacent pulses have a degree of coherence. It would however be interesting to know how the degree of coherence varies for non adjacent pulses from adjacent pulses.

Although the use of a lock in amplifier removed external sources of noise and allowed fringes to be visible when the light in both arms of the interferometer was not balanced, the time average that it performed will have decreased the observed fringe visibility because of the unavoidable drift in the alignment of the interferometer over time. It should be noted though that without the use of a lock in amplifier averaging of multiple shots occurs because of the response time of the detector diode, though this response time is negligible compared to the time scale that drift occurs over.

4.4 Chapter Summary

This chapter detailed efforts by the author towards a better understanding of HC-PCF for soliton propagation applications. Although the work in section 4.1, detailing the fabrication of a 3-cell HC-PCF designed to have a lower soliton threshold energy, had only limited success because of the increased dispersion of the 3-cell fibre, a deeper understanding of the subject was gained. In section 4.2, how the air filling fraction of a 7-cell HC-PCF affects the soliton energy of guided light was investigated through modelling. Section 4.3 reported an improved dispersion measuring technique that allowed the first measurement of the ratio of dispersion to dispersion slope of a HC-PCF.


5 Pulse Compression in Tapered HC-PCF

5.0 Chapter overview

This chapter details two separate experiments where pulses are compressed in tapered HC-PCF.

- In the first experiment (section 5.1) transform limited 2.5ps and 1.2ps pulses were compressed to 21.5fs and 17.5fs respectively, using a 33m taper.
- The second experiment (section 5.2) details the compression of chirped pulses with an autocorrelation width of 8ps to a temporal FWHM of 190fs, using a HC-PCF with both linear and tapered sections.

The author took the leading role in all the work presented in this section.

5.1 Picosecond Pulse Compression

This section details the compression of near transform limited picosecond pulses to femtosecond solitons in a HC-PCF with decreasing dispersion along its length (section 3.3.2). This decrease in dispersion was achieved by increasing the outer diameter of the HC-PCF as a function of its length, which shifts the bandgap and ZDW to longer wavelengths.

Working at Bath, Gérôme et al. (2007) showed that it was possible to compress adiabatically 19.5fs time bandwidth limited pulses to durations less than 90fs in an 8m HC-PCF taper. Compression of pulses to durations less than 100fs is difficult because of the effects of TOD (Ouzounov et al. 2005) and Raman gain (Gorbach and Skryabin 2008). The next logical step is to try to compress picosecond pulses to this limit. This is not only desirable to achieve large headline compression factors; commercial laser systems exist that output transform limited picosecond pulses (Neodymium and Ytterbium glass solid state lasers), the compression of these pulses to femtosecond durations by passing them through a single piece of HC-PCF would be highly desirable. However because the soliton period scales with the square of the soliton temporal duration (equation 1.30), such a taper must be much longer than that demonstrated by Gérôme et al. (2007) if the compression is to remain adiabatic. The taper reported in this section is 33m long.

5.1.1 Taper Fabrication

The outer diameter of an optical fibre created during a draw can be calculated from the diameter of the jacketed cane by conserving the volume entering and exiting the furnace (assuming no inflation or deflation of air holes occurs). Hence for a set diameter of jacketed cane the fibre size can be manipulated by controlling the feed rate and capstan speed. To create the tapered fibre reported in this section, the feed rate was kept constant throughout the drawing process and only the capstan speed was varied in a predetermined quadratic manner to create a taper that had a linear gradient (Fig 5.1); to create a fibre with a linear gradient the capstan speed must vary quadratically if the volume is conserved. For clarity, no feedback system was used; the capstan speed was determined by reading values out of a pre-populated database.

The full section of tapered fibre produced was longer than that used experimentally and was cut back at both ends before it was used. To achieve optimal compression the dispersion at the input of the taper should be high whilst keeping TOD low and the dispersion affecting the soliton at the output of the taper should be close to zero. From SEMs and optical micrographs of the taper it was found that the diameter of the cladding scaled directly proportionally to the outer diameter of the taper. Therefore the pitch and central wavelength of the bandgap was assumed to also vary directly proportionally to the outer diameter of the fibre.
The fabrication of a tapered region of HC-PCF does not require vastly more expertise than required to fabricate a linear HC-PCF; provided that one has the facility to control the speed of the capstan in a programmable manner. However expertise are required to select the required part of taper from a long tapered section. Multiple dispersion measurements are time consuming and are imprecise when measured from small lengths of fibre (section 4.3). Transmission measurements of short off-cuts contain light from cladding modes as well as higher order core guided modes (Petrovich et al. 2008) and therefore are little use for ascertaining the attenuation of the fundamental mode. If the attenuation of the fundamental mode was known, prior knowledge of how the dispersion of HC-PCFs typically varies with attenuation could have been used to predict the actual dispersion of the fibre.

To aid the cut back of the manufactured taper to the required length an attenuation measurement of a piece of linear fibre taken from the front of the taper was performed. By comparing the outer diameter of this linear fibre to the outer diameter profile of the taper an attenuation profile for the taper was predicted. This attenuation profile being used to predict the dispersion profile of the taper using prior knowledge of how the dispersion of HC-PCFs typically varies with attenuation. From this dispersion profile the required outer diameters at the output and input of the taper were predicted and the manufactured taper was cut back*. The dispersion of the input and output of the taper was then measured from off cuts to obtain a measured dispersion profile for the taper (Fig. 5.2). This dispersion measurement was however time consuming and imprecise (as discussed in section 5.1.4).

At the input of the taper and at the 802nm pump wavelength the dispersion was 1.50ps/nm/km. At the output the ZDW was 830nm. The output dispersion was minimized at 830nm rather than 802nm as the effect of the soliton self frequency shift is to increase the wavelength of the propagating soliton (Mitschke and Mollenauer 1986, Gérôme et al. 2007). The measured ZDWs of the input and output are approximately 65nm apart; this is in agreement with a rough calculation from the change in outer diameter of the fibre that predicts the ZDW to change by 69nm.

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* In section 5.2 a commercial autocorrelator is used to aid the cut back process; the output pulse duration was measured as the taper was cut back. The only autocorrelator available for this experiment was a homemade autocorrelator based upon the nonlinear response of a LED (Reid et al. 1997). Using this autocorrelator to measure picosecond pulses was extremely difficult, therefore it was not a practical tool to use to optimise the output of the taper, as when the taper is too long and the soliton is affected by normal dispersion the output pulse duration rapidly broadens (Fig. 5.7). The GRENOLLE (O’Shea et al. 2001) was not rated for pulses longer than 300fs.
Linear fibre drawn at the same time as the taper had a loss of 125dB/km at 800nm giving the 33m taper an estimated loss of 4dB. This loss sets the limit on the maximum length of the taper, as any loss has to be compensated by a decrease in dispersion, or it will result in an increase in the output pulse duration (equation 1.37).

5.1.2 Experimental Setup

The laser system used was a regeneratively amplified mode-locked Ti:Sapphire laser (coherent Mira 900 and a Rega 9000), which gave transform limited 200fs pulses centred at 802nm with a repetition rate of 250kHz and up to 4.4µJ pulse energy; corresponding to an average power of 1.1W. These pulses were then processed using a Fourier plane pulse shaper (Weiner et al. 1988), where the spectral bandwidth was reduced using an adjustable opaque slit placed in the Fourier plane; temporally broadening the pulse. This simple filtering keeps the time bandwidth product small, though loses much of the input power. The propagation of two pulse durations (1.2ps and 2.5ps) through the taper was characterised temporally and spectrally at the output. Pulses much longer than 2.5ps had energies that were too small to work with.

These pulses were coupled into the fibre with an estimated efficiency of 65%, and the polarization was rotated until optimal compression was observed, corresponding to the black dispersion curves in Fig. 5.1. Although it was not confirmed experimentally that the black curves were used, the dispersion of the grey curve is low at the input of the taper and highly normal at the output, and therefore not appropriate for these experiments.

5.1.3 Results

For both input pulse durations, the input energy was increased whilst observing the output pulse duration until the minimum pulse duration was observed. Fig. 5.3. shows the spectral and temporal changes for separately launching 80nJ (20mW), 1.2ps and 55nJ (14mW), 2.5ps pulses into the taper. For these two different input pulses deconvolved output pulse durations of 17.5fs and 21.5fs respectively were recorded on an autocorrelator based on two photon absorption in an LED (Reid et al. 1997).
Observing the output pulses on a (Swamp Optics) GRENOMILL (O’Shea et al. 2001), confirmed the presence of compressed pulses at 812nm and 820nm respectively (Fig. 5.4.) hence it is believed that both these spectral peaks correspond to the solitons. The GRENOMILL computed pulse durations of 138fs and 265fs respectively for these traces. Spectrally both peaks are broader than the input pulse, both having a bandwidth of approximately 4.3nm and hence setting a lower bound on the pulse duration of 1.5fs at this wavelength. The output traces for the 1.2ps pulse from both the GRENOMILL and the autocorrelator are clean. However for the 2.5ps input pulse it is evident that the output pulse sits on a pedestal. This pedestal is more visible if the interferometric autocorrelation in Fig. 5.3.b is converted into an intensity autocorrelation by numerically filtering out the fringes in the frequency domain Fig. 5.3.d; the deconvolved FWHM of this trace is 225fs. The deconvolved FWHM of the Fourier filtered autocorrelation generated from Fig 5.3.a is larger at 240fs, Fig. 5.3.c. The oscillations in the input pulse spectra, Fig.5.2.c and Fig.5.2.f, are an effect of the Fourier plane pulse shaper.

Fig. 5.3. (a)&(b). Input and output interferometric autocorrelations for the 1.2ps and 2.5ps input pulses respectively, the output pulse energies were 19.4nJ (4.83mW) and 15nJ (3.25mW) respectively. (c)&(d) The respective intensity autocorrelations to these interferometric autocorrelations created by apply a numerical Fourier filter. (e)&(f) The respective corresponding spectra.

For all cases the input pulse is grey and the output pulse is black.
Compression ratios of 7 times and 12 times for the 1.2ps and 2.5ps input pulses respectively were achieved in a taper that is only a few dispersion lengths long. In the case of the 1.2ps input pulse it can be estimated from the spectrum that roughly 50% of the output energy is in the compressed pulse. For this case, the input soliton period was 14m, and this length decreases as the pulse is compressed (equation 1.30), therefore it is expected that adiabatic compression was significant. For the 2.5ps pulse the input soliton period (60m) is longer than the entire taper, and spectrally it is obvious that less of the input pulse has been converted into the compressed pulse (around 37%). Based on analysis of the spectrum one can determine the compressed output energies as 9.4nJ (2.35W) and 5nJ (1.25mW) respectively. Assuming a coupling efficiency of 60%, the input pulse energies in the two cases are 47nJ (11.75mW) for the 1.2ps pulses and 33nJ (8.25mW) for the 2.5ps pulses. Based on the fibre parameters, one can estimate the input soliton numbers as N=2 for the 1.2ps pulse and N=2.4 for the 2.5ps pulse. As with work reported in conventional (Pelusi and Liu 1997) and solid-core photonic crystal fibres (Travers et al. 2007), optimum compression with short tapers is found to be in the regime somewhat above the fundamental soliton energy, although the soliton numbers for experiments performed in this section are slightly higher than those reported.
5.1.4 Discussion

A tapered HC-PCF has been demonstrated that compresses 1.2ps and 2.5ps time bandwidth limited pulses by 7 and 12 times respectively. Long pass filters could have been used to filter the dispersive radiation in Fig. 5.3.e&f, to produce spectrally clean solitons and pedestal free autocorrelations, however they were not available at the time.

A cut back measurement of a taper showing the output pulse duration decreasing and increasing as the dispersion at the output of the taper changes from normal to anomalous would have provided further information. Unfortunately this was not completed, though spectral and temporal evolutions are presented in section 5.2 for a different HC-PCF taper.

The measured dispersion is normal at the output of the taper for the 1.2ps input pulse and is slightly normal at the output for the 2.5ps pulse. When the taper output was cut back by 5m the measured output pulse duration increased. This discrepancy may be due to the difficulty of taking accurate measurements of the dispersion profiles of rapidly tapered HC-PCFs. The use of a phase retrieval measurement such as FROG or SPIDER (Trebino et al. 1997, Iaconis and Walmsley 1998 respectively), could have been used to determine the precise spectral temporal profile of the output pulses from the taper. This would determine whether the output pulses were unchirped solitons propagating in anomalous dispersion or chirped pulses propagation in normal dispersion, it would also allow the size of any pedestal to be quantified effectively.

One method that may have avoided this discrepancy in the dispersion of the output of the taper, would have been to fabricate multiple sizes of linear fibre at the same time as the taper, then to infer the dispersion of the taper from measuring the outer diameter and the dispersion of these lengths of linear fibre. Measurements of the dispersion of these linear lengths of fibre would be more accurate than measurements of the tapered sections as longer lengths of fibre can be used; the longer lengths creating a larger variation in group delay with respect to changes in wavelength, section 4.3. Moreover this technique of comparing the outer diameter could be used to create replicas of an already functioning taper.

This experiment built upon the work of Gérôme et al. (2007) and showed that it is possible to fabricate HC-PCFs that compress multi-picosecond optical pulses to durations of a few hundred femtoseconds. HC-PCF tapers are robust, have a small footprint and deliver spatially clean Gaussian-like output beams, making them useful components for “upgrading” picosecond laser systems.
5.2 Compression of Solitons Created from Chirped Laser Pulses

Most pulses directly outputted from fibre amplified laser systems are chirped. They acquire this chirp in the amplifying fibre both from its normal dispersion and from the occurrence of SPM. Working at Bath, Gérôme et al. (2008) showed that chirped picosecond pulses from such a source could be compressed to time bandwidth limited femtosecond solitons by propagating them through the anomalous dispersion of a length of HC-PCF. It has also been shown by Gérôme et al. (2007) that it is possible to compress femtosecond solitons propagating in a HC-PCF, using an adiabatic decrease in dispersion. (For further details see sections 3.3 and 3.2.2 respectively).

Section 5.2 details an experiment using a single piece of HC-PCF that combines these two processes. The input of the HC-PCF has a constant outer diameter along its length. In this linear fibre the anomalous dispersion of the HC-PCF compensates the chirp of the laser pulses, increasing their peak power and allowing the formation of solitons. The outer diameter of the fibre then increases along its length. This increase in the outer diameter shifts the bandgap to longer wavelengths, decreases the dispersion at a fixed wavelength and compresses the soliton. To achieve the minimum pulse duration the soliton should exit the fibre at a wavelength with nearly zero anomalous dispersion. For clarity the fibre described in this paragraph will henceforth be described as the “linear then tapered fibre” (LTT fibre).

It is important that this LTT fibre be fabricated as a single piece of fibre and is not created by splicing a piece of linear fibre to a piece of tapered fibre as any loss at the splice will cause the soliton to broaden (Equation 1.31). This loss occurs due to the modal mismatch and the deformation of the fibre structure that occurs in a splice. In addition the polarisation axes of both fibres would need to be exactly aligned so that a clean polarisation state of the tapered fibre is excited.

5.2.1 Laser System and Experimental Setup

The amplified fibre laser system used in the following experiments output chirped 500mJ pulses, centred at 1064nm, with a recorded autocorrelation FWHM of 8ps, a repetition rate of 20MHz and an average power of 10W (Fig. 5.5.). These pulses were transmitted through freespace power and polarisation controls before being launched into the LTT fibre. Pulses from the output of the fibre were fed into a commercial (APE Pulse Check) autocorrelator and an OSA (Ando AQ6315A) for characterisation.

In Fig. 5.5.f the leading and trailing edges of the pulse are sharp: a characteristic of pulse self steepening (Grischkowsky and Balant 1982, Nakatsuka et al. 1981). This effect occurs in waveguides with SPM and normal dispersion (the amplifier stage of the fibre laser in this case) and can be visualised in the following manner. Through SPM the leading edge of the pulse becomes red shifted, in material with normal dispersion these red shifted components travel away from the centre of the pulse, with the most red shifted components travelling at the fastest rate. As the most red shifted components are generated at the steepest part of the leading edge of the pulse which then travels away from the centre of the pulse at the fastest rate it leads to a steeping of the leading edge of the pulse. The trailing edge of the pulse becomes self steepened in the same manner but for blue shifted components.

The chirp on the pulses arises as consequence of SPM and normal dispersion in the amplifier stage of the fibre. In Fig 5.5.e & 5.5.f the laser pulses have a clear quadratic phase shift (linear chirp) making them ideal for being compressed in a linearly dispersive element; however the chirp from SPM alone is far from being linear (section 1.3.5). This linear chirp arises as a consequence of the pulses being self steepened; SPM generates new frequencies primarily at the steep edges of the pulse which gradually move apart in time through normal dispersion, developing a linear frequency shift across the pulse (Tomlinson et al. 1984).
Fig. 5.5. The characteristics of the amplified fibre laser system used. (a) & (b) A measured spectrum and intensity autocorrelation of the laser. (c), (d), (e) & (f) A measured second harmonic FROG trace, corresponding retrieved trace, spectrum and temporal profile respectively. The dotted lines in (e) & (f) are quadratic phase and have been included to demonstrate how closely the chirp on the laser pulses approximates a linear chirp. Note: The wavelength scale in the FROG measurements is set by the phase matching angle of a second harmonic generating crystal. The offset between the FROG and OSA spectra in wavelength is due to the angle of the crystal not being calibrated to an absolute value (the rate of change of wavelength on the scale is correct).
5.2.2 Fabrication

It is important that the dispersion at the input of the LTT fibre is highly anomalous but has low TOD at the pump wavelength so that the chirp the on laser pulses can be compensated effectively. Unlike in section 5.1 the dispersion of the linear fibre at the input of the fabricated tapered section must be correct; this complicates the fabrication process greatly. To achieve this several HC-PCF fabrication draws were undertaken until the criteria of high anomalous dispersion but low TOD at 1064nm could be fulfilled reliably. Then during a repeat fabrication of this linear HC-PCF, the capstan speed was linearly decreased and then linearly increased with respect to the drawn fibre length to create a fibre in which the outer diameter varied as shown in Fig. 5.6. This fibre was then cut in the middle to yield two LTT fibres.

![Fibre Outer Diameter profile](image)

Fig. 5.6. The measured outer diameter profile of a section of fabricated fibre from which the LTT fibre was made.

To optimise the length of linear fibre at the input of the LTT fibre, a separate experiment was done using lengths of linear fibre from the same fabrication draw. It was found that 14m of linear fibre was sufficient to compensate the chirp of the laser pulses and form spectrally separated solitons. The linear fibre at the input of the LTT fibre was therefore cut back to this length.

To achieve optimal compression the soliton must exit the tapered part of the LTT fibre at a wavelength with nearly zero anomalous dispersion. To affect this optimisation the output of the taper was cut back by small lengths whilst measuring the output pulse duration. As lengths of fibre with normal dispersion at the soliton wavelength were removed, the measured output pulse duration shortened. This process of cutting back the output of the fibre continued until the measured output pulse duration began to increase. This temporal broadening corresponding to the dispersion felt by the soliton becoming increasingly anomalous Fig. 5.7. The outer diameter of the completed LTT fibre is shown in Fig. 5.8, the linear fibre was 14m long, the taper section was 13m long.
Fig. 5.7. The variation in output pulse duration as a function of removed fibre length from a manufactured LTT fibre, it can be seen that the soliton rapidly broadens if the taper is too long and is affected by normal dispersion. Notes: 1. Zero removed fibre is arbitrary defined. 2. This cutback was not performed on the same LTT fibre which is reported in the rest of this section, though it was created using the same variation in capstan speed.

Fig. 5.8. The measured outer diameter profile of the completed LTT fibre. The linear fibre was 14m long, the taper section was 13m long. The “steps” visible are an artefact of how the outer diameter was recorded and logged.

The dispersion of the input and output of the LTT fibre was measured from removed off cuts and is displayed in Fig. 5.9.a. At the pump wavelength (of 1064nm) the dispersion of the input was highly anomalous (9.5ps/nm/km). The ZDW of the output was measured to be 1110nm. Throughout the following experiments only the displayed polarisation state of the LTT fibre was excited; the dispersion of the other polarisation state was not appropriate for this experiment and therefore femtosecond pulses were not observed.
5.2.3 Results: Linear Fibre

Fig. 5.10 shows results for the transmission of the aforementioned laser pulses through the 14m of linear fibre comprising the input of the LTT fibre. Although these results are presented before the response of the LTT fibre is characterised, they were taken last to allow the LTT fibre to be cut up into its constituent linear and tapered parts. In these results the power launched at the fibre was 9.5W (47.5nJ), this was coupled into the fibre with an estimated efficiency of 70%, the power through the fibre was 5.3W (265nJ). The output spectrum has a clearly defined soliton which has self frequency shifted away from the pump wavelength to 1092nm, spectrally 49% of the output power is in this soliton. Blocking the residual pump spectrum with a long pass filter yields a pedestal free pulse with an autocorrelation FWHM of 600fs (corresponding to an intensity FWHM of 390fs if a sech² shape is assumed). The transmitted power through this filter was 2.3W (11.5nJ), the pulse having a spectral bandwidth of 4.5nm and hence being time bandwidth limited to 270fs.

At the end of the linear section of the LTT fibre the soliton period can be calculated as approximately 1m. Along the tapered section this will become shorter as the dispersion and soliton’s duration decreases. The tapered section of the LTT fibre is therefore many soliton periods long and the change in dispersion can be considered adiabatic.
5.2.4 Results: LTT Fibre

The spectral and temporal response of the full 27.1m LTT fibre as a function of input power is shown in Fig. 5.11. As the input power and hence the energy of the soliton is reduced the observed self frequency shift is reduced. This ability to tune the self frequency shift of the soliton and therefore the dispersion felt by it, was used to optimise the compression. The observed output autocorrelation FWHM reached a minimum value of 323fs (210fs deconvolved) for a power prior to the coupling optics of 7.88W (394nJ) and an output power of 2.88W (144nJ).

It should be noted that the measured dispersion at the output of the taper (Fig. 5.9.a) was normal at the soliton wavelength, and as such the dispersion should not be optimised. This measurement is believed to be in error due to the difficulty in measuring the dispersion of tapered HC-PCF and that in fact the dispersion was optimised.

![Spectral and temporal response of LTT fibre](image)

*Fig. 5.11: The spectral and temporal evolution observed launching the chirped laser pulses into the LTT fibre, recorded as a function of output power. (a) shows the recorded spectra (these are independently and arbitrary normalised). (b) shows the corresponding autocorrelation FWHMs (grey) and the calculated intensity FWHMs (black).*

Using an 1100nm long pass filter, tuned in wavelength by rotating it away from normal incidence it was possible to suppress the non solitonic radiation in a controlled manner. By doing so an autocorrelation FWHM of 292fs (190fs deconvolved) was observed, Fig. 5.12. The output power after this filtering was 1W (50nJ), with approximately 50% of the output power being in this filtered spectrum. The spectral bandwidth of the soliton is at least 8.2nm (compared to 4.5nm at the start of the taper), and therefore is time bandwidth limited to 150fs. It is evident from the spectral and temporal shape of the filtered pulse that some non solitonic radiation remained.

Through the use of a saturable absorber it should possible to suppress this pedestal and improve the performance of the LTT fibre. The filtered pulses were frequency doubled using a non critically phase matched LBO crystal in manner described in section 6.1.1. the aforementioned 1W (50nJ) of filtered power yielding 0.63W (32nJ) of frequency doubled power, with the pulses having a deconvolved duration of 170fs. This conversion efficiency is higher than that achieved with the spectrally filtered 400fs pulses from the linear fibre which were frequency doubled with efficiencies of less than 55%. This high efficiency (in both cases) implies that there was a small percentage of the power in the pedestal.
Fig. 5.12. Shows the improvement in the performance of the LTT fibre if spectral filtering is employed. (a) shows the spectral response of the LTT fibre for an output power of 2.88W (144nJ), also shown is a filtered part of the spectrum. (b) Shows the corresponding autocorrelations for the whole spectrum (grey) and the filtered part of the spectrum (black), also shown is the autocorrelation of a numerically generated 190fs soliton (dashed black).

5.2.5 Side-Scan Measurement

Due to the high average power of the laser source it is possible to record the spectral evolution along the length of the fibre by collecting light lost through the side of the fibre. This light was collected by placing a multimode fibre with a core diameter of 200µm against the side of the LTT fibre; the output of the multimode fibre being fed to an OSA. Spectra were collected at 0.5m intervals and were normalised such that each recorded spectrum had the same integrated spectral power. A composite of these spectra is shown in Fig. 5.13, the unfiltered output pulse duration was 210fs and the output power was 2.88W (144nJ). Care must be taken in interpeting this data because the loss of the LTT fibre is highly wavelength dependent and therefore the relative brightness of different spectral components should be scaled by the wavelength dependent loss.

Observing Fig. 5.5 it can be noted that the spectral phase varies across half of the pulse by 17 radians in 6 nm. Modelling this chirp as being canceled by solely second order dispersion ($\beta_2$), a rough calculation ($17\text{ radians} = \beta_2\Delta x \equiv \frac{1}{2}\beta_2(\omega_0 - \omega)^2 x$) predicts the chirp will be cancelled after approximately 2m. This is in agreement with Fig. 5.13 where a spectrally separate soliton can clearly be seen after 5m. For more details on the soliton formation process from chirped laser pulses see section 3.3.1

The soliton self frequency shifts to longer wavelengths with a rate that decreases with length until it reaches the tapered part of the LTT fibre. This decrease is caused by the soliton broadening and losing energy from the effects of TOD, Raman gain and waveguide loss as it propagates. In the tapered region the rate of self frequency shift increases along the length of the fibre. This occurs because the pulse becomes compressed by the decreasing dispersion, increasing the peak power and spectral bandwidth of the pulse.
Fig. 5.13.(a&b) The spectral evolution of the LTT fibre for an output power of 2.88W (144µJ). The results were recorded using the aforementioned “side-scan” method and are displayed in arbitrary units without and with interpolation.
5.2.6 Discussion

The LTT fibre decreased the duration of pulses from the fibre amplified laser system by approximately 40 times compared with a 20 times compression from a solely linear fibre. The 90% power loss that occurred with this compression is not particularly economically disadvantageous because fibre amplifiers (unlike solid state systems) have high gain efficiencies due to the long interaction lengths possible (Limpert et al. 2003) and therefore this lost power is not costly.

Freespace polarization and power controls were used as well as freespace optics to launch light from the fibre laser into the LTT fibre; however the fibre could have been spliced directly onto the output of the amplified fibre laser. This could potentially have increased the coupling efficiency into the LTT and would have acted as a robust alignment-free compression element.

In Fig 5.13, it appears that the linear fibre at the input of the LTT is longer than necessary, as a spectrally separate soliton can be seen after approximately 5m. The soliton in this length of linear fibre self frequency shifts at decreasing rate with respect to the length of the fibre, indicating the pulse energy is decreasing and/or the temporal width of the soliton is increasing. Shortening the length of linear fibre may have decreased the output pulse duration from the LTT fibre; this was not explored fully. Investigating it would have been a time consuming and complicated process as the output of the LTT fibre must also be cut back so that the ZDW matches the reduced self frequency shift of the soliton. Having a longer “than necessary” length of linear fibre at the input of the LTT fibre has advantages. If the output of the taper is not spectrally filtered then a long length of linear fibre will disperse non solitonic radiation so that the soliton will sit on a “smaller” pedestal at the output. Furthermore the cumulative dispersion of the tapered section of HC-PCF will be zero at a single wavelength: in a similar manner to a dispersion mapped fibre (Agrawal, 2007, pp144). This zero cumulative dispersion wavelength will be frequency shifted away from 1064nm where dispersive radiation is present by having a long anomalously dispersive section at the input of the LTT fibre. It is also possible that the LTT fibre will be more stable to variations in the chirp of pulses from the fibre laser because the rate which the soliton self frequency shifts decreases proportionally to the propagation length.

The ability to use the “side-scan” technique to non-destructively measure the spectral evolution along a fibre is a one of the advantages of using high average power laser systems. Further work on this technique, exploring ways to gain higher resolution and to calibrate for wavelength dependent loss could save time in future experiments and the need to cut up carefully created fibres.
5.3 Chapter Summary

The work presented in this chapter built upon the work done at Bath by Gérôme et al. (2007) and Gérôme et al. (2008) (section 3.2.2 and section 3.3 respectively), and has shown the ability of tapered HC-PCF to compress temporally picosecond and femtosecond pulses, by over 10 times as well as how variations in input power can be used to optimise the compression.

Broadly speaking two quality factors are important in the compression of pulses; firstly the compression ratio (which can be limited by the minimum pulse duration (Gorbach and Skryabin 2008)) and secondly the amount of the input power that is contained within the compressed output pulse (how adiabatic the compression is). A full analysis of how these quality factors vary with changes in; fibre length, tapering rate, HC-PCF type, bandgap width, attenuation and the gas that fills the HC-PCF’s core (to name a few variables) does not exist and would be highly interesting and beneficial to the design of future tapers. It would also be interesting to know how the position of the ZDW at the output of the taper affects compression ratio and the amount of the input power that is contained in the compressed pulse.
6 Soliton Effect Compression of Green Solitons in HC-PCF

6.0 Chapter Overview

This chapter details the propagation and soliton effect compression of pulses centred at 539nm wavelength in HC-PCF.

The author took a leading role in the collection of the experimental results shown in this section. He also performed the numerical simulations in Fig. 6.10. The green guiding HC-PCFs were manufactured by a colleague who also performed the loss measurements, and recorded the scanning electron micrographs (B. J. Mangan), the dispersion measurement of HC532 was performed by the author, the dispersion measurement of “HC508” was performed by a colleague (P. J. Mosley).

6.1 Introduction

This section details the compression and delivery of 300fs input pulses centred at 539nm, through green guiding HC-PCFs. Over a 1m fibre length the 300fs pulses were compressed to 109fs; the longest fibre length investigated was 3m for which a minimum output pulse duration of 230fs was observed.

HC-PCFs can deliver light from a laser source to an experiment/device with peak powers that would lead to nonlinearity induced problems if solid core fibres were used (section 1.3). Unlike the only other viable alternative: freespace optics, they are robust, maintenance free and have a very small footprint. The shortest reported wavelength at which solitons have been reported in HC-PCF is 800nm (Luan et al. 2004). There are however multiple applications where the delivery of light at a shorter wavelength is necessary/advantageous:

- Short wavelength (visible/UV) photons are more effective than IR photons at writing waveguides in glasses because of their higher photon energies (Davis et al. 1996).
- Multiple fluorescent proteins used with techniques such as laser scanning confocal microscopy have peak excitation wavelengths which are at UV-visible wavelengths (Miyawaki et al. 1997).
- The minimum spot diameter which light can be focused to using a lens is set by the diffraction limit which scales proportional to wavelength. Therefore the delivery of solitons at short wavelengths has applications in laser machining/nanostructuring (Korte et al. 2003).

Moreover for laser machining applications the use of temporally short pulses such as those outlined in this section create cleaner structures than picosecond pulses. Damage from picosecond pulses occurs through energy transfer to the lattice of the material, in a thermal process where material is melted and boiled away leaving a residue of material around the machined area. Femtosecond pulses have sufficiently high peak powers to ionise the atoms away from the material on a time scale too short for significant energy transfer to the lattice to occur, the removed material being directly linked to the spatial profile of the beam (Perry et al. 1999).
6.1.1 Experimental Setup

The 539nm light used to excite solitons in green guiding HC-PCFs was generated from a laser centred at 1064nm. A diagram of the experimental setup used is shown in Fig. 6.1.

![Diagram of experimental setup](image)

**Fig. 6.1.** A schematic of the experimental setup used to investigate the propagation and compression of solitons at 539nm.

Key: PBS = polarising beam-splitting cube, \( \lambda/2 \) = half waveplate, OSA = optical spectrum analyser, LBO = lithium triborate.

Note: The 1100nm long pass filter was placed at a non normal incidence angle to allow its cut off wavelength to be tuned.

The laser used had a repetition rate of 20MHz and an average power 10W (500nJ), it emitted highly chirped picosecond pulses centred at 1064nm, (this was the same laser source used in section 5.2; a complete spectral and temporal characterisation can be seen in Fig. 5.5). The laser pulses were passed though power and polarisation controls before being launched into 15m of HC-PCF with an estimated efficiency of 55%. The power at the output of this length of fibre was 4.5W (225nJ).

*(As explained in sections 3.3 & 5.2 the chirped picosecond pulses from this source can be compressed to femtosecond solitons by propagating them through the anomalous dispersion of a HC-PCF.)*

The solitons at the output of this fibre were centred at 1078nm (because of the occurrence of a self frequency in the HC-PCF) and had a temporal FWHM of 400fs. These pulses were then passed through a long pass filter to remove the non self frequency shifted dispersive radiation. The power after this filtration was a 2W (100nJ). These spectrally clean pulses centred at 1078nm were frequency doubled to 539nm using a 3mm long non-critically phase matched lithium triborate (“LBO”) crystal tuned in an oven (Velsko et al. 1991).
To maximise (and tune) the conversion efficiency of the frequency doubling process a half waveplate was used to orientate the polarisation of the input 1078nm beam to the principle axis of crystal, a dichroic beam splitter was placed after the crystal to remove the residual 1078nm light. The maximum conversion efficiency achieved was 50%; and corresponded to 1W(50nJ) of spectrally clean 300fs pulses, which had a spectral width of 1.3nm Fig. 6.2. These pulses had the low time bandwidth product of 0.4.

A non-critically phase matched crystal was chosen to frequency double the 1078nm beam, to preserve beam quality during the frequency doubling process (Gérôme et al. 2008), and therefore increase the coupling efficiency for launching into the green guiding HC-PCFs. The generated 539nm wavelength light was launched separately into two different green guiding HC-PCFs which had measured ZDWs of 532nm and 508nm Fig. 6.3; these fibres will henceforth be referred to as HC532 and HC508. The outputs of these fibres were fed to an autocorrelator (APE Pulse Check) and an OSA (Ando AQ6315A) for temporal and spectral characterisation.

Fig. 6.2. A recorded spectrum (a) and autocorrelation (b) of the frequency doubled 1078nm light.
Fig. 6.3. The measured attenuation and dispersion of HC508 and HC532 shown in (a) and (b) respectively. The points on the dispersion curves represent the two point difference between measured values of group delay. Shown in (c) and (d) are scanning electron micrographs of HC508 and HC532 respectively, the deformation of the core walls of both fibres occurred when the end faces of the fibres were prepared/cleaved.

The birefringent nature of HC-PCFs should be noted, and that only one dispersion curve is presented for each fibre in Fig. 6.3. The proximately of the pump wavelength to the bandgap edge and ZDW of HC532 means that rotating the input polarisation between the polarisation axes of the fibre results in a large relative change in the dispersion and attenuation (Wegmueller et al. 2005, Poletti et al. 2005). The polarisation mode of HC532 used in the experiments had significantly lower attenuation than the orthogonal polarisation allowing it to be identified when the dispersion of the fibre was measured. During soliton propagation experiments there was little difference identifiable between the responses of the two polarisation axes of HC508 and therefore only one is presented in Fig. 6.3.

It should also be noted that there was a large difference in the attenuation of HC532 dependent on the input polarisation. A polarisation dependent loss measurement would have been interesting as it might have explained the discontinuity in the gradient of the attenuation curve that can be seen in Fig 6.3.b. at 530nm. The attenuation of HC-PCFs is inherently larger at visible frequencies than in the near infrared (section 2.2.5), the minimum attenuation of both HC532 and HC508 is \( \sim 1000 \text{dB/km} \). This compares with typical attenuations of 200dB/km at 800nm, 50dB/km at 1064nm and 15dB/km at 1550nm (Roberts et al. 2005).
6.1.2 Results HC508

Launching the aforementioned 1W(50nJ) of 539nm light at the input of 1.35m of HC508 a power of 385mW(19.25nJ) was measured at the output. The output pulse duration of the 320fs was close to that of the 300fs input pulses. Spectrally, a self frequency shift of 1nm was observed, Fig. 6.4, and the spectral bandwidth of the output pulses (1nm) was slightly less than the input pulses (1.3nm). The pulses are spectrally clean apart from the presence of dispersive radiation at shorter wavelengths created by the perturbative effects of TOD and Raman gain acting on the soliton.

![Graph showing spectral response and autocorrelation of HC508](image)

Fig. 6.4. (a) The spectral response of 1.35m of HC508 for an output power of 385mW(19.25nJ). (b) Shows the corresponding recorded autocorrelations. Black is the output pulse, grey is the input pulse.

This maintenance of pulse duration and spectral width during propagation is characteristic of pulses with soliton numbers close to N=1. Furthermore the time bandwidth product of these pulses can be calculated to be 0.33 which is close to the value expected for a fundamental soliton (0.315). If it is assumed that the output pulse is a fundamental soliton then the Kerr nonlinear coefficient of the HC508 can be estimated to be \( \gamma \approx 3.5 \times 10^{-7} \text{W}^{-1}\text{m}^{-1} \) at this wavelength. This compares to previously reported values at 800nm and 1500nm of \( \gamma \approx 2.4 \times 10^{-7} \text{W}^{-1}\text{m}^{-1} \) and \( \gamma \approx 2.1 \times 10^{-6} \text{W}^{-1}\text{m}^{-1} \) respectively (Luan et al. 2004 and Ouzounov et al. 2003).

The dispersion felt by the input pulse was 350ps/(nm.km), increasing to 400ps/(nm.km) at the output of the fibre (from the 1nm self frequency shift). The 1.35dB/m attenuation of the 1.35m fibre results in 35% of the input light being lost. Spectrally, 93% of the output power remains within the solitonic peak. Assuming that the output pulse is a fundamental soliton the input soliton number must be N>1.3 to compensate for these effects (equation 1.37). The soliton period of the input pulse can be calculated to be approximately 0.8m.

An evolution of the output spectrum and temporal duration as a function of output power is shown in Fig 6.5. As the output/input power is decreased the soliton number decreases and the output pulse duration increases. This decrease in the pulse duration and pulse peak power reduces the observed self frequency shift. Assuming the input pulse was a bandwidth limited Gaussian shaped 300fs pulse, under purely linear conditions it can calculated that the output pulse duration would be 8106fs (section 1.2.3). Unfortunately it was not possible to verify this experimentally as the autocorrelator used was not sensitive enough to measure pulses with powers lower that those presented in Fig. 6.5.
6.1.3 Results HC532

Launching light into HC532 it was possible to excite higher order solitons than in HC508, because of the proximity of the ZDW to the pump wavelength. The dispersion of HC532 at the pump wavelength was measured to be 65ps/(nm.km), compared to 350ps/(nm.km) for HC508.

The evolution of pulses through a 3m and a 1m length of HC532 was characterised. The results for the 3m length are shown first:

3m HC532

The input and output spectral and temporal profiles for launching 1W(50nJ) of power at the input of 3m of HC532, with a power of 300mW(1.5nJ) being measured at the output of the fibre are shown in Fig. 6.6.

For this power soliton effect compression was observed with a temporal FWHM of 230fs being measured at the output of the fibre. This evidence of temporal compression is supported by the bandwidth of the pulse broadening from 1.3nm to 1.8nm. A self frequency shift of 4nm was also observed, this shift corresponds to the dispersion felt by the soliton increasing from 65ps/(nm.km) to 105ps/(nm.km). Spectrally 85% of the output power remains in the soliton peak. The attenuation of the 3m length of fibre corresponds to ~3.3dB.

Assuming the soliton number of the output pulse is N>1, then to compensate for the aforementioned; fibre attenuation, the dispersive radiation created, the increased dispersion felt by the output pulse and the temporal compression, then the input soliton number must be N>2.3. The soliton period can be calculated to be 4.6m for the input pulse. Approximating the nonlinearity response γ of HC532 at this wavelength to be the same as that calculated for HC508 (γ=3.5×10⁻⁶ W⁻¹m⁻¹) then input and output soliton numbers of N=3.2 and N=1.7 respectively can be calculated.
A spectral and the temporal evolution of the output pulse as a function of output power can be seen in Fig. 6.7. Assuming the input pulse was a time bandwidth limited Gaussian shaped 300fs pulse, under purely linear conditions it can calculated that the output pulse duration would be 430fs (section 1.2.3). All of the observed output pulse durations were less than or equal to the 300fs input pulse. With the input pulse duration being equal to the output pulse duration for an output power of 34mW(1.7μJ). If this output pulse is assumed to be a fundamental soliton then the input pulse must be an N=1.4 soliton to compensate for the attenuation of the fibre. This calculation is in agreement with the nonlinear response of the fibre being $\gamma \approx 3.5 \times 10^{-21}$ W m$^{-1}$ as assumed previously.

Fig 6.7. Results showing the temporal and spectral evolution of pulses though 3m of HC532 as a function of output power, shown in (a) and (b) respectively. The intensity map in (b) was formed by normalising recorded spectra to the output power.
**1m HC532**

Temporal narrowing from soliton effect compression is much more apparent in the 1m length of HC532. Results for the maximum obtainable output power of 670mW (33.5nJ) are shown in Fig. 6.8. In these results significant broadening of the 1.3nm input pulse occurs; the main peak in the output pulse has a spectral width of 2.8nm. The measured output pulse duration was 109fs.

If the nonlinear response of the fibre is assumed to be $\gamma \approx 3.5 \times 10^5$ W⁻¹m⁻¹ as assumed previously, the input soliton number at this power can be calculated to be $N = 3.6$. Neglecting, higher order dispersion, Raman gain and the fibre attenuation, the optimal compression length for this soliton is approximately 0.8m (0.17% of the 4.5m soliton period), (Fig. 3.7.). It is evident in Fig 6.8, that the output pulse sits on a pedestal, as explained in section 3.2.1 this is the main disadvantage of the soliton effect compression. As the temporal characterisation was performed using an autocorrelation the size of the pedestal was not quantified. The use of a technique such as FROG or SPIDER (Trebino *et al.* 1997, Iaconis and Walmsley 1998) would have allowed a full characterisation of the output pulses.

![Fig. 6.8. (a) The spectral evolution of pulses through 1m of HC532 for an output power of 670mW (33.5nJ). (b) Shows the corresponding recorded autocorrelations, the dotted line is a sech² pulse of the same duration as the output pulse. Black is the output pulse and grey is the input pulse.](image)

**Temporal and spectral evolutions of the output pulse as a function of power are shown in Fig. 6.9.** As the power and the input soliton number increases, the output pulse duration reduces and the size of the pedestal increases (Fig. 3.7.). Notably the output pulse duration is always less than the input pulse duration for all measured output powers. Assuming the nonlinearity of HC532 to be $\gamma = 3.5 \times 10^5$ W⁻¹m⁻¹ the soliton number at the lowest input power can be calculated to be $N = 0.9$, and therefore a slight temporal broadening is expected. The author attributes this discrepancy to the noise present on low power autocorrelation traces (Fig 6.9. b).
Fig 6.9. The characteristics of pulses through 1m of HC532, all shown as a function of the power measured at the output. (a) The deconvolved temporal FWHM. (b) An intensity plot of the recorded autocorrelation traces, the individual traces were normalised to have a peak value of 1, interpolation between the recorded traces was applied. (c) An intensity plot of the recorded spectra, the individual spectra were normalised to the output power.

Split step Fourier modelling of this propagation through 1m of HC532 for an output power of 670mW was undertook (equation 1 from Gorbach and Skryabin 2008) and agreement with the experimental results was found Fig. 6.10. The nonlinearity was modelled to be $\gamma = 3.5 \times 10^{-3}$ W$^{-1}$m$^{-1}$ as previously assumed and was modelled to arise half from the guided mode’s overlap with the glass and half from its overlap with air as this is typical for a 7-cell HC-PCFs (Luan et al. 2004). A constant attenuation of 1.1dB/m was assumed. The input pulse was an N=3.6, 300fs soliton and the output power was 670mW(33.5mJ). $\beta_2$ and higher order dispersive terms were taken from a polynomial fit to Fig. 6.3.b.
The modelled spectrum is slightly broader than that observed experimentally; the main peak has a FWHM of 3.2nm compared to 2.8nm seen experimentally. When modelling the propagation of higher order solitons in a material with solely Kerr nonlinearity and second order dispersion (no higher order effects), the minimum pulse duration in their periodic evolution arises from all the frequency components that constitute the pulse. However when higher order solitons are perturbed by TOD and Raman Gain they evolve into fundamental solitons as they propagate, with dispersive spectral components being visible to shorter wavelengths as the soliton self frequency shifts to longer wavelengths. In the modelled spectrum shown in Fig 6.10.a removing light at shorter wavelengths than the main peak temporally broadened the pulse, however this behaviour was not explored in the experimental result through the use of a filter. The modelled output pulse duration was shorter than that observed experimentally, 40fs compared to 109fs; the calculated time bandwidth products of the pulses using the bandwidth of the main spectral peak are 0.13 and 0.31 respectively (the time bandwidth limit of a fundamental soliton is 0.315).

### 6.1.4 Conclusion

Launching pulses into (1.35m of) HC508 soliton numbers of N<1.3 were generated at the input of the fibre and no compression was observed at the output of the fibre because of the effects of loss, higher order dispersion and Raman gain. This contrasts with the soliton numbers of N< 3.6 that were generated in (1m of) HC532, where a temporal compression of almost a third (300fs to 109fs) was observed. The reason for this difference in response is due to the reduced dispersion (at 539nm) of HC532 compared to HC508: 65ps/(nm.km) compared to 350ps/(nm.km). Although nanometre scale changes in the structure of HC-PCFs can cause substantial changes in their properties, HC532 and HC508 were designed to have the same cladding/core structure (scaled in size), therefore it is reasonable to assume that their nonlinear responses (γ) are similar.

A more thorough study of how compression varies as a function of detuning of the green pump wavelength from the green guiding fibre’s ZDW would be interesting. This could be achieved by fabricating multiple green guiding fibres with different ZDWs or by tuning the wavelength of the input green light. If the length of the 1064nm guiding fibre was adjusted it would change the magnitude of the self frequency shift that occurs and hence the wavelength of the generated green light. Experimentally this would be easier than fabricating multiple fibres and would remove any inaccuracies that may arise from unintentional changes in the HC-PCF’s structure between fibre draws.
It would be interesting to have completed an experimental investigation of multiple lengths of HC532 to find the length at which the output pulse duration was minimised for a fixed input power, (section 3.2.1). Modelling predicted that 1m of this fibre corresponded to the optimal fibre length, for lengths longer than this length soliton fission occurred and multiple (temporally broader) pulses where generated.

As previously explained the attenuation of HC532 and HC508 is approximately 200 times greater than what obtainable at 1550nm (Amezcue-Correa et al. 2008). This increased loss arises primarily from two mechanisms, firstly surface scattering losses which scale proportional to $\lambda^{-3}$ (section 2.2.5). Secondly the fabrication of visible guiding HC-PCFs is technically more difficult than IR guiding fibres, because (in theory) the size of the pitch, apex sizes and strut sizes scales proportional to the centrally guided wavelength (for a set fibre design). Therefore the microstructure of visible guiding fibres is usually less regular that IR guiding fibres and as such fibre attenuation increases.

In the IR, solitons are primarily perturbed by third order dispersion and Raman gain (Ouzounov et al. 2003). However as the attenuation of HC-PCF’s at visible wavelengths is much greater, the observed soliton dynamics are perturbed greatly by loss. The longest fibre length explored in this section was 3m and which approximately corresponds to the 3dB attenuation length, a study of how the output pulse evolves for longer fibre lengths would be interesting.

It should be noted that coupling efficiency of the two lengths of HC532 differed and therefore it is difficult to perform comparisons. Using the measured loss of HC532 of $\sim$1.1dB/m, the maximum output power obtainable of 670mW(33.5$n$J) through 1m of HC532 would correspond to a launched input power of 860mW(43$n$J) and 400mW(20$n$J) after a further 3m of fibre. This is in contrast to the maximum power of 300mW(15$n$J) that was obtainable through 3m of HC532 which corresponds to an extrapolated launched power of 640mW(32$n$J) and 500mW(25$n$J) after 1m. A measurement of the spectral and temporal response of different lengths of HC532 whilst maintaining the same input coupling would be highly interesting.
Conclusion

The author started his Ph.D. in 2006, three years after the first demonstration of the propagation of femtosecond solitons in HC-PCF by Ouzounov et al. (2003). Since this demonstration much progress has been made into propagating and manipulating solitons in HC-PCF. Work carried out by the author towards this progress was outlined in sections 4, 5 & 6.

Chapter 4 detailed work carried out into understanding and improving HC-PCF for soliton propagation applications it was broken up into three bodies of work.

1. A 3-cell HC-PCF was fabricated which had a higher nonlinearity $\gamma$ than the usual 7-cell design. Solitons were generated in this fibre with extremely good agreement being found to numerically generated results. The guided modes of a modelled 3-cell and a 7-cell HC-PCF were computed and it was shown that although the 3-cell design has a higher nonlinearity $\gamma$ the waveguide dispersion of the smaller core is larger. It was for this reason the 3-cell design did not fulfil its original aim of supporting lower energy femtosecond solitons than possible in 7-cell HC-PCF, despite the fibre’s increased nonlinearity.

2. Modelling was undertaken into how variations in the pitch/air filling fraction of a 7-cell HC-PCF effected its dispersion and nonlinearity $\gamma$. For all the fibres modelled; 10nm from the ZDW there was a negligible change in the soliton threshold energy, further from the ZDW the soliton threshold energy decreased for fibres with larger pitches.

3. A technique for accurately measuring the dispersion of optical fibres was outlined and applied to measure the dispersion of a 7-cell HC-PCF with greater accuracy than any previously published data. The accuracy of the recorded data allowed the dispersion slope of the fibre to be known: which is an important quantity in soliton propagation and dispersion compensation experiments/applications. The technique used an interferometer to calculate the group velocity of a test fibre for different frequencies filtered from a supercontinuum. The measurement was highly accurately as pulses passing through the test fibre were interfered with supercontinuum pulses generated from the next shot of the laser. This allowed a long length of test fibre to be used and therefore a large variation in the balancing position of the interferometer was observed. The wavelength range this technique can be used over is however limited by the shot to shot coherence of supercontinua.

Chapter 5 detailed the compression of pulses in dispersion decreasing tapered HC-PCFs it was broken up into two bodies of work.

1. As the minimum pulse duration of solitons in HC-PCF is limited to $\sim$100fs by TOD and Raman gain, the compression of picosecond solitons in a dispersion decreasing taper was explored; in a manner similar to the work of Gérôme et al. (2007) with femtosecond solitons. As the soliton period scales to the square of the temporal duration, a taper that compresses picosecond pulses must be much longer than that used for femtosecond pulses. The taper created was limited by loss to 33m long. The maximum observed compression of 12 times (2.5ps pulse being compressed to 215fs), occurred for an input soliton number of $N\approx2$ with the compression incorporating elements of soliton effect compression.
2. Building upon the work of Gérôme et al. (2008) and Gérôme et al. (2007) which detailed the compression of chirped amplified laser pulses in a HC-PCF and the compression of femtosecond solitons in a dispersion decreasing taper, a fibre that had both linear and tapered sections, combining these two processes was created. The high average power of the laser source used was taken advantage of to record the spectral evolution of pulses propagating in the fibre by collecting light scattered from the fibre along its length.

Chapter 6 detailed the propagation and soliton effect compression of pulses centred at 539nm wavelength in HC-PCF.

- Solitons were generated in two fibres with differing ZDW, strong compression being observed in one of the fibres because of the locality in frequency of the input pulses to the fibre’s ZDW. In the second fibre compression was not observed as the input pulses where launched into the fibre further from the ZDW in a region of high dispersion. The maximum observed compression was from 300fs to 109fs, with good agreement being found between experimental results and numerical modelling. The attenuation of HC-PCFs is much greater in the visible than in the IR and the attenuation of the fibres used was approximately 1dB/m. This meant that fibre attenuation played a larger perturbative effect on the observed soliton dynamics than for IR solitons.

The Future: 7-cell HC-PCF is starting to find commercial applications that involve the delivery of femtosecond solitons; one such application being in laser machining where light from a (inexpensive fibre) laser is delivered to the end of a robotic arm. However as the power of fibre lasers increase HC-PCF must adapt to guide pulses that have energies well above the soliton threshold energy of the current generation of 7-cell HC-PCFs. Whether this is through the development of surface mode free 19-cell HC-PCFs or the development of lower nonlinearity 7-cell fibres, there is plenty of room for innovation in this field. The compression of solitons in tapered HC-PCF is unlikely to find commercialisation as tapered HC-PCFs are not stock products and the creation of custom fibres is unlikely to be economically viable. There are no such obstacles in the use HC-PCF based soliton effect compression in commercial systems and it is likely to find uses as an inexpensive and simplistic means of improving the performance of laser pulses.
List of Publications

Journal papers


Conference papers

## List of Acronyms

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<th>Acronym</th>
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<tr>
<td>PCF</td>
<td>Photonic crystal fibre</td>
</tr>
<tr>
<td>HC-PCF</td>
<td>Hollow core - photonic crystal fibre: (7-cell unless otherwise stated)</td>
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<tr>
<td>ESM fibre</td>
<td>Endlessly single mode fibre</td>
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<tr>
<td>GVD</td>
<td>Group velocity</td>
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<tr>
<td>ZDW</td>
<td>Zero (group velocity) dispersion wavelength</td>
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<tr>
<td>TOD</td>
<td>Third order dispersion</td>
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<tr>
<td>OPA</td>
<td>Optical parametric amplifier</td>
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<tr>
<td>IR</td>
<td>Infra-red</td>
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<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>LTT</td>
<td>Linear then tapered (fibre)</td>
</tr>
<tr>
<td>OSA</td>
<td>Optical spectrum analyser</td>
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<td>FWHM</td>
<td>Full width (at) half maximum</td>
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<td>SEM</td>
<td>Scanning electron micrograph</td>
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<tr>
<td>FROG</td>
<td>Frequency resolved optical gating</td>
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<tr>
<td>NLSE</td>
<td>Nonlinear Schrödinger equation</td>
</tr>
<tr>
<td>SPM</td>
<td>Self phase modulation</td>
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<tr>
<td>DOS</td>
<td>Density of states</td>
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Appendix 1: Nonlinear Refractive Index

The purpose of this appendix section is to show in detail the derivation of equation (1.23):

\[ n(\omega, |E|^2) = n(\omega) + n_2 |E|^2 \quad \text{where} \quad n_2 = \frac{3}{8\pi} Re \left( \chi^{(3)} \right) \]

which expresses the refractive index of a Kerr nonlinear medium in terms of linear and nonlinear refractive indices.

The induced polarisation of a Kerr nonlinear medium by an incident field is defined in equation (1.21) as:

\[ P = \varepsilon_0 \left( \chi^{(1)} E + Re \left( \chi^{(3)} \right) E^3 \right) \]

Defining an incident continuous wave electric field as: \( E = E_0 \cos(\beta x - \omega t) \) yields:

\[ P = \varepsilon_0 \left( \chi^{(1)} E_0 \cos(\beta x - \omega t) + \left( \chi^{(3)} \right) E_0^3 \cos^3(\beta x - \omega t) \right) \]

Applying the identity: \( \cos^3(\phi) \equiv \frac{3}{4} \cos(\phi) + \frac{1}{4} \cos(3\phi) \) yields:

\[ P = \varepsilon_0 \left( \chi^{(1)} E_0 \cos(\beta x - \omega t) + \left( \chi^{(3)} \right) E_0^3 \left[ \frac{1}{4} \cos(3\beta x - 3\omega t) + \frac{3}{4} \cos(\beta x - \omega t) \right] \right) \]

In almost all cases the \( \cos^3(3\beta x - 3\omega t) \) harmonic is not phase matched with the driving field and thus integrates to zero over multiple wavelengths:

\[ P = \varepsilon_0 \left( \chi^{(1)} E_0 \cos(\beta x - \omega t) + \frac{3}{4} \left( \chi^{(3)} \right) E_0^3 \cos(\beta x - \omega t) \right) \]

Substitution into a definition of refractive index \( \varepsilon_0 n^2 E = \varepsilon_0 E + P \) yields:

\[ \varepsilon_0 n^2 E_0 \cos(\beta x - \omega t) = \varepsilon_0 E_0 \cos(\beta x - \omega t) + \varepsilon_0 \left( \chi^{(1)} E_0 \cos(\beta x - \omega t) + \frac{3}{4} Re \left( \chi^{(3)} \right) E_0^3 \cos(\beta x - \omega t) \right) \]

Cancelling and rearranging, yields the following expression for refractive index:

\[ n = \sqrt{1 + \chi^{(1)} + \frac{3}{4} Re \left( \chi^{(3)} \right) E_0^2} \]
Recalling how the refractive index of a linear material is defined \((n_0^2 = 1 + \chi^{(1)})\) yields:

\[
n = \sqrt{n_0^2 + \frac{3}{4} Re(\chi^{(3)}_{xxxx})}
\]

This can be simplified using a binomial approximation as \(\left(\frac{3}{4} Re(\chi^{(3)}_{xxxx}) E_0^2\right) \ll n_0^2\) to yield:

\[
n = n_0 + \frac{3 Re(\chi^{(3)}_{xxxx})}{8 n_0} E_0^2
\]

Which can be expressed as:

\[
n = n_0 + n_2 E_0^2 \quad where \quad n_2 = \frac{3}{8 n_0} Re(\chi^{(3)}_{xxxx})
\]
Appendix 2: Dispersion measurements

The aim of this appendix section is to outline the mathematics required to calculate the dispersion of a fibre using the experimental setup shown in Fig A1.

For interference between pulses from both arms to occur then both arms must arms of the interferometer have the same effective length:

\[ 2L_{\text{air2}} + n_g L_{\text{fibre}} = 2\Delta x(\lambda) + L_{\text{air1}} \rightarrow n_g = \frac{2\Delta x(\lambda) + L_{\text{air1}} - 2L_{\text{air2}}}{L_{\text{fibre}}} \]

Recalling how dispersion (D) is defined

\[ D = \frac{d\beta}{d\lambda} = \frac{1}{v_g} \frac{d}{d\lambda} \]

Then the dispersion of a fibre can be calculated from the wavelength dependence of the effective arm lengths of the interferometer.

\[ D = \frac{1}{c L_{\text{fibre}}} \frac{d}{d\lambda} \left( 2\Delta x(\lambda) + L_{\text{air1}} - 2L_{\text{air2}} \right) = \frac{2}{c L_{\text{fibre}}} \frac{d\Delta x(\lambda)}{d\lambda} \]

The key point that should be noted is that to know the dispersion of a test fibre, it is not necessary to know the absolute lengths of the air sections of the interferometer, it is only necessary to know the change in the lengths of the air sections.
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