LIMIT EQUILIBRIUM ASSESSMENT OF DRYSTONE RETAINING STRUCTURES

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**ABSTRACT:** A limit equilibrium analysis program has been developed as part of an investigation into the stability of drystone retaining structures. Initial verification of the program’s function was in relation to field trials conducted in 1834 by Lieut-General Burgoyne, which have been the main reference to date for checking numerical modeling of drystone retaining walls. Parametric studies and investigations of bulging mechanisms are reported and analysed. Program predictions have been compared with the initial results from new small scale and full scale drystone retaining wall tests carried out at the University of Bath in 2008.

1 INTRODUCTION

Drystone technology is an ancient form of construction which relies for its integrity on careful construction giving an appropriate degree of overlap between stones, which are held in position through interlock and friction. Used extensively around the world wherever suitable building material is to be found, the technique is most commonly used for boundary walls, but is also used for earth-retaining structures. The stone is generally used as it comes, either as it is broken from quarrying, or simply picked up from fields, though some minimal shaping may be applied to make a piece fit in a particular position. The aim of the masons is to select a stone and place it in an appropriate position straight away, with as little trial and error as possible. Together with the need to ensure appropriate overlaps, and the challenge of maintaining stability if the stone does not tend to have parallel faces, this requires considerable skill on the part of the masons.

In the UK alone there are estimated to be 9000 km of drystone retaining structures lining the road and rail networks (Powrie et al. 2002), mostly dating to the 19th and 20th centuries. Though poorly constructed walls presumably collapsed shortly after their construction, the majority of walls have remained perfectly stable over decades of usually steadily increasing loading and weathering of the constituent stone. However, many walls have deformed or bulged and are regarded as potentially unstable. Because little guidance is currently available to assist engineers in the assessment of these structures (O'Reilly and Perry, J 2009), they are often replaced, at great cost. They are very rarely rebuilt in drystone, as the dimensions required by current design practice make this substantially more expensive than a concrete replacement. It has been estimated that the total replacement cost for the drystone walls lining the UK’s highways would be over £10 billion (O'Reilly et al. 1999). Indeed, internationally accepted design practice would deem most existing structures to be inadequate.
There is therefore a clear requirement to have means of assessing existing structures that is realistic. There are substantial difficulties in obtaining information about individual walls, especially their effective thickness and backfill properties, but there is also considerable uncertainty regarding the appropriateness of current design methods for such structures, and research has been carried out at the Universities of Bath and Southampton to address this. The main focus has been on model and full scale testing linked to advanced numerical modeling. However valuable such computational techniques are for research, they are not suitable for routine work by local engineers around the world, who simply do not have the appropriate expertise and resources. A part of the work at Bath has therefore been to develop a simple computer program, which can be distributed freely and used easily to explore the stability of drystone retaining structures.

2 DRYSTONE CONSTRUCTION

If the nature of the stone allows it, drystone walls are typically built in horizontal layers or ‘courses’, with each course ideally consisting of stones of a ‘uniform’ thickness, retaining a straight and level appearance. Walls usually consist of a tightly-packed outer face and a core of smaller random material packed behind, sometimes followed by a more roughly built inner face. ‘Tie-stones’ span from the outer to the inner face or into the backfill, binding the wall together. Coping stones can act in a similar manner, spanning the entire width of the wall at the crest (Fig. 1), whilst their greater concentrated mass stabilises the stones in the upper levels of the wall.

It is usually necessary to wedge small pieces of rock known as pins underneath many stones to prevent them rocking, though these will eventually result in deformation of the structure as their small size allows them to weather away more quickly than the larger main stones. Pins are often used extensively to tilt stones to make the face appear more even and assist in drainage. A tightly constructed and planar appearance can therefore conceal a construction which will weather and deform relatively quickly. Though the timescale may still be in years or decades, depending on the quality of rock used for the pins, this is quick relative to the normal lifespan of drystone structures.

Fig. 1 – Comparison of single and double faced walls.
The overall density achieved in drystone wall construction varies considerably with the skill of the builders and the speed of construction. Experiments at the University of Bath have demonstrated a range of voidage from 20% to 40% to be possible with the same ideal stone, and larger variation is probable given the range of stone types that can be used. As well as reducing the weight of the wall, to which its ability to resist earth pressures is directly related, increased voidage considerably increases the deformability of the wall. Whilst ductility is in general good, as it allows differential settlement to be accommodated, concentrated loadings to be distributed, and weak areas of construction to be unloaded, excessive deformation can result in a geometry that is no longer stable. Assessing the consequences of deformation has therefore been an important goal in the development of the software reported here.

3 ANALYTICAL APPROACHES

Drystone walls, including earth-retaining structures, were traditionally built following guidance based on experience, rather than being designed. When earth retaining structures have been designed it has usually been on the basis of treating them as monolithic gravity walls, with the degree of friction that can be mobilised between the stone and the soil substantially underestimated, resulting in structures much wider than those that have been built previously. Consequently, there is a belief that existing structures are at best marginally stable. In addition, designers would normally apply the 'middle third' rule. This states that the resultant force at any level of the wall should act through the middle third of the cross-section, so that the back of the structure is not brought into tension. This rule incidentally ensures a good margin of safety against toppling failure, and helps guard against bearing failure in the foundations. It is normally used for gravity retaining structures, but is not strictly necessary. The stone itself is usually strong compared with the stresses on it, and some concentration of load towards the face of the wall arising from a resultant in front of the middle third can be tolerated. This has been amply demonstrated by testing of model and full-scale walls at the University of Bath, to be reported in full elsewhere. However, once the resultant reaches the face of the structure the stress would in theory become infinite. Crushing would of course take place before this point was reached, and overturning very shortly afterwards. Assessment of the location of the resultant relative to the face of the wall is therefore a fundamental requirement for the design of drystone retaining walls (Cooper 1986).

Drystone walls may also fail by sliding on near-horizontal surfaces within the wall, which will usually be followed by a toppling instigated by the stones at the face tipping forwards as they project too far beyond the stones they rest on. Assessment of sliding stability is relatively simple, as normal methods for gravity retaining walls can be applied, the difference being that the sliding surface is within the structure rather than at its base. This means that the relevant properties are much better controlled, and the normal reduction factors applied to frictional strength are not required. However, if the courses slope down towards the face, which can be determined by careful inspection, then the sliding resistance will be reduced. This is easily accounted for.

Bulging deformations are very frequently observed in drystone retaining walls, in which the face of the wall bulges outwards, with the maximum displacement typically being at about a third of the height. Bulging may be localised, associated with weaknesses in the construction, but may also occur along considerable lengths of wall. This has not been understood. As it has been seen as a displacement phenomenon rather than a global instability, it has been investigated using discrete element modeling. UDEC (Universal Distinct Element Code) has been used by various authors to test both the validity of various modes of analysis, and to study further the various parameters at work within drystone
structures (e.g., (Harkness et al. 2000), (Powrie et al. 2002)). Although highly informative, these investigations are both complex and time consuming, often requiring several hours to run a single cycle of analysis. Work is currently being carried out at Southampton University to develop a three dimensional model of the full scale tests being carried out at the University of Bath.

Current analysis techniques for drystone walls are either simplistic, considering the static equilibrium of the wall as a monolithic structure, or too complicated, using time consuming numerical packages to model each element within the wall and backfill. Numerical packages such as UDEC (Universal Distinct Element Code) may provide precise details regarding wall stability and the potential failure mechanisms given sufficient data and careful modeling, but the analysis can take several hours, making parametric studies of any particular structure a lengthy and expensive process.

The computer program reported here was developed to allow limit equilibrium analysis of drystone retaining structures to be carried out which takes account of their particular nature, rather than treating them as conventional gravity walls. A most important aspect is their ability to deform, and to develop an understanding of the consequences of deformations.

4 PROGRAM OPERATION

To begin with, each wall within the model is comprised of a number of blocks, representative of the courses found within an in-situ drystone wall. The wall geometries, number of blocks, general material properties and friction angles are all user-defined, allowing representation of any wall constructed of any material. For each block, the active pressures from the backfill are calculated using the Coulomb earth pressure coefficients as demonstrated by Cooper (Eq. 1) (Cooper 1986). The stabilising forces arising from both the wall’s self weight and any vertical frictional forces acting at the wall/backfill interface are similarly calculated, and an overall thrust is determined (Fig. 2).

\[
P_{hi} = \frac{1}{2} K_A H^2 \cos(\alpha + \delta - 90) \\
P_{iv} = \frac{1}{2} K_A H^2 \sin(\alpha + \delta - 90)
\]

where:

\[
K_A = \left[ \frac{\sin(\alpha - \phi)/\sin(\alpha)}{\sqrt{\sin(\alpha + \delta)} + \sqrt{\sin(\phi + \delta) \sin(\phi)/\sin(\alpha)}} \right]^2
\]

\[
H = \text{block height} \\
\phi = \text{friction angle of soil} \\
\alpha = \text{angle of back face to horizontal} \\
\delta = \text{friction angle between soil and wall}
\]
This process is repeated through every block element in the wall, generating a thrust for the entire wall, giving the local eccentricities (i.e. distance from the neutral axis) as well as an eccentricity for the entire structure. Figure 3 shows an example of a typical wall structure with a retained fill.

To simulate vehicle and other imposed loads, an additional surcharge may be applied via a patch load on the surface of the backfill. The area, location and magnitude of this patch are all user-defined, with the pressure spreading throughout the backfill three dimensionally by a ratio of 1H:2V. This additional pressure does not cause any extra loads to be applied to the wall until this expanding boundary reaches the back face of the wall. Whilst simplistic, this method overcomes the potential problems encountered with more complicated stress distributions due to uncertainties regarding wall and backfill stiffness.

Generally for gravity retaining walls, the convention is to design the structure such that the eccentricity remains within the middle third at the base. If the structure is monolithic, and the stress is distributed in a linear manner, this ensures that the heel of the wall is never
subjected to tensile stresses. However, as there is no physical bond between the blocks in a
drystone wall, there can be no tensile stresses induced, and instead the rearmost stones
simply carry less of the overall loads. As the thrust line moves further and further away from
the middle third and towards the toe, more and more of the heel becomes unloaded, and
consequently the blocks at the toe are subjected to greater loads and stresses.

Typically, the compression strength of

drystone masonry is relatively high compared
to stresses being applied, even as the thrust
line moves closer to the toe. However, there is
scope for one or several other failure
mechanisms to develop as a consequence of
these increased compressive stresses.
Localised crushing of the pins which help to
maintain the wall blocks’ positions could lead
to individual block rotation. This in turn could
instigate a rotational failure of the whole wall,
or redistribute the loads in such a way that
causes flexural fracturing of blocks, deforming
the wall further. Alternatively, the increases in
compressive stress could lead to settlement of
the foundations, causing either significant
defformations or failure.

One of the main advantages of the program is the ability to instantly deform the wall and
create user-defined bulges. This is done either by using the mouse to drag blocks in either
direction, or by typing in the desired co-ordinates. Recalculation of the thrust line is
instantaneous, and allows walls with any profiles to be recreated with ease (e.g. Fig. 4). This
can be used both to allow greater understanding of how bulges affect stability, but also by
engineers in the field, attempting to ascertain the safety of an in-situ wall.

5 PROGRAM VALIDITY CHECK

Mathematical checks were initially made, comparing the results from simple test walls with
hand calculations. Once completed, the program was used to recreate the physical tests
recorded by Burgoyne in 1853. These tests – almost the only physical drystone wall tests
conducted until very recently – consisted of the construction of four walls of identical
volume but with varying cross-sections (Fig. 5).
The geometries and material properties entered into the program were identical to those reported during the original tests (Burgoyne 1853), with backfill heights initially set to zero. As with Burgoyne’s tests, the level of fill behind the wall was then systematically increased 300mm at a time until failure occurred (indicated by the thrust line moving beyond the position of the toe). The final heights recorded by Burgoyne, the LE program and a separate numerical analysis (Harkness et al. 2000) are presented in table 1.

Both the first and second of Burgoyne’s test walls were backfilled to their full height without excessive movement, and by using the LE program it can be demonstrated that the thrust line lies within the boundaries of the wall. It should be noted that for both these walls the eccentricity is outside of the middle third at the base, indicating uplift at the heel. The third and fourth walls both fell before full height of retention was achieved. For both these wall geometries, the LE program predicted failure at a height similar to that found by Burgoyne, although it has been demonstrated that consideration of individual block rotation gave a tighter correlation with actual failure heights (Claxton et al. 2005). To allow this to be seen in the program, the direction of the resultant force at each level is also shown at the point at which it acts. A resultant which points in front of the toe at any level could result in rotation of a block of stone at the face, so instigating overall failure. With regards to the results shown in table 1, this would indicate a failure at 5.2m for Wall C, bringing it in line with the observed results.

Table 1 – Comparison of Burgoyne tests

<table>
<thead>
<tr>
<th>Wall geometry</th>
<th>In situ observations</th>
<th>UDEC analysis</th>
<th>Limit equilibrium analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum fill height</td>
<td>Maximum fill height</td>
<td>Maximum fill height</td>
</tr>
<tr>
<td>Wall A</td>
<td>Full height</td>
<td>Full Height</td>
<td>Full height</td>
</tr>
<tr>
<td>Wall B</td>
<td>Full height</td>
<td>Full Height</td>
<td>Full height</td>
</tr>
<tr>
<td>Wall C</td>
<td>5.2m</td>
<td>5.2m</td>
<td>5.5m</td>
</tr>
<tr>
<td>Wall D</td>
<td>5.2m</td>
<td>5.2m</td>
<td>5.2m</td>
</tr>
</tbody>
</table>

6 PARAMETRIC ANALYSIS

The program is intended for use by engineers in the field, giving them a grasp of the forces acting within any given structure. Due to the almost instantaneous generation of results, it is possible to perform parametric analyses very quickly, determining the critical forces at work within any system.

By analysing the effect of each variable for a given wall geometry, it is apparent that there are several parameters which dominate stability. For example, due to the 1H:2V surcharge spread, the generated wall is only affected by a surcharge which is applied at a distance of half the total backfill height from the back face of the wall. Furthermore, due to the dissipation of this load as it spreads, to have any substantial effect on the wall it must be relatively large and close to the back face. These results correspond with anecdotal evidence, and are also confirmed by numerical modeling studies9, showing that there is a relationship between increasingly heavy traffic and failures of walls which had been safe for many years.
The various angles of friction relating to the backfill are also critical for stability, determining both the magnitude of the loads generated and how they affect the wall. The friction angle of the backfill itself determines the coefficient of active pressure ($k_a$), which in turn determines the magnitude of the horizontal forces upon the retaining wall. Whilst a backfill might be extremely dense, if it has a comparatively high angle of friction it will exert a relatively low horizontal pressure. Conversely, a loose material might have a much lower angle of friction, providing higher horizontal forces and hence be a more critical case in terms of wall stability.

There is also the friction between the wall and backfill to consider. Due to the nature of drystone construction, the wall faces are generally rough, which allows the back face adjacent to the retained material to attract some of the vertical load from the backfill. As this vertical force acts against the overturning forces and stabilises the wall, this is a value which would ideally be as high as the interface allows, although in reality it is not always guaranteed that the full friction angle will be achieved.

The general exception regarding the importance of friction angle relates to the friction angle of the stone itself, which determines the sliding between courses. It has been found that for most walling materials, the vertical forces generated by both the wall and the backfill are enough to inhibit movement due to the horizontal forces. However, should the walling material have a low enough friction angle, it is possible that the principle failure mechanism could be via sliding.

One of the most variable and difficult to ascertain parameters is the density of the walls themselves. NDT (ground radar) or horizontal coring can be used to give some indication of wall depth, profile and even voidage. Whilst the density of the rock will not vary greatly, its age and the construction style, and the skill of the mason will all affect a wall’s overall density and hence the total volume of voids within. Whilst this voidage has little impact on wall stability when changed by a few percent, this value may vary by much more than this. Low density reduces the wall’s stability in terms of both sliding and overturning. Perhaps most critically, a reduction in density allows easier movement and rotation of the individual blocks, determining the flexibility of the wall and the amount of bulging that may occur.

7 BULGING INVESTIGATION

In many walls, distinct bulges form either due to aging or general deterioration. These bulges usually form roughly a third to half the height of the wall, and can become extremely pronounced. These bulges are generally associated with instability. However, by using the program it has been determined that a bulged form may in some cases aid stability against certain failure modes.

Bulging begins when the loads behind the wall cause blocks or entire sections of wall to move, and the resulting movement causes both the forces acting on the wall and the equilibrium of its own mass to change, such that a new equilibrium position is found. Were this not the case, the wall would continue to move resulting in collapse.

In addition to the wall’s own centre of mass changing, once a bulge is formed the pressures acting upon the wall must also change in response to the new geometry. A section of a typically bulged wall is shown in Fig. 7, highlighting the common features. Above the bulge, the wall is leaning back somewhat, having a twofold effect. Firstly, it stabilises the wall by moving its centre of gravity away from the toe of the wall, which is usually the overturning point. Secondly, it also reduces the magnitude of the forces applied to the wall by the backfill.
Below the bulge, the wall is leaning forwards, causing the active pressures within the backfill to have a much greater effect upon this portion of the wall. The magnitude of the force will be greater, but the downwards component will be most increased, so increasing the stability of this portion of the wall, provided that the face has not moved so far forwards that individual blocks are no longer supported. Overall, these changes tend to be in favour of increasing wall stability, having the effect of moving the thrust line away from the wall toe.

Due to the flexible nature of drystone walls, significant movements may take place before a failure occurs, giving visible warning signs. Final collapse can occur either by toppling or bursting, but is usually a combination of both.

8 SMALL SCALE TESTING

In order to further test the accuracy of the program, a series of small scale tests were conducted, using concrete blocks housed in a steel box (Fig. 8). Walls of 500mm in height, 500mm in width and 100mm deep were constructed within the box, which was lined with a double layer of plastic sheeting, to help reduce friction at the edges and hence minimise end effects (Bailey 2008).

To generate stresses at a level to compare with the forces usually generated by a retained fill, small pellets (2-3 mm diameter) of lead shot were used as backfill to induce sufficient pressures to cause deformations and failures. The lead shot used has an uncompacted unit weight of 50 kN/m³ and an internal friction angle of 31°, allowing the generation of sufficient lateral pressures to overcome the stabilising forces within the test walls.

Each wall consisted of blocks which spanned almost the whole width of the test box, ensuring that no arching occurred between the steel side walls. Both timber and concrete blocks were independently used to construct the walls, however the timber did not have sufficient mass to realistically replicate full scale behavior (5.5 kN/m² as opposed to 24 kN/m² for the concrete blocks). However, whilst the timber blocks were not representative of drystone walls, the data still proves useful for reproduction within the LE program environment.
For each test, the scale walls were fully constructed without any retained backfill, and then slowly backfilled. Results from the small scale tests are shown in table 2 together with the backfill heights predicted by the LE program.

Table 2: Small scale testing results

<table>
<thead>
<tr>
<th>Cross-section Profile</th>
<th>Wall Material</th>
<th>Failure Details</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Softwood Timber</strong></td>
<td>Recorded backfill height at failure via toppling: 245mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Predicted backfill height at failure: 240mm</td>
</tr>
<tr>
<td>100mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Concrete Blocks</strong></td>
<td>Recorded backfill height at failure via toppling: 350mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Predicted backfill height at failure: 350mm</td>
</tr>
<tr>
<td>100mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Concrete Blocks</strong></td>
<td>Recorded backfill height at failure via toppling: 300mm</td>
</tr>
<tr>
<td></td>
<td>(10mm chamfer)</td>
<td>Predicted backfill height at failure: 315mm</td>
</tr>
<tr>
<td>100mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 2 it is clear that the program is accurately predicting the collapse heights of these small scale tests. It was assumed that the interface friction between the wall blocks and the backfill was \( \frac{2}{3} \) the full value of the backfill’s internal friction angle. Evidence gathered by the small scale tests supports this assumption, although in practice it is difficult to ascertain precisely how much of the backfill’s full friction angle has been mobilised against the wall. This obviously has a consequently large impact upon wall stability, although it is expected that ground settlement over time and the rough nature of drystone structures results in the full friction angle being mobilised for in-situ walls.
9 CONCLUSIONS
The limit equilibrium analysis program described in this paper has enormous potential compared to numerical analysis packages; its simplicity allows any engineer with a basic knowledge of a wall’s geometry and material properties to undertake an accurate estimate of its stability without resorting to complex and expensive numerical analyses. Flexibility allows walls of any geometry with variable backfills to be analysed, and the application of surcharging can be applied to represent circumstances such as new constructions in the proximity of the wall or increased vehicle loading.

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