Displacement based multiple wedge slope stability analysis
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Abstract

A method of slope stability analysis based upon multiple wedges is developed, accounting for kinematics in a similar way to proposals by Srbulov (1997) and Donald and Chen (1999). Instead of relying on almost arbitrary assumptions about stresses on wedge or slice boundaries to demonstrate an equilibrium of both forces and moments, the method works from simple assumptions about the kinematics of movement, increasing displacements and hence resisting forces iteratively until force equilibrium is attained. The procedure is simple and efficient, and ensures that inter-wedge forces developed by the movement are consistent with the shape of the sliding surface.

Keywords: slope stability analysis; reinforced soil.

1. Introduction

Stability analysis of purely translational slope failure mechanisms is straightforward - they can be considered as infinite slope problems, and there is neither rotation nor distortion of the sliding mass of soil. Analysis of mechanisms which can be represented by a segment of a circle on a cross-section can be carried out satisfactorily by a range of methods which give very similar answers (Zhu, Lee and Jiang, 2003; Michalowksi, 1995) as no distortion of the sliding mass is required. So-called rigorous methods use a variety of assumptions to arrive at a modelling of stresses that satisfies both moment and force equilibrium, but the widely used non-rigorous Bishop's simplified method gives results which are at least as reliable in practice. The method has the important advantage that the inherent assumptions are less likely to result in serious misrepresentation than those which may be made using 'rigorous' methods. Bishop's simplified method can account for all the actions resisting failure, but the precise distribution of normal stress on the failure surface may be inaccurately described, particularly if there is a complex geometry or soil layering, or high imposed loadings. In such cases the critical slip surface is likely to be non-circular.

Conventional analysis of non-circular slip surfaces is either by methods of slices (e.g. Janbu, 1957, Morgenstern and Price,1965, or Sarma, 1973) or by multiple wedges (e.g. Sarma,1979, or Donald and Chen, 2001). In methods of slices, assumptions must be made relating to the distribution of stresses within the sliding mass of soil to allow a factor of safety to be determined – the shear forces on the side of the slices affect the distribution of normal stress on the slip surface, which may alter the proportion of the load carried by stronger materials, and so the total available shear strength. Detailed assumptions about the distribution of stress on each face of a slice are required if the analyst for some reason wishes to demonstrate that moment equilibrium may be satisfied, though given the difficulty of proving the correctness of those assumptions, such a demonstration may have little real meaning. A distinction may be drawn between an academic desire for a complete 'solution', and the practical need to obtain a reliable assessment of margin of safety against failure. The simplest assumptions are most commonly expressed in terms of the inclination of interslice forces, and often appear to be reasonable if there is little distortion required and the soil is homogeneous – circumstances for which a circular analysis would be realistic. Multiple wedge methods also require assumptions to be made to ensure a satisfactory analysis – Sarma (1979), for example, follows the methods of slices and recommends simple assumptions about stress distribution. Multiple wedge methods require the orientations of inter-wedge boundaries to be defined as well as the position of the slip surface, and these are normally found by trial and error, as is the position of the critical slip surface. However, an interesting approach to this problem is presented by Sarma and Tan (2006), in which the shape of the slip surface is determined on the basis of minimising the critical acceleration for each wedge in turn. This is a logical progression from Sarma’s ealier work.

Non-circular failures require internal deformation of the sliding mass of soil if significant movement is to take place. Energy is required to produce this deformation, in addition to the energy which must be expended in moving against the shear resistance on the slip surface itself. All the energy expended must come from the loss of gravitational potential energy as the sliding mass moves downwards, and if the rate at which this energy is generated cannot match the rate at which it must be expended, then the movement will not take place. This principle is used in the upper bound plasticity method (Donald and Chen, 1997), but is fundamental and its use need not be restricted to plasticity approaches. The comparison is equivalent to a limit equilibrium analysis provided that the mechanism is kinematically admissible, the problem is statically determinate, and the inclination of forces is compatible with the yield conditions of the materials (Drescher and Detournay, 1993). If any of these conditions are not satisfied, either the movement cannot take place, or the calculation of energy expended in movement against the forces will be incorrect. In most ‘rigorous’ methods the inclination of interslice forces is assumed in order to reduce the number of unknowns and enable the pretence that moment equilibrium has been assessed and demonstrated, rather than for compatibility with the yield conditions and kinematics. For example, in Spencer's method (Spencer, 1967) the inclination is constant, and inter-wedge boundaries are always vertical, whilst in Morgenstern and Price's method (1965), the inclination is normally dictated by a function which is probably reasonable for nearly circular surfaces.

For routine work, it is necessary to provide an automated routine for searching for the critical slip surface. If it is necessary for the engineer to make decisions regarding wedge boundary positions and functions for defining interslice forces, then automation is not possible, and the search slows from hundreds of analyses per second to hundreds of seconds per analysis. If the slope being analysed includes soil reinforcement, concentrated loading, or weak layers, then the conventional assumptions are much less likely to be appropriate, as the distribution of stress within the slope will be markedly altered. It is therefore necessary to find a method of analysis that works from assumptions which are more likely to be accurate for the range of circumstances being analysed before an automated search routine can be reliable. This was the aim of the present work.

2. Displacements in a multiple wedge system

In a simple translational or rotational failure, displacements at every point along the slip surface will be the same, unless compression of the sliding mass of soil can take place at the same time as the sliding. In traditional methods of slope stability analysis, the local factor of safety at any point along the slip surface is taken to be the same as the overall factor of safety. The relationship between these two presumptions is the implication that either the shear stress vs displacement characteristic of the soil is uniform along the slip surface, and the initial conditions similar, or that the entire length of the slip surface has reached a state of plastic deformation, which would be regarded as a failure.

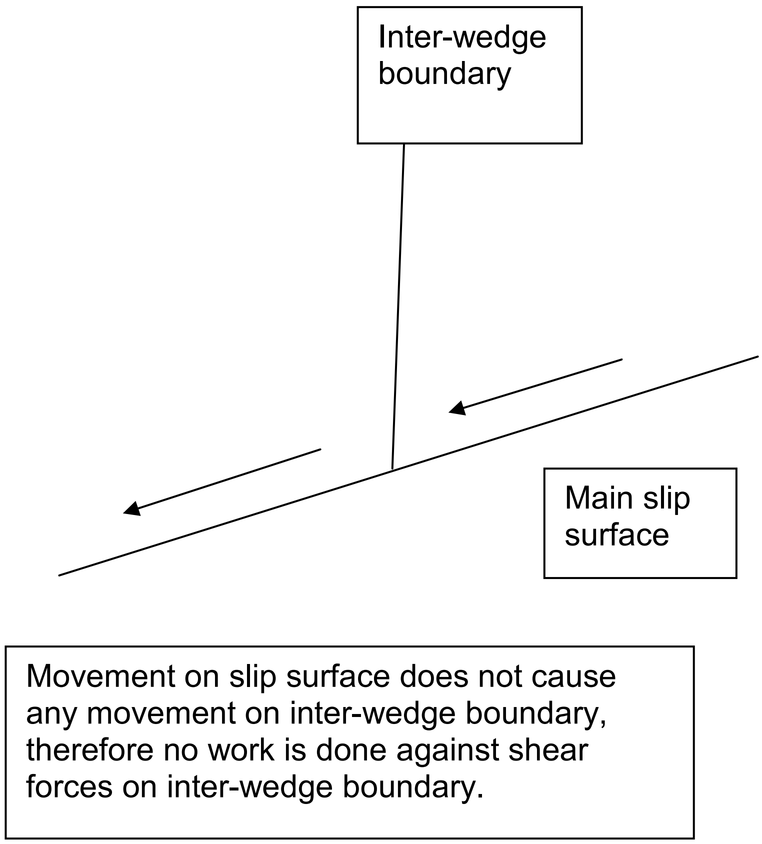

Figure 1 Relative displacement between wedges

The relationship between displacements of each wedge in a multiple wedge system is determined primarily by the change in angle of the slip surface at the boundary between the wedges. If there is no change in angle there will be no relative movement, and the two will behave as a single wedge (Figure 1). If dilation occurs as shearing takes place along a boundary, this will also affect the relative movement across a boundary.

Given a change in angle at the junction between adjacent wedges, the precise relationship of the velocities will be dependent upon the orientation of the inter-wedge boundary. This relationship may be examined using a hodograph (see Donald and Chen, 1997), which is a simple graphical means of showing velocity compatability, i.e. that the adjacent wedges remain in contact without overlapping. Hodographs for a series of wedges may be shown on the same diagram (figure 2), the origin, or zero velocity, representing the underlying ground. The vectors for each wedge relative to the ground will all start from the origin, and will be in the same direction as the orientations of the corresponding sections of slip surface, unless there is dilation taking place. The relative velocities between the wedges are found by joining the ends of these vectors. $V_{1..4}$ are the velocities of the wedges relative to the underlying soil, while $V_{12..34}$ are the relative velocities between the wedges. The hodograph on the left is for zero dilation. It can be seen that if the speed of movement along the failure surface is uniform, then the direction of the relative velocities between adjacent wedges is fixed as the bisector of the orientations of the slip surface at the corresponding junction. Unless a gap is to open between the wedges, or they are to be squeezed into the same space so that they overlap, then the orientation of the wedge boundaries must correspond to the direction of relative movement between the wedges.

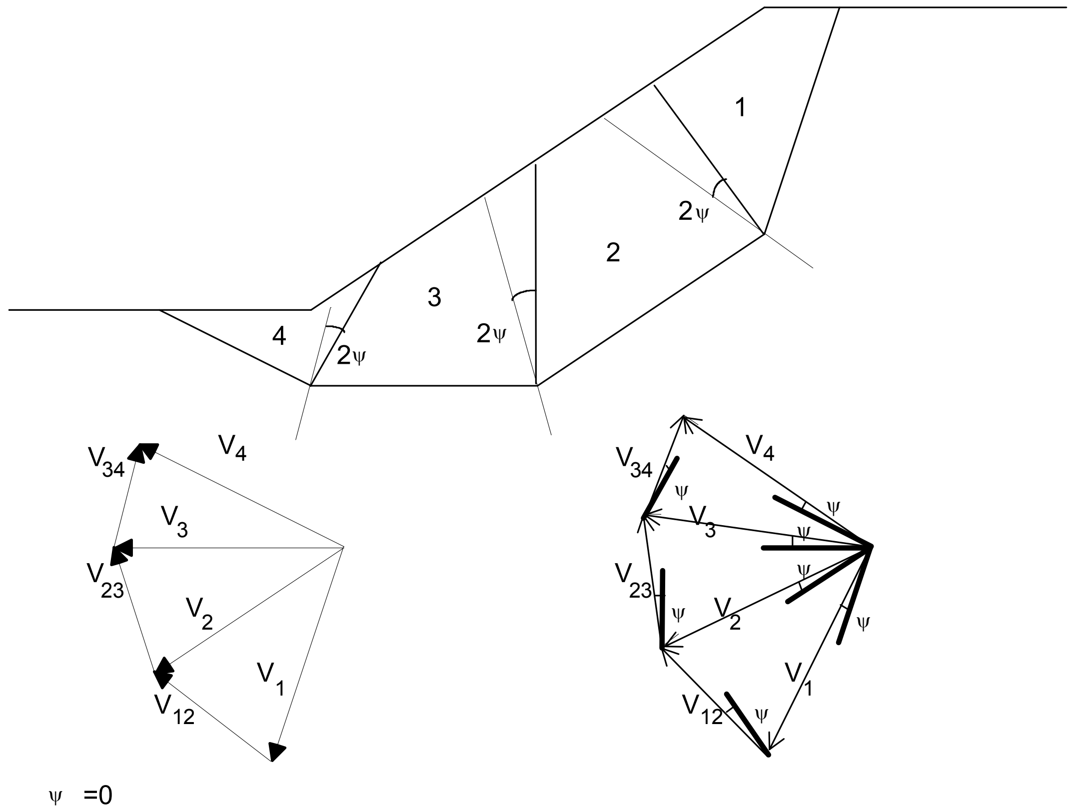

Figure 2 Hodograph for a four wedge mechanism. The thick lines in the hodograph on the right are the orientations of the surfaces.

It should be noted that the assumption of uniform velocity along the slip surface corresponds to the discussion above regarding uniform displacement – it will be correct for failures which involve no distortion or compression of the sliding mass of soil, and will result in uniform mobilisation of soil strength, and so allows the full strength of the soil to be mobilised in the ultimate condition when the factor of safety falls to 1. If one section of the slip surface were tending towards being overloaded, and starting to yield, load would if possible be shed onto adjacent less heavily loaded sections. There must therefore be a tendency towards a uniform factor of safety along the slip surface, and the positions of boundaries between masses of soil which move as intact units (wedges) would be expected to arise from this. If there is significant distortion of the sliding mass then velocities may no longer be uniform, and this effect will be identified below.

Intuitively, given the discussion above, it would be expected that the interwedge boundaries in nature would tend to be orientated to allow uniform displacement, and perhaps hence nearly uniform degree of mobilisation of shear strength. Ambraseys and Srubulov (1995) gave some consideration to the orientation of the interwedge boundary for simple failure mechanisms, particularly in relation to the orientation required for uniform velocity. They tended towards using an orientation arising from uniform displacement along the slip surface, but iterated through different orientations to find the lowest overall factor of safety.

If dilation takes place with shearing along the surfaces, then all the velocities must take this into account. The direction of movement is rotated from the orientation of the surface being considered by the angle of dilation ψ. In conventional plasticity analysis the angle of dilation is taken to be equal to the angle of friction, for the convenience of using an associative flow rule. Whilst real soils may behave in this way for very small strains, they certainly do not do so for large strains. For analysing translational failure mechanisms, the implied non-associative flow rule does not invalidate the analyses carried out (Drescher and Detournay, 1993). For the implementation of this method in a computer program, ψ may be taken to be defined according to Bolton (1986) as 0.8 of the difference between peak and critical state angles of friction. It can be seen from examination of the hodograph on the right of figure 2 that the consequence of introducing dilation is that the inter-wedge boundaries must be rotated backwards by 2ψ.

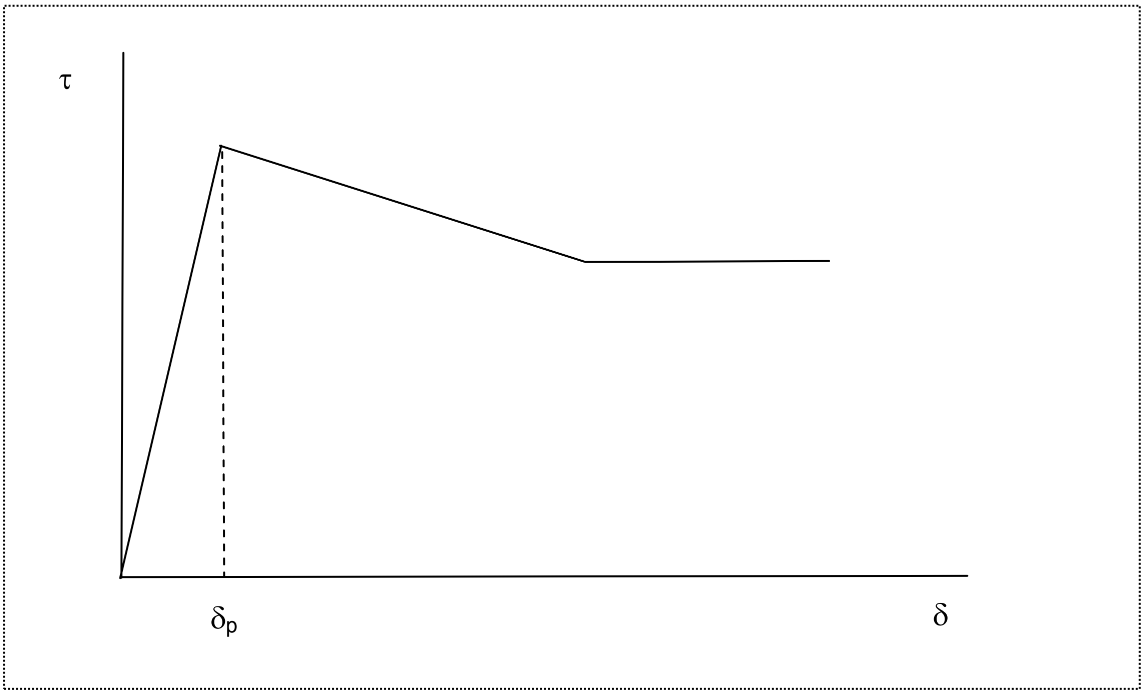

Figure 3 Linearised model for displacement to peak and critical state

The relationship between mobilised shear strength and displacement (as opposed to strain) has been well explored in direct shear testing. Srbulov (1995, 1997) explored the effects of soil brittleness in a multiple wedge analysis, using a detailed model of the stress-displacement relationship. This was used indirectly by defining varying local factors of safety (see also Srbulov, 2001). It is simpler to prescribe a set of compatible displacements, using hodographs to define the necessary relationships. For the present work, a simple model is used as shown in figure 3, which shows the essential characteristics necessary to describe the behaviour of a sliding mass of soil. Peak frictional strength is mobilised at δ_p, following Srubulov's notation, and all displacements will be expressed in terms of this displacement. If the data are available, more complex models might be used, for example as proposed by Srbulov (1997), at a cost of increased computing time, but the effects on the final factor of safety would normally be slight. Different relationships could also be used for different soil types, though this could have an effect on the starting presumption of uniform velocity along the slip surface. For example, if there was a horizontal surface on which the shear strength could be mobilised very quickly, then a step could form in the shear surface. However, this would still be picked up in an iterative search using the procedure described here, as if a difference in velocities was called for by the geometry of the shear surfaces, it would be taken into account in the analysis.

Srbulov (1999, 2001) appears to use an elaborate solution method which considers moment as well as force equilibrium, in the manner of Sarma (1979), even though this depends upon the prediction of stress distributions, or points of actions of equivalent forces, which really calls for a time-stepping finite element modelling of the development of the slope through its formation, supported by knowledge of the appropriate material properties. Local factors of safety are iterated until equilibrium is obtained.

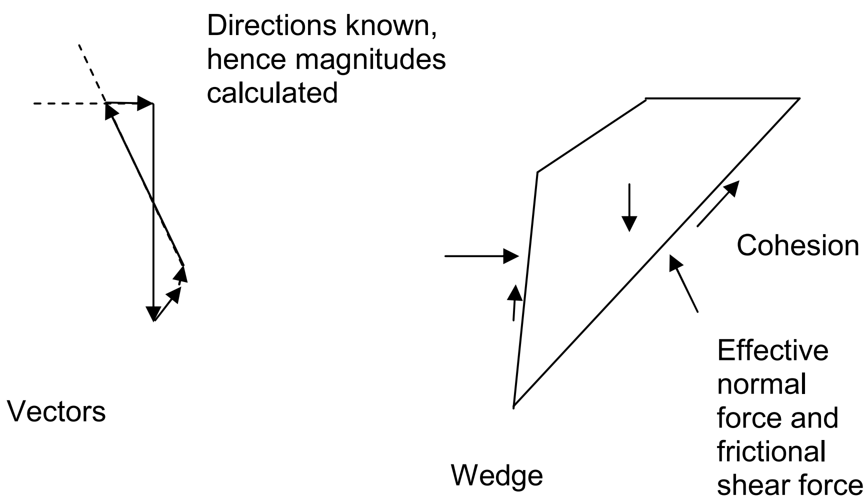

Figure 4 Determination of forces on first wedge

If an assumption can be made about the initial mobilisation of shear strength before the set of displacements defined by the hodographs takes place, then the angle of friction mobilised at any given displacement can be determined. Though the magnitudes of the forces acting on a wedge are not known initially, the self weights are, and the direction of the resultant forces on the boundaries are set by the mobilised angles of friction. Starting from a wedge at one end of the surface, there are two unknowns: the magnitude of the resultant force on the base of the wedge, and the magnitude of the resultant force on the next wedge. These two magnitudes can be determined by finding the position of their intersection in a force vector polygon (figure 4), which is equivalent to resolving forces in two directions. Then each successive wedge has two similar unknowns which are determined in the same way, until for the last wedge the second interwedge force is replaced by an unknown additional ‘out of balance’ horizontal force, which is needed to maintain equilibrium. The displacements, and hence mobilised friction angles, may then be scaled until the ‘out of balance’ force is zero, and equilibrium has been demonstrated.

The force polygon may be expressed algebraically as follows:

$$C_{i,i-1}+C_i+C_{i,i+1} + T_{i,i-1}+T_i +T_{i,i+1} + N_{i,i-1}+N_i+N_{i,i+1} + W_i + P = 0$$

In which all the quantities are vectors:

C = cohesion shear force
T = frictional shear force
N = normal force
W = self weight
P = out of balance force on the last wedge only

and the subscripts denote the boundary being considered:

i,i-1 forces on the upslope boundary (= 0 for the top wedge)
i forces on the base of the wedge
i,i+1 forces on the downslope boundary (=0 for the bottom wedge)

T is then related to (N – U), where U is the proe water pressure force on the given boundary, by the tangent of the mobilised angle of friction, which is a function of the displacement along the boundary concerned. This function could take any desired form, and may include an initial value at zero displacement.

The removal of the need for guessed assumptions to give the appearance of satisfying moment equilibrium is a big advantage to the method proposed here, especially compared with methods which are critically dependent such assumptions. However, in place of this, the engineer needs to make a guess about the initial mobilisation of shear strength. For example, if the slope is made by excavation, an initial profile of K_o may be determined, allowing the mobilised angle of friction to be obtained which would be dependent upon the orientation of the surface being considered. If the slope is made by the placing and compaction of clay fill, an intitial state might be that there is a low mobilisation of shear strength due to negative pore pressures in the expanding fill. In either case, if the state of the soil is at all stages in an approximately linear pre-peak condition, then it makes no difference what the initial condition was. However, if the slope is near or past failure, then the stage at which different parts of the surface are taken past their peak strength has an important influence on the solution – one which is ignored, however, in conventional methods. Therefore even a somewhat erroneous assumption about the initial mobilisation of shear strength is likely to result in a more realistic assessment of the condition of a slope than a conventional method. Srbulov (1997) gives detailed consideration to the effects of brittleness, hence allowing this effect to be taken into account, but does not give guidance about how initial conditions and the process by which the slope is made should be allowed for. Investigation of different initial assumptions by the author suggests that in general there may not be a large effect on the final factor of safety, and that the assumption of starting from zero, apparently implicit in Srbulov’s work, is likely to be suitable in practice, especially for fill slopes. If there is a better understanding of initial conditions this can of course be incorporated into the analysis.

3. Relationship between wedge geometry and displacements

If a non-circular slip surface goes around a tight corner the displacements, and hence local factors of safety, might be expected to be non-uniform. The approach taken here is to start with an assumption of uniform F along the slip surface, and hence uniform displacement. This then defines the orientations of the interwedge boundaries, taking account of dilation if necessary, as shown in figure 2. If this results in overlapping wedge boundries, as in the exaggerated surface in figure 5, then the assumption of uniform displacement must be in error. The boundaries must then be rotated until they do not overlap. The kinematic compatibility of the wedge displacements then determines the variation in displacement along the slip surface. The only aspect which cannot be logically determined is the way in which the wedge boundaries should be rotated in such circumstances. It may be observed that such surfaces are less likely to be critical than others, but it is nevertheless important that they are assigned an appropriate factor of safety for search routines to give correct results, and for the overall problem to be understood correctly.

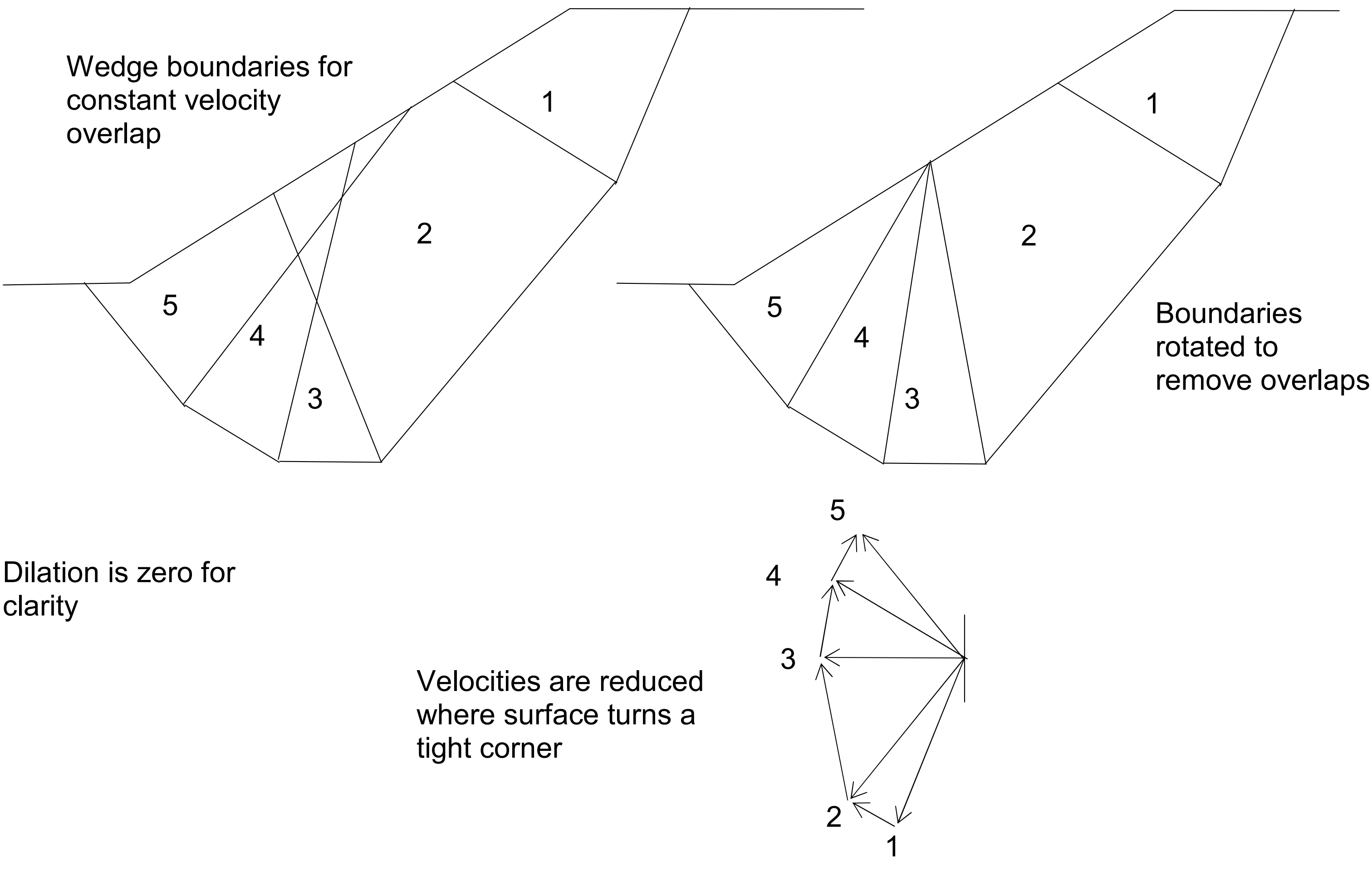

Figure 5 Overlapping inter-wedge boundaries

Once it is evident that uniform velocity along the slip surface is not possible, then any orientation of boundaries might be accepted, as long as they do not actually overlap. However, an adjustment of boundaries as shown in figure 5 results in a region in which internal shearing of the sliding soil dominates. In situations where there is a substantial weak layer, as in the example in section 8 below, the critical slip surface may involve substantial internal shearing, and variation in velocity along its length. This method allows such effects to be taken into account automatically if they are present, correctly identifying the critical slip surface.

4. Solution

In conventional methods the failing mass of soil is usually divided by vertical boundaries; a distribution of forces is determined on the basis of simple or simplistic assumptions; then a factor of safety is found which enables force and/ or moment equilibrium to be obtained. Methods which demonstrate both a force and a moment equilibrium are dubbed ‘rigorous’, and are often considered to be superior to methods which only demonstrate one or other equilibrium, regardless of the arbitrariness of the assumptions made. Because of this misleading use of the term ‘rigorous’, priority has often been given to satisfying both force and moment equations as the route to determining equilibrium. In contrast, in the proposed method the boundaries are determined to ensure compatibility as the mass of soil starts to move, allowing for dilation, and assuming uniform velocity along the slip surface. If this is not possible, then the generation of feasible boundaries allows the change in velocity along the slip surface to be determined. Forces are then determined that are consistent with a compatible set of displacements, the magnitude of which is incrementally increased until equilibrium is obtained, as described above.

An overall factor of safety can then be obtained by comparing the total shear resistance available along the slip surface with that which is mobilised at equilibrium. The total available may be limited by one section of the surface going past peak, and so reducing in strength, while another section is approaching peak. This potentially important issue is usually ignored in limit equilibrium analysis, leading to misleadingly high factors of safety for some surfaces close to failure, though the issue has been investigated (e.g Chugh, 1986). If equilibrium cannot be obtained, a factor of safety below one is determined by setting the displacement to give the maximum resistance, then scaling up the shear resistance until equilibrium is reached.

The variation in displacement along the slip surface is equivalent to variation in the local factor of safety, something generally believed to occur but not accounted for in conventional methods. Variation in mobilisation of shear strength due to the geometry of the surface does not correspond to what is normally meant by ‘progressive failure’, which arises from compression of the soil along the slip surface, allowing displacements at the top of the slip surface to exceed those nearer the toe. Such progressive failure could be included in the analysis if desired, as the detailed information obtained about the lateral forces on each wedge would allow compressions to be determined if appropriate modulii values were available.

Srubulov (1995, 1999, 2001) takes into account variation in displacement along the slip surface, but does so starting from variation in local factor of safety, setting up a large number of simultaneous equations to be solved iteratively. However, it is much more straightforward to start with a set of compatible displacements derived from the hodographs, then iterate for the displacement. In the simple scheme used for the present study, displacements are expressed as a proportion of that required to generate peak strength (δ_p). Given enough information to support it, a different scheme might be used based upon actual displacements to reach peak strength for different materials and locations wthin the slope. It is appropriate to start with a relatively small proportion, so that an additional horizontal force will be required to maintain equilibrium. From the displacement on each portion of the slip surface or inter-wedge boundary, a mobilised shear strength can be derived, using the relationship and initial conditions as discussed above. Determination of forces acting on each wedge then follows the procedure described in section 2 above. The direction of any cohesive component is fixed as acting along the boundary, while the relationship between the effective normal and shear force on a boundary is controlled by the mobilised angle of friction, though the magnitude of the resultant force is unknown. The weight of the wedge is known, and pseudo-static forces may be applied to represent seismic conditions. Thus for the first wedge, force equilibrium alone is required to determine the magnitude of the resultant combination of normal force and frictional shear force on each of the two boundaries (figure 4). The forces acting on the next wedge at the interwedge boundary are of course equal and opposite to those acting on the first wedge, so for the next wedge the only unknowns are again the magnitude of the resultant effective normal force and frictional shear force, and these may be solved in the same way as the first wedge. Working in a similar way through each wedge in turn, an 'out of balance' horizontal force will be required to hold the last wedge in equilibrium. It is likely that the first value for this force will be helping prevent sliding, and the displacements will need to be incrementally increased until the out of balance force reduces to zero and equilibrium is obtained. Moment equilibrium need not be considered.

It is possible that the displacement reaches a point where the peak strength on a section of the slip surface has been passed, and the strength starts to reduce towards the critical state value, while the force on other sections continues to increase. Once all sections of the surface have reached peak strength, then the resisting force can only reduce, indicating a brittle failure, so the factor of safety is less than one. Alternatively, if none of the soil on the slip surface shows brittle behaviour, then a state of plastic deformation has been reached, also indicating a factor of safety less than or equal to one, because if the disturbing forces exceed the resisting forces then the movement will accelerate. In order to complete the analysis and determine by how much the factor of safety is less than one, the displacement is set to the value which requires a minimum additional force to maintain equilibrium. All the strengths are then scaled up by an 'overload factor' until an equilibrium can be found.

The procedure is necessarily iterative, to determine the precise displacement or 'overload factor' that is required for equilibrium. As with the iteration in conventional methods such as Bishop's, much of the calculation is carried out before iteration takes place.

5. Moment equilibrium

The method does not require the determination of moment equilibrium to arrive at a solution for factor of safety. In a similar context, Sarma (1979) pointed out correctly that moment equilibrium played no part in the determination of K_c, though it was essential for a complete solution. As K_c is the result the engineer wishes to know, there is therefore no need for the complete solution to be determined, provided that the asusmptions that have been made in determining K_c (or factor of safety) are realistic and appropriate. In the present context, the requirement to satisfy kinematic compatibility is a more onerous and relevant requirement than the notional satisfaction of moment equilibrium, and usually produces a set of forces that allows moment equilibrium to be determined. In conventional 'rigorous' methods which make assumptions about the inclination of forces on boundaries, assumptions are usually also made about the positions of their points of action, or about the distribution of stress on the boundaries which amounts to the same thing. Except in the case of cohesive soils, the resultants may be expected to be acting somewhere within the middle third of a boundary, and probably below the midpoint of the boundary. The actual points of action will depend upon the orientation of the boundaries, and the way in which stress is distributed in the mass of soil being analysed. If the actual stress distribution is needed, then finite element analysis is the appropriate tool to use, and testing must be carried out to determine the necessary parameters. 'Rigorous' methods which purport to demonstrate moment equilibrium, and in so doing make simplistic assumptions about the stress distribution, are not necessary to assess the translational stability of the body of soil.

6. Programming

With the support of Tensar International, this method has been implemented in their program TensarSlope, an interactive, graphically based tool for the design of reinforced soil slopes using Bishop's or Janbu's simplified methods. A Simple Genetic Algorithm (SGA) search tool originally developed for circular failure surfaces (McCombie and Wilkinson, 2002) has been incorporated into the program (McCombie, Zolfaghari and Heath, 2005). The SGA could not be used with Janbu's method as the internal deformation is not accounted for, and unrealistic results are obtained. An additional form was provided to display details of the analysis, such as the components of individual forces, enabling the calculated equilibrium conditions to be checked independently in a spreadsheet.

The weight of individual wedges is determined by division into forty vertical slices above each boundary; the same division is used for obtaining average cohesion, pore water forces, and friction on each wedge boundary. The centroid of each wedge is shown on the screen. A vector diagram of the forces on the wedge may be shown at this point, allowing inspection of the relative importance of different factors. The forces on the boundaries of each wedge are also shown at their supposed points of action, estimated using similarly simplistic assumptions to those used in 'rigorous' methods, with some iteration to refine those points of action on the basis of moment equilibrium of each wedge. However, this latter operation is purely for presentational purposes, having no bearing on the solution, and need not be described here.

7. Comparison with other limit equilibrium methods

Comparisons with other methods of analysis were carried out, for example with the results reported by Zhu, Lee and Jiang (2003) in which a range of methods were applied to a limited number of examples, mostly with unusually high cohesions. The proposed method gives consistently lower factors of safety - for example 1.82 compared with 2.07 for the circular analysis of the example from Fredlund and Krahn (1977). This is in line with the differences expected between factors of safety based upon force equilibrium and those based upon moment equilibrium. However, given the failings of conventional methods outlined earlier, it is necessary to test the proposed method on a thoroughly investigated real failure, in which there can be considerable confidence in the soil parameters and in the actual factor of safety.

8. Analysis of Carsington failure

The failure of Carsington embankment has been subject to very detailed investigation and study, and has been used as an ideal test case by Srubulov (1995) and Donald and Chen (1999). For the present study, the conclusions regarding soil parameters and failure geometry presented by Potts et al (1990) have been used; these values were supported by very thorough finite element analyses. The cross-section through the initial failure before movement was entered into the program, together with the values of parameter set A (Table 1), which gave the best fit to incipient failure in the finite element analysis. As the objective of the present work is to produce a method that can be used efficiently to search for the critical surface, the actual failure surface was not entered into the program. The SGA was used to find the critical surface, using a population of 100 surfaces and two point crossover. Minor refinement of the resulting surface was then made by automated stepwise movement of the points defining the surface, to obtain the surface with the lowest factor of safety as shown in figure 6. The factor of safety produced was 1.000, consistent with the observations and the findings of the finite element analysis. The actual position of the failure surface differed slightly from that sketched from the investigations, but agreed well with the vectors of incremental displacement from the finite element analysis. It should be noted that the resulting displacements vary along the length of the slip surface, with soil in the core and ‘boot’ having sheared beyond peak, by percentages shown beneath each wedge in figure 6.

	γ: kN/m^3	c'_P/c'_R : kPa	ϕ'_P/ϕ'_R
Yellow clay	-	6 / 0	19 / 12
Core and boot	18.5	42 / 30	0 / 0
Zone I mudstone fill	20.6	10 / 0	22 / 14.5
Zone II mudstone fill	22.0	15 / 0	25 / 14.5

Table 1 Soil properties for Carsington analysis (Potts et al.).

In terms of the time required, entering the cross section took under thirty minutes, while the search took under five minutes. This performance was in line with the requirements for a method suitable for practical use, while the accuracy achieved, both in terms of surface definition and factor of safety, were completely satisfactory.

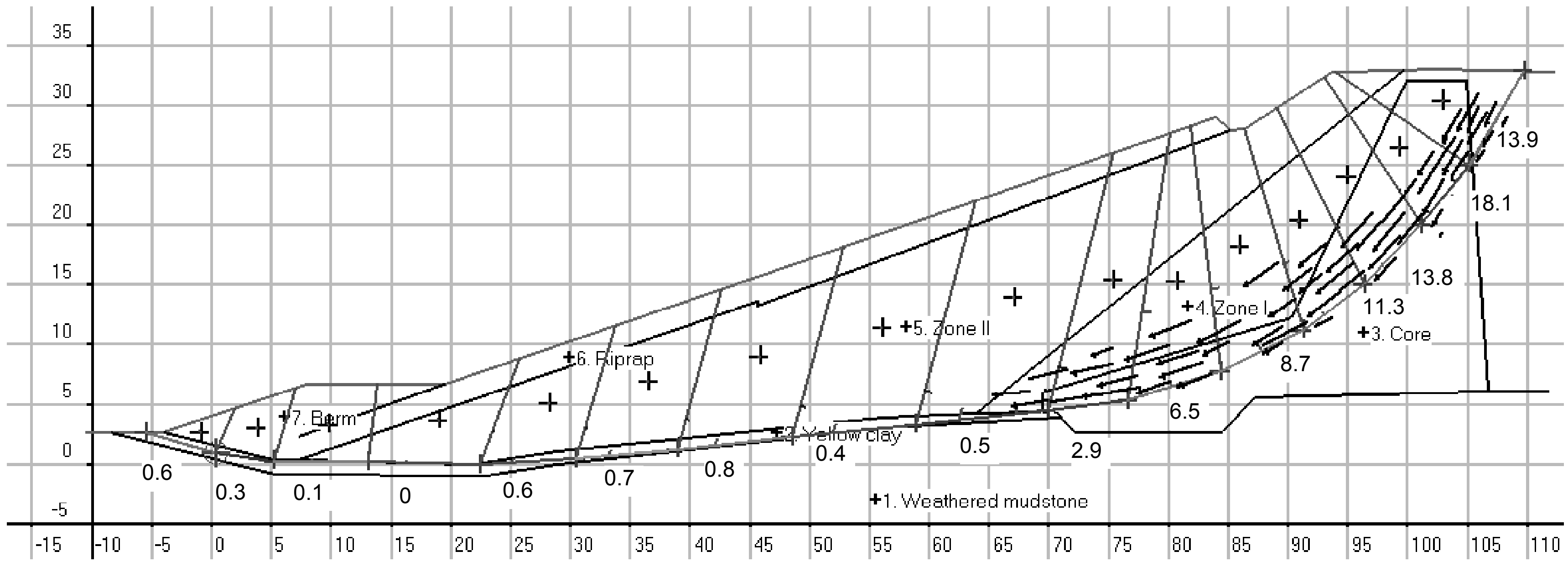

Figure 6 Analysis of Carsington embankment from the computer screen, with vectors from finite element analysis [Potts et al.] added to show shape of failure surface through the core and the 'boot'. The numbers beneath the slip surface indicate the displacement beyond peak, as a percentage of the displacement to peak strength.

9. Soil reinforcement

A main objective of the development of the method was to allow the inclusion of the effect of soil reinforcement in the analysis of non-circular failure surfaces, as the simplistic assumptions made in conventional methods, though practicable for failure surfaces which do not depart too much from circular or planar, are likely to be seriously in error for a slope which contains significant concentrated reinforcement.

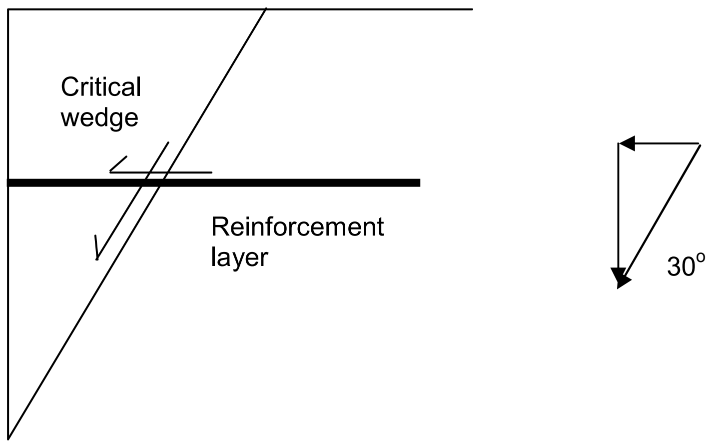

Figure 7 Strains in reinforcement in a vertical reinforced soil wall

It is not straightforward to relate the relative displacements along the boundaries determined from the kinematic analysis to the loads transmitted to the reinforcement. The stiffness of the reinforcement might be defined, but there is no gauge length to allow the derivation of reinforcement strain from displacement on the surface, which is in any case only expressed in terms of the displacement to peak strength, δ_p. Allen and Bathhurst (2003) in an extensive study concluded that for geosynthetic reinforced soil walls, a peak soil strain of 2-3% corresponded to a reinforcement strain of 3-4% or more for high shear strength granular soils, and observed that "the relationship between reinforcement strain and the soil shear strain at peak strength needs to be investigated for a wider range of soils". Most of the walls studied were shown to be severely over-designed. The finding is compatible with the working assumption that for orientated high density polyethylene geogrids used in reinforced soil retaining walls, the strain undergone by the geogrids at working load are compatible with the strains undergone by the soil. The geogrids are laid horizontally, while maximum shear in the soil is mobilised on surfaces at angles of $(45-\phi'/2)$ to the vertical. Thus, the horizontal component of strain within the soil across shear surfaces as peak strength is reached results in full mobilisation of the design strength of the reinforcement. For an angle of friction of 30^o, this would indicate that the peak load in the reinforcement is reached at a horizontal component of displacement equal to half δ_p (figure 7). The accuracy of this finding will clearly vary somewhat from situation to situation, but it provides a basis for predicting mobilised reinforcement loads in terms of the displacements across a surface. If the movement results in shortening of the reinforcement, then the reinforcement load will be zero, otherwise it can be taken to be in direct proportion to the horizontal component of displacement on the boundary, reaching a maximum when the horizontal component reaches $\delta_p/2$.

If the reinforcement stiffness varies from this ideal, then the distribution of load between soil and reinforcement will vary, but as the reinforcement continues to carry load at larger strains, the overall margin of safety will probably vary only slightly. For heavily reinforced slopes the result will be misleading for very stiff, and especially brittle, or very stretchy reinforcement, unless the factor relating reinforcement force to displacement is changed. Also, care will be needed if the reinforcement is both stiffer than ideal, and ruptures rather than stretches at a load not much higher than the design load.

It is also necessary to calculate the anchorage available for reinforcement, as short lengths, slippery materials or elements with small surface area in proportion to their design loading will pull out of the soil on one side of the boundary before the full strength of the reinforcement is mobilised. It has been assumed that once the peak design strength has been reached, the reinforcement can stretch plastically up to a limiting strain at which rupture would occur. In practice, the plastic strain would result from an increasing length of reinforcement reaching yield on either side of the surface. This would certainly happen, as the effect of the presence of the reinforcement would be to create a zone of soil either side of the critical failure surface which has very similar conditions as failure approaches. In a properly designed reinforced soil slope with an adequate factor of safety, neither soil nor reinforcement will be near a limit state as equilibrium is reached. It should be noted that reinforcement forces acting across inter-wedge boundaries ought to be assumed equal to their design working load, allowing for factor of safety, and allowing for anchorage if necessary. Even if the mechanism being considered results in low tensions in the reinforcement, other mechanisms are likely to result in tensions approaching the design load, so affecting the equilibrium of the wedge. Reinforcement forces acting across inter-wedge boundaries should therefore be based upon the design load of the reinforcement, but those acting across the slip surface at the base of a wedge should be determined using the method described above, to ensure that the results are realistic.

10. Discussion

The method as described above gives fast predictions of factors of safety, taking proper account of distortions required for significant movement to take place, and allowing a shear load-displacement model to be incorporated into the analysis. However, the initial conditions are treated as zero mobilisation of shear strength. The actual initial conditions will be much more complex than this, and the slope will have been made in stages by excavation or placing of soil. The effect of assuming some kind of K_o conditions, in which the initial mobilisation of shear strength was a function of orientation of the surface being considered, was investigated and found to produce little difference in the results. Other initial conditions could be investigated by programming them into the analytical procedure. It may be noted that the simple assumption of starting from zero may be reasonably appropriate for unsaturated compacted fills, or for relatively rapidly excavated cut slopes, in both of which pore water suction will ensure that shear stresses start out low compared with effective normal stresses. In the case of construction from very wet fills, a shear strength mobilisation approximated by K_o for normally consolidated soils may be more appropriate. If more detailed knowledge is available this could be programmed into the analysis.

The method as presented automatically allows for progressive failure arising from the geometry of the slip surface. Given appropriate data, it would be relatively simple to make a crude allowance for lateral compression of the wedges based upon the forces on their boundaries, so that displacements become progressively less from the top of the slope to the bottom. The method takes account of differences in displacement in determining the mobilised shear stresses, and can account for a reduction from peak to critical state strength. By entering an appropriate shear stress-displacement model for the soil, residual strength could also be used if appropriate. Whilst it may be argued that a finite element analysis would be more appropriate in such a case, the quality of data available to enter into the analysis would be unlikely to warrant using such a method in place of the very fast limit equilibrium approach described above.

Body forces from pseudo-static seismic loading are easily incorporated into the method at the stage of determining the equilibrium of each wedge.

11. Conclusions

A method of slope stability analysis using non-circular surfaces has been developed which takes proper account of the distortion required for movement to take place on surfaces which are neither circular nor infinite planar. The inclination of interwedge forces takes proper account of the kinematics of movement, rather than being based upon an arbitrary assumption as in most so-called ‘rigorous’ methods of analysis. The method is shown to predict both the shear surface location and the factor of safety when applied to a thoroughly investigated and well-documented case.

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