Catastrophic Health Expenditure and Household Well-Being

Ramses H. Abul Naga and Karine Lamiraud

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Ramses H. Abul Naga * and Karine Lamiraud †

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Abstract

According to the catastrophic health expenditure methodology a household is in catastrophe if its health out-of-pocket budget share exceeds a critical threshold. We develop a conceptual framework for addressing three questions in relation to this methodology, namely: 1. Can a budget share be informative about the sign of a change in welfare? 2. Is there a positive association between a household’s poverty shortfall and its health out-of-pocket budget share? 3. Does an increase in population coverage of a health insurance scheme always result in a reduction of the prevalence of catastrophic expenditures?

Keywords: Catastrophic health expenditure, welfare change, poverty, performance of health insurance schemes.

JEL codes: I1, I3.

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*Department of Economics and International Development (University of Bath).
†Faculty of Business and Economics, and Institute for Health Economics and Management (Université de Lausanne).
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1. Introduction

Risk averse individuals will appeal to insurance mechanisms as a means of diversifying their risks. This diversification of risk is important and takes on many forms, institutional and informal. In developing countries such as India, Townsend (1995) finds that via informal mechanisms individuals are able to absorb some health related risks. However, for more serious and chronic illnesses, Gertler and Gruber (2002) find that health shocks can have a major impact on consumption and can severely disrupt household welfare. There is similar evidence about the effect of health shocks in relation to developed countries such as the United States where health insurance is not mandatory. There, it has been documented (cf. Feen- berg and Skinner, 1994; Waters et al., 2004) that illness can cause households to reallocate substantial shares of their spending to out-of-pocket (OOP) health expenditures.

Thus it has been proposed to ascribe to a situation where health OOP expenditures exceed a critical share of the household’s total outlay the state of catastrophic health expenditure (Xu et al 2003; Wagstaff and van Doorslaer 2003). There is no exact consensus about the critical threshold level. Some studies choose values of 5% (Berki, 1986), 10% (Waters et al., 2004) and up to 40% of non-subsistence spending (Xu et al., 2003).

In this growing literature, the measurement of catastrophic health care payments appears to serve three main objectives surveyed below: (i) to identify changes in levels of well-being, (ii) to assess the extent of poverty / low levels of living at the household level and (iii) to assess the performance of existing health insurance schemes. There is empirical evidence regarding each of these issues, though an economic conceptual framework appears to be missing. Our aim is to try to fill this gap in the literature, by attempting to provide satisfactory answers to the following three questions:

1. Can a budget share be informative about the sign of a change in welfare?
2. Is there a positive association between a household’s poverty shortfall and its health out-of-pocket budget share?
3. Does an increase in population coverage of a health insurance scheme always result in a reduction of the prevalence of catastrophic expenditures?

The plan of the paper is the following. Section 2 surveys the literature in relation to the three questions stated above. The following three sections deal with each of questions 1 to 3. The final section contains a summary and concluding comments.
2. What does catastrophic expenditure aim to measure?

To date, we can distinguish three major purposes in relation to the measurement of catastrophic health expenditure. First, interest in the measurement of catastrophic health payments stems from the fact that in the absence of health insurance, high expenditures on health care can severely disrupt household living standards. For instance Berki (1986) states that “An expenditure for medical care becomes financially catastrophic when it endangers the family’s ability to maintain its customary standard of living.” Ideally, this change in welfare would be assessed with longitudinal data through examination of how health shocks disrupt consumption paths (Gertler and Gruber, 2002; Wagstaff, 2007). In the absence of longitudinal data, OOP health payments in excess of a threshold budget share have been used as a proxy for severe disruptions to household living standards. Regarding this point, Van Doorslaer et al. (2007) write “We focus on payments that are catastrophic in the sense of severely disrupting household living standards, and approximate such payments by those absorbing a large fraction of household resources.” Thus it may be argued that a catastrophic situation may be used to capture a change in household welfare.

Second, our reading of the literature suggests an implicit association between the state of poverty and the state of health catastrophic expenditure. In the economic literature on poverty, one distinguishes an ethical approach from a levels of living approach (Atkinson, 1987). In the former, an ethical position is used to argue that every member of society should be entitled to a minimum level of resources. In the levels of living approach, poverty is associated with insufficient consumption resulting in a low level of welfare. We find parallels to these two approaches in the health payments literature. In presenting the methodology on the measurement of catastrophic expenditure, Wagstaff and Van Doorslaer (2003) write “The ethical position is that no one ought to spend more than a given fraction of income on health care” \(^1\). There are also authors who suggest that catastrophic health expenditures are associated with low levels of living. Referring to the costs of health services, Xu et al (2003) write ”However accessing these services can lead to individuals having to pay catastrophic proportions of their available income and push many households into poverty.” More explicitly, in

\(^1\)The authors also propose measures of catastrophic expenditure drawing on similar tools as in the poverty literature. This leads to ”By analogy with the poverty literature, one could define not just a catastrophic payment headcount but also a measure analogous to the poverty gap, which we call the catastrophic payment gap”.
defining the medical poverty trap, Whitehead et al. (2001) state that "Rises in OOP costs for public and private health-care services are driving many families into poverty, and are increasing the poverty of those who are already poor." Finally, Flores et al. (2008) examine how households finance OOP payments (the problem of coping with health care costs) and the implications of coping strategies for the measurement of poverty. Clearly, in order to understand the overall relation between the incidence of poverty and catastrophic expenditure one needs to explore, at the micro-level, the relation between the budget share Engel curve for health OOPs and the poverty shortfall. This is what we set out to do in Section 4 below.

Catastrophic health care payments are also used to measure the performance of prevailing health insurance schemes. The understanding is that a large fraction of individuals experiencing catastrophic health payments is associated with an insufficient coverage in relation to health insurance contracts. According to Waters et al. (2004), "One rationale for health insurance coverage is to provide financial protection against catastrophic health expenditures." By insufficient coverage researchers most often refer to the small percentage of the population in benefit of any health insurance scheme (Scheil-Adlung et al., 2006). But it may also refer to the lack of generosity of the health insurance scheme, with respect to copayments and the levels at which benefits are capped. A Mexican study (Knaul et al. 2006) concludes that the prevalence of health catastrophic expenditure is reduced by an increased coverage of the population by health insurance schemes. Likewise a joint ILO, WHO and OECD study covering three developing countries (Scheil-Adlung et al., 2006) finds that membership in health insurance schemes contributes to reducing the probability of incurring catastrophic health expenditures. Nonetheless this study shows that the protective effect of being insured is not general: in South Africa it only concerns the richest quintile of the population who is able to afford more comprehensive packages (Lamiraud et al., 2005). The study of Waters et al. (2004) also reveals that low income and the occurrence of multiple chronic conditions, alongside the lack of health insurance, increase the probability of catastrophic health payments. These findings are to be contrasted with those of Wagstaff and Lindelow (2008) who conclude that in three Chinese provinces health insurance increases the risk of catastrophic spending.

In the sections below we develop an economic framework for addressing the questions stated in the Introduction in relation to this literature. We begin with asking to what extent a budget share can be informative about a change in welfare.
3. Catastrophic expenditure: a measure of change in welfare?

When panel data on a household’s consumption are not available, it has been suggested to proxy disruptions in household welfare via the use of the level of the budget share of health OOP expenditure. Our first question, formulated in general terms therefore is: can a budget share be informative about the direction of a change in household welfare?

We consider the following problem: by choice of quantities \( q_1 \) and \( q_2 \) of two goods, a household maximizes its utility \( u(q_1, q_2) \) subject to a budget constraint \( p_1 q_1 + p_2 q_2 = m \). We let \( \lambda \) denote the Lagrange multiplier, \( p = [p_1, p_2] \), and we denote the consumer’s indirect utility function by \( v(m, p) \). In this conceptual framework, a change in welfare arises from either a change in household income \( m \), a change in one or more prices, or finally a change in prices and income. The literature most often focuses on income shocks as a source of welfare disruptions, since these are household specific, whereas changes in prices are often perceived to affect all individuals alike.

Let \( m_0 \) denote the household’s base period income, and let \( m_1 \) denote the household’s current period income. Consider then an income change \( \Delta m = m_0 - m_1 \) (which need not be a negative quantity). Approximating the resulting change in welfare using a first-order Taylor approximation, we have:

\[
v(m_0, p) \approx v(m_1, p) + \frac{\partial v}{\partial m}(m_1, p)[m_0 - m_1] \tag{3.1}
\]

From the envelope theorem we have that \( \frac{\partial v}{\partial m}(m_1, p) = \lambda > 0 \), since the marginal utility of income is always positive. Letting \( \Delta v = v(m_0, p) - v(m_1, p) \), we can write (3.1) in a more compact fashion as

\[
\Delta v = \lambda \Delta m \tag{3.2}
\]

Since the marginal utility of income \( \lambda \) is a positive quantity, we can state the following preliminary result:

**Lemma** The change in welfare \( \Delta v \) is always of the same sign as the change in income \( \Delta m \).

Let \( w_i = p_i q_i / m_1 \) denote the budget share for good \( i \) in the current period. Our next purpose is to write (3.1) in terms of \( w_i \). We first use Roy’s identity to write the marginal utility of income as :

\[\text{The results below easily generalize in the context of } n > 2 \text{ goods.}\]
\[ \lambda = -\frac{\partial v/\partial p_i}{q_i} \quad (3.3) \]

and since \( q_i = w_i m_1 / p_i \), we have that

\[ \lambda = -p_i \frac{\partial v/\partial p_i}{w_i m_1} \quad (3.4) \]

Replacing (3.4) in (3.2) we have that

\[ \Delta v = -p_i \frac{\partial v/\partial p_i}{w_i m_1} \Delta m \quad (3.5) \]

Observe that \( \partial v/\partial p_i < 0 \) (the indirect utility function is decreasing in prices), so that, as stated in the Lemma, \( \Delta v > 0 \) if and only if \( \Delta m > 0 \). Finally, rearranging terms, we obtain the desired relation between the budget share \( w_i \) and the welfare change \( \Delta v \):

\[ w_i = -p_i \frac{\partial v/\partial p_i \Delta m}{m_1 \Delta v} \quad (3.6) \]

Because \( \Delta v \) has the same sign as \( \Delta m \), a high level of the budget share \( w_i \) is equally compatible with a scenario where \( \Delta v \leq 0 \) (for which \( \Delta m \leq 0 \)) and with a situation such that \( \Delta v > 0 \) (corresponding to a \( \Delta m > 0 \)). Accordingly, the answer to our first question is the following:

**Proposition 1** Without additional information about the sign of the income change, the level of a budget share cannot be informative about the sign of the change in welfare.

The scope therefore for identifying households who experience a severe decline in their levels of living using a budget share is limited, unless the data analyst is sure that the household has experienced an income drop. Such information about changes in income is however not always available in cross-section type household surveys. The main problem is that it is hoped to identify a change in a variable (household welfare) by means of another variable (a budget share) measured in levels.

### 4. Catastrophic expenditure and poverty

We now turn to our second question, where we investigate the household level relation between the poverty shortfall and the budget share for health OOPs. Our
purpose here is to inquire as to the existence of a positive association between these two variables. The answer to this question would appear to be straightforward if we were willing to assume the existence of a decreasing Engel curve relation between the health budget share and income. With this assumption we could certainly conclude that catastrophic expenditure rises with poverty and thus we could argue that to the extent that economic development reduces poverty, it would also reduce the incidence of catastrophic expenditure.

However, the assumption of a decreasing Engel curve, or more specifically of a monotonic Engel curve is not as natural as it seems. Modern empirical research as well as economic theory of consumer choice highlights the importance of non-linearities in the Engel curve relation. Stated differently, economic theory does not rule out that a good could be a necessity at some income intervals and a luxury at others. This is where we begin our investigation of our second question.

Let \( z \) denote the poverty line and define \( \pi(m, z) \) as a household’s poverty shortfall from the poverty line. Because poverty measures respect the Pareto principle (see Atkinson, 1987) the function \( \pi(m, z) \) is monotonically decreasing in \( m \), for \( m < z \) and is further assumed to be zero for \( m \geq z \). The function \( \pi(m, z) \) is thus invertible and we define

\[
m = h(\pi, z)
\]

(4.1)

to be the resulting inverse function. We also write the general Engel curve relation for health OOP as \( w = \psi(m) \), where we observe once again that this relation need not be monotonic.

Substituting for \( m \) in the Engel curve relation using (4.1), we have

\[
w = \psi[h(\pi, z)]
\]

(4.2)

Accordingly, in the region \( \pi > 0 \) we have

\[
\frac{dw}{d\pi} = \frac{d\psi}{dh} \frac{dh}{d\pi}
\]

(4.3)

Because \( m = h(\pi, z) \) is decreasing in \( \pi \), the second right-hand-term is non-positive and it follows that in \((\pi, w)\) space the function (4.2) will indeed have a

\footnote{That is, a poverty measure \( P(m, z) \) relates to the sum of individual family level poverty shortfalls via the identity \( P(m, z) = \int_0^z \pi(m, z) dF \), where \( F(m) \) is the distribution of income.}

\footnote{At \( \pi = 0 \), the function \( h(\pi, z) \) rises to infinity, and accordingly the derivative \( \frac{dh}{d\pi} \) is no longer defined.}
positive slope for all $\pi > 0$ provided the Engel curve relation $\psi(m)$ is monotonically decreasing at all income levels. In such a situation, it is the case that for any budget share threshold defining the state of catastrophic expenditure, increases in income will simultaneously result in a decline in the intensity of catastrophic expenditure and poverty.

Now consider the general case where the Engel curve is non-monotonic. The simple relation (4.3) informs us that the non-linearities in the Engel curve will be depicted by the function (4.2). To illustrate our point, we may borrow the specification of quadratic logarithmic demands from Banks et al. (1997) \(^5\). In a cross-section environment where prices are taken to be constant, the resulting budget share relation is of the form

$$w(m) = \phi_0 + \phi_1 \ln m + \phi_2(\ln m)^2$$

(4.4)

where $\phi_1$ and $\phi_2$ are coefficients that may be of opposite or identical signs. For the individual poverty shortfall function consider the simple specification based on Watts (1968):

$$\pi(m, z) \begin{cases} \log(z/m) & m < z \\ 0 & m \geq z \end{cases}$$

(4.5)

The resulting relation between $\pi$ and $w$ is given by

$$w = \phi_0 + \phi_1 \ln z + \phi_2(\ln z)^2 - [\phi_1 + 2\phi_2 \ln z]\pi + \phi_2\pi^2$$

(4.6)

Preliminary research by Lamiraud et al. (2008) using South African data suggests that the Engel curve for health OOP is $U-$shaped. Accordingly, in Figure 1 we plot a hypothetical situation where we set $\phi_0 = 0.40$, $\phi_1 = -0.80$ and $\phi_2 = 0.50$. We also set the poverty line at $z = 4.5$, and consider a range of income values in the interval $m \in [1; 4]$ pertaining to individuals experiencing poverty. The NorthEast quadrant plots the Engel curve relation which, given the parameter values, is $U-$shaped, and reaches its minimum at $m^* = \exp(0.80) = 2.23$. The SouthEast

\(^5\)The budget shares underlying quadratic logarithmic demands are of the form

$$\bar{w}(m, p) = \bar{\phi}_0(p) + \bar{\phi}_1(p)[\ln m - \ln a(p)] + \bar{\phi}_2(p)[\ln m - \ln a(p)]^2$$

where each of the functions $\bar{\phi}_1$ and $\bar{\phi}_2$ may either be both positive or negative over the price space. If we abstract from price variations across family units, the resulting function $w(m)$ is of the form (4.4).
quadrant plots the relation (4.5) between income and the poverty shortfall. To obtain the relation between $\pi$ and $w$, we draw a 45° degree line in the SouthWest quadrant of the diagram. Finally, in the NorthWest quadrant we obtain the desired relation (4.6). The graph illustrates in a simple fashion (4.3), that is, the fact that the relation between $w$ and $\pi$ will reflect the curvature properties of the Engel curve.

Algebraically, the relation drawn in the NorthWest quadrant, given our choice of functional forms and parameter values, is readily obtained from (4.6) as $w = 0.33 - 0.7\pi + 0.5\pi^2$ such that the curve reaches its minimum at $\pi^* = 0.7$.

As an answer to our second question, the preceding discussion is summarized by means of the following proposition:

**Proposition 2** (a) Let the Engel curve for health out-of-pocket expenditure be a non-monotonic function of income below the poverty line. Then the resulting relation between the budget share and the poverty shortfall is also non-monotonic.

(b) A sufficient condition to obtain a positive association between the budget share and the poverty shortfall is that the Engel curve for health OOP be a decreasing function of income below the poverty line.

It is therefore an empirical question to ask as to whether the Engel curve for OOP health expenditures is indeed a declining function of income so as to guarantee a positive association between poverty and catastrophic expenditure at the economy-wide level.

5. **Catastrophic expenditure: a measure of performance of health insurance systems?**

As discussed in Section 2, it has been advocated that the intensity of catastrophic health care payments may be used as an index of the performance (i.e. under-coverage) of prevailing health insurance schemes. On the other hand, the empirical findings of Wagstaff and Lindelow (2008) from Chinese data suggest, in contrast to this common belief, that health insurance may increase the intensity of catastrophic expenditure because the take-up of insurance encourages individuals to consume additional health resources when stricken by illness. In this section therefore we construct a simple model for the demand of health insurance in the developing country context in order to address our third question, namely: Does an increase in population coverage of a health insurance scheme always result in a reduction of the prevalence of catastrophic expenditures?
Let $\theta$ be a random variable in relation to the health state of an individual. The variable $\theta$ takes the value 1 with probability $\kappa$ when the individual falls ill, and takes the value 2 in case the individual does not fall ill. Below, we modify the classic treatment of health insurance (Culyer, 1989) to take into account several features of the decision to purchase health insurance in the developing country context. We firstly define the individual’s preferences over two goods; a consumption aggregate denoted $C$ and a health care aggregate good denoted $H$. Conditional on being in the health state $\theta$, the individual’s utility function has a quasi-linear form

$$u(C, H; \theta) = \theta \gamma \ln C + H (2 - \theta)$$

(5.1)

such that health consumption is assumed to provide zero utility if the person is in good health. Also, given the parameter $\gamma > 0$ the marginal utility of consumption becomes infinite when $C$ approaches zero.

The specification is relevant in the developing country context, since (5.1) entails that individuals initially allocate their spending to the consumption good, and upon reaching a certain threshold, start consuming the health good. To be more specific, the consumer maximizes their ex-ante expected utility

$$Eu(C, H; \theta) = \kappa (\ln C + H) + (1 - \kappa) 2\gamma \ln C$$

$$= (2 - \kappa) \gamma \ln C + \kappa H$$

(5.2)

where

$$H \geq 0$$

(5.3)

The individual may subscribe to a simplified health insurance scheme characterized by a premium $P$ and a coverage $d$. That is, we assume that the scheme offers a capped reimbursement, in other words a 100% coverage rate for expenditure levels up to $d$. Additional health expenditures above this level are however totally paid out-of-pocket by the enrollee $^6$. Normalize the price of the health care good

$^6$Note that in the context of many developing countries this assumption is quite realistic (e.g. Gertler, 1998). In South Africa for instance, in parallel with the public health care sector there exists a private health sector, which users access mainly via their own funds or through private medical insurance packages known as medical schemes. While there is a list of services that any contract must cover, known as the Prescribed Minimum Benefits [PMB], medical schemes still have differing degrees of coverage on other services not covered by PMBs. For this reason, low cost options (mainly chosen by low income beneficiaries) will not insure beneficiaries from catastrophic health expenditures, to the extent made possible by more costly options. Hence, three quarters of low-cost options offered by medical schemes to people with lower levels of income rely on monetary limits, levies and co-payments to curb the use of top-up hospital benefits (Doherty and McLeod, 2002).
at unity, and let $H_o$ denote OOP health expenditures. Then $H$ and $H_o$ relate in the following manner: $H = H_o + d$ when the individual is insured, and $H = H_o$ in case the individual does not enroll in a health insurance plan.

If the health insurance contract is actuarially fair then $\Pi$, $\kappa$ and $d$ relate in the following manner

$$\Pi = \kappa d \quad (5.4)$$

while the consumer’s budget constraint takes two distinct forms depending on whether she / he chooses to take up health insurance. As above, we let $m$ denote the individual’s income and let $p_c$ be the price of the consumption good. First, if the individual does not take up health insurance, the budget constraint is given by

$$p_c C + H = m \quad (5.5)$$

On the other hand, when the individual is insured

$$p_c C + (H - d) = m - \kappa d \quad (5.6)$$

since the person pays a premium $\kappa d$ and receives capped benefits $d$.

The consumer’s problem is whether or not to enroll in the health plan and to determine the optimum amounts of $C$ and $H$. As the purchase of health insurance is a discrete choice good, we solve the consumer’s programme in two steps (Small and Rosen, 1981). In the first step, we solve two different problems: (i) we maximize (5.2) over consumption $C$ and healthcare $H$ subject to (5.5) and (5.3) when the person does not buy insurance, and (ii) we maximize (5.2) subject to (5.6) and (5.3) when the person is insured. Let $v^0(m, p_c)$ denote indirect utility (that is, maximized expected utility) when the person does not buy insurance, and let $v^I(m - \kappa d, p_c)$ denote indirect utility when the person is insured. The second step involves a comparison of $v^0(m, p_c)$ and $v^I(m - \kappa d, p_c)$: the person will choose to enroll in the health scheme if $v^I(m - \kappa d, p_c) \geq v^0(m, p_c)$.

Define $A = \gamma(2 - \kappa)/\kappa$. As $\kappa$, $d$ and $A$ are all exogenous parameters, the model can be solved distinguishing two cases according to whether $\kappa d$ is greater or smaller than $A$. Here we discuss the case $\kappa d < A$, which can be argued to be the more relevant case in the developing country context 7.

It is a property of quasi-linear preferences, that the consumer considers spending on $H$ only when $m \geq A$. Thus the indirect utility function for an individual

\footnote{The assumption $\kappa d < A$ in effect means that the health insurance premium is set at a level which is lower than the threshold at which the individual begins to consume health-care; see below.}
who does not take up insurance is of the form

\[ v^o(m, p_c) = A \ln \left( \frac{m}{p_c} \right) \quad m \leq A \]  
(5.7)

\[ v^o(m, p_c) = A \ln \left( \frac{A}{p_c} \right) + m - A \quad m > A \]  
(5.8)

while for a person who takes up an insurance policy the indirect utility function is of the form

\[ v^I(m - \kappa d, p_c) = A \ln \left( \frac{m - \kappa d}{p_c} \right) + d \quad m - \kappa d \leq A \]  
(5.9)

\[ v^I(m - \kappa d, p_c) = A \ln \left( \frac{A}{p_c} \right) + d + m - \kappa d - A \quad m - \kappa d > A \]  
(5.10)

From the above four equations, we note that for \( m \leq \kappa d \) a poor person would never buy health insurance (the marginal utility of consumption in (5.9) is infinite at \( m = \kappa d \).) Conversely, a rich person \((m > \kappa d + A)\) would always buy health insurance since the first \( d \) units of health spending are purchased at a cost of \( \kappa d \) (the difference between (5.10) and (5.8) is positive when \( m > \kappa d + A \).) Furthermore, there are two important income thresholds which determine the decision to take up health insurance. First consider a person with income \( \hat{m}_I \) such that

\[ A \ln \left( \frac{\hat{m}_I}{p_c} \right) = A \ln \left( \frac{\hat{m}_I - \kappa d}{p_c} \right) + d \]  
(5.11)

At \( \hat{m}_I, \) a person who does not incur health OOPs is indifferent between taking up insurance or being uninsured. Clearly, for such a case to be pertinent, we must have \( \hat{m}_I < A \). This inequality defines a first regime of our model, which we shall label Case I sub-model. This is the case where a person who does not subscribe to an insurance policy will never consume health. If on the other hand \( \hat{m}_I \geq A \), there exists an income threshold \( \hat{m}_{II} \) such that

\[ A \ln \left( \frac{\hat{m}_{II}}{p_c} \right) + \hat{m}_{II} - A = A \ln \left( \frac{\hat{m}_{II} - \kappa d}{p_c} \right) + d \]  
(5.12)

where \( A < \hat{m}_{II} < A + \kappa d \). Under this second Case, some individuals with incomes between \( A \) and \( \hat{m}_{II} \) do not take up health insurance but consume health, thus incurring OOPs. Below we refer to this second case as the Case 2 sub-model.\(^8\)

\(^8\)The two regimes of the model Case I and Case II are mutually exclusive. A proof of this result (Proposition 4) is provided in the Appendix.
5.1. Case 1 sub-model

To spell out the effect of a change in population coverage of a health insurance scheme on the incidence of catastrophic expenditure, we first derive the Engel curve for OOPs $H_o$ under the Case 1 scenario. We denote this Engel curve $w(m; d)$:

\begin{align*}
  w(m; d) &= 0 & m < \hat{m}_I \\
  w(m; d) &= 0 & \hat{m}_I < m < A + \kappa d \\
  w(m; d) &= 1 - \left(\frac{A + \kappa d}{m}\right) & m \geq A + \kappa d
\end{align*}

(5.13) (5.14) (5.15)

For $m < \hat{m}_I$, the consumer will never buy health insurance, and also does not spend on health care. When $\hat{m}_I < m < A + \kappa d$, the consumer buys insurance and consumes an amount $H = d$. This amount is fully covered, and accordingly $H_o = 0$ so that in this second case $w(m; d)$ is also equal to zero. Finally, when $m \geq A + \kappa d$, all additional spending on health is incurred out-of-pocket. Both relations, that is the relation between $m$ and $H$, and between $m$ and $H_o$ are sketched in Panel $a$ of Figure 2.

In Figure 3 Panel $a$ we sketch the Engel curve for health OOPs. The graph depicts the implications of the Case 1 sub-model for the relation between catastrophic expenditure and the extent of health insurance. Note that here, catastrophic expenditure (a budget share in excess of some arbitrary threshold $w(z; d)$) only occur when $m > A + \kappa d$. The share of the population insured is $1 - F(\hat{m}_I)$ where $F(.)$ is the cumulative distribution of income. Solving for $\hat{m}_I$ in (5.11) we have that

\begin{align*}
  \hat{m}_I &= \exp(d/A) \cdot \kappa d \\
  \frac{\partial}{\partial d} \hat{m}_I &= 0 \quad \text{when } m > A + \kappa d
\end{align*}

(5.16)

In turn this implies that $\partial \hat{m}_I/\partial d > 0$, that is the share of the population covered by the health insurance scheme decreases when $d$ is larger. On the other hand $\partial w/\partial d = 0$ when $m < A + \kappa d$ and $\partial w/\partial d < 0$ when $m \geq A + \kappa d$. That is, while there is a resulting decrease in the share of the population covered via an increase in $d$, the incidence of catastrophic expenditure is reduced. The Case 1 sub-model therefore provides a counter-example to the hypothesis that catastrophic expenditures are associated with an insufficient population coverage of a given health insurance scheme.
5.2. Case 2 sub-model

Unlike the Case I sub-model where OOP expenditure is only incurred by the rich ($m > A + \kappa d$), the Case 2 sub-model where individuals may be incurring health expenditures without being insured, presents a more varied configuration of OOPs by income groups. For $m < A$, the consumer allocates the total amount of her budget to $C$ and does not spend on health. Accordingly, both $H$ and $H_o$ are equal to zero. For $A < m < \tilde{m}_{II}$, the consumer does not take up insurance, but spends $m - A$ on health. Therefore, both $H$ and $H_o$ are equal to $m - A$. For $\tilde{m}_{II} < m < A + \kappa d$, the individual chooses to buy insurance and consumes the amount $H = d$ which is fully reimbursed by the insurance scheme. Hence in this case also $H_o$ is equal to zero. Finally, for $m > A + \kappa d$, $H = m - A - \kappa d + d$. All expenditures then are financed out-of-pocket. We sketch the resulting relations between $m$, $H$ and $H_o$ in Panel b of Figure 2. The related health OOP budget shares are readily derived as follows:

$$w(m; d) = \begin{cases} 0 & m < A \\ 1 - \left( \frac{A}{m} \right) & A < m < \tilde{m}_{II} \\ 0 & \tilde{m}_{II} < m < A + \kappa d \\ 1 - \left( \frac{A + \kappa d}{m} \right) & m \geq A + \kappa d \end{cases}$$

We may note from Panel b of Figure 3 that catastrophic expenditure may now concern two population groups: those with resources $m \in (A, \tilde{m}_{II})$ (who do not purchase health insurance) and those for which $m \geq A + \kappa d$.

For the latter group, an increase in $d$ (and hence a decrease in the share of the population covered) results in a decline of catastrophic health expenditure (as was the case in the Case 1 sub-model.) On the other hand, we show in Proposition 5 of the Appendix that the effect of an increase in $d$ on $\tilde{m}_{II}$ is ambiguous. Consequently, the effect of a change in $d$ on the share $1 - F(\tilde{m}_{II})$ of the population which is insured is also undetermined.

If the overall effect of an increase in $d$ is that $\tilde{m}_{II}$ is increased, the incidence of catastrophic expenditure rises for individuals in the income range $A < m < \tilde{m}_{II}$, while it still falls for individuals with the higher incomes $m \geq A + \kappa d$. Thus an increase in the share of the population covered would result in an ambiguous overall effect on the incidence of catastrophic expenditures.
5.3. Change in population coverage: overall effect

The Case I sub-model entails that the incidence of catastrophic expenditure is reduced when population coverage of the health insurance scheme declines, and conversely that increased coverage increases the incidence of catastrophic expenditure. The Case II sub-model generates catastrophic expenditures for both the insured and uninsured populations. For this reason, there the overall effect of a change in population coverage on the incidence of catastrophic expenditure is ambiguous. We may summarize our above discussion with the following Proposition:

**Proposition 3** (a) Under the case I sub-model \((\bar{m}_I < A)\), a reduction in the share of the population insured results in a decline in the incidence of catastrophic expenditure.

(b) Under the case II sub-model \((\bar{m}_I > A)\), a reduction in the share of the population insured has an undetermined effect on the incidence of catastrophic expenditure.

6. Concluding comments

We may summarize our answers to the three questions raised in the paper as follows:

1. An observed budget share level for a given good is compatible with both a drop or a rise in levels of living. Thus, without additional information about the direction of income change, the level of a budget share cannot be informative about the sign of the change in welfare.

2. In the general case where below the poverty line the Engel curve for OOPs is a non-monotonic function of income, the resulting relation between the budget share and the poverty shortfall is also non-monotonic. However, if below the poverty line the income elasticity for OOPs is smaller than one (i.e. the Engel curve is decreasing), this will indeed entail a positive association between the poverty shortfall and the budget share for health OOPs.

3. An increase in population coverage of a health insurance scheme need not always result in a reduction of the prevalence of catastrophic expenditures.

In relation to our first question, we may conclude therefore that the scope for using cross-section data to identify households who experience a severe decline in their levels of living using a budget share is considerably limited, unless the data analyst is sure that the household has experienced an income drop. Such
information about changes in income is readily available from panel data, but is rarely encountered in cross-section type household surveys.

We note from our discussion in relation to the second question that we have addressed in the paper, that it does not follow that catastrophic health expenditure increases with poverty. Empirical work is therefore needed in order to further explore the curvature properties of the Engel curve for health care spending.

Finally, with the help of a simple model, we have shown that catastrophic expenditure could well decline when the share of the population covered by a health insurance scheme also falls. Thus, more work is needed in order to better understand how the overall performance of a health insurance scheme relates to the incidence of catastrophic expenditures.

7. Appendix

This appendix gathers proofs of two results stated in the paper.

**Proposition 4** There exists \( \hat{m}_{II} \in [A, A + \kappa d] \) such that for any \( m \geq \hat{m}_{II} \) the consumer always chooses to buy insurance.

**Proof:** Define \( \hat{m}_I \) by the equality

\[
A \ln \left( \frac{\hat{m}_I}{p_c} \right) = A \ln \left( \frac{\hat{m}_I - \kappa d}{p_c} \right) + d
\]

This yields \( \hat{m}_I = \frac{\exp(d/A) \cdot \kappa d}{\exp(d/A) - 1} \). Now, if \( \hat{m}_I < A \), the consumer switches to insurance at an income level \( m \) in the interval \([\kappa d; A]\), and \( \hat{m}_{II} = A \). If \( \hat{m}_I > A \), then at \( m = A \),

\[
A \ln \left( \frac{m}{p_c} \right) + m - A > A \ln \left( \frac{m - \kappa d}{p_c} \right) + d
\]

i.e. \( v^o(m, p_c) > v^f(m - \kappa d, p_c) \). Also, at \( m = \kappa d + A \), \( v^f(m - \kappa d, p_c) - v^o(m, p_c) = d - \kappa d > 0 \).

Since the indirect utility function is continuous in \( m \), there must exist an \( \hat{m}_{II} \) in the interval \([A, A + \kappa d]\) such that \( v^o(\hat{m}_{II}, p_c) = v^f(\hat{m}_{II} - \kappa d, p_c) \).

**Proposition 5** The effect of a change in \( d \) on \( \hat{m}_{II} \) is undetermined.

Define \( G(\hat{m}_{II}, d) = A \ln A + \hat{m}_{II} - A - A \ln (\hat{m}_{II} - \kappa d) - d \). By definition, \( G(\hat{m}_{II}, d) = 0 \). Let \( \Delta x \) denote an infinitesimal change in \( x \). It follows from the
implicit function theorem that

\[
\frac{\Delta \hat{m}_{11}}{\Delta d} = \frac{\partial G / \partial d}{-\partial G / \partial \hat{m}_{11}} = \frac{\kappa d + \kappa A - \hat{m}_{11}}{\kappa d + A - \hat{m}_{11}}
\]

The denominator of this last ratio is positive in virtue of Proposition 5. However the numerator of the ratio has an undetermined sign. Thus, the sign of \(\Delta \hat{m}_{11}/\Delta d\) is undetermined.

8. References


Figure 1: Relationship between health care budget share and poverty shortfall
Figure 2: Health care consumption in Case 1 and Case 2 submodels

Panel a: Case 1 submodel

Panel b: Case 2 submodel
Figure 3: Engel curves for OOP spending in Case 1 and Case 2 submodels

Panel a: Case 1 submodel

Panel b: Case 2 submodel