PHD

# Will international trade between two economies improve their well-being and environmental quality?

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# Will international trade between two economies improve their well-being and environmental quality?

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# Acknowledgment

In my life I have always thought knowledge is what characterize our species. The best representation is in the Divine Comedy of Dante Alighieri: ”Consider your origin; you were not born to live like brutes, but to follow virtue and knowledge”. I consider obtaining a PhD the greatest achievement of my life that gave me skills I have never thought I could possess which will be useful in my future challenges.

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# Summary

This thesis is divided in five chapters. The first chapter is the introduction that explains the historical background and the development of environmental economics.
The second chapter is a preliminary description of the North-South Directed Technical Change (DTC) with environmental externalities model using exogenous taxation.
The third chapter explores new externalities with a focus on a paternalistic optimal policy and the different solutions to each new constraint.
The fourth one focuses on trade and its effect on innovation. There I propose two different models depending on the type of trade concluding with a general model.
Finally, the last chapter is the conclusion with policy implications and ideas for further research.

# Contents

# Chapter 1

# Introduction

## 1.1 Research Background

At the end of 20th century climate change started to be concerning for the public opinion and policy makers. It is evident the effect of climate change on our lives from health to economic production. The by-product of the use of carbon fuels generates CO2 which can increase the temperatures when its accumulation exceeds the natural decay.When the temperature will increase 1.5 Celsius degree the immediate result will be a growth in skin cancer, spread of mosquitoes and diseases and lack of drinkable water. On the production side, agriculture and fisheries would be severely affected resulting in large losses that are still now difficult to estimate. The ecosystems are in a fragile equilibrium and a small change creates a new transition towards new equilibriums difficult to forecast. The first international body that recognizes the issue was the Club of Rome, which addresses the limit to growth linked to the limited resources of our planet. In 1972 they released the report The Limits to Growth [1] where they estimate possible scenarios showing a trade off between industrial production and population growth. This report starts the debate of exhaustible resources that follows modern Malthius laws.

In the 1988 the Intergovernmental Panel on Climate Change (IPCC)[2] starts a methodology to analyse the yearly change of the state of the planet. It is a complex report that covers all the aspects of the environment. It analyses the change in CO2 levels, the growth and reduction of forests, the sea level rise and biospheres change giving different forecasting scenarios depending on the commitment to avoid environmental degradation.

In 1992 the Kyoto protocol, which is considered the first most important climate conference addresses important issues to reach legal commitments and cooperation to avoid an increase in the CO2 levels. One of the main results is the decision to reduce the cost of mitigation for developing countries. This is the main point of my thesis, because I am referring to this principle using unilateral policy. This concept requires the developed countries to sustain the majority, if not all the cost to reduce environmental damages following the principle of who polluted pays. One of the main issues to address climate issue is that some countries have enjoyed a long growth polluting in the 18th and 19th century. Since CO2 has a lifespan of 120 years therefore we still have the carbon dioxide (CO2) generated in the last century.

In October 2006 the British government released the Stern review, the first extensive guide of climate change around environmental issues ever released by a government. After this report, it starts to be mandatory for countries to highlight the importance of climate change on their economy and to start environmental accounting. This leads to a larger dataset that can be used to compare the progress of different countries investigating the cost of adaptation and mitigation.

During the Paris Agreement of 2015 the majority of the recognized countries agreed to coordinate to keep the global warming below 2 Celsius degrees compared to pre-industrial levels by 2050. In order to do so, they proposed different strategies. The most popular one is the Net zero emissions target. It requires the agreeing party to have an overall balance between the emission taken from the atmosphere and released to it by 2050 such that the environment can slowly recover. To reach this targets there are different tools such as carbon storage and sequestration (CSS) that absorb CO2 from the atmosphere sealing it in the ground and filters to cap the emissions. The main problem is that there is a need to innovate in order to increase the efficiency of these tools.

Thanks to these events, researchers are incentivised to innovate for the purpose to address climate change. Today it is a broader subject that affects all fields from Engineering and Medicine to Humanities. The main results of the scientists are collected every year in the IPCC report in order to improve our understanding of the environment and develop new adaptation and mitigation tools.

## 1.2 Research questions

### Is it possible to avoid an environmental disaster using unilateral policy?

The first and main research question is whether or not it is possible to avoid an environmental disaster applying a policy only in one region. An unilateral policy is a tool applied only in one region that has the ability to solve a local problem and thanks to spillovers to affect also other regions. I solve this problem in Chapter 3 where there is no trade thanks to the imitation of technology. In Chapter 4, I introduce trade of inputs showing that it is still possible to avoid an environmental disaster contrarily to Acemoglu et al 2014.

### What is the effect of one country's unilateral environmental policies on the other country's economy and welfare?

The elimination of the externalities depends by the nature of the environmental problem. In fact, when we consider trans-boundary pollution the carbon tax has two

components. A proper carbon tax and a compensation for the other country. The trans-boundary pollution affects the country that generate it and the other that sustains it.
In the chapters I define and apply a net zero emissions target that aim at stopping the environmental degradation. It has different results depending on the presence of an abatement technology. When it is applied to soon it leads to a loss of welfare larger than a carbon tax.

### Are discretionary environmental policies effective when countries trade inputs?

When we talk about international trade and environment the most common results are pollution havens which is a consequence of comparative advantage. When a country specializes in one sector the other one specializes in the other to be competitive. In this framework it results in a country specializing in clean technologies and the other in dirty one. In Chapter 4 I show that when there is trade of inputs between symmetric economies it is not happening and trade has a small influence on the demand of a region which is amplified by the elasticity of substitution between the clean and dirty intermediate output.

# Bibliography

[1] Behrens III J., Meadows D H., Meadows D. L., Randers J., 1972 *The Limits to Growth, A report for the Club of Rome's project on the predicament of mankind,* New York, Universe Books

[2] IPCC, 2022: *Climate Change 2022:Impacts, Adaptation, and Vulnerability.* Contribution of Working Group II to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [H.-O. Pörtner, D.C. Roberts, M. Tignor, E.S. Poloczanska, K. Mintenbeck, A. Alegría, M. Craig, S. Langsdorf, S. Löschke, V. Möller, A. Okem, B. Rama (eds.)]. Cambridge University Press. Cambridge University Press, Cambridge, UK and New York, NY, USA, 3056 pp., doi:10.1017/9781009325844.

# Chapter 2

# Directed technical change towards net zero emissions

## 2.1 Introduction

In 1997 during Kyoto conference representatives of all nations tried to solve global warming through international policy with few results. The main reason it was unsuccessful is due to the lack of coordination between major countries. The last attempt was made in 2015 during the Paris agreement, but it seems clear that it is almost impossible to have the same policy for all countries with different characteristics due to free-riding and asymmetric information. Instead, there is consensus to use unilateral policies[1], usually in OECD countries, in order to change the direction of climate-change globally. This last concept is highlighted by Khan (2015) [16] who builds a two region economy with unilateral policy showing that when there is no trade an environmental disaster is impossible to avoid. He concludes his work saying that "Research could also consider cases in which countries may run trade deficits or build on the research done in other countries", which is exactly what I develop in this model thanks to the imitation process of a less developed country such that the two economies are linked. On January 2020 European Union established the goal to reach carbon neutrality by 2050, which is also called Net zero emission target [2]. The EU policy makers believe that "according to the Commission's estimates, EU GDP is expected to increase more under zero-emissions scenarios than in scenarios with smaller emissions reductions, with

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[1] Hemous (2016) states: "unilateral environmental policies should aim at developing clean technologies, which have the potential to reduce emissions in the North, but also in the South either through technology diffusion or by slowing down the move of polluting industries there. These policies should be thought of as transitory until a satisfactory global agreement is reached"[13].

[2] "Net zero emissions are achieved when anthropogenic emissions of greenhouse gases to the atmosphere are balanced by anthropogenic removals over a specified period. Where multiple greenhouse gases are involved, the quantification of net zero emissions depends on the climate metric chosen to compare emissions of different gases (such as global warming potential, global temperature change potential, and others, as well as the chosen time horizon)".( https://www.ipcc.ch/sr15/chapter/glossary/)

the effects in both cases being spread unevenly across the EU as a result of differences among Member States, inter alia in terms of GDP per capita and the carbon intensity of the energy mix; considers that inaction would be by far the costliest scenario and would not only result in significant GDP loss in Europe, but also further increase economic inequalities between and within Member States and regions, as some are expected to be hit harder than others by the consequences of inaction"[3].

In my work I investigate the effect of unilateral policies applied in a better developed economy to see if they can change the path of less developed one to green growth avoiding environmental disasters. The main framework of my work is the so-called AABH model of Acemoglu et al. (2012)[1] which represents a model of Directed Technical change with environmental externalities where the main target is to avoid an environmental disaster through clean technologies. The extension of that model is the North-South AABH model of Acemoglu et al (2014)[2] where the authors study the same problem in a two country world with international trade. Their main result is that global policy is the best way to avoid an environmental disaster while under autarky unilateral policy could work because the South cannot be a pollution haven. Here I built a North-South model with two sector, clean and dirty, that use the respective inputs with elasticity of substitution greater than one [3] which are utilized to assemble the final good. While the clean inputs have no effect on environment, the dirty ones degrade it. The profits of monopolists are the incentives for scientists engaged in dirty and clean research in the North, while in the South scientists imitate the existing technology. The DTC framework allows the study of alternative technologies that deliver growth to an economy. In fact, the scientists move from a developed sector to an undeveloped one that does not generate externalities. The main characteristic is that we enter transitional dynamics that generates unbalanced growth. The consequence of this behaviour combined with the lack of an interest rate is the absence of constant growth rates. The only situation that allows constant growth rates and therefore a balanced path is the scenario where both technologies grow at the same rate because each variable is influenced by both technologies simultaneously.

Meanwhile Acemoglu et al (2014) uses a mix of carbon taxes on dirty intermediate inputs and subsidies on clean research using the same environmental constraint, my analysis involves carbon taxes on the use of dirty machines and dirty intermediate output and by an extension in the environmental law of accumulation into an environmental target in the North that represents the Net zero emission target. In fact, their work is not able to answer my research question due to the trade assumption on the intermediate production meanwhile I am doing it applying net zero emission targets on the technological leader creating the bases to analyse trade of inputs. In this chapter I describe the main drivers of the DTC framework applied to the environmental issues. In fact, the main goal is to analyse the framework of Acemoglu et al. (2014) and to lay

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[3] Article 10, European Parliament resolution of 14 March 2019 on climate change – a European strategic long-term vision for a prosperous, modern, competitive and climate neutral economy in accordance with the Paris Agreement.

[3] Acemoglu et al (2012) solved their model assuming an elasticity of substitution greater than one while Papageorgiou et al (2017) verified it through a panel of 26 countries finding that assumption consistent between clean and dirty energy inputs.

down the foundation of my next two chapters. Here through taxes on dirty inputs and output I show how the innovation is directed to new clean technologies decreasing the demand of dirty input to Net zero emission target and consequently changing the path of technology in the South, throughout imitation, avoiding an environmental disaster. The main point is that without trade a Net Zero Emission target is an unilateral policy strong enough to boost the demand of clean research to a point where even the South follows the same clean path thanks to knowledge spillovers. The North considers it as a terminal point but, on the other side of the coin, it could be seen as a starting point to increase clean production continuing international trade for the South due to the raised demand of clean goods. Also, it is possible that thanks to trade a net zero target transforms less developed countries in pollution havens because they are willing to obtain a comparative advantage in the production of dirty output[4]. To eliminate this ambiguity under free trade we need more research to establish the effect of a net zero emission under free trade.

## 2.2 Relation to literature

### 2.2.1 Main framework

In international trade the normal approach is given by proxies ,as the ecological footprint, to analyse differences between countries taking into account the trade agreements and consequently the openness of their markets using econometric analysis as Kolcava et al. (2019)[17] show. My study belongs to an increasing literature around the relationship between international trade and growth economics called North-South model. In fact, Bretschger and Suphaphiphat (2014) [6] studied a "two-region endogenous growth model with climate change affecting the countries' capital stocks negatively comparing two different climate policies" using a North-South model measuring the effect of unilateral policies as climate mitigation with financial aid discovering that the first method is preferable. Another example is the model of Amigues and Durmaz [4] which is a directed technical change model with only one country and pollution accumulation. The final product has a double utilization, consumption and pollution abatement. Also, there are two types of scientists, the one engaged in pollution abatement and the one in resource saving technology. It is plenty of studies around the effect of green policies on clean research as Naqvi and Stockhammer (2018), "a post-Keynesian ecological macromodel, which is stock-flow consistent, and incorporates directed technological change" in one country. They solve it with two different approaches: "a market-based Resource tax increase, and a centralized green policy, where public R&D budget is

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[4] This consequence depends by the definition of Net zero emissions. In fact the European target allows to buy polluting goods from another country. We should divide it in two different targets, a relative Net zero emission that deals only with the domestic production, and an absolute one, where it is incorporated the use of foreign goods. This means that the country which is following this path is accounting the use of foreign goods like they are produced in their country. Just to simplify, an absolute Net zero emission should reflect the Ecological Footprint of imports and exports. In this model I try to create an absolute Net zero emission target.

shifted towards Resource-saving technologies". And as a result, they "highlight that in the presence of labour market institutions, which give rise to hysteresis, and limited R&D budgets, a policy of continuous Resource tax growth is needed to induce Resource-saving technological change to achieve a greener economy" [19].

There is consensus in literature to prefer unilateral policy rather than multilateral. Hemous (2016) [13] has expressed a similar concept saying that "the economic literature on unilateral environmental policies has largely ignored the role played by innovation". An example of unilateral policy is the work of Chackraborty and Chatterjee (2017)[9] where they analyse the effect of a ban in Germany against polluted input imported from India explaining the indirect effect of environmental regulation on a foreign economy.

### 2.2.2 Other models

Di Maria and an der Werf (2008)[10] suggests that "current economic analyses of climate policy tend to over-estimate the degree of carbon leakage, as they abstract from the effects of induced technological change", Their results are "that the leakage rates reported in the literature may be too high, as these estimates neglect the effect of price changes on the incentives to innovate".
Di Maria and Smulders (2004)[9] is focused on the environmental problem of the pollution havens. The authors study "the role of endogenous technology and technology spillovers in explaining cross country differences in pollution and the pollution haven effect of international trade". Their results suggest that "without goods trade and in the absence of technology subsidies, the North imposes more stringent environmental regulation than the South. When opening up to trade, the South experiences a rise in prices for pollution-intensive goods and tends to raise pollution as in a standard trade model". Meanwhile, "Induced technical change, however, may reverse this pollution haven effect."

In 2021 Hemous and Olsen[14] show the links between environment and labour in a directed technical change model trying to understand when the direction of technological change is efficient. Gerlagh and Kuik (2014) "studies the effect of endogenous technical change and international technology spillovers on carbon leakage" developing "a simple mathematical model of carbon leakage and technological spillovers and perform numerical simulations with an adjusted CGE model to illustrate the potential importance of international technology spillovers".
To conclude and summarize, in this study I modify the equation which describes the quality of the environment to reflect the Net zero emission target using it as a unilateral policy. Then, I describe how it is possible to avoid an environmental disaster showing that the country which is not following the policy will adopt clean technology thanks to the imitation spillovers.

## 2.3 Model

This is a discrete-time, infinite-horizon model where the world is divided in two, a North and a South that exchange the intermediate output between them. The only difference between North and South is in technology. The North represents the technological leader while South is the follower who exploits northern knowledge spillovers. There are four sectors in each country. The first transforms the intermediate output in final output. The second who create intermediate goods from machines. The third is composed by a monopolist for each sector who maximizes the profit to pay researchers engaged in the fourth sector.
This model strictly follows the methodology of the paper of Acemoglu et al. (2012)[1] and its extension Acemoglu et al. (2014)[2]. In these two models there is no intertemporal choice, the final output is consumed by the households and it is not invested. Also, the machines fully depreciate at the end of each period therefore cannot be invested. These two conditions imply there is no interest rate. The main reason we are not seeing interest rates is linked to the absence of a return of investments. In the literature, the monopolists are owned by the households that receive a share at the end of every period from their profits which can be linked to the development of a new technology. In my main literature each scientist that can generate a new technology obtains the property rights which are used to start a monopolist firm becoming entrepreneur. The cost to produce a machine is a fraction of the final output that is parametrized to a finite constant value per unit of machines. It is noteworthy that differently from other models, Acemoglu et al (2012)[1] is not using any government and household budget constrain due to the absence of investments. In fact they are using only the market clearing condition because the final household is using everything that is produced in the economy without investment decisions. To summarize this point, given the simple households behaviour the market clearing condition coincides with the final household budget.

### 2.3.1 Preferences

In each region there is a representative consumer. The lifetime utility of the representative consumer is represented by the following utility function

$$\sum_{t=1}^{\infty}(\frac{1}{1+\rho})^t u_i(C_{it}, S_t) \tag{2.1}$$

here $i = N, S$ where $N$ refers to the north and $S$ to the south. In the function $C_{it}$ represents the consumption of the final goods respectively in north and south at time t, $S_t$ denotes the quality of the global common environment at time t, and $\rho \geq 0$ is the discount rate. As in Acemoglu et al (2012)[1] the environmental quality $S \in (0, +\bar{S})$ where $\bar{S}$ is the quality of the environment without any pollution, while 0 implies that an environmental disaster occurs.
The instantaneous utility function $u_i(C_{it}, S_t)$ is increasing in $C_{it}$ and $S_{it}$, differentiable and concave. The following Inada-type conditions are verified

$$\lim_{C\to 0}\frac{\vartheta u(C,S)}{\vartheta C}=\infty \text{ and } \lim_{S\to 0}\frac{\vartheta u(C,S)}{\vartheta S}=\infty \tag{2.2}$$

$$\lim_{S\to 0}u(C,S)=-\infty \tag{2.3}$$

which implies that when the environmental quality reaches 0 the consequences are serious for the representative household.

### 2.3.2 Production technology

In this model there is a final output $Y_t$ which is produced competitively using CES production function with two inputs $Y_{ct}$ and $Y_{dt}$ made by the intermediate firms. To be clear, $Y_{ct}$ and $Y_{dt}$ are semi finished products that need to be assembled by the final producer. The final good is used only for consumption and it is decided as numeraire with price normalized to 1. The final good producer maximization problem is represented as follow

$$\max \Pi_i = p_Y(Y_{ict}^{\frac{\epsilon-1}{\epsilon}}+Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}-p_{ct}Y_{ict}-p_{dt}Y_{idt} \tag{2.4}$$

where $p$ is the final price normalized to 1, and $p_{ct}$ and $p_{dt}$ are the prices of the intermediate inputs, that in this problem are also the costs of the final producers. Usually the elasticity of substitution is $\epsilon \in (0,+\infty)$ allowing intermediate inputs to be substitutes, when $\epsilon > 1$, or complementary, when $0 < \epsilon < 1$. But in this model we assume the inputs are gross substitutes in the final production function, which means $\epsilon > 1$ or, using the notation above, $\epsilon \in (1,+\infty)$[4].
The final output $Y_{ct}$ and $Y_{dt}$ are produced using a Cobb-Douglas production function using labour and a continuum of sector specific machines. Here the firms use specific intermediate machines as input in their production.

$$\max_{x_{ijt},L_{ijt}}\{p_{jt}L_{ijt}^{1-\alpha}\int_0^1 A_{jt}^{1-\alpha}x_{ijkt}^{\alpha}dk-W_{it}L_{ijt}-\int_0^1 p_{jkt}x_{ijkt}dk\} \tag{2.5}$$

where $\alpha \in (0,1)$ , $A_{jkt}$ is the quality of machine of type $k$ used in sector $j \in \{c,d\}$ at time $t$, $x_{ijkt}$ is the quantity of this machine in country $i \in \{1,2\}$, $L_{ijt}$ is the quantity of labour used in each sector $j \in \{c,d\}$. and $p_{jkt}$ is the cost to buy a machine of type $k$ from the monopolist producer. In fact machines $x_{ijkt}$ are supplied by monopolistic firms.

### 2.3.3 Monopolist production

Endogenous growth models use monopolists firms to endogenize the production of new technologies and also here these firms have the same objective. In both countries the

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[4] Papageorgiu et al. (2017)[15] show that the elasticity of substitution between clean and dirty energy inputs is greater than 1 implying they are gross substitutes.

clean and dirty machines are produced by monopolists who gains the profits. The maximization problem of the monopolist firm is given by

$$\max_{x_{ijkt}}(p_{jkt} - \psi)x_{ijkt}$$

where $p_{jkt}$ is the selling price of a machine $x_{ijkt}$ and $\psi$ the cost to produce one[5]. The scientists want to obtain patents to became new monopolists and these profits incentive new discoveries. In fact, researchers will decide in which sector, clean or dirty, work comparing the expected profits, which are given by the equation above times the probability to discover a successful innovation.

### 2.3.4 Innovation

Following Acemoglu et al (2014) [2], at the beginning of every period each researcher decides to work in a sector and then they are randomly allocated to a type of machine. When their innovation is successful they obtain a one time patent becoming entrepreneur and producer of that machine, and if there is not a successful innovation the property right is allocated to a random possible entrepreneur. "This assumption allows us to model in the simplest way a 'building on the shoulder of giants' externality, whereby innovation generates future social benefits that are not captured by innovator"[2].

The average productivity in sector $j$ evolves following the difference equation below

$$A_{Njt} = (1 + \gamma\eta_j s_{Njt})\, A_{jt-1} \tag{2.6}$$

where $\eta_j \in (0, 1)$ is the probability of a successful innovation, $(1 + \gamma)$ is the increased amount of machine's quality with $\gamma > 0$ and $s_{Njt}$ is the number of scientist in the North working in clean and dirty sectors with $j = c, d$. Here each new technology is used by monopolists to create better machines that will be sold to intermediate firms. The main idea is that the knowledge is a public good[6] and must be adapted by the South through imitation, but only the scientists can do it and they are collecting the profits of patents as a reward for their work.

The difference equation that represents imitation in the South has the following specification

$$A_{Sjt} = \kappa_j s_{Sjt} A_{Njt} + (1 - \kappa_j s_{Sjt}) A_{Sjt-1} \tag{2.7}$$

The parameter $\kappa_j \in (0, 1)$ expresses the probability to copy an innovation in the North. This equation says that if is not possible to copy an innovation in the North,

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[5] Following Acemoglu et al (2012) construction $p_{jkt}$ is equal to $\alpha$ and $\psi$, the cost to produce a machine, is equal to $\alpha^2$ meaning that the profits of the monopolists are determined by $\alpha(1-\alpha)x_{ijkt}$

[6] When I state public good I mean that scientists in the South do not have to pay to imitate northern technology. It allows to have free knowledge diffusion in the follower country. The use of a technology to produce a machine of type $k$ is protected by patents, but this property right do not cover the imitation in another part of the world and so, it is excludable only locally but not globally. To be precise, property rights work only in the country where they are issued and have no effect in other countries.

the property right will remain to an existing technology in the South.
The environmental quality decreases due to dirty production emissions generated in the North and South. The difference equation that represents the quality of the common environment is given by

$$S_{t+1} = -\xi \left( Y_{Ndt} + Y_{Sdt} \right) + (1 + \delta) S_t \tag{2.8}$$

where $\xi$ is the environmental degradation parameter while $\delta$ is the environmental regeneration one. $S_{t+1} \in \left(0, \bar{S}\right)$ where $S_{t+1} = \bar{S}$ means there is no pollution, while $S_{t+1} = 0$ means that an environmental disaster occurs. It is clear that the state of the environment strongly depends by the global dirty production levels. Also, it is important to notice that when an environmental disaster occurs, $S_t = 0$, there is no more possibility to recover the damage. It is important to explain that $\bar{S}$ is the initial and maximum level of the environmental quality.

## 2.4 Taxation

In this section I introduce a taxation in the North. It is decided to use carbon taxes and a Net zero emission target to avoid the possibility of an environmental disaster. In this stage the money collected with the taxation can be used to subsidy the clean research. I am ignoring subsidies to concentrate on the taxation effect on the demand of clean technologies because the revenue from the taxation could be allocated to increase the welfare of the households as well as other objectives. This model is innovative also in the use of taxes. Usually there is a combination of taxes on dirty inputs and subsidies on clean research, while here we are obtaining the same result using only taxes on all dirty inputs. This behaviour reflects the current trend where only governments are interested in avoiding further environmental degradation. In fact, as an example, of the oil companies only BP and Shell had little commitment to that target, firms will never reduce their emissions without public opinion and institutional obligations[7]. Nonetheless this kind of tools is effective only in an organized society with a low degree of corruption and this has been outlined by the report of Black et al. (2021) [8] where "poor governance increases the risk that targets are missed".

### 2.4.1 Green policies

I begin adding taxes to the dirty production functions then in the next section I will describe the equilibrium. The government add a tax $\tau_O \in (0, 1)$ to the dirty intermediate output used by the final producer such that now it solves the following maximizing problem with unilateral policies

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[7] "Companies with net zero commitments together represent sales of nearly \$14 trillion – 33% of total sales across the top 2,000 public companies – with wide annual variations between sectors. Countries with net zero targets together represent 61% of global emissions, 68% of global Gross Domestic Product (in PPP terms) and 52% of the global population" [8].

$$\max \Pi_i = p_Y(Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - p_{ct}Y_{ict} - (1+\tau_O)p_{dt}Y_{idt} \tag{2.9}$$

and a tax on the use of machine $\tau_I \in (0,1)$ to increase its cost such that the intermediate good producer solves the following maximization problem

$$\max_{x_{idt}, L_{idt}} \{p_{dt}L_{idt}^{1-\alpha}\int_0^1 A_{dt}^{1-\alpha}x_{idkt}^{\alpha}dk - W_{it}L_{idt} - (1+\tau_I)\int_0^1 p_{dkt}x_{idkt}dk\} \tag{2.10}$$

These taxes have the purpose to diminish the use of dirty products in the society helping a shift from carbon intensive economy to a greener one.
The government acknowledges that the environmental quality does not depends only by the pollution generated in her country, but also in others. So, the environmental quality in the North is decreasing by the dirty emissions generated globally[8] and locally. It is defined as

$$S_{Nt+1} = -[\zeta_G(Y_{Ndt} + Y_{Sdt}) + \zeta_L Y_{Ndt}] + (1+\delta)S_{Nt} \tag{2.11}$$

which means that environment in the North is affected by the global, $\zeta_G$, and local $\zeta_L$, pollution and it has a regenerative component given by $(1+\delta)$ [9]. This new equation will allow us to build the Net zero emission target in section 7.

## 2.5 Equilibrium

In this section I present the equilibrium and, since it is quite similar in North and South, I focus on the North, especially on the dirty sector showing the effects on the demand of new technologies. I use the same assumption of Acemoglu et al (2012) which expresses the clean technology enough backward compared to the dirty one because if the technologies start at the same level they would grow at the same rate

**Assumption 1** $\frac{A_{ic0}}{A_{id0}} < \min\{(1+\gamma\eta_c)^{\frac{-\varphi+1}{\varphi}}\frac{\eta_c}{\eta_d}^{\frac{1}{\varphi}}, (1+\gamma\eta_d)^{\frac{\varphi+1}{\varphi}}\frac{\eta_c}{\eta_d}^{\frac{1}{\varphi}}\}$

Following Acemoglu et al (2012) notation I introduce the following definition $\varphi \equiv (1-\alpha)(1-\epsilon)$.

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[8] This is a consequence of the fact that we suffer not only by the co2 generated in our country but also by the one produced in other parties of the world due to one of the properties of this gas, which allows it to be instantaneously in each part of the world after its emission.

[9] This last equation is consistent with equation (2.8) because it takes into account the components of pollution as a public good and private good. In fact, equation(2.8) can be rewritten as
$S_{t+1} = -[\zeta_G(Y_{Ndt}+Y_{Sdt}) + \zeta_L(Y_{Ndt}+Y_{Sdt})] + (1+\delta)(S_{Nt}+S_{St})$. This means that the environmental quality of tomorrow depends negatively from the effects of global pollution faced in North and South, from the local one produced in each country and it increases depending the regeneration rate.

### 2.5.1 Market clearing condition

In this model market clearing condition requires that in each country the amount of workers is equal to one

$$L_{Nct} + L_{Ndt} \leq 1 \text{ and } L_{Sct} + L_{Sdt} \leq 1 \tag{2.12}$$

also, I normalize the number of scientists $s$ to 1 denoting the total amount of researchers working on machines in sector $j \in c, d$ at time $t$ by $s_{ijt}$. The market clearing condition for scientists engaged in research are

$$s_{Nct} + s_{Ndt} \leq 1 \text{ and } s_{Sct} + s_{Sdt} \leq 1 \tag{2.13}$$

In each country the final consumer uses the final output minus its cost such that the market clearing condition for the final goods is given by

$$C_{it} = Y_{it} - \psi \left( \int_0^1 x_{ickt} dk + \int_0^1 x_{idkt} dk \right) \tag{2.14}$$

which means that the final consumers in North and South use the output of both countries. The equilibrium definition holds in each country

**Definition 1** *An equilibrium is given by sequences of wages* $(w_{it})$ *, prices for inputs* $(p_{jt})$*, prices for machines* $(p_{jkt})$*, demands for machines* $(x_{idkt})$*, demands for inputs* $(Y_{ijt})$*, labour demands* $(L_{ijt})$ *by input producers* $j \in c, d$*, research allocations* $(s_{idt}, s_{ict})$*, and quality of environment* $(S_t)$ *such that, in each period* $t$*: (1)* $(p_{jkt}, x_{ijkt})$ *maximizes profits by the producer of machine* $i$ *in sector* $j$*; (2)* $L_{ijt}$ *maximizes profits by producers of input* $j$*; (3)* $Y_{ijt}$ *maximizes the profits of final good producers; (4)* $(s_{idt}, s_{ict})$ *maximizes the expected profit of a researcher at date* $t$*; (5) the wage* $(w_{it})$ *and the prices* $p_{jt}$ *clear the labour and input markets respectively; and (6) the evolution of* $S_t$ *is given by (2.8).*

### 2.5.2 Characterization of the equilibrium

To characterize the equilibrium I start from the derivatives of $Y_t$ in equation (2.9) with respect to $Y_{ct}$ and $Y_{dt}$, which are

$$\frac{\partial Y_{it}}{\partial Y_{ict}} = \frac{\epsilon}{\epsilon - 1} [Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} (\frac{\epsilon - 1}{\epsilon}) (Y_{ict})^{\frac{-1}{\epsilon}} = p_{ct} \tag{2.15}$$

$$\frac{\partial Y_{it}}{\partial Y_{idt}} = \frac{\epsilon}{\epsilon - 1} [Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} (\frac{\epsilon - 1}{\epsilon}) (Y_{idt})^{\frac{-1}{\epsilon}} = p_{dt}(1 + \tau_O) \tag{2.16}$$

The result is that the marginal product of clean production is equal to its price and the marginal one of dirty output is equal to the new price post taxes which is the old price plus the tax on dirty output. The final producer has a greater marginal cost using dirty input compared to clean one and, since the price of the final output is normalized to one is less convenient to use it. Dividing line (2.15) with line (2.16) we have the relative prices with respect to intermediate output

$$\frac{p_{ct}}{p_{dt}(1+\tau_O)} = (\frac{Y_{ict}}{Y_{idt}})^{\frac{-1}{\epsilon}} \tag{2.17}$$

It means that the relative prices of cleans goods with respect to dirty ones post taxes are decreasing in their relative supply and that the elasticity of the relative price response is the inverse of the elasticity of substitution between the two inputs which is the same result as in Acemoglu et al. (2012).
The intermediate firm pay a tax $\tau_I$ on dirty inputs maximizing the following profits

The wages are obtained from the derivative of (2.9) with respect to the labour

$$w_{Nt} = (1-\alpha)p_d L_{Ndt}^{-\alpha} A_{Ndkt}^{1-\alpha} x_{Ndkt}^{\alpha} \tag{2.18}$$

and also, the derivative of (2.9) with respect to the machine leads to the demand of dirty machines[5].

$$x_{Ndkt} = (\frac{p_{dt}}{1+\tau_I})^{\frac{1}{1-\alpha}} A_{Ndkt} L_{Ndt} \tag{2.19}$$

It says that the demand of dirty machines is increasing by the price, the average productivity and the amount of labour, while it is decreasing by the tax on machines. To define the price of the intermediate dirty output we use the first derivative of (2.10) with respect of labour substituting the demand of machines inside it obtaining:

$$p_{dt} = (\frac{w_{Nt}}{A_{Ndkt}} \frac{1}{1-\alpha})^{1-\alpha}(1+\tau_I)^{\alpha} \tag{2.20}$$

The term $(1+\tau_I)^{\alpha}$ slightly increases the price of dirty intermediate output meaning that it becomes less competitive than clean one. The price of intermediate output strictly depends by the ratio between salary $w_{Nt}$ and the average productivity in each sector $A_{Njt}$.
The price of the intermediate clean output follows the same methodology giving

$$p_{ct} = (\frac{w_{Nt}}{A_{Nckt}} \frac{1}{1-\alpha})^{1-\alpha} \tag{2.21}$$

Now dividing the clean price with the dirty one we have the relative prices

$$\frac{p_{ct}}{p_{dt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}(\frac{1}{1+\tau_I})^{\alpha} \tag{2.22}$$

The relative prices $\frac{p_{ct}}{p_{dt}}$ strongly depend by the ratio of the present technologies, $\frac{A_{Nct}}{A_{Ndt}}$, and are decreasing by the tax $\tau_I$. This implies that the first effect of a tax on dirty machines is to make dirty intermediates more profitable, but on the other side less convenient than clean ones.

To establish the direction of technical change, and, consequently the profitability of new research, I use the demand of machines obtaining the profits of monopolist machine

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[5] Note that the demand for clean machines is the same without $1+\tau_I$.

producers which are given by $\pi = (p_{jkt} - \psi)x_{ijkt}$[6]. This construction is defined as follow by:

$$\pi_{Nd} = \alpha(1-\alpha)(\frac{p_d}{1+\tau_I})^{\frac{1}{1-\alpha}} A_{dkt} L_{Ndt} \tag{2.23}$$

and

$$\pi_{Nc} = \alpha(1-\alpha)(p_c)^{\frac{1}{1-\alpha}} A_{ckt} L_{Nct} \tag{2.24}$$

Using the last two equations and the probability to achieve a successful innovation we obtain the scientists expected profits

$$\Pi_{Nc} = \eta_c(1+\gamma)\alpha(1-\alpha)(p_c)^{\frac{1}{1-\alpha}} A_{ckt-1} L_{Nct} \tag{2.25}$$

and

$$\Pi_{Nd} = \eta_d(1+\gamma)\alpha(1-\alpha)(\frac{p_d}{1+\tau_I})^{\frac{1}{1-\alpha}} A_{dkt-1} L_{Ndt} \tag{2.26}$$

Then dividing (2.25) with (2.26) we obtain the final expected profits by sector of scientists engaged in research in the North which is represented by

$$\frac{\Pi_{Nc}}{\Pi_{Nd}} = \frac{\eta_c}{\eta_d}(\frac{p_c(1+\tau_I)}{p_d})^{\frac{1}{1-\alpha}} \frac{A_{Nckt-1}}{A_{Ndkt-1}} \frac{L_{Nct}}{L_{Ndt}} \tag{2.27}$$

which is the benefit from undertaking research in sector $c$ relative to sector $d$. When the ratio of the expected profits is greater than 1, $\frac{\Pi_{Nc}}{\Pi_{Nd}} > 1$, the clean sector is more profitable than the dirty one and researchers are willing to develop new clean technologies to became entrepreneur. Innovation incentives are represented by three forces: (1) the price effect $(\frac{p_c(1+\tau_I)}{p_d})^{\frac{1}{1-\alpha}}$ which pushes innovation to the sector with the highest price; (2) the direct productivity effect $\frac{A_{Nckt-1}}{A_{Sdkt-1}}$ which increases the demand of technology in the sector with larger productivity; (3) the market size effect $\frac{L_{Nct}}{L_{Ndt}}$ which increases the demand of innovation in the sector that employs more workers.
The tax on dirty input $\tau_I$ increases the price of clean machines by a factor $1+\tau_I$ meaning that a greater tax demands more clean research. [10]
The relative labour is represented by

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}(1+\tau_O)^{\epsilon} \tag{2.28}$$

Substituting the relative prices and labour we can express the direction of technological change in terms of technology and taxes obtaining

$$\frac{\Pi_{Nc}}{\Pi_{Nd}} = \frac{\eta_c}{\eta_d}(\frac{A_{ct}}{A_{dt}})^{-1-\varphi}\frac{A_{ct_{-1}}}{A_{dt_{-1}}}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}} \tag{2.29}$$

Here the price $p_{dt}$ is equal in both countries and directly affects the profit of researchers and due to the tax $\tau_O$ it increases the incentive to clean research with $\epsilon \geq 1$. From

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[6] To understand why it is equal to Acemoglu et al (2012) construction see Appendix at the end of document.

[10] ”When the two inputs are substitutes ($\epsilon \geq 1$), this is also the sector with the higher aggregate productivity”. Acemoglu et al (2012)[1]

equation(2.29) is evident that a tax on final good producers can directly affect the direction of technological change.
Expanding equation (2.29) with (2.6) we obtain the direction of technical change in terms of scientists and taxes

$$\frac{\Pi_{Nc}}{\Pi_{Nd}} = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}]^{-1-\varphi}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}} \tag{2.30}$$

The direction of technical change strongly depends by, the probability of success, the number of scientists, the level of the past aggregate productivity and taxes added to intermediate output and dirty machines.
The expected profits in the South are represented as

$$\frac{\Pi_{Sc}}{\Pi_{Sd}} = \frac{\kappa_c}{\kappa_d}(\frac{p_{ct}}{p_{dt}})^{\frac{1}{1-\alpha}}\frac{L_{Sct}A_{Nct}}{L_{Sdt}A_{Sdt}} \tag{2.31}$$

If the intermediate prices are international and decided by the North. It means that

$$\frac{\Pi_{Sc}}{\Pi_{Sd}} = \frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-1}(\frac{A_{ct}}{A_{dt}})^{-(\varphi-1)}(1+\tau_I)^{-\frac{\alpha\varphi}{1-\alpha}} \tag{2.32}$$

Now substituting the technological levels we can express the expected profits in the South in terms of scientists.

$$\frac{\Pi_{Sc}}{\Pi_{Sd}} = \frac{\kappa_c}{\kappa_d}(\frac{\kappa_c s_{Sct}A_{Nct}+(1-\kappa_c s_{Sct})A_{Sct-1}}{\kappa_d s_{Sdt}A_{Ndt}+(1-\kappa_d s_{Sdt})A_{Sdt-1}})^{-1}(\frac{1+\gamma\eta_c s_{Nct}}{1+\gamma\eta_d s_{Ndt}})^{-(\varphi-1)}(1+\tau_I)^{-\frac{\alpha\varphi}{1-\alpha}} \tag{2.33}$$

### 2.5.3 Equilibrium allocation of scientists

To characterize the scientists equilibrium I represent $\frac{\Pi_c}{\Pi_d} = f(s)$. Assuming that all scientists engaged in the clean sector are represented by s and the one in the dirty sector as $(1-s)$, with $s \in (0,1)$

$$f(s) = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s}{1+\gamma\eta_d(1-s)}]^{-1-\varphi}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}[1+\tau_O]^{\epsilon}\ (1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}} \tag{2.34}$$

Given the above informations, an innovation occurs in the clean sector when $\eta_c A_{ct_{-1}}^{-\varphi} \geq \eta_d(1+\gamma\eta_c)^{1+\varphi}(A_{dt_{-1}})^{-\varphi}(\frac{1}{1+\tau_O})^{\epsilon}(1+\tau_I)^{-\frac{1-\alpha-\alpha\varphi}{1-\alpha}}$, in the dirty sector only when $\eta_c(1+\gamma\eta_d)^{1+\varphi}(A_{ct_{-1}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}} \leq \eta_d(A_{dt_{-1}})^{-\varphi}$ and in both sectors when $\eta_c A_{ct_{-1}}^{-\varphi}(1+\gamma\eta_d S_{dt})^{1+\varphi} = \eta_d(A_{dt_{-1}})^{-\varphi}(1+\gamma\eta_c S_{ct})^{1+\varphi}(\frac{1}{1+\tau_O})^{\epsilon}(1+\tau_I)^{-\frac{1-\alpha-\alpha\varphi}{1-\alpha}}$.
To understand the equilibrium amount of scientists engaged in clean research I solve equation (2.34) with $f(s)=1$ obtaining

$$s_{Nct} = \frac{1+\gamma\eta_d - [\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_i)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}}{\gamma\{\eta_d+\eta_c[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_i)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}\}} \tag{2.35}$$

and consequently the number of scientists engaged in dirty research is given by $1-s$. (To better understand it look appendix).
The number of scientists engaged in research strongly depends by $\varphi$ which should be less than 2, meaning that $\epsilon \geq 2,857$ otherwise the equality is not satisfied and the number of scientists is greater than 1. Greater is the amount of taxes and more it incentives clean researchers.The number of clean scientist depends positively by the increase of quality of the new dirty machine. Meanwhile in the South the number of clean scientists is given by

$$s_{Sct} = \frac{\frac{\kappa_c}{\kappa_d}(\frac{1+\gamma\eta_c s_{Nct}}{1+\gamma\eta_d(1-s_{Nct})}\frac{A_{Nct-1}}{A_{Ndt-1}})^{-(\varphi-1)}(1+\tau_I)^{-\frac{\alpha\varphi}{1-\alpha}}(\kappa_d(A_{Ndt}-A_{Sdt-1})+A_{Sdt-1})-A_{Sct-1}}{[\kappa_c(A_{Nct}-A_{Sct-1})+\kappa_d(A_{Ndt}-A_{Sdt-1})\frac{\kappa_c}{\kappa_d}(\frac{1+\gamma\eta_c s_{Nct}}{1+\gamma\eta_d(1-s_{Nct})}\frac{A_{Nct-1}}{A_{Ndt-1}})^{-(\varphi-1)}(1+\tau_I)^{-\frac{\alpha\varphi}{1-\alpha}}]} \tag{2.36}$$

### 2.5.4 Steady state

In this subsection I add the steady state for the northern region. This model accepts steady state solutions when both technologies grow at the same rate, or when the scientists engage research in the two sectors allowing the relative technology to be constant over time. Since in steady state the technologies grow at the same level each period $(\frac{A_{ct}}{A_{dt}}) = \frac{A_{ct_{-1}}}{A_{dt_{-1}}} = \frac{A^*_{ct}}{A^*_{dt}}$ and the expected profits $\frac{\Pi_{Nc}}{\Pi_{Nd}} = 1$ it implies that eq(2.29) becomes

$$(\frac{A^*_c}{A^*_d}) = (\frac{\eta_c}{\eta_d})^{\frac{1}{\varphi}}(1+\tau_O)^{\frac{\epsilon}{\varphi}}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{\varphi(1-\alpha)}} \tag{2.37}$$

Until now, I show the technological steady state of the North. Here I develop the discussion using the equation above to express all relative variables in steady state as follows

$$(\frac{p^*_c}{p^*_d}) = (\frac{\eta_c}{\eta_d})^{\frac{-(1-\alpha)}{\varphi}}(1+\tau_O)^{\frac{-\epsilon(1-\alpha)}{\varphi}}(1+\tau_I)^{\frac{\alpha-1}{\varphi}} \tag{2.38}$$

$$(\frac{L^*_c}{L^*_d}) = (\frac{\eta_c}{\eta_d})^{-1}(1+\tau_I)^{-1} \tag{2.39}$$

$$(\frac{Y^*_c}{Y^*_d}) = (\frac{\eta_c}{\eta_d})^{\frac{\epsilon(1-\alpha)}{\varphi}}(1+\tau_O)^{\frac{\epsilon^2(1-\alpha)+\epsilon\varphi}{\varphi}}(1+\tau_I)^{\frac{-\epsilon(\alpha-1)}{\varphi}} \tag{2.40}$$

The scientists engaged in clean research in steady state are represented as follow

$$s^* = \frac{\eta_d(\frac{\eta_c}{\eta_d})^{\frac{1}{\varphi}}(1+\tau_O)^{\frac{\epsilon}{\varphi}}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{\varphi(1-\alpha)}}}{\eta_c+\eta_d[(\frac{\eta_c}{\eta_d})^{\frac{1}{\varphi}}(1+\tau_O)^{\frac{\epsilon}{\varphi}}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{\varphi(1-\alpha)}}]} \tag{2.41}$$

Nonetheless, in the long run this solution does not avoid an environmental disaster unless we allow the use of abatement technology that reduces the effect of pollution on the environment. The reason is given by the steady state assumption that all the variables are growing at the same time and therefore the dirty technology is growing at the same rate of the clean one. This means that the environmental damage is still increasing. To simplify, without an abatement technology it is impossible to avoid an environmental disaster when both technologies are growing at the same rate.

## 2.6 Net zero emissions

Net zero emissions or carbon neutrality is the last policy I introduce in this study and the most innovative. The best definition is given by IPCC[8]

**Definition 2** *Net zero emissions are achieved when anthropogenic emissions of greenhouse gases to the atmosphere are balanced by anthropogenic removals over a specified period. Where multiple greenhouse gases are involved, the quantification of net zero emissions depends on the climate metric chosen to compare emissions of different gases (such as global warming potential, global temperature change potential, and others, as well as the chosen time horizon).*

Net zero emission targets has been enshrined by law in many different countries as Sweden, Uk, European Union, the State of New York. It has temporal goals, usually initial ones by 2030 and to reach carbon neutrality around 2050 in order to keep an increase of the temperature at a maximum of 1.5 Celsius degree[11]. There are two ways to obtain net zero emissions: carbon removals[12] and carbon reduction[13]. At this stage I focus only on carbon reduction because it can affect the decisions of other countries, while carbon removals need to be built as an abatement technology that can decrease the amount of pollution directly from the environment. However it has been explored especially in the context of closed economy and its effect on open economy is not clear. Here I modify the law of environmental quality accumulation assuming absolute Net zero emissions because it embodies not only the co2 of the North, but also the one of the South.

To model this tool the first step consists in building a terminal time condition where the target must be satisfied. Then I decide that this policy is the situation where the environment should not change during time, or, in a simple way, that the environmental quality of today should be equal to the one of tomorrow. This implies that to achieve carbon neutrality northern government decides it is necessary that at a given time $T$ $S_{NT+1} - S_{NT} = 0$[14] which leads to

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[11] "In scenarios limiting warming to 1.5 degrees C, carbon dioxide (CO2) needs to reach net-zero between 2044 and 2052, and total GHG emissions must reach net-zero between 2063 and 2068. Reaching net zero earlier in the range avoids a risk of temporarily overshooting 1.5 degrees C. Reaching the top of the range almost guarantees surpassing 1.5 degrees C for some time before it eventually drops down"[18].

[12] They are the tools used to remove the CO2 from the atmosphere. As examples, to plant new trees or to capture and deposit greenhouse gases in reservoirs

[13] It means to diminish the emissions of pollutants compared to the past time

[14] Note that this is not a steady state but an engineered condition.

$$Y_{NdT} = \frac{\delta S_{NT} - \zeta_G Y_{SdT}}{\zeta_G + \zeta_L} \tag{2.42}$$

Equation (2.42) describes net zero emission production of dirty input in the North fixing its optimum level in relation to environmental quality and dirty output of the South. This policy should be used as a last tool to switch the regime of a carbon economy because we do not know its effect on the short-run. Nonetheless, it could be necessary to avoid an environmental disaster because at least it gives policy makers more time to develop new technologies.

## 2.7 Conclusions

In this chapter I built an endogenous model to introduce the effects of carbon taxes and net zero emissions that will be explained in details in the successive chapters. The long run effects of the taxes depends on the elasticity of substitution. Also, a correct taxation should activate an immediate regime switch, going from an intensive carbon economy to a clean one. The new technologies will be adopted by the rest of the world in short time. The taxes and new technologies will allow the transition to net zero emission to be ”less painful” for the representative household.

This work continues an existing framework which addresses the dilemma between growth and environment hoping to give an exhaustive point of view for policy makers and researchers. I recognize that this kind of model is quite far from the real world and it needs some extensions. The first one is to allow international trade of machines, substituting monopolies with oligopolies, which will reduce dead weight loss and, consequently, positive research spillovers. The second one is to build the model such that there is international trade of all goods, from machines to intermediate and final goods to make the model more consistent with our economies, otherwise we could underestimate the effect of production to environmental quality. The last extension is the use of abatement technology that decreases the emissions from dirty production, which could reduce the need of taxes and other policy tools. This abatement technology could be financed by the taxes collected by the government or through the use of capital.

## 2.8 Appendix

### 2.8.1 Normalization of the monopoly prices

To normalize the monopoly price $p_{dkt}$ I start from the derivative in terms of machines of equation (2.10)

$$p_{dkt} = (L_{idt}A_{dt})^{1-\alpha}\ x^{\alpha-1}\frac{\alpha\ p_{dt}}{1+\tau_I} \tag{2.43}$$

Applying it in the monopolist profit function $(p_{dkt} - \psi)\ x_{idkt}$ we obtain

$$\pi_{dt}^m = [x_{idkt}^{\alpha-1}\ \alpha\ (L_{idt}A_{dt})^{1-\alpha}\ \frac{p_{dt}}{1+\tau_I} - \psi]\ x_{idkt}$$

$$\frac{\partial \pi_{dt}^m}{\partial x_{idkt}} = \alpha^2\ x_{idkt}^{\alpha-1}\ (L_{idt}A_{dt})^{1-\alpha}\ \frac{p_{dt}}{1+\tau_I} - \psi = 0$$

$$x_{idkt}^{\alpha-1} = \frac{\psi\ (1+\tau_I)(L_{idt}A_{dt})^{\alpha-1}}{p_{dt}\ \alpha^2}$$

$$x_{idkt} = [\frac{\psi\ (1+\tau_I)}{p_{dt}\ \alpha^2}]^{\frac{1}{\alpha-1}}\ L_{idt}A_{dt}$$

then, using this last equation in the equation that describes the dirty monopolist profits we obtain the relationship between $p_{dkt}$ and $\psi$

$$p_{dkt} = (L_{idt}A_{dt})^{1-\alpha}\ [[\frac{\psi\ (1+\tau_I)}{p_{dt}\ \alpha^2}]^{\frac{1}{\alpha-1}}\ L_{idt}A_{dt}]^{\alpha-1}\frac{\alpha\ p_{dt}}{1+\tau_I}$$

$$p_{dkt} = \frac{\psi\ (1+\tau_I)}{p_{dt}\ \alpha^2}\ \frac{\alpha\ p_{dt}}{1+\tau_I}$$

which leads to

$$p_{dkt} = \frac{\psi}{\alpha} \tag{2.44}$$

Now I normalize as in Acemoglu (2012) $\psi = \alpha^2$ and as a result, $p_{dkt}$ became equal to $\alpha$.

### 2.8.2 Relative labour

In order to obtain the relative labour in te North I start from the derivatives of the final producer.

$$\frac{\epsilon}{\epsilon-1}[Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}}(\frac{\epsilon-1}{\epsilon})(Y_{Ndt})^{\frac{-1}{\epsilon}} = p_{dt} + \tau_O \tag{2.45}$$

Substituting $Y_{idt}$ with its optimum production and imposing the assumption $\varphi \equiv (1-\alpha)(1-\epsilon)$ we have

$$p_{dt}(1+\tau_O) = [(\frac{p_{dt}}{1+\tau_I})^{\frac{\alpha}{1-\alpha}}A_{Ndt}L_{Ndt}]^{-\frac{1-\alpha}{1-\alpha-\varphi}}[Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}}$$

I do not solve $[Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}}$ because when I divide the clean labour with the dirty one it will disappear, so I will call it $D$.

$$p_{dt}(1+\tau_O) = (\frac{p_{dt}}{1+\tau_I})^{-\frac{\alpha}{1-\alpha-\varphi}}(A_{Ndt}L_{Ndt})^{-\frac{1-\alpha}{1-\alpha-\varphi}}D$$

$$\frac{p_{dt}(1+\tau_O)}{p_{dt}} = (A_{Ndt}L_{Ndt})^{-\frac{1-\alpha}{1-\alpha-\varphi}}\frac{(p_{dt})^{-\frac{\alpha}{1-\alpha-\varphi}}}{p_{dt}}(\frac{1}{1+\tau_I})^{-\frac{\alpha}{1-\alpha-\varphi}}D$$

$$(1+\tau_O) = (A_{Ndt}L_{Ndt})^{-\frac{1-\alpha}{1-\alpha-\varphi}}p_{dt}^{\frac{\varphi-1}{1-\alpha-\varphi}}(\frac{1}{1+\tau_I})^{-\frac{\alpha}{1-\alpha-\varphi}}D$$

and finally we obtain the optimal amount of workers in the dirt sector.

$$L_{Ndt} = A_{Ndt}^{-1}p_{dt}^{\frac{\varphi-1}{1-\alpha}}(\frac{1}{1+\tau_I})^{-\frac{\alpha}{1-\alpha}}(1+\tau_O)^{\frac{1-\alpha-\varphi}{1-\alpha}}(D)^{\frac{1-\alpha-\varphi}{1-\alpha}}$$

Now, the amount of workers employed in the clean sector is

$$L_{Nct} = A_{Nct}^{-1}p_{ct}^{\frac{\varphi-1}{1-\alpha}}(D)^{\frac{1-\alpha-\varphi}{1-\alpha}}$$

Dividing the last two equations we obtain

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-1}(\frac{p_{ct}}{p_{dt}})^{\frac{\varphi-1}{1-\alpha}}(1+\tau_I)^{-\frac{\alpha}{1-\alpha}}(1+\tau_O)^{\epsilon}$$

Which is the relative supply of labour. To obtain it in terms of technology and taxes we apply equation (2.22) obtaining

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}(1+\tau_O)^{\epsilon} \tag{2.46}$$

Here $(\frac{\tau_O+p_{dt}}{p_{dt}})$ is raised to the power of $\epsilon$ because it is equal $\frac{1-\alpha-\varphi}{1-\alpha}$. To verify it I solved $\varphi \equiv (1-\alpha)(1-\epsilon)$.

### 2.8.3 Number of scientists

To find the number of scientists I use equation (2.34) and I substitute $f(s)$ with 1

$$f(s) = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s}{1+\gamma\eta_d(1-s)}]^{-1-\varphi}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}\ (1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}$$

$$\frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s}{1+\gamma\eta_d(1-s)}]^{-1-\varphi}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}\ (1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}} = 1$$

$$\frac{\eta_c}{\eta_d}(1+\gamma\eta_c s)^{-1-\varphi}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}\ (1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}} = [1+\gamma\eta_d(1-s)]^{-1-\varphi}$$

$$1+\gamma\eta_d(1-s) = (1+\gamma\eta_c s)[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}\ (1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}$$

$$1+\gamma\eta_d-\gamma\eta_d s=$$

$$=[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}+\gamma\eta_c s[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}$$

$$\gamma s(\eta_d+\eta_c[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}})=$$

$$=1+\gamma\eta_d-[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}$$

$$\gamma s=\frac{1+\gamma\eta_d-[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_i)^{\frac{1-\alpha-alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}}{\eta_d+\eta_c[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_i)^{\frac{1-\alpha-alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}}$$

and finally we obtain the number of scientists employed in clean research.

$$s=\frac{1+\gamma\eta_d-[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_i)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}}{\gamma\{\eta_d+\eta_c[\frac{\eta_c}{\eta_d}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon}(1+\tau_i)^{\frac{1-\alpha-\alpha\varphi}{1-\alpha}}]^{\frac{1}{-1-\varphi}}\}} \tag{2.47}$$

By construction, to find the amount of scientists engaged in dirty research we solve $1-s$.

# Bibliography

[1] Acemoglu D, 2009, *Introduction to Modern Economic Growth,* Princeton: Princeton University Press

[2] Acemoglu, D., Aghion, P., Bursztyn, L., Hémous, D. 2012a *'The Environment and Directed Technical Change',* American Economic Review, 102(1), 131–66.

[3] Acemoglu D.,Aghion P., Hémous D, 2014 *The environment and directed technical change in a North–South model,* Oxford Review of Economic Policy, Oxford University Press, Volume 30, Number 3 pp. 513–530

[4] Amigues J., Durmaz T., 2019 *A Two-Sector Model of Economic Growth with Endogenous Technical Change and Pollution Abatement,* Environmental Modelling & Assessment, Volume 24, pp 703-725

[5] Bretschger L., Suphaphiphat N., 2014 *Effective climate policies in a dynamic North–South model,* European Economic Review, Volume 69, pp 59-77

[6] Bretschger L, Valente S, 2012 *Endogenous growth, asymmetric trade and resource dependence,* Journal of Environmental Economics and Management, Volume 64, pp 301-311

[7] Black, R., Cullen, K., Fay, B., Hale, T., Lang, J., Mahmood, S., Smith, S.M., 2021 *Taking Stock: A global assessment of net zero targets,* Energy & Climate Intelligence Unit and Oxford Net Zero

[8] Chackraborty P., Chatterjee C., 2017, *Does environmental regulation indirectly induce upstream innovation? New evidence from India* Research policy, Volume 46, pp 939-955

[9] *Trade Pessimists vs Technology Optimists: Induced Technical Change and Pollution Havens,* Advances in Economic Analysis and Policy, Volume 4, Issue 2, 2004.

[10] *Trade pessimists vs technology optimists: Induced technical change and pollution havens,* Advances in Economic Analysis and Policy, Volume 4, Issue 2, 2004.

[11] Grossman G. M., Helpman E., Kircher P., 2017, *Matching, Sorting, and the Distributional Effects of International Trade* Journal of Political Economy, Volume 125, pp 224-264

[12] Hemous D., 2016, *The dynamic impact of unilateral environmental policies,* Journal of International Economics, Volume 103, pp 80-95

[13] Heomus D., Olsen M., 2021, *Directed Technical Change in Labor and Environmental Economics* Annual Review of Economics, Volume 13, pp 571–597

[14] IPCC, 2018: Annex I: Glossary [Matthews, J.B.R. (ed.)]. In: *Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*[Masson-Delmotte, V., P. Zhai, H.-O. Pörtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I. Gomis, E. Lonnoy, T. Maycock, M. Tignor, and T. Waterfield (eds.)]

[15] Khan A., 2015. *Unilateral Environmental Policy in a Two-Country World with Directed Technological Progress.* Thesis(M.Sc). Northwestern University

[16] Kolcava D., Nguyen Q., Bernauer 2019 *Does trade liberalization lead to environmental burden shifting in the global economy,* Ecological Economic, Volume 163, pp 98-112

[17] Kelly Levin K., Taryn Fransen T., Clea Schumer C, Davis C., 2017. *What Does "Net-Zero Emissions" Mean? 8 Common Questions, Answered.* Washington, DC. World Resource Institute. Available from: https://www.wri.org/insights/net-zero-ghg-emissions-questions-answered.

[18] Naqvi A, Stockhammer E., 2018, *Directed Technological Change in a Post-Keynesian Ecological Macromodel,* Ecological Economics, Volume 154, pp 168-188

[19] Naito T., 2017, *Growth and welfare effects of unilateral trade liberalization with heterogeneous firms and asymmetric countries,* Journal of International Economics, Volume 109, pp 167-173

[20] Papageorgiou C., Saam M., Schulte P., 2017, *Substitution between clean and dirty energy : A macroeconomic perspective,* The Review of Economics and Statistics, Volume 99(2), pp 281–290

[21] Peretto P., Valente S., 2011, *Resources, innovation and growth in the global economy,* Journal of Monetary Economics, Volume 58, pp 387-399

[22] Proops J., 2004 *The growth and distributional consequences of international trade in natural resources and capital goods: a neo-Austrian analysis,* Ecological economics, Volume 48, pp 83-91

[23] Zhang J., 2020 *International production fragmentation, trade in intermediate goods and environment,* Volume 87, pp 1-7

# Chapter 3

# Directed technical change local. trans-boundary pollution and net zero emissions

## 3.1 Introduction

In this chapter I extend the article of Acemoglu et al 2014[2] modifying the environmental constraints obtaining a framework more comparable with different kind of environmental damage introducing the concept of trans-boundary pollution to better address an unilateral policy finding more detailed policy tools such as compensations to the other country. Then I add another variable represented by the abatement technology that decreases the effective emissions generated by the pollutant production. Also, I add a new type of environmental target that represents net zero emission target[1]. In this model there is no trade, the channels through which the countries affect each other are the technological spillovers from North to South and the interaction between the externalities via global pollution affecting both countries' environmental quality. In this study I investigate the effect of different taxes on the final firms, finding different consumption growth rate and labour force compositions. Then I show what are the implications to apply a net zero emissions target as an environmental target showing mechanisms that affects the maximum amount of production admitted in the economy. In the end, through a simulation I try to quantify the shock generated by this switch in the dirty production. These implications are represented using a North-South Directed technical change model with different environmental constraints as transnational pollution, abatement technology and net zero emission target. Each one of the constraints mentioned above brings different results as a compensation for the pollution generated in the North that affects the South, the introduction of a quota on the use of dirty machines and the cap on the optimal dirty production.
Nowadays the developed countries use different systems to regulate the pollution pro-

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[1] Net zero emissions refers to achieving an overall balance between greenhouse gas emissions produced and greenhouse gas emissions taken out of the atmosphere.

duced by the firms and other nations such as trade emission system, price cap and carbon taxes. Some of the clean projects, as an example REDD+[2] aim to buy carbon credit through regeneration projects in other parts of the world, which means to obtain a credit compensating the emissions that affects other countries. A good way to improve this mechanism could be through subsidizing the imitation of technology in the South, which would allow the country to reduce its future emissions. In my model I represent these mechanism through compensations that are justified by a Social planner that operates only in the North.

The COP26 in Glasgow renewed the commitment to reach the objective of net zero emissions by 2050 meanwhile the United Kingdom and the EU are the international leaders and the most committed countries to reach this goal. In June 2019 the UK government signed its 2050 net zero commitment into law and as an example, it decided to satisfy this requirement banning the sale of new petrol and diesel cars by 2030 [8]. In March 2019 the EU recognized that to have zero emissions it should be more energy independent cutting the imports from 55% to 20% by 2050. Another drastic change will affect the transport sector going from oil to electric fuel[3]. The main areas that are outlined to reach Net zero are transport and energy since both can continue operating with clean energy. The model we will represent is consistent with the energy market as explained by Papageorgiu et al. (2017) [15]. In fact our society is so energy-dependent that living without electricity would imply a strong decrease in our utility due to a lower level of consumption and as a consequence it would mean to reach the levels of a pre-industrial economy. Electricity is seen as the engine of the second industrial revolution which it can be made by dirty inputs, as coal, oil or gas, now it can also be obtained by renewable resources and nuclear energy. It is important to explain that 40% of the global emissions of CO2 come from the power sector, 23% from transport, 23% from industry, 10% from buildings and 5% from other sources. This means that the electrical sector is the most important when we discuss about CO2 generation. Since the policy makers started discussing about green economy and sustainable development we are seeing increasing subsidies in the clean sector creating a market for new clean innovation. Thanks to new technologies we can use electric cars that can substitute the oil one diminishing its 23% emissions contribution [4]. It is clear from the policies I cited at the beginning that the countries who agreed to reach the net zero want to do it when their economies are ready to sustain the shock from the technical change. In fact their preparation involved different steps to reach it first decreasing the damage dealt to the environment over time and second to decrease the shock that policy would introduce.
Between policy makers it is clear that the best way to decrease the environmental degradation is through global coordination. This is unlikely for different reasons, including free riding, it is difficult to coordinate different countries with the same policy, therefore I use a policy that affects directly only one country which is called unilateral.

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[2] REDD+ stands for "Reducing Emissions from Deforestation and forest Degradation"

[3] European Parliament resolution of 14 March 2019 on climate change

[4] https://www.iea.org/data-and-statistics/charts/global-energy-related-co2-emissions-by-sector

The importance of this policy is well described in Hemous (2016) stating: "unilateral environmental policies should aim at developing clean technologies, which have the potential to reduce emissions in the North, but also in the South either through technology diffusion or by slowing down the move of polluting industries there. These policies should be thought of as transitory until a satisfactory global agreement is reached" [13]. The main point is that the net zero emission is a unilateral policy that can improve the clean sector in the south implicitly.

## 3.2 Literature

In the last decade there is a growing interest in endogenous models with environmental externalities. For example, Hassler et al (2021) [12] is a Directed technical change model with input saving technology where the authors try to understand its relationship with scarce resources. This model takes inspiration from the model of Acemoglu et al (2012) [1] where a directed technical change one country model needs to avoid an environmental disaster. In their fundamental work the authors found that an environmental disaster is inevitable in a decentralized economy. They solve this problem with a combination of carbon tax and subsidy on research. From this model we learn the main points of this problem. "First, there is path dependence in the development of clean versus dirty technologies. Second, taking into account the endogeneity of innovation calls for earlier action. Third, in addition to Pigovian carbon taxation, the optimal policy includes research subsidies specifically devoted to clean innovation".
The extension of that model is with two countries, a North-South model. In their model Acemoglu et al (2014)[2] used a different evolution of technology in the South allowing it to grow through imitation. They discovered the conditions that allow to solve the model in two different ways, with global and local coordination. It depends by the elasticity of substitution that influences the effect of the tax on the economy. They outlined also when the South enters a pollution haven specializing in the dirty sector. The idea of the imitation process and unilateral policy is a key point in my model and it is explored with different environmental constraints. There is a larger interest around unilateral policy, which is the scenario where the optimal policy is applied only to one country, usually in the North, seeing how this decision influences the other country. The main model to describe the unilateral policy is Hemous (2016) [13] where the author "builds a two-country, two-sector (polluting, nonpolluting) trade model with directed technical change, examining whether unilateral environmental policies can ensure sustainable growth".
Another research that discussed the importance of trade agreement is Harstad (2016) [11] where the author analyses" a framework in which countries over time pollute and invest in green technologies. The study highlights the effect of a climate treaty on the investment in new clean technologies showing the agreement can increase the demand of new environmental friendly technologies.
Amigues and Durmaz (2019) [4] built a directed technical change model with only one country and pollution accumulation. The final product has a double utilization,

consumption and pollution abatement. Also,there are two types of scientists, the one engaged in pollution abatement and the one in resource saving technology. The main objective is to find a way to reach full abatement called FA, and from the study emerges an Environmental Kuznet curve. Once full abatement is reached the economy completely switch to clean production and, as usual, "researchers are optimally allocated to the sector with sufficiently high innovative capacity." In 2021 Hemous and Olsen show the links between environment and labour in a directed technical change model trying to understand when the direction of technological change is efficient. I try to explain how the two countries can reach a sustainable growth using unilateral policy including the presence of transboundary pollution, abatement technology and using the net zero emissions. The results in all the scenarios will be similar. In fact the Social planner always requires a policy that influences the demand of labour moving the workers to the sector non pollutant and allowing the economy to reallocate the scientists.
In the next section I show the general framework of the model, from the utility function to the environmental constraint used by Acemoglu. Then, in section 4 I show the Laissez-faire equilibrium with all the variable expressed in terms of technology. The 5th section explains all the environmental constraints that will be used in the 6th section. After that part I show the numerical simulation of the optimal policies ending with the result and the conclusion.

## 3.3 The model

This section outlines the main characteristics of the model. The world is divided in two economies, North and South, in an infinite discrete time horizon. The only differences between the North and the South are given by the environmental policy and by technologies. The North engages a unilateral policy to avoid an environmental disaster in the North which will affect the South due to the imitation process. Also, the North represents the technological leader meanwhile the South is the follower who exploits northern knowledge spillovers. The difference in initial technological levels allows to represent a wealthier country that has enjoyed the gains of industrial revolution before the other poorer nation justifying the unilateral policy. In this model there is no capital, instead it is substituted by machines as an input that fully depreciates at the end of each period. The machines combined with technology and the labour force generate intermediate output which is assembled by the final producer.

### 3.3.1 Preferences

In the model of Acemoglu et al (2014) [2] the utility function of the representative household is shown as follow

$$\sum_{t=1}^{\infty}(\frac{1}{1+\rho})^t u_i(C_{it}, S_t) \tag{3.1}$$

where $i = N, S$ with $N$ referring to the north and $S$ to the south. $C_{it}$ represents the consumption level of the final goods respectively in north and south at time t, $S_t$ denotes the quality of the global common environment at time t, and $\rho \geq 0$ is the discount rate. The reason behind this utility function derives from a common environment that affects everybody representing the environmental damage generated by the emission of co2.

In this model the utility function will differ in the environmental component. In fact the utility function will be

$$\sum_{t=1}^{\infty}(\frac{1}{1+\rho})^t u_i(C_{it}, S_{it}) \tag{3.2}$$

in most of the scenarios. This allow us to have different utilities depending in which region the representative household lives which is a consequence of having two different environments. Under this new utility function, $S_{it}$ denotes the quality of the local environment at time $t$ in country $i$. The main idea behind this utility function is that environmental quality is affected in different ways. As in Acemoglu et al (2012)[1] the environmental quality is $S \in (0, \bar{S})$ where $\bar{S}$ is the quality of the environment without any pollution, while 0 implies that an environmental disaster occurs.

The instantaneous utility function $u_i(C_{it}, S_t)$ is increasing in $C_{it}$ and $S_{it}$, differentiable and concave. The following Inada-type conditions are verified in both countries

$$\lim_{C\to 0} \frac{\partial u(C,S)}{\partial C} = \infty \text{ and } \lim_{S\to 0} \frac{\partial u(C,S)}{\partial S} = \infty \tag{3.3}$$

$$\lim_{S\to 0} u(C,S) = -\infty \tag{3.4}$$

which implies that when the environmental quality reaches 0 the consequences are serious for the representative household. In fact once the environmental quality reaches zero it is impossible to enjoy the final good and the utility goes to zero. The economic intuition behind this concept is that it does not matter how many goods you can consume in a world that cannot sustain life.

### 3.3.2 Production technology

The production side of the economy is composed by a final producer, an intermediate and a monopolist producer. The final producer operates competitively through a CES production function[5] in both countries maximizing her profits as follows.

$$\max \Pi_i = p_{Yi}(Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - p_{ict}Y_{ict} - p_{idt}Y_{idt} \tag{3.5}$$

The final good is sold at the price $p_{Yi}$, which is normalized to 1, the elasticity of substitution is represented by $\epsilon \in (1, +\infty)$ [6]. Meanwhile the intermediate inputs $Y_{ijt}$,

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[5] The CES production function is fundamental to represent a firm that assembles two intermediate goods to create a final good.

[6] In this model the elasticity of substitution is supposed to be greater than 1 implying perfect substitution between the intermediate input following Acemoglu et al (2012) [1] framework. With

with $i = N, S$ indicating the country, and $j = c, d$ representing the sector, $c$ clean and $d$ dirty, are produced competitively. The final producer purchases the intermediate output, which is her input, at the price $p_{ijt}$. Then, the intermediate producer sells the intermediate output at the price $p_{ijt}$ using the labour $L_{ijt}$, a continuum[7] of technologies $A_{ijkt}$ and the machines $x_{ijkt}$ sustaining the salaries $W_{it}$ and the cost of machines $p_{ijkt}$.

$$\max_{x_{ijt}, L_{ijt}} \{p_{ijt} L_{ijt}^{1-\alpha} \int_0^1 A_{ijkt}^{1-\alpha} x_{ijkt}^{\alpha} dk - W_{it} L_{ijt} - \int_0^1 p_{ijkt} x_{ijkt} dk\} \tag{3.6}$$

where the input share is represented as $\alpha \in (0, 1)$. The firm uses a continuum of machines, $x_{ijkt}$ ,and each one of them is used with its own specific technology , $A_{ijkt}$. In fact $\int_0^1 A_{ijkt}^{1-\alpha} x_{ijkt}^{\alpha} dk$ represents this continuum as the sum of every machine of type $k$ employed in the production. Also, it is important to highlight that there is no accumulation of machines, instead they fully-depreciate at the end of each period. The intermediate firm produces competitively using a Cobb-Douglas production function. Finally, the machines employed in the intermediate production are made under monopolist competition as follows

$$\max_{x_{ijkt}} (p_{jkt} - \psi) x_{ijkt}$$

where the price ,$p_{jkt}$, is equal to $\alpha$ and the production cost is represented by $\psi = \alpha^2$.[8] This construction represents the endogenous generation of technology increasing their quality where the profits are used to generate new technology. The average productivity, represented by the technology $A_{ijt}$, in each sector satisfies

$$A_{ijt} \equiv \int_0^1 A_{ijkt} dk$$

In this model researchers discover new technologies obtaining a one time patent becoming entrepreneur and producer of the machine related to that technology, and if there is not a successful innovation the property right is allocated to a random possible entrepreneur. Without this assumption the sector that stops innovating has no more property rights over the technologies that already exists, meaning no more profits. But without profits, that sector will never innovate again so we must have property rights that allow the existence of monopolies in the sector with no innovation. In fact this will allow us to have expected profits at every period. In other words, without assuming the property rights are allocated to someone we deny the possibility to switch to a new technologies. Given those informations the technology in the North evolves as

$$A_{Njt} = (1 + \gamma \eta_j s_{Njt}) A_{Njt-1} \tag{3.7}$$

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complementarity there is innovation in both sectors over time, which implies that an environmental disaster is always possible. Meanwhile, we should see this model applied to the energy sector which is the best market where there is perfect substitution, and it is one of the most pollutant.

[7] We talk about a continuum since there are different type of machines used in the production. We represent all these type $k$ using the integral $\int_0^1$. In fact this machines are produced by the monopolist using a specific technology for each type of machine. The correct interpretation is that there exists a monopolist, for each type of machines that is created using the connected technology protected by the $k$ property right.

[8] To find the demonstration why $\psi = \alpha^2$ please refer to the appendix in chapter 1

where $\eta_j \in (0,1)$ is the probability of a successful innovation, $(1+\gamma)$ is the increased amount of machine's quality with $\gamma > 0$ and $s_{Njt}$ is the number of scientist in the North working in clean and dirty sectors with $j = c, d$. Here each new technology is used by monopolists to create better machines that will be sold to intermediate firms.The use of a technology to produce a machine of type $k$ is protected by patents, but this property right do not cover the imitation in another part of the world and so, it is excludable only locally but not globally. To be precise, property rights work only in the place where they are issued and have no effect in other regions.
The difference equation that represents imitation in the South has the following specification

$$A_{Sjt} = \kappa_j s_{Sjt} A_{Njt} + (1 - \kappa_j s_{Sjt}) A_{Sjt-1} \tag{3.8}$$

The parameter $\kappa_j \in (0,1)$ expresses the probability to copy an innovation in the North. The meaning of this equation is that the scientists in the South will imitate the technology from the North and if it is not possible they will keep using the one already existing in the South. When a sector in the North stops innovating the technology in the South cannot become larger than the one in the North. To be more precise, let's assume that the scientists in the North starts researching clean technology in all the future periods, this implies that $A_{Ndt} = A_{Ndt-1}$ for $t = t^*..\infty$ and consequently $A_{Sdt} \leq A_{Ndt}$ for $t = t^*..\infty$. This is a consequence of the imitation process, otherwise they would innovate instead of imitating.

### 3.3.3 Environmental constraint

In this subsection we start characterizing the environmental constraint as in Acemoglu et al 2014.

$$S_{t+1} = -[\zeta(Y_{Ndt} + Y_{Sdt})] + (1+\delta)S_t \tag{3.9}$$

where $\zeta$ is the emission rate and $\delta$ is the regeneration rate. When the difference between the total emissions produced and the natural regeneration is negative the quality decreases. The first effect of a lower environment is a lower ability to absorb future emissions which is represented by the positive component $(1+\delta)S_t$ where a lower $S_t$ reduces the regeneration capacity [9].The second effect is a minor utility for the representative household because we have designed the utility function to be complementary in the environment. But, the main problem appears when the environmental quality reach the value of "0" meaning that the economy face an environmental disaster and the utility goes to the value of "0" with no possibility to recover.

This equation expresses pollution as a global problem that affect the environmental quality of the world because it has been thought to represent the co2 emissions. In my opinion it is good to represent a globalized world where the decision of a country

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[9] As an example the sea has the capacity to absorb co2, but more greenhouse gasses are incorporated in the sea, it will sequestrate less in the future, meaning a lower regeneration capacity

affects another country for that specific environmental problem.
In this paper I try to expand this formulation having two separate environments that are degraded by local pollution, such as the toxic waste of production, and transboundary pollution as co2 [10].

In the next sections we will analyze different environmental laws trying to embody all the possible kinds of pollution finding what is the best policy to adress these externalities expecially dividing the total environment used by Acemoglu et al (2014)[2] in two subsets. Nonetheless, dividing the environment in two will not imply that an environmental disaster in one region does not bring a disaster in the other.

In the next section we represent the equilibrium forces that decide in which sector the scientists are going to innovate and we will see the outcome of a decentralized economy.

## 3.4 Laissez-faire equilibrium

In the section before we have described the main characteristics of the model such as the production functions, how technology evolves and how the environment degrades. This section will show what happens in a decentralized equilibrium from the market clearing conditions, showing what are the strongest forces that move the direction of technological change. Then I show the equilibrium values of the main variables ending with the representation of the imitation process in the south.

**Definition 3** *An equilibrium is given by sequences of wages* $(w_{it})$ *, prices for inputs* $(p_{jt})$*, prices for machines* $(p_{jkt})$*, quantities of machines* $(x_{idkt})$*, quantities of inputs* $(Y_{ijt})$*, quantities of labour* $(L_{ijt})$ *by input producers* $j \in c, d$*, research allocations* $(s_{idt}, s_{ict})$*, and quality of environment* $(S_t)$ *such that, in each period* $t$*: (1)* $(p_{jkt}, x_{ijkt})$ *maximizes profits by the producer of machine* $i$ *in sector* $j$*; (2)* $L_{ijt}$ *maximizes profits by producers of input* $j$*; (3)* $Y_{ijt}$ *maximizes the profits of final good producers; (4)* $(s_{idt}, s_{ict})$ *maximizes the expected profit of a researcher at date* $t$*; (5) the wage* $(w_{it})$ *and the prices* $p_{jt}$ *clear the labor and input markets respectively; and (6) the evolution of* $S_t$ *is given by (3.9).*

Such that

$$L_{Nct} + L_{Ndt} \leq 1 \text{ and } L_{Sct} + L_{Sdt} \leq 1 \tag{3.10}$$

also, I normalize the total mass of scientists in each country $s_i$ to 1, denoting the total amount of researchers working on machines in sector $j \in c, d$ at time $t$ by $s_{ijt}$. The market clearing condition for scientists engaged in research are

$$s_{Nct} + s_{Ndt} \leq 1 \text{ and } s_{Sct} + s_{Sdt} \leq 1 \tag{3.11}$$

The resource constraint is represented as

$$C_{it} = Y_{it} - \psi \left( \int_0^1 x_{ickt} dk + \int_0^1 x_{idkt} dk \right) \tag{3.12}$$

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[10] The main idea is to represent a part o pollution static and another dynamic to analyze the best answer to this kind of problem without the assumption of having only one type of pollution.

where what I consume must be equal to the produced output minus the cost of production of the machines. In this model market clearing condition requires that in each country the amount of workers is equal to one.[11]
Before going deeply in the characterization of the equilibrium it is worthy to say that the equilibrium definition holds in each country. Also, most of the equations I show are the same in the North as well in the South, the main difference will be given by the presence of northern or southern technology. I will refer with the index $i$ when equations and variables have the same form in both country, otherwise I will use the letters $N$ and $S$ when they are different.

The direction of technical change is decided by the profits of the monopolists in each sector. We derive the profits from the demand of machines that we derive from eq (3.6), then we evaluate the expected profits. In fact, the scientists will allocate to the sector with the larger expected profit defined as

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}(\frac{p_{Nct}}{p_{Ndt}})^{\frac{1}{1-\alpha}} \frac{A_{Nckt-1}}{A_{Ndkt-1}} \frac{L_{Nct}}{L_{Ndt}} \tag{3.13}$$

where on the left side of the equation we have the relative expected profits and on the right there are the three forces that move the direction of innovation. We have respectively $(\frac{p_{Nct}}{p_{Ndt}})^{\frac{1}{1-\alpha}}$, which is the price effect, $\frac{A_{Nckt-1}}{A_{Ndkt-1}}$ the productivity effect and the market size effect $\frac{L_{Nct}}{L_{Ndt}}$. "When the two inputs are substitute, the price effect is weaker and innovation tends to be directed toward the most advanced sector exhibiting path dependence " [14]. This implies that in our model the main force is the productivity effect induced by the market size effect as it will be outlined in the optimal policy section.
In equilibrium all the variables can be defined in relative productivity terms. Inserting the demand of machines inside the derivative of eq (3.6) with respect of labour we find the prices of the intermediate good. Then dividing by sector we obtain the relative prices which are described by

$$\frac{p_{ict}}{p_{idt}} = (\frac{A_{ict}}{A_{idt}})^{-(1-\alpha)} \tag{3.14}$$

where $i = N, S$. In fact this equation take the same form in both countries, implying that the relative price depends by the relative productivity. From the optimum production levels, obtained inserting the demand of machines inside the production function, we find the relative labour [12]

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-1}(\frac{p_{Nct}}{p_{Ndt}})^{\frac{\varphi-1}{1-\alpha}} \tag{3.15}$$

which depends by the relative productivity and prices. To simplify the notation as in Acemoglu et al (2012)[1], $\varphi \equiv (1-\alpha)(1-\epsilon)$. Then using equation (3.14) and applying

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[11] In this model the amount of population engaged in work and research activity is represented as a percentage. Also, there is no population growth to avoid scale effects. The maximum value is normalized to 1 which represent 100% of the total population.

[12] To see the proof look chapter 1

eq(3.10) in the previous one the clean labour force is given by

$$L_{Nct} = \frac{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} + 1} \tag{3.16}$$

and consequently since the total amount of workers is normalized to 1 the labour force employed in the dirty sector can be defined as, $L_{Ndt} = 1 - L_{Nct}$ Now, substituting the main factors in terms of technology using eq (3.14) and eq(3.15) inside eq(3.13) and expanding it we can express the direction of technical change in productivity terms.

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}]^{-1-\varphi}(\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}})^{-\varphi} \tag{3.17}$$

Under lassiz-faire the scientists will always move to the most advanced sector as we said before. In our framework this sector is the dirty one leading to an environmental disaster.[13]

Dividing the marginal production of clean output in the final producer, eq(3.5),by the one of the dirty sector we find the relative production of intermediate goods expressed in term of relative prices

$$\frac{Y_{ict}}{Y_{idt}} = (\frac{P_{ict}}{P_{idt}})^{\frac{-1}{\epsilon}} \tag{3.18}$$

Now, applying eq(3.14), (3.18) inside the derivatives of eq(3.5) allows us to find the equilibrium prices which are given by

$$p_{ict} = [(\frac{A_{ict}}{A_{idt}})^{\varphi} + 1]^{\frac{1}{\epsilon-1}} \tag{3.19}$$

and

$$p_{idt} = [(\frac{A_{ict}}{A_{idt}})^{-\varphi} + 1]^{\frac{1}{\epsilon-1}} \tag{3.20}$$

Given all the information above, substituting these variables in the optimum production we can find the equilibrium intermediate production in productivity terms as

$$Y_{idt} = [(A_{ict})^{\varphi} + (A_{idt})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} A_{ict} A_{ict}^{\alpha+\varphi} \tag{3.21}$$

$$Y_{ict} = [(A_{ict})^{\varphi} + (A_{idt})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} A_{ict} A_{idt}^{\alpha+\varphi} \tag{3.22}$$

The equilibrium intermediate production in the South is characterized by the same equations with the southern variables. Despite the fact that there is perfect substitution, as outlined before, the model still shows a degree of complementarity, since both technologies contribute in both the equilibrium productions. In fact when only the dirty technology grows, the clean production grows slower than the dirty one, but it still increases. We can easily say that the sector that expands more its productivity

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[13] To demonstration is in Acemoglu et al (2012)[1] otherwise you can see it in the numerical simulation how the environment quickly goes to an environmental disaster

enjoys an exponential growth over time, meanwhile the other sector has a logarithmic growth.

Solving the model in southern terms we can find the expected profits for an imitator and substituting it with the technological levels we can express the expected profits in the South in term of relative productivity. The expected profits in the South are represented as in Acemoglu et al (2014)

$$\frac{\Pi_{Sc}}{\Pi_{Sd}} = \frac{\kappa_c}{\kappa_d}(\frac{p_{ct}}{p_{dt}})^{\frac{1}{1-\alpha}} \frac{L_{Sct}A_{Nct}}{L_{Sdt}A_{Ndt}} = \frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi-1} \frac{A_{Nct}}{A_{Ndt}} \tag{3.23}$$

The South is moved by the same forces of the North, the price effect, market size effect and the productivity effect. But, since we are deciding the expected profit to imitate, the decision is made from the relative productivity of the North.
The result of the lassaiz-faire equilibrium is that an environmental disaster is impossible to avoid due to the main factors that feeds the creation of new technology: the price effect, the market size effect and the productivity effect. These factors together build the demand of new technology moving all the scientist to research only in one sector.[14] The main driver is the market size effect which is the ratio between two inputs used in the production. In facts, the use of more machines a in one sector imply a growing need of labour force and subsequently a larger market for innovation. In fact, the expanding markets are the one that receive more investments. The characteristics of this kind of model show us how scientists decide in which sector work. Considering that the dirty productivity is larger than the clean, the decentralized equilibrium bring the economy to the same result, an environmental disaster. In Acemoglu et al 2012 it is shown why, saying

**Proposition 1** *When the clean and dirty intermediate output are gross substitute and the dirty sector is the most advanced it exists only a solution where the dirty technology grows leading to an environmental disaster.*

This result is identical to the one of Acemoglu et al. (2012)[1] and the proof is shown in the appendix. The main point is that the dirty sector becomes more productive compared to the clean one. This implies that more people decide to work in the dirty sector every period. The firms produce more polluting goods because of the two effects,more productivity and more labour force, not internalizing the environmental damage they create leading to a disaster. In the next section we will see different ways to represent environmental qualities. Then, we will see different optimal policies, one for each environmental constraint and after that we will discuss the results.

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[14] In the real world a part of what is studied and researched by scientists depends by the profitability in the private sector. A pharmaceutical firm will invest its money on a research that will bring profits to pay the stakeholders and cover the cost of the next research. This profitability is determined by the three forces that direct the technical change.

## 3.5 Environmental constraints

In this section I formally introduce the concepts of trans-boundary pollution, abatement technology and net zero emission in different environmental constraints highlighting the type of pollution and the key differences. Then, in the next section I will show for each constraint a different policy decided by the social planner. In Acemoglu et al. 2014[2] the world share the same environment. In fact that model is built to solve an environmental problem with emissions that represent greenhouse gasses. Instead of that version, the best way could be represented by a model where the world is shown with different environments connected by transnational pollution. This allow us to better represent different kind of pollution as CO2 emissions and toxic waste which are by-product of the manufacture process in the same equation where the trans-boundary pollution is represented as a public good. Also, there is another reason why it could be useful to separate the environment and it is because of the perception the environmental degradation varies country by country and the marginal utility of the environment depends on your local environment. As an example, people care more about their shores quality compared to the north pole melting. This is a consequence of the fact that we better understand what we see with our eyes everyday compared to what is reported from other people. Also, following the environmental kuznet curve, a richer country care more about its environment compared to a poorer one. To summarize, the environment is divided in two subsets and then analysed.

$$S_{Nt+1} = -\{(\zeta_L + \zeta_G)(Y_{Ndt}) + \zeta_G Y_{Sdt}\} + (1+\delta)S_{Nt} \tag{3.24}$$

where $S_{it} \in (0,1)$ is the country specific environmental quality at time $t$,where $\zeta$, our initial emission rate, is divided in its two components, $\zeta_L$ which is the local pollution and $\zeta_G$ which represents the transboundary pollution. The one in the South is similar, and it takes the form of

$$S_{St+1} = -\{(\zeta_L + \zeta_G)(Y_{Sdt}) + \zeta_G Y_{Ndt}\} + (1+\delta)S_{St} \tag{3.25}$$

where the variables have the same meaning of the equation before.

### 3.5.1 Abatement technology

In this section I consider abatement technology as any mechanism, process or method that has the potential to reduce the emission of pollution[15]. It is built as an investment done by an authority using final output and it is done only in the North.[16] Now we add an abatement technology transforming our market clearing condition in

$$C_{Nt} = Y_{Nt} - \psi\left(\int_0^1 x_{Nckt}dk + \int_0^1 x_{Ndkt}dk\right) - \iota Y_{Nt} \tag{3.26}$$

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[15] Using this general definition I incorporate the carbon storage and sequestration (CSS), filters and all the technologies that allow to produce the same amount of output with less pollution emitted.

[16] This decision reflect the fact that the South is less willing to invest compared to the North, and we want the South to be in lassaiz-faire with no policy.

where a portion of the final output is invested to develop new abatement technologies. The law that represents the evolution of the abatement technology can be expressed as

$$\zeta_{at} = (1 + \iota Y_{Nt-1})\zeta_{at-1} \tag{3.27}$$

which implies that the abatement technology is a function of the invested final product. In fact, $\iota \in (0, 1)$ is the percentage of final output $Y_{Nt}$ that instead of being consumed is invested in the development of an abatement technology. Also, we must assume that the difference between the emissions and the abatement technology must be positive, or in other words, $\zeta - \zeta_{At} > 0$ otherwise the dirty production can remove pollution meanwhile producing goods. Instead the abatement technology must have an upper limit such that $\zeta_{At} \in (0, \zeta)$. When the abatement technology reaches its highest level the investment in new technology becomes a maintenance cost to keep the structures and practices working. An abatement technology change the environmental quality law in

$$S_{Nt+1} = -\{(\zeta_L + \zeta_G - \zeta_{At})(Y_{Ndt}) + \zeta_G Y_{Sdt}\} + (1 + \delta)S_{Nt} \tag{3.28}$$

This last equation means that greater is the investment lower becomes the net emissions produced by the dirty industry, where net emission represents the difference $\zeta_L + \zeta_G - \zeta_{At}$. Meanwhile, since the South does not invest in abatement technology her environmental quality law is the same described in eq(3.25). The reason behind this is given by the assumption that the South is not rich as the North and prefers to consume all its production instead of investing it. The effect of the abatement technology is to reduce the impact of the dirty production over time.[17]

Abatement technology is fundamental to introduce the net zero emissions target, because to reach it the economy needs to remove the pollution from the environment and the net emission have a central role defining a new production function that satisfies the condition.

### 3.5.2 Net zero emissions

In this section we define what a net zero emission means and how I use it in the environmental quality law. The best definition is given by IPCC[8]

**Definition 4** *Net zero emissions are achieved when anthropogenic emissions of greenhouse gases to the atmosphere are balanced by anthropogenic removals over a specified period. Where multiple greenhouse gases are involved, the quantification of net zero emissions depends on the climate metric chosen to compare emissions of different gases (such as global warming potential, global temperature change potential, and others, as well as the chosen time horizon).*

To better represent the anthropogenic removals we need an abatement technology that can remove the emissions, which we introduced in the last section. It is noteworthy

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[17] This could be represented by the use of filters in the cars. Through an investment the car pollutes less, as well as the industries that use river water release less waste.

to say that I define inside the net zero emission all types of pollution, from co2 to production waste, meaning that the economy must remove all of them to reach this condition. As we said before a net zero condition requires that the emission $\zeta$ must be removed by abatement technology and carbon reservoirs which are represented by $\zeta_{at}$, and by the natural regeneration rate so we obtaining

$$S_{t+1} \geq S_t \tag{3.29}$$

as our terminal condition.

This is a consequence of the net zero which requires that at least $\delta S_t - (\zeta - \zeta_{at})Y_{dt} = 0$ is respected. Now in our setting a balance between the produced emissions and the emissions taken from the atmosphere should have the form of

$$\zeta Y_{Nd} \leq \delta S_N + \zeta_{at} Y_{Nd} \tag{3.30}$$

where on the left side we have the pollution that degrades the environment meanwhile on the right side we have the reductions that improve it. This is a modification of eq(3.28) without transboundary pollution applying the conditions I state before. Solving it as an equality requires us that the production of dirty input be at least equal to the relative regeneration of the environment plus the relative pollution of the dirty production

$$Y_{Nd} = \frac{\delta S_N}{\zeta} + \frac{\zeta_{at}}{\zeta} L_{ijt}^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{ijkt}^{\alpha} dk \tag{3.31}$$

This equation must be interpreted as a boundary to the production implying its limit. In fact, if the dirty production is higher than the one established by this equation, the environment degrades and consequently the relative regeneration decrease. This effect would limit the maximum amount of dirty production allowed in the economy.

Now, if we apply the net zero condition when the environment has been degraded to a certain level the equation before can be approximated by

$$Y_{Nd} = \frac{\zeta_{at}}{\zeta} L_{ijt}^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{ijkt}^{\alpha} dk \tag{3.32}$$

Also, to satisfy the condition above, $S_{t+1} \geq S_t$, we can require the production to be eq(3.32) such that the environment will still grow obtaining what is called net negative emissions[18]. In the decentralized equilibrium the intermediate producer does not internalize the environmental damage represented by $\frac{\delta S_N}{(\zeta_G + \zeta_L)} + \frac{\zeta_{at}}{(\zeta_G + \zeta_L)}$.

In the next section we will see the different policies to avoid environmental disaster without global policy coordination. This implies that the Southern economy will grow under laissez-faire meanwhile the North will implement different environmental policies.

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[18] This is also a way to avoid using the environment regeneration since it is difficult to evaluate. Meanwhile, the abatement should be easier since we know how much co2 we create from the production and how much we take from the atmosphere.To find the demonstration look the appendix

If we consider the existence of transboundary pollution, to reach zero emission I need to recognize the pollution coming from the other country that is still damaging its environment. Therefore applying the same logic I use before with the environmental constraint eq(3.24) the eq (3.31) becomes

$$Y_{Nd} = \frac{\delta S_N - \zeta_G Y_{Sdt}}{(\zeta_G + \zeta_L)} + \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L_{ijt}^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{ijkt}^{\alpha} dk \tag{3.33}$$

Where the relative regeneration, $\frac{\delta S_N - \zeta_G Y_{Sdt}}{(\zeta_G + \zeta_L)}$, which represents the ratio between regeneration and environmental damage is decreasing in the transboundary production from the South. Since $Y_{Sdt}$ grows over time $\frac{\delta S_N - \zeta_G Y_{Sdt}}{(\zeta_G + \zeta_L)}$ becomes closer to zero, and if the dirty production in the South is enough large $\frac{\delta S_N - \zeta_G Y_{Sdt}}{(\zeta_G + \zeta_L)}$ becomes negative decreasing the maximum amount of dirty production required. This enforce the assumption to consider the first fraction equal to zero solving the problem with the quota $\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}$.

## 3.6 Optimal policy

As we discussed at the end of section 4 we already know that the economy will face an environmental disaster under lassaiz-faire.[19] In the literature it is clear that the optimal solution is through global coordination[20] where both countries collaborate to achieve the best path through optimal policy. In this work we assume that the South still works under lassaiz-faire and the explanation is due to different factors. The first is that the South is less advanced and cares less about the environment which implies a delay in the response. The second is that decisions made in the North still influence the imitation process in the South as described in eq(3.23), implying that it is more important an optimal policy in the North compared to the South. In fact the social planner will include the importance of the imitation process inside the social gain of an innovation in the North. The last decision is more ethical and concerns the fact that the South should not be responsible because its emissions level and welfare are lower compared to the North and should improve its growth. This kind of policy is called unilateral and it will be assumed in each policy. so we try to characterize an unilateral solution. This reflect the fact that a poor country prefers to increase its consumption instead of improve its environmental disutility.

It is important to explain that in this Thesis when I discuss about optimal policy I intend a paternalistic one. Im fact, I do not have an optimal policy by which a government can decentralize the socially optimal allocation into market economies.

So, given all the motivations above, in this section I characterize the optimal policy chosen by a social planner which maximizes the utility of the representative household in the North meanwhile the South operates under lassaiz-faire.[21]

The social planner corrects the externalities that result from the structure of this problem. They are the monopolist distortion that affects the price of the demanded

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[19] The proof of this statement is in Acemoglu et al 2012[1]

[20] The proof is in Acemoglu et al 2014[2]

[21] This reflects the idea of unilateral policy.

machines, the knowledge externality which depends by the profits instead of the social gain, and the environmental externality created from the dirty sector emissions. The answer to most of these externalities are already outlined in Acemoglu et al 2012([1]) and Acemoglu et al 2014 [2]. We focus on the different answers on the environmental externality. Especially with transboundary pollution, the social planner introduces new taxes to solve the problem. Also, in the last subsection it is outlined the policy chosen by the social planner to obtain and sustain a net zero emission target.
Before starting I describe the utility function as a CRRA,

$$u(C_{it}, S_{it}) = \sum_{t=1}^{\infty}(\frac{1}{1+\rho})^t u_i \frac{(C_{it}S_{it})^{1-\sigma} - 1}{1-\sigma} \tag{3.34}$$

where $\sigma$ is the risk aversion. The utility shows complementarity in both goods and it reaches its maximum when the environment has fully capacity, or $S_{it} = 1$. This function will be used in the next subsections to find the marginal utility of consumption and of the environment.
In all the different scenarios I will analyse the two economies enter a transition to a long-run equilibrium where every variable reaches asymptotically their steady states. In fact the economy moves from dirty technologies to clean. Also, given the nature of the DTC model with environment, the only situation where there is a balanced growth path occurs when both technologies grow at the same rate.

### 3.6.1 Carbon tax applied to a common environment

In this part I refer to the solution of the optimal problem with a common global environment as outlined in equation (3.9). Under this scenario the social planner will implement the same policies as in Acemoglu et al 2014. Using their words "In the two-country case when $\epsilon \geq 1/(1-\alpha)$ a policy $\{\tau_{Nt}, q_{Nt}\}$ in the North that would direct innovation towards clean technologies only is sufficient to avoid a disaster without taxation in the South provided that $\overline{S}$ is sufficiently high, if $1 < \epsilon < 1/(1-\alpha)$ then such a policy cannot prevent a disaster".
The value of the elasticity os substitution $\epsilon \geq 1/(1-\alpha)$, given the value adopted in this model of $\alpha = 0.33$ implies that $\epsilon$ must be at least equal to 1.515 to have enough elasticity of substitution. Otherwise it is impossible to obtain a technological switch under unilateral policy[22], leading to an environmental disaster. In fact, even under perfect substitution there is a degree of complementarity, as we outlined describing equation (3.21) and equation (3.22), and more importantly affects our identity $\varphi \equiv (1-\alpha)(1-\epsilon)$. I will use this restriction in my model through all the passages, and in the numerical simulation its value is represented as $\epsilon = 3$. Still, solving the model we obtain under unilateral policy the same result of Acemoglu et al 2012 [23], which is a tax on the output

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[22] The proof is in Acemoglu et al 2014

[23] We obtain the same result because in this scenario the environment is represented in the same way, meaning that the Social planner needs to solve the same issue using the same method, which results in a tax on the intermediate dirty output. The appendix E.1 describes in detail the maximization

of value

$$\tau_O = \frac{\omega_{Nt+1}\zeta}{\lambda_{Nt}\hat{p}_{Ndt}} \tag{3.35}$$

which is increasing by, the shadow value of the future environment $\omega_{Nt+1}$, and the emission rate $\zeta$ and decreasing by the marginal utility of consumption $\lambda_{Nt}$ and by the optimal dirty price decided by the social planner $p_{\hat{Ndt}}$. Quoting the authors "this tax reflects that at the optimum, the marginal cost of reducing the production of dirty input by one unit must be equal to the resulting marginal benefit in terms of higher environmental quality in all subsequent period"[1]. To demonstrate it we develop the first order condition of the objective function with respect to the dirty intermediate output obtaining

$$\frac{\partial L}{\partial Y_{Ndt}} = Y_{Ndt}^{\frac{-1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}} - \frac{\zeta\omega_{Nt+1}}{\lambda_{Nt}} = \hat{p}_{Ndt} \tag{3.36}$$

where the last part of the left hand is the wedge between the marginal productivity and the price. Then applying the tax in the decentralized equilibrium we obtain the new allocation of scientists

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}]^{-1-\varphi}(\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{3.37}$$

where the clean sector increases its expected profits thanks to the tax $\tau_O$ with an effect amplified by the value of the elasticity of substitution. The tax has a positive effect on the demand of clean technologies increasing the market size effect and subsequently reducing the gap between clean and dirty technologies to the point where the incentive to use the clean one is larger thanks to the higher cost of the intermediate dirty output. The scientists recognize this mechanism and engage research in the clean sector that will be more profitable compared to the dirty one.

To summarize, greater is the value of the elasticity of substitution, greater is the effect of the tax. When there is complementarity,$\epsilon \in (0,1)$ the tax has ambiguous effects.
In this study to avoid an environmental disaster through unilateral policy the South need to be sufficiently less advanced in both sectors compared to the North This depends on the imitation process that uses the technology of the North. Assuming it is less developed give the North enough time to increase its relative productivity to the point where we can observe a switch in the imitation in the south. In fact the allocation of scientists in the North is decided by

$$\frac{\eta_c\gamma\mu_{Nct}A_{Nct-1}}{\eta_d\gamma\mu_{Ndt}A_{Ndt-1}} \tag{3.38}$$

which represents the ratio between the social gain of an innovation in the clean and dirty sector. Here $\mu_{Nct}$ and $\mu_{Ndt}$ are respectively the shadow prices of the technological levels in the North. In the South there is not the social gain of an imitation, instead

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problem subject to the constraints. In the next scenarios we will see how it differs with alternative environments.

we use the expected profits in the clean and dirty sector because it operates under lassaiz-faire.The expected profits are represented by

$$\frac{\Pi_{Sc}}{\Pi_{Sd}} = \frac{\kappa_c}{\kappa_d}(\frac{p_{ct}}{p_{dt}})^{\frac{1}{1-\alpha}}\frac{L_{Sct}A_{Nct}}{L_{Sdt}A_{Ndt}} = \frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi-1}\frac{A_{Nct}}{A_{Ndt}} \tag{3.39}$$

When the ratios are greater than one, the scientists are allocated to the clean sector. These two equations allow us to introduce the next proposition

**Proposition 2** *In order to prevent an environmental disaster with an unilateral policy the tax needs to increase depending on the productivity levels of the South*

This is a consequence of eq(3.39) where the allocation of southern scientists depends by the productivity levels of the two countries and the goal of the North is to affect the imitation of the South before it will reach an environmental disaster. By assumption when an environmental disaster reach a country it has global effects. The imitation process create a delay between the two countries to adopt the clean technologies. In fact, the North will do it first thanks to the unilateral policy, then the southern scientists will engage in clean research after the northern clean technology is larger than the dirty one. Therefore the North should take into consideration the productivity of the South to close the gap between the two sectors as fast as possible.

### 3.6.2 Carbon tax applied to transboundary pollution

In the scenario with transboundary pollution the social planner needs to address the global and local emissions. She will do it taxing the emissions that stays in the country and compensating the pollution that affect the South. This tax is a direct consequence of the social planner maximization problem where the first order condition with respect to dirty intermediate output internalize the cost of the pollution that affects South. The first order condition of the intermediate dirty output is described as follows

$$\frac{\partial L}{\partial Y_{Ndt}} = [Y_{Ndt}^{\frac{1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] - \frac{[\zeta_G + \zeta_L]\omega_{Nt+1}}{\lambda_{Nt}} - \frac{\zeta_G\omega_{St+1}}{\lambda_{Nt}} = \hat{p}_{Ndt} \tag{3.40}$$

where the marginal productivity of the intermediate output is equal to its price. The marginal productivity is subtracted by the future damage on the environment of each new emission. We can rearrange the equation above to express the tax in an explicit form

$$\frac{\partial L}{\partial Y_{Ndt}} = [Y_{Ndt}^{\frac{1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] = \hat{p}_{Ndt}(1 + \frac{[\zeta_G + \zeta_L]\omega_{Nt+1}}{\lambda_{Nt}p_{Ndt}} + \frac{\zeta_G\omega_{St+1}}{\lambda_{Nt}p_{Ndt}}) \tag{3.41}$$

The meaning of the equation is that the marginal product of the final output with respect to dirty inputs is equal to its price plus the taxes that eliminate the future externalities. Like in Acemoglu et al (2012) the Social planner introduces a wedge, $\frac{[\zeta_G+\zeta_L]\omega_{Nt+1}}{\lambda_{Nt}}$, which is the future local and global environmental destruction of an unit of

dirty production today. Also, we have $\frac{\zeta_G \omega_{St+1}}{\lambda_{Nt}}$, which is the global cost of an additional unit of dirty output produced in the North and sustained in the South.
Now we assume that in the decentralized economy it is introduced a tax $\tau_O \in (0,1)$ to the dirty intermediate output used by the final producer such that now it solves the following maximizing problem with unilateral policies

$$\max \Pi_i = p_Y(Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - p_{ct}Y_{ict} - (1+\tau_O)p_{dt}Y_{idt} \tag{3.42}$$

Solving the problem with the new tax leads to the relative labour in the North that now depends by

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{3.43}$$

which now it is increasing by the value of the term $(1+\tau_O)^{\epsilon}$. The tax on the intermediate output $\tau_O$ changes the composition of the labour force enough to avoid the degradation of the environment. Also, the perfect substitution increases the effect of the tax demanding more workers in the clean sector, meanwhile under complementarity it would be less relevant compared to the relative productivity.
The prices are increased by the tax on the dirty intermediate output as follows

$$p_{ict} = [(\frac{A_{ict}}{A_{idt}})^{\varphi}(1+\tau_O)^{\epsilon-1} + 1]^{\frac{1}{\epsilon-1}} \tag{3.44}$$

and

$$p_{idt} = [(\frac{A_{ict}}{A_{idt}})^{-\varphi}(1+\tau_O)^{\epsilon-1} + 1]^{\frac{1}{\epsilon-1}} \frac{1}{1+\tau_O} \tag{3.45}$$

The relative production in the lassaiz faire production satisfies the condition $(\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{-1}{\epsilon}} = \frac{p_{Nct}}{p_{Ndt}}$ but since this is the optimal solution that incorporates the compensation process, the relative production is equal to $\frac{p_{Nct}}{p_{Ndt}(1+\tau_O)}$ which implies that the relative prices are equal to the relative productivity corrected by the tax, $(\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{-1}{\epsilon}} = \frac{p_{Nct}}{p_{ndt}(1+\tau_O)} = (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}\frac{1}{(1+\tau_O)}$.
In this model there are two reasons to use a tax on the market size. The first is that it increases the the demand of clean technology until it is more productive than the dirty one allowing the economy to reach a sustainable path in the long-run. The second reason is to decrease the amount of pollution generated by the dirty sector avoiding an environmental disaster in the short-run. This is a consequence of the fact that the labour force is an input in the production and if we design a function that represents the externality generated by the dirty production, it would be positively dependent by the dirty labour force. Also, the productivity, or the technological level, cannot decrease, so the only factors that can be easily changed are the relative labour and the relative prices and as we said in section 4 citing Hemous (2021) the market size effect has a stronger effect on the demand of technology compared to the relative price. The introduction of a tax on the intermediate dirty output would change the expected profit as they are represented by the next equation

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}]^{-1-\varphi}(\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{3.46}$$

where the tax can be used to finance the subsidy to the clean scientists in the North, and a share of it of value $\frac{\zeta_G \omega_{St+1}}{\lambda_{Nt} p_{Ndt}}$ could fund the clean imitation in he South. Also, one of the things I found solving this model is that under unilateral policy the North would not be compensated for the pollution created by the South. The reason is quite simple and depends by the fact that the unilateral policy requires that the Southern economy operates under lassaiz-faire, implying that it maximizes the dirty production function, eq (3.6), not introducing any tax. In fact this framework requires that the North is the only country sustaining the cost to eliminate the pollution which has largely generated in the past.[24]

**Proposition 3** *Taking into consideration the transboundary pollution, a tax on intermediate dirty output to be optimal must embody the pollution received by the other country with value* $\frac{\zeta_G \omega_{St+1}}{\lambda_{Nt} p_{Ndt}}$*, such that the new tax is represented as* $\tau_O = \frac{\omega_{Nt+1}(\zeta_L + \zeta_G) + \omega_{St+1}\zeta_G}{\lambda_{Nt} P_{Ndt}}$

The part of the tax $\tau_O$ that depends by the shadow value of the environment in the south, is the compensation for the damage created in the other country by the production in the North. This tax embodies the environmental principle that who pollutes pay. Also, this tax can be used to subsidy the scientists engaged in the imitation of clean technology. This is a consequence of the transboundary nature of the pollution. In fact the social planner recognizes that the pollution created in the North affects southern utility and she tries to fix it using the southern environmental shadow value corrected by its marginal utility.
In the next section I introduce the optimal policy using abatement technology only in the North.

### 3.6.3 Abatement technology

So far we have seen the optimal policies with two different environmental constraints, one with a common environment, which is the most used in literature, and one with transboundary pollution. I introduce the abatement technology in this subsection because in the next part where I discuss about Net zero it will have a key role.
This is an informative part to understand what could be the effect of an abatement technology. The reason to do it is the fact that one of the tools that is debated is the possibility to use mitigation tools to avoid environmental disasters instead of clean technology. Also, this abatement technology is exogenous to reduce the complexity of the model.
We solve the maximization problem obtaining the shadow value of an investment in abatement technology that takes the form of

$$\frac{\partial L}{\partial \zeta_{at}} = (1 + \sigma Y_{Nt})\mu_{at} + \omega_{Nt+1} Y_{Ndt+1} = \mu_{at} \tag{3.47}$$

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[24] In the numerical simulation it is clear that given different initial technological levels the South is not able to degrade its environment at the same rate of the North. In fact one of the goals of this model is to reproduce the pollution generated in the past by western countries giving them the responsibility to solve that issue under the European concept of "polluter pays principle".

which is the first order condition with respect of the abatement technology. Here we can see that the shadow value of the investment in pollution reduction increases with the amount of the present final production plus the environmental price of the the future dirty production. In the next equations, which are the marginal productivity of the intermediate output, we have the subsidy on the production,

$$\frac{\partial L}{\partial Y_{ict}} = [Y_{ict}^{\frac{1}{\epsilon}}(Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}][1+\sigma(\frac{\mu_{at+1}\zeta_{at+1}}{\lambda_{Nt}} - 1)] = \hat{p}_{ict} \tag{3.48}$$

$$\frac{\partial L}{\partial Y_{Ndt}} = [Y_{Ndt}^{\frac{-1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}][1+\sigma(\frac{\mu_{at+1}\zeta_{at+1}}{\lambda_{Nt}} - 1)] - \frac{[\zeta_G + \zeta_L - \zeta_{at}]\omega_{Nt+1}}{\lambda_{Nt}} - \frac{\zeta_G \omega_{St+1}}{\lambda_{Nt}} = \hat{p}_{Ndt} \tag{3.49}$$

where, when $\frac{\mu_{at+1}\zeta_{at+1}}{\lambda_{Nt}} - 1 > 0$ it take the form of a subsidy, otherwise as a tax. The reason of this policy is that the social planner can increase the abatement technology producing more, or decrease it if it is excessive depending on a mix of factors as the social gain of a future emission reduction compared to the present marginal utility. In fact, when the marginal utility of consumption is lower than the social gain of a future emission reduction the social planner introduces a subsidy to increase the amount of production that would be invested. Also, since the abatement technology is funded with final output, the social planner subsidize both sectors simultaneously. Also, the tax on the emissions produced by the dirty sector is

$$\tau_O = \frac{\omega_{Nt+1}(\zeta_L + \zeta_G - \zeta_{At}) + \omega_{St+1}\zeta_G}{\lambda_{Nt} P_{Ndt}} \tag{3.50}$$

which implies that the value of the tax decreases with the evolution of the abatement technology. To obtain the corresponding equation that defines the allocation of scientists in the decentralized economy we apply the new relative labour which take the form of eq (3.43) and the relative prices, $\frac{p_{\hat{N}ct}}{p_{\hat{N}dt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}$, in the pre-tax expected profits (3.32) which gives us

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}]^{-1-\varphi}(\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{3.51}$$

Now I summarize the effect of the abatement technology in the following proposition:

**Proposition 4** *The social optimum can be implemented through a combination of carbon taxes, as in equations (3.49) and an abatement subsidy on the intermediate output as it is shown in (3.48) and (3.49).*

The presence of the abatement technology reduces the tax required in the production of dirty output since it pollutes less over time.[25]

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[25] The evidence of this is shown in the numerical simulation where the scenario with abatement technology has a lower tax on the intermediate dirty output compared to the one without abatement technology because the environmental degradation is perceived lower over time.

### 3.6.4 Carbon tax and quotas under Net zero emissions target

Before discussing the policy implication is worth considering the marginal utility of environment under net zero emission. In fact assuming that the environment is constant over time implies that there is no environmental marginal utility in the North because the environment is fixed, and as a consequence there is no variation of our environmental pleasure. This is represented by the condition $S_{Nt+1} = S_{Nt} = S_{Nt^*}$. Now, in this model we assume that the environment can increase its quality, implying that $S_{Nt+1} = S_{Nt}(1+\delta)$. This is derived by the fact that we are solving the environmental constraint with $\zeta Y_{Nd} = \zeta_{At} Y_{Ndt}$ meaning that the emissions that are degrading the environment are balanced by an abatement technology.
In this subsection I first highlight the the net zero target under the scenario of local pollution in two different environment, to finish with trans-boundary pollution. Now, the social planner maximizes the production function with respect to the machines obtaining the new policy

$$x_{idt} = (\frac{\zeta_{at}}{(\zeta_G + \zeta_L)} \frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt} \tag{3.52}$$

We can impose a tax $\tau_I$ that increase the cost of the machine input such that $\frac{1}{1+\tau_I} = \frac{\zeta_{at}}{(\zeta_G+\zeta_L)}$ the intermediate dirty firm now maximizes

$$\max_{x_{idt}, L_{idt}} \{p_{dt} L_{idt}^{1-\alpha} \int_0^1 A_{dt}^{1-\alpha} x_{idkt}^{\alpha} dk - W_{it} L_{idt} - (1+\tau_I) \int_0^1 p_{dkt} x_{idkt} dk\} \tag{3.53}$$

$$x_{Ndkt} = (\frac{p_{Ndt}}{1+\tau_I})^{\frac{1}{1-\alpha}} A_{Ndkt} L_{Ndt} \tag{3.54}$$

It says that the demand of dirty machines is increasing by the price, the average productivity and the amount of labour, while it is decreasing by the tax on machines. Since the social planner requires a constant environment she establish the production quota of machines that needs to be used in the production. As an example, we could see it as the production of oil. Since we know that each barrel of oil has an effect on the environment that we can quantify, we can easily impose to the producer the quota of barrel that keep the amount of CO2 in the atmosphere constant over time. In environmental economics quotas are used to establish the maximum amount of resources that can be extracted from an open access market as fisheries or hunting. It is interesting that under net zero with an abatement technology the social planner use the same policy used for open access markets.

**Proposition 5** *Given net zero assumption, the social planner introduces a quota on the demand of dirty machines with a value that depends from the ratio* $\frac{\zeta_{at}}{\zeta_G+\zeta_L}$.

The proof of this proposition is in the appendix (E.3) The social planner is adding a quota on the inputs because he requires a constant environment over time and she knows that the only way to obtain it is fixing the demand of inputs. In fact, Net zero emissions implies that the environment must be constant and as a consequence, it substitutes the tax on the tax intermediate output in the case of local environment. Another interesting aspect is given by the marginal utility of environment, which now depends by the shadow price of the dirty input, $\lambda_{Ndt}$ because assuming a constant environment implies that its utility is a function of the dirty production and assuming that the environmental quality law still holds implies

$$\omega_{it} = \frac{1}{(1-\rho)^t}\frac{C_{it}^{1-\sigma}}{1-\sigma} + (1+\delta)\omega_{it+1} + \frac{\lambda_{Ndt}\delta}{\zeta} \tag{3.55}$$

In fact, since the environment goes inside the production function, the shadow value of the intermediate dirty production affects the marginal utility of the environment. Larger is th sum of the emission rates lower is the contribute of the shadow price of the dirty production. Also, the regeneration rate positively affects the effect on the marginal utility.

Now, assuming the pollution created in the North expands to the South, the social planner impose a wedge on each unit of pollution created in the North that affects the utility in the South. This implies that to fulfil a net zero emissions target the social planner need to introduce a tax on the intermediate output, $\tau_O$ which can be represented as

$$\tau_O = \frac{\omega_{St+1}\zeta_G}{\lambda_{Nt}\hat{p_{Ndt}}} \tag{3.56}$$

which means that the North has to compensate the disutility of pollution that affects the other country. This depends by the fact that the North is correcting its externality with the quota but it is not affecting the damage generated in the South. It is noteworthy that since the dirty production in the North is larger than the one in the South, the damage generated by its dirty production is greater compared to the one created in the South by the local firms. This is well represented by the CO2 that has been generated in the last century by the western country and that is now affecting the rest of the world. In fact the pollution that is produced by the South of the world is not comparable by the one that we are currently making now in the North. The social gain of an innovation in the dirty sector is different from the scenario we saw in the previous subset. In the one with abatement technology it depends by

$$\frac{\partial L}{\partial A_{Ndt}} = \mu_{Ndt} = \lambda_{Ndt}(1-\alpha)(L_{Ndt}^{1-\alpha}A_{Ndt}^{-\alpha}x_{Ndt}^{\alpha}) + \mu_{Ndt+1}(1+\gamma\eta_d s_{Ndt+1}) + \mu_{Sdt}(\kappa_d s_{sdt}) \tag{3.57}$$

It says that the shadow value of an innovation in the dirty sector depends by the marginal utility of consumption, $\lambda_{Nt}$, times the marginal product of the productivity plus the futures shadow values of innovation and the shadow price of an innovation in the south. The main factor is the marginal product of the productivity. Meanwhile in the net zero scenario by

$$\frac{\partial L}{\partial A_{Ndt}} = \mu_{Ndt} = \frac{\zeta_{At}}{\zeta_G+\zeta_L}\lambda_{Ndt}(1-\alpha)(L_{Ndt}^{1-\alpha}A_{Ndt}^{-\alpha}x_{Ndt}^{\alpha}) + \mu_{Ndt+1}(1+\gamma\eta_d s_{Ndt+1}) + \mu_{Sdt}(\kappa_d s_{sdt}) \tag{3.58}$$

where the marginal product of the average technology is weighted by the quota which reduces the most important component of the shadow price of a dirty innovation in

the North represented by its marginal product.
To better represent the effect of the quota on the demand of research it is worthy to define that the scientist engaged in research are allocated in the clean sector when

$$\frac{\eta_c \gamma \mu_{Nct} A_{Nct-1}}{\eta_d \gamma \mu_{Ndt} A_{Ndt-1}} > 1 \tag{3.59}$$

because the social planner allocates the scientists in the sector that has the higher social gain from an innovation.

**Proposition 6** *Given the net zero assumption, a sufficiently small relative pollution is strong enough to redirect the innovation without a subsidy on the clean research.*

This means that the quota affect the demand of dirty labour to a point where there is no more social gain in an innovation in that sector, because there is a higher productivity for a sector that has not the potential to attract new workers combined with the quota applied to the shadow price of a dirty innovation. Also, we should discuss the fact that a net zero emission applied only on local pollutants needs a tax on the machines used on the production instead of the intermediate output leading to the demand of machine that we have seen before, eq (3.54).
Solving the model from the derivatives of the intermediate output and applying the optimal production we obtain the relative labour which is represented as

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1 - \frac{\delta S_N}{(\zeta_G + \zeta_L)}(Y_{Ndt})^{-\epsilon})(\frac{\zeta_{at}}{(\zeta_G + \zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}} \tag{3.60}$$

which implies that the demand of labour depends by two forces, the first is the relative regeneration/damage that increases the demand of dirty workers raised to $\epsilon$, on the other hand the relative emissions that decreases the demand of dirty workers.

If we apply the net zero when the environment is sufficiently low, the equation above can be rewritten as

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{\zeta_{at}}{(\zeta_G + \zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}} \tag{3.61}$$

where the demand of labour in each sector is described by

$$L_{Nct} = \frac{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}}}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}} + 1} \tag{3.62}$$

and the demand of dirty labour by consequence is defined by $1 - L_{Nct}$ or

$$L_{Ndt} = \frac{1}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}} + 1} \tag{3.63}$$

Here the quota positively affects the demand of clean labour in the North, redirecting the workforce to the less pollutant sector. In the equation above the demand of labour depends more by the relative emission than the relative productivity. The social planner corrects the monopoly distortion in the quota meanwhile, applying a tax of the same value on the demand of machine, bring to a different demand of worker having $(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}$. Substituting the relative prices and labour we can express the direction of technological change in terms of technology and taxes obtaining

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}(\frac{A_{Nct}}{A_{Ndt}})^{-1-\varphi}\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}}(\frac{\zeta_{At}}{\zeta_G+\zeta_L})^{\frac{\alpha\varphi-1}{1-\alpha}} \tag{3.64}$$

0r better

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}(\frac{A_{Nct}}{A_{Ndt}})^{-1-\varphi}\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}}(\frac{\zeta_G+\zeta_L}{\zeta_{At}})^{\frac{1-\alpha\varphi}{1-\alpha}} \tag{3.65}$$

The last equation shows the effect of the tax on the input to the allocation of scientists. Since by construction $\frac{1-\alpha\varphi}{1-\alpha}$ is always greater than one[26] the tax positively affects the allocation of scientists in the clean sector. This will is a strong effect to the expected profits of the scientists and it has a stronger effect compared to the tax we saw in the subsections before. The incentive depends by the inverse relative emission rate which decreases with the evolution of the abatement technology. We can see that larger is the abatement technology, smaller is the incentive to innovate in the clean sector due to the reduction of the pollution potential.

## 3.7 Net zero without Abatement

In this section we consider the social planner solution under net zero emissions target without abatement technology. The main reason is the fact that in this model the removal technology is exogenous and brings distortions in the optimal production affecting the marginal productivity and consequentially the consumption grows slower, and at the beginning it decreases.. Also, I am adding the trans-boundary pollution in this analysis to increase the complexity and to compensate the quota effect on the social value of a dirty innovation in the North. [27]

We start from the environmental constraint in eq(3.24) without trans-boundary pollution, concluding the section with a generalization. Now to satisfy a net zero emission target the environmental constraint takes the form of

$$(\zeta)Y_{Ndt} \leq \delta S_{Nt} \tag{3.66}$$

where the total emissions that affect the North must be not greater than the regeneration rate of the northern environment. Now, in northern terms the environmental target can be represented as

$$Y_{Ndt} \leq \frac{\delta S_{Nt}}{\zeta} \tag{3.67}$$

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[26] To show this we see that $-\alpha\varphi > 0$ for $\epsilon > \frac{1}{1-\alpha}$ which implies that $1-\alpha\varphi > 1-\alpha$

[27] In the previous section I highlight that the quota implicitly reduces the demand of dirty imitation in the South. Without that mechanism I need to introduce a new process that speed up the imitation transition before the South gets too developed contrasting the unilateral policy effect. In fact, the trans-boundary pollution introduces the compensation process that subsidizes the imitation of clean imitation in the South.

where the dirty production must not be greater then the relayive pollution represented as the ratio between environmental regeneration and the total emission rate. It can be seen as a cap on the maximum amount of emissions allowed in the economy where lower is the environmental quality $S_{Nt}$ implies a lower cap.

To study the problem we relax the target focusing on the critical point where the relationship is satisfied such that

$$Y_{Ndt} = \frac{\delta S_{Nt}}{\zeta_G + \zeta_L} \tag{3.68}$$

This relationship implies that the maximum amount of production must satisfy the cap established by the Net zero emission. This formulation allow the intermediate output to not degrade the environmental quality avoiding a disaster [28]. This implies that the optimum production takes the form of

$$Y^*_{Ndt} = \frac{\delta S_{Nt}}{\zeta} = (\frac{\alpha p_{dt}}{\psi})^{\frac{1}{1-\alpha}} A_{Ndt} L_{Ndt} \tag{3.69}$$

In other words, the optimum production must be fixed to the net zero emission target[29]. As a consequence the demand of worker in the dirty sector is defined as

$$L_{Ndt} = \frac{\delta S_{Nt}}{\zeta} A^{-1}_{Ndt} (\frac{\alpha p_{dt}}{\psi})^{\frac{-\alpha}{1-\alpha}} \tag{3.70}$$

Now comparing the last equation with the demand of clean labour we obtain

$$L_{Nct} = (\frac{\alpha}{\psi})^{\frac{-1}{1-\alpha}} \hat{p}_{Nct}^{\frac{\varphi-1}{1-\alpha}} A^{-1}_{Nct} ((Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}})^{\frac{1-\alpha-\varphi}{1-\alpha}} \tag{3.71}$$

which is the standard form of the demand of workers. It is clear that the main difference between the two sectors is given by the Net zero emission target that substitutes the production side and the higher price effect in the clean sector. Now it can be easily demonstrated that the equilibrium relative demand of worker in net zero takes the same form of the scenario without it [30].

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} \tag{3.72}$$

Implementing the policy in the market equilibrium we obtain

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d} [\frac{1 + \gamma \eta_c s_{ct}}{1 + \gamma \eta_d s_{dt}}]^{-1-\varphi} (\frac{A_{Nct_{-1}}}{A_{Ndt_{-1}}})^{-\varphi} (1 + \tau_O)^{\epsilon} \tag{3.73}$$

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[28] In fact, this policy can be applied at every environmental quality stopping it to reach the disaster. In most of these critical scenarios the social planner would require to stop the dirty production.

[29] This relationship is always satisfied by the fact that the only variable that is fixed is the productivity $A_{Ndt}$, meanwhile the other can be changed as the demand of workers and the price to satisfy this condition. In fact, assuming that the intermediate dirty production in the South is growing, or the environmental quality is sufficiently small, the social planner can reduce the amount of workers to balance these effects.

[30] To see the demonstration see the appendix

The reason of this solution derives from the fact that the externalities has been eliminated requiring a cap on the production of dirty intermediate output such that the economy cannot be degraded. The result of this cap is that the economy follows the same rules in the planned and market equilibrium with an upper limit on the dirty output.

In the next part I generalize this framework adding transboundary pollution where we see the effect of the tax on the relative demands.

### 3.7.1 Net zero with transboundary pollution

Now I extend the framework above adding transboundary pollution. This implies that we are rewritting the environmental constraint in eq() to satisfy our condition. Now to satisfy a net zero emission target the environmental target takes the form of

$$(\zeta_G + \zeta_L)Y_{Ndt} + \zeta_G Y_{Sdt} \leq \delta S_{Nt} \tag{3.74}$$

where the total emissions that affect the North must be not greater than the regeneration rate of the northern environment. Now, in northern terms the environmental target can be represented as

$$Y_{Ndt} \leq \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} \tag{3.75}$$

where the dirty production must be lower than the difference between environmental regeneration and transboundary pollution generated by the South with respect to the emission rate, which is again our cap on the dirty production.

To study the problem we relax the target focusing on the critical point where the relationship is satisfied such that

$$Y_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} \tag{3.76}$$

This relationship implies that the maximum amount of production must satisfy this equality. This formulation allow the intermediate output to not degrade the environmental quality avoiding a disaster [31]. This implies that the optimum production takes the form of

$$Y^*_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} = (\frac{\alpha p_{dt}}{\psi})^{\frac{1}{1-\alpha}} A_{Ndt} L_{Ndt} \tag{3.77}$$

In other words, the optimum production must be fixed to the net zero emission target[32]. As a consequence the demand of worker in the dirty sector is defined as

$$L_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} A^{-1}_{Ndt} (\frac{\alpha p_{dt}}{\psi})^{\frac{-1}{1-\alpha}} \tag{3.78}$$

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[31] In fact, this polic can be applied at every environmental quality stopping it to reach the disaster. In most of these critical scenarios the social planner would require to stop the dirty production.

[32] This relationship is always satisfied by the fact that the only variable that is fixed is the productivity $A_{Ndt}$, meanwhile the other can be changed as the demand of workers and the price to satisfy this condition. In fact, assuming that the intermediate dirty production in the South is growing, or the environmental quality is sufficiently small, the social planner can reduce the amount of workers to balance these effects.

Now comparing the last equation with the demand of clean labour we obtain

$$L_{Nct} = (\frac{\alpha}{\psi})^{\frac{-1}{1-\alpha}} \hat{p}_{Nct}^{\frac{\varphi-1}{1-\alpha}} A_{Nct}^{-1}((Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}})^{\frac{1-\alpha-\varphi}{1-\alpha}} \tag{3.79}$$

The social planner requires again that the marginal production of the intermediate output must be

$$\frac{\partial L}{\partial Y_{Nct}} = [Y_{Nct}^{\frac{1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] = \hat{p}_{Nct} \tag{3.80}$$

$$\frac{\partial L}{\partial Y_{Ndt}} = [Y_{Ndt}^{\frac{1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] = \hat{p}_{Ndt}(1 + \frac{\zeta_G \omega_{St+1}}{\lambda_{Nt} p_{Ndt}}) \tag{3.81}$$

Where the tax is represented as follows

$$\tau_O = \frac{\zeta_G \omega_{St+1}}{\lambda_{Nt} p_{Ndt}} \tag{3.82}$$

which is the compensation for the transboundary pollution that has affected the South.

The relative labour force is represented as

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1 + \tau_O)^{\epsilon} \tag{3.83}$$

To see the proof refer to the appendix (H)
Given the information above it is clear that the demand of labour in the clean sector is given by

$$L_{Nct} = \frac{\frac{A_{Nct}}{A_{Ndt}}^{-\varphi}(1 + \tau_O)^{\epsilon}}{\frac{A_{Nct}}{A_{Ndt}}^{-\varphi}(1 + \tau_O)^{\epsilon} + 1} \tag{3.84}$$

and by consequence, the demand of dirty labour, which is defined by $1 - L_{Nct}$, is as follow

$$L_{Ndt} = \frac{1}{\frac{A_{Nct}}{A_{Ndt}}^{-\varphi}(1 + \tau_O)^{\epsilon} + 1} \tag{3.85}$$

Which is the same we see in the previous sections. The main difference is that the tax is lower due to the fact that the local component of the tax, $\frac{\omega_{Nt+1}(\zeta_L+\zeta_G)}{\lambda_{Nt} P_{Ndt}}$ is embodied inside the net zero.
To summarize, we the main findings of this paper are: 1) when the clean and dirty intermediate output are gross substitute and the dirty sector is the most advanced it exists only a solution where the dirty technology grows leading to an environmental disaster; 2) In order to prevent an environmental disaster with an unilateral policy the tax needs to increase depending on the productivity levels of the South; 3) a tax on intermediate dirty output to be optimal must embody the pollution received by the other country; 4) the social optimum can be implemented through a combination of

carbon taxes and an abatement subsidy on the intermediate output;5) given net zero assumption, the social planner introduces a quota on the demand of dirty machines; 6) given the net zero assumption, a sufficiently small relative pollution is strong enough to redirect the innovation without a subsidy on the clean research.

## 3.8 Numerical simulation

### 3.8.1 Methodology

In this section I run a simulation under four different scenarios to see the effect of different characteristics on the welfare of the representative household. As in Acemoglu et al (2012) [1] in every simulation each period correspond to 5 years and the other variable are defined as: the quality of the innovation $\gamma = 1$, the share of inputs $\alpha = 0.33$, the probability of innovate in each sector $\eta_c = \eta_d = 0.02$,the risk adversion $\sigma = 2$, the elasticity of substitution $\epsilon = 3$ and the cost of machines $\psi = \alpha^2$. Following Nordhaus assumption, the discount rate $\rho = 0.015$ following Nordhaus (2007)[14] specification.

Meanwhile, I decided that the probability to imitate in the South is $\kappa_c = \kappa_d = 0.04$[33]. In fact I assume that the probability to copy is at least the double the one to create a new technology following the assumption of Barro and Sala I Martin in Economic Growth[5]. The initial value of the technologies, which are the same for every simulation, are represented as follow: $A_{Nd0} = 0.5, A_{Nc0} = 0.4, A_{Sd0} = 0.4, A_{Sc0} = 0.3$. In the paper of Acemoglu et al 2012[1], the productivity levels are different from mine, but, since I am doing a qualitative analysis on the main variables of the economy that are in an interval between 0 and ?1 I chose values that give me consumption and production not to far from 1. Otherwise we could not be able to see the effect of taxation simultaneously on consumption and environment because the latter would be too close to the x-axis. The values of the average technologies depends by the fact that the dirty sector must be more advanced than the clean one, otherwise the scientists innovate only in the clean sector avoiding the pollutive production. Also, since there is no capital in this model, I impose that the North is more advanced than the South to represent the difference between a developed country and a developing one.
The initial environmental quality is at full capacity, $S_{it} = 1$ and $S_{i0} = 1$. Also, in this section I substitute the upper value of the environmental quality, $\bar{S}$ with 1 implying that $S \in (0, 1)$. This simplification satisfies the Inada-conditions at the beginning of the model. Also, the simulation runs for 100 periods, and since the tax is established by the environmental marginal utility of the successive period, at time $t = 100$ it is set to zero. Also, as we have seen in the past sections, most of the shadow prices recall in their equations their future values[34], so I impose their terminal values equal to zero. The respectively are, the shadow value of an innovation in the sector $j = c, d$ in country $i = N, S$ $\mu_{ij100} = 0$ and the shadow value of the environment $\omega_{i100} = 0$.

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[33] This decision depends by the fact that it is easier to copy and adapt an existing technology instead of creating a new one.

[34] In fact they are iterative functions and they depends by the expectations of the future.

The global and local emission rates are respectively $\zeta_G = 0.2$ and $\zeta_L = 0.3$ meanwhile on the other side the regeneration rate is $\delta = 0.25$ and the initial abatement technology is $\zeta_{A0} = 0.05$. I used the software Maple generating two loops to solve the system of equation that describe the economy. The first loop runs the first period where the social plan maximizes the economy without the tax. The second loop last 9 periods where the tax is in place, meanwhile the final one runs for the remaining 90 periods. The main difficulty of this simulation is given by the high recursion of the variables. In fact, the tax $\tau_O$ depends by the marginal utility, which is influenced by the consume of the final output generated with the labour which varies with the value of the tax. To avoid this problem I ask the software to solve the second loop without the tax on the labour at the first attempt. This allows the software to find the terminal points that do not change because at time $t = 100$ the shadow value are not influenced from their future values since they are equal to zero. Then, since we have some terminal points I insert the tax in the labour asking the two loops to repeat in sequence until they have fixed all the values. Doing this way the values are fixed going backward from the terminal condition allowing us to obtain the optimal path. I could say that given initial conditions I am asking the simulation to find a path that goes to some condition and then I require it to adjust it to the optimal trajectory. In this model, as the one of Acemoglu, each period represents 5 years.

In the next subsections I describe the plots from three different scenarios. In order the scenarios are: 1)with abatement technology ;2) without abatement technology ; and 3) with net zero and the abatement technology. They are shown for the first ten periods, $t = 1...10$ to show the short run transition and for $t = 1...100$ to show the long run economy.

In the final part I analyse the scenarios with abatement technology trying to eliminate the distortions generated by the exogenous abatement technology working on the final output invested focusing on the net zero emission target in full detail. In this part of the analysis I use a mix of policy, without trans-boundary pollution and with it to understand what is the best transition to prepare the economy to enter the emission target. Therefore, I conclude the numerical simulation with the net zero emission target removing the abatement technology to understand the welfare difference given by an exogenous technology.

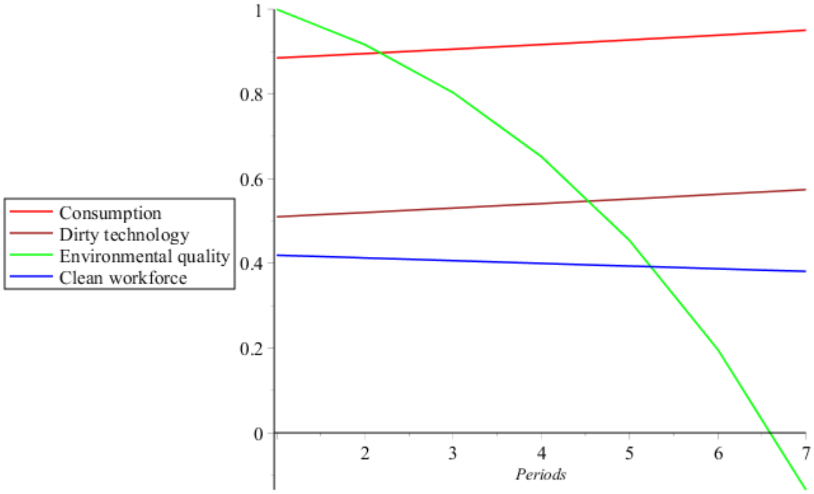
The aim of the numerical simulation is to show the transition from a dirty economy to a sustainable one where the consumption and production growth rate change at every period. In fact we are studying the entire equilibrium time paths generated by different policy regimes for each scenario.

| Summary | | | |
|---|---|---|---|
| Scenarios | environmental constraint | policy target | policy instruments |
| I Lassaiz-faire | Transboundary pollution | None | None |
| II Abatement technology | Transboundary pollution | Restoring full environmental quality | Carbon taxes and abatement technology |
| III Transboundary pollution | Transboundary pollution | Restoring full environmental quality | Carbon taxes and compensations |
| IV Net zero emission | Transboundary pollution | Net zero emissions | Quotas and compensation |

### 3.8.2 Scenario I: Lassaiz-faire equilibrium

In this subsection I show the numerical simulation under laissez-faire to show the direction of the economy and the inevitable environmental disaster. This effect is presented in the next three plots that reflect respectively the northern economy, the southern and the growth of the dirty technology in both countries.

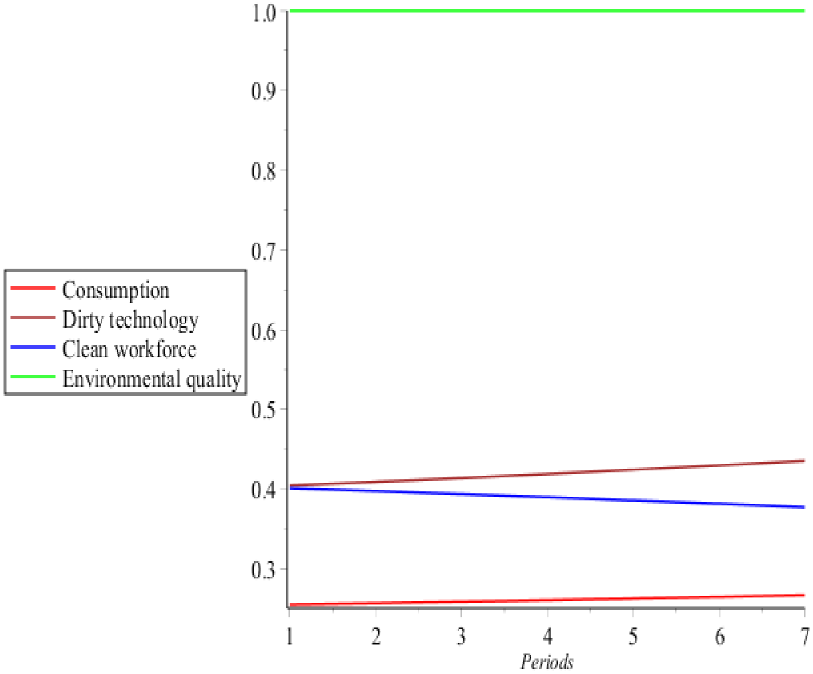
Figure 3.1: The red curve represents the consumption, the brown one the evolution of the dirty technology over time, the green curve is the environmental quality and the blue line is the labour force engaged in the clean sector. This plot represents the evolution of the economy in the North until it meet an environmental disaster.



We can see from the figure(3.1) that the economy increases the consumption of final goods over time thanks to the dirty sector that becomes more productive. In fact the brown line that represents the innovation in the dirty sector grows over time at the same rate, $\gamma\eta_d$. Meanwhile, an increase in the dirty technology implies a decrease in

the relative productivity $\frac{A_{Nct-1}}{A_{Ndt-1}}$ which is reflected in the relative demand of labour $\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}$ diminishing the demand of clean labour as it is shown by the blue curve[35]. Since the relative productivity goes to zero over time the relative prices are affected by the relation $\frac{p_{Nct}}{p_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}$ which is also going to zero enhancing the direction of technology to the dirty sector. The environmental quality goes to zero at $t = 7$, [36] and the economy faces an environmental disaster. As we can see once the environment starts degrading more than its regenerating the quality quickly decreases to a non turning point. Also, due to the high damage only a draconian policy could change this direction.
In the South, figure( 3.2), we have the same behaviour, but delayed due to the lower initial technology. Also, we can see that the environment in the South does not degrade due to the lower productivity. In fact, $Y_{Sdt}$ is not enough large to overtake the regeneration rate. Just to compare, in the North the dirty production at time $t = 6$ is equal to 0.6719 meanwhile in the South is equal to 0.299. This difference is also expressed by the differences in the consumption level.

Figure 3.2: The red curve represents the consumption, the brown one the evolution of the dirty technology over time, the green curve is the environmental quality and the blue line is the labour force engaged in the clean sector. This plot represent the evolution of the economy under lassaiz-faire in the South



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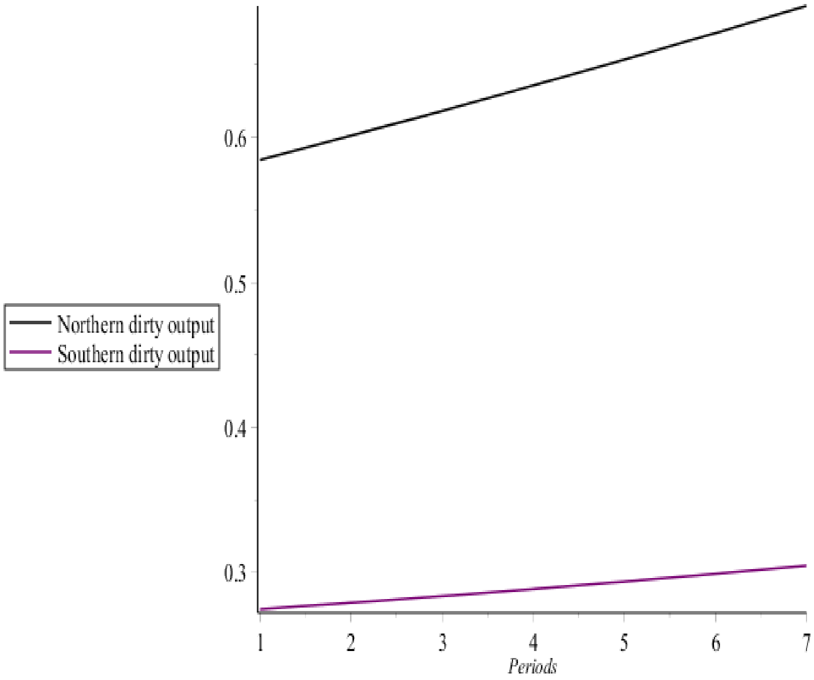
[35] In laissez-faire the relative demand of labour is defined by $(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}$ where $\varphi$ is negative for $\epsilon > 1$. This is a consequence of the construction of $\varphi$ chosen by Acemoglu et al 2012.

[36] To be precise, at $t = 7$ the environmental quality is equal to $-0.13$ but by construction the environmental quality cannot be below zero, or more formally $S_{it} \in (0, 1)$. This is the explanation of the change of slope between time $t = 6$ and $t = 7$.

This delay in the environmental degradation allows us to switch the direction of technical change before it is to late avoiding an environmental disaster in the South.[37] In both plots we can see that the dirty sector is getting more productive and as a result the percentage of clean worker is reducing. This difference of the initial technological values $A_{Nd0} = 0.5$ and $A_{Sd0} = 0.4$ is reflected in the dirty production by the following plot, figure 3.3.

Figure 3.3: The black curve represents the dirty production in the North under lassaiz-faire meanwhile the purple one represents it in the South.



These production levels explain why the environment in the South is not damaged in the plots before. In fact once the technological level reach an enough large level the production of intermediate output exhibits exponential behaviour. To see it we should run the simulation for other periods but this would not be possible due to the environmental disaster that the North is facing at time $t = 7$. Also, this lower level of production in the South is the reason why we have enough time to implement an unilateral policy in the North.

In the next subsection we will see the transitions and the long run that avoid an environmental disaster.

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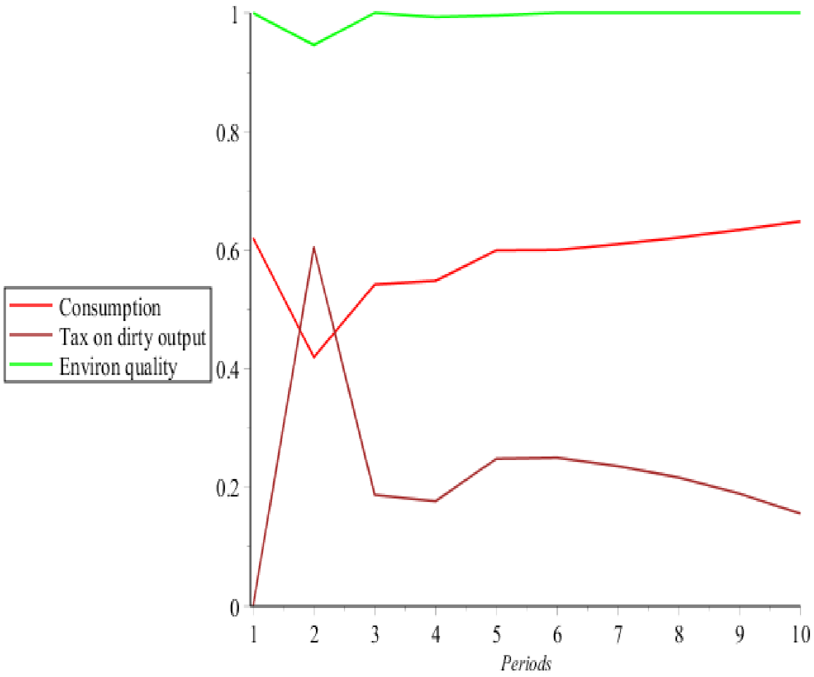
[37] As it is explained in Acemoglu et al 2014 [2] and in equation 3.73, the direction of technical change in the South depends by the relative profits of an imitation $\frac{\Pi_{Sc}}{\Pi_{Sd}} = \frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi-1}\frac{A_{Nct}}{A_{Ndt}}$. When this ratio is bigger than 1 the South imitate clean technology and it is clear from the equation we have seen before that it strongly depends from the northern relative technology. This means that to avoid an environmental disaster through unilateral policy we need to increase the relative productivity of the North before the dirty sector in the South becomes to advanced degrading its own environment.

### 3.8.3 Scenario II: Abatement technology

We correct the externality created by the dirty production introducing a tax that increases the price of the intermediate dirty input. In figure (3.4) the consumer will enjoy a lower consumption compared to the one without abatement technology (Scenario III). This is a consequence of the nature of the abatement that is seen as a fixed cost even if it is an investment. I am assuming that the North invest up to the 20% of its resources in abatement technology, and once its value is equal to the emission coefficient,$\zeta_G + \zeta_L$, it invest the 10% of its output in maintenance costs. The first evidence is that the economy that invest in abatement technology has a lower consumption but also a lower tax compared to the scenario without the abatement technology (Scenario III). It means that investing in the reduction of the pollution decreases the necessity of a tax on the final output.

Figure 3.4: The green curve represent the environmental quality, the red one the consumption, the brown one the tax on the output. In this plot there is abatement technology and trans-boundary pollution. This plot represents the transition until the economy has the abatement technology that eliminates almost all the emissions



The social planner sees that the environment decreases and apply a tax, which decreases the consumption by 0.094%. This recession depends by the switch to a less productive sector. Once the environment has been restored to its maximum level the tax is decreased to a level where the percentage of workers in the clean sector is 3/5 of the total workforce. The social planner keeps this level to avoid damages to the environment and implicitly she is increasing the social gain of an innovation in the clean sector. The tax grows slowly thanks to the abatement technology that reduces

the need of the tax at every period.[38] At period $t = 2$ the tax reaches its maximum level due to a lower utility of consumption and a higher environmental shadow value which is described as $\omega_{it} = \frac{1}{(1-\rho)^t} C_{it}^{1-\sigma} S^{-\sigma} + (1+\delta)\omega_{it+1} v$ where $v$ is a dummy of value 0 when the environment is fully restored, and of value 1 when it is damaged[39]. At period $t = 3$ the environment is restored and the consumption increases of a 5% due to the tax decreasing to a substantial level, from 60% to 20%.
The figure (3.5) shows the evolution of the economy during all the periods. Since I already discussed what happened in the first 10 periods, I will discuss what happens from period $t = 10$ to period $t = 100$. The abatement technology reach its maximum level at $\zeta_{At} = 0.49$[40], and the economy stop investing in new technology. Instead the investment is reduced to a 10% in maintenance cost. As a result the consumption increases of a 21% continuing to grow over time. This jump implies a substantial reduction of the marginal utility of consumption. We must remember that the tax is defined as $\tau_O = \frac{\omega_{Nt+1}(\zeta_L + \zeta_G - \zeta_{At}) + \omega_{St+1}\zeta_G}{\lambda_{Nt} P_{Ndt}}$, where $\lambda_{Nt}$ is the marginal utility of consumption. The marginal utility of the environment of both countries represented on the top of the fraction by $\omega_{Nt+1}$ and $\omega_{St+1}$ are decreasing over time, but not as fast as the marginal utility and as a consequence the value of the tax increases slowly over time. Also, the price is slowly increasing over time but it does not compensate the decrease of the marginal utility. As well as the difference between the emission rate and the abatement technology. This behaviour create the necessity of a tax even when it is not needed any more, in fact the direction of technology has been established to be clean and it cannot be reversed without a new policy that affect the clean sector.

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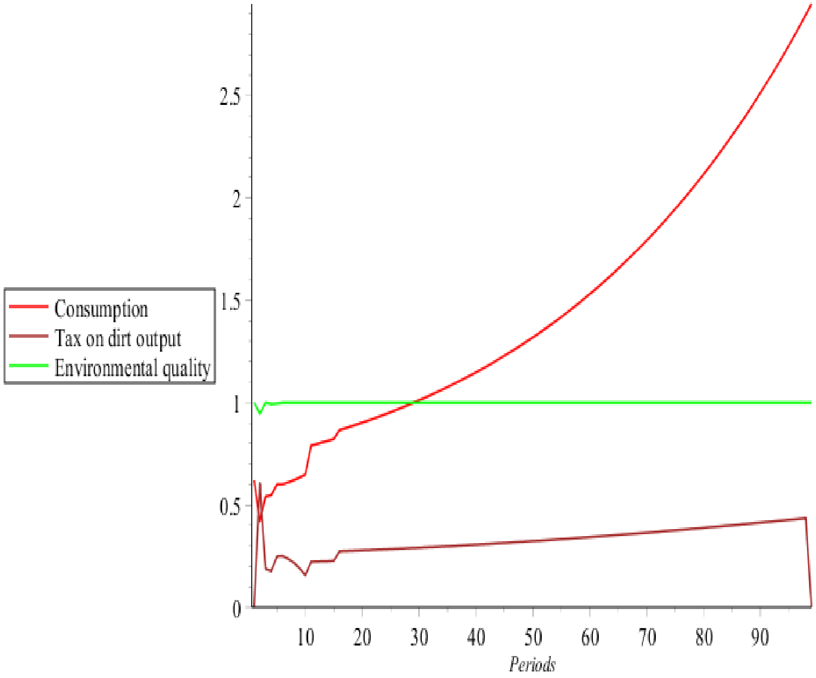
[38] This effect depends by $\tau_O = \frac{\omega_{Nt+1}(\zeta_L + \zeta_G - \zeta_{At}) + \omega_{St+1}\zeta_G}{\lambda_{Nt} P_{Ndt}}$ where higher is the value of the abatement technology $\zeta_{At}$ smaller is the effect of the emissions decreasing the necessity of a tax.

[39] This formulation is first introduced by Acemoglu et al 2012 [1]. It implies that when the environment is damaged the representative household is worried of its future quality meanwhile when the quality is at its maximum level she is not worried about the future.

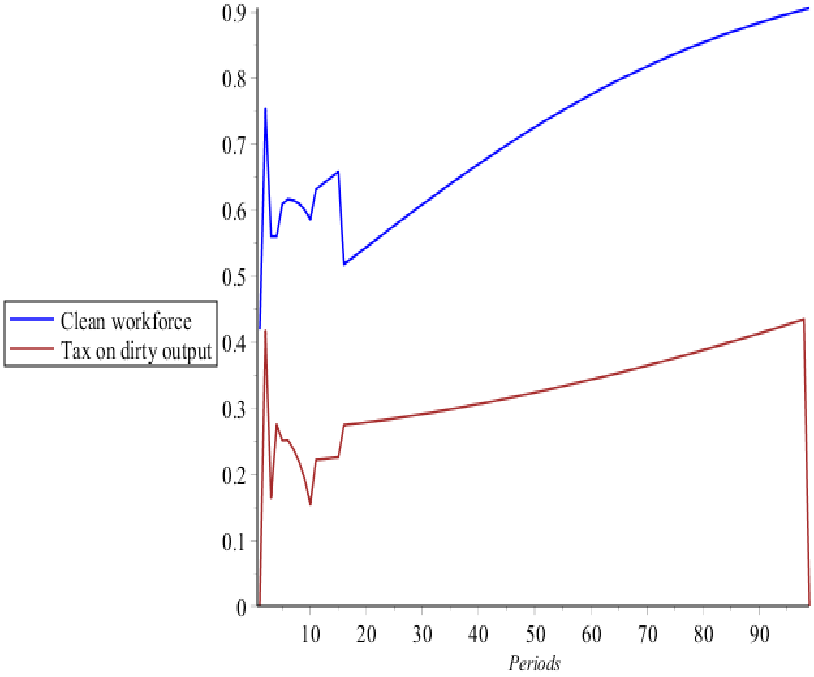
[40] I assumed before that the abatement technology cannot eliminate all the emissions since it is something physically impossible.

Figure 3.5: The green curve represents the environmental quality, the red one the consumption, the brown one is the tax on the intermediate dirty output in the North. In this plot there is abatement technology and transboundary pollution



In figure (3.6) we can see that at the highest tax level corresponds to the highest level of worker engaged in the clean sector and when the tax is equal to zero at time $t = 10$, the clean workforce is reduced to 0.47 which is greater than its initial value 0.4193at time $t = 1$ due to the increased productivity in the clean sector over the time horizon.

Figure 3.6: The blue curve represents the percentage of workers employed in the clean sector having an abatement technology in the North. The brown curve represents the tax applied to the dirty intermediate production.



The clean labour forces is clearly influenced by the tax and apparently it mimics its behaviour. In fact when the tax decreases there is a corresponding decrease in the percentage of workers in the clean sector. Also, at the final time when the tax $\tau_O$ is equal to zero the percentage of worker is around the 90% due to the innovations that the clean sector has enjoyed.

### 3.8.4 Scenario III: Transboundary pollution without abatement technology

In this subsection we run the simulation with the same variables as before, except that now there is no abatement technology. The first thing is the higher consumption due to the absence of the investment, but at the same time the lack of a reduction in the emission production implies a lower environmental level because it is more damaged. This requires an higher tax on the intermediate output. In figure (3.7) the consumption decreases of 14% from $t = 1$ to $t = 2$. At time $t = 2$ when the tax is applied the percentage of workers in the clean sector reaches its higher level of 72%. Then the positive change of the environment decreases the tax to 0.51 and is almost stable over time.[41]

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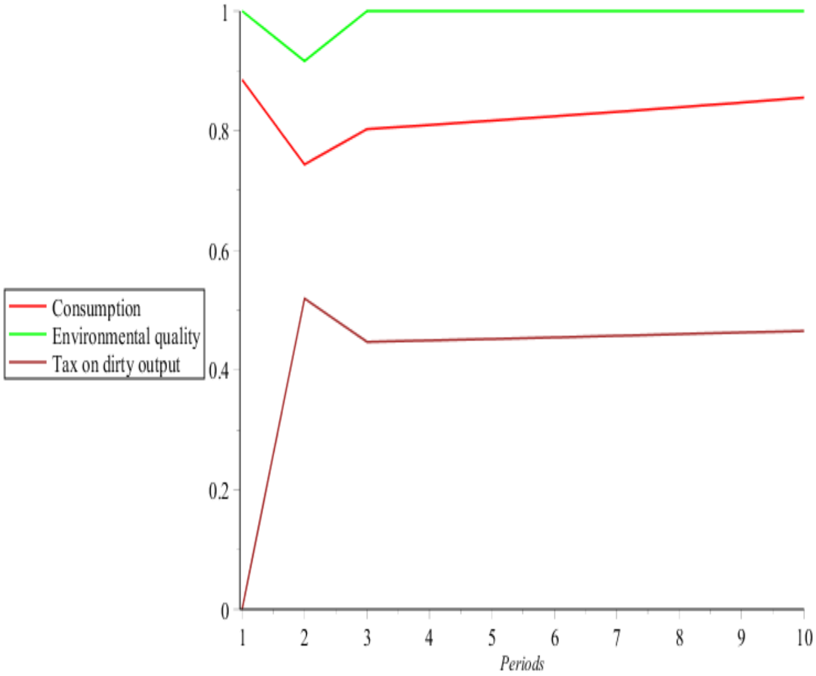
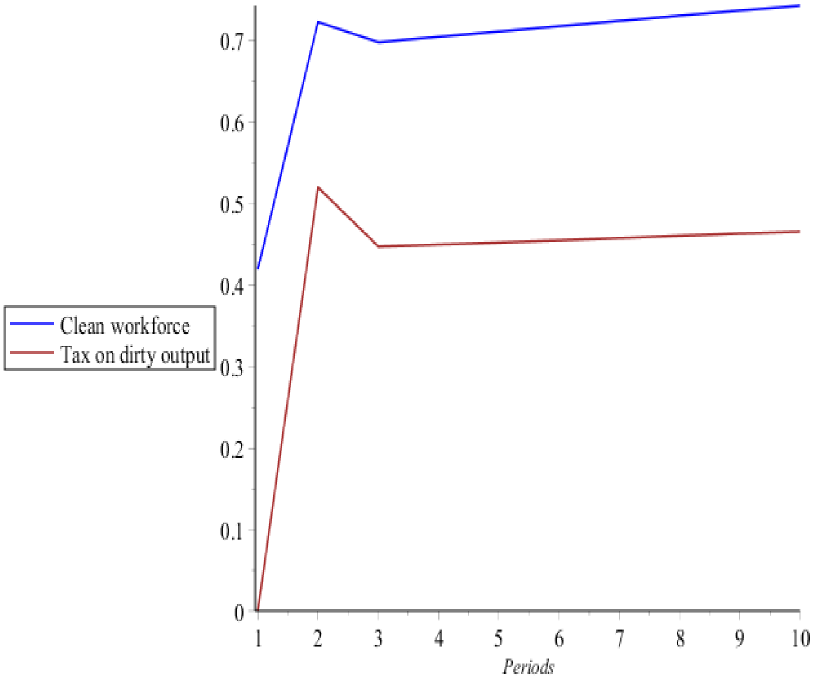
[41] It changes over time in the range of $\pm 0.01$ from the range 0.57 ending at time $t = 9$ with value 0.568

Figure 3.7: The green curve represents the environmental quality, the red one the consumption, the brown one the tax on the intermediate output in the North. There is trans-boundary pollution but no abatement technology



The next plot, figure(3.8) represents the demand of clean labour in the short run

Figure 3.8: The blue curve represents the clean workforce and the brown one the tax on the intermediate output in the North. There is trans-boundary pollution but no abatement technology



where the clean workforce perfectly mimics the evolution of the tax. The demand of workers grows at a faster rate due to the increased productivity in the clean sector. Then, considering the range $t = 10...100$ we can see in figure (3.9) that the consumption grows exponentially over time without the jump we saw in the last subsection. This is a consequence of the absence of an abatement technology and the change of investment from 20% to 10%. What it is important is the tax that increases the cost of the intermediate dirty output compared to the lassaiz-faire (Scenario I). This is a consequence of the shadow value of the environmental quality that decreases slower than the marginal utility of consumption. The explanation of this effect is given by the growth of the dirty production that grows at a logarithmic rate due to the evolution of the clean technology.[42] It means that the tax needs to reduce the pollutant input even when it is more competitive to use clean inputs. In this scenario, the clean output is growing exponentially and is used by the final producer to assemble the final good. On the other hand, the final producer still needs a small amount of dirty input, and this tax reduces the use of it. Imagine we are making a table using two components, wood and plastic. Increasing the clean technology implies that we use less plastic over time per unit of final good, but creating more goods over time requires us to use more plastic than before due to the higher number of tables produced. In other words, a larger economy requires less dirty input for each unit of final output but more in absolute term due to an higher production of final output.

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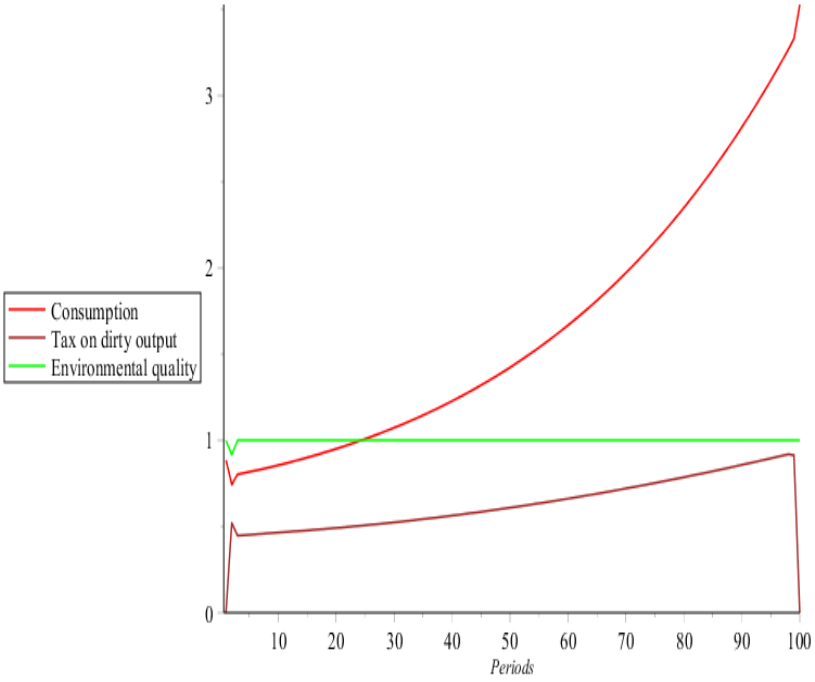
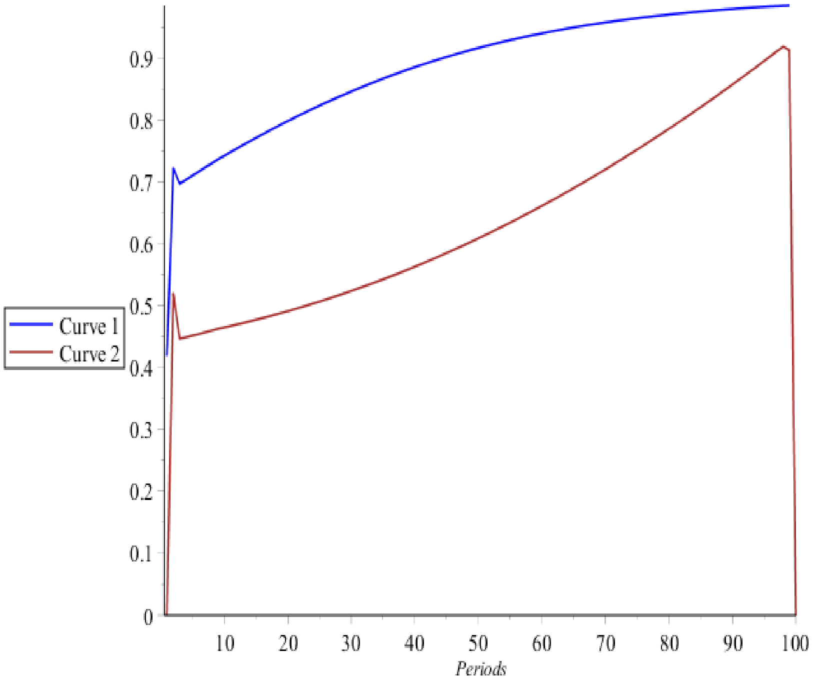
[42] In fact the equation that represents the equilibrium intermediate production shows limited complementarity in both technology even when we assume the perfect substitution of the inputs.

Figure 3.9: The green curve represents the environmental quality, the red one the consumption, the brown one the tax on the intermediate output in the North. There is no abatement technology



In figure (3.10) the labour force is increasing over time because the clean sector is getting more productive and there is no abatement technology that decreases the tax. At time $t = 3$ the tax decreases because the environment has been fixed but, the social planner keeps it to avoid a possible future damage and grows over time to keep high the workforce in the clean sector.

Figure 3.10: The blue curve represents the percentage of workers employed in the clean sector in the North. The brown curve represents the tax applied to the intermediate dirty output. We have no abatement technology.



It is worth noting how the demand of labour mimics the evolution of the tax over time. Still, the tax in Scenario II strongly decreases the consumption, the percentage variation is greater compared to the other one.

### 3.8.5 Scenario IV: Net zero and no abatement

In this subsection we run the numerical simulation with the variables I have used before, except that now there is not an abatement technology. The first thing is the higher consumption due to the absence of the investment, but at the same time the lack of a reduction in the emission production implies a lower environmental level because it is more damaged. The consumption decreases less from period $t = 1$ to period $t = 2$ because the local tax is embodied inside the production function, which requires that the optimum dirty output must be not greater than $\frac{\delta S_{Nt}-\zeta_G Y_{Sdt}}{\zeta_G+\zeta_L}$ [43]. This implies that the tax on the intermediate output has a larger value compared to the scenario without net zero (Scenario III). In fact, the social planner applies a cap on the maximum dirty production allows the economy to stop degrading the environment, which combined with the tax that takes the form of a compensation reorganizes the economy restoring

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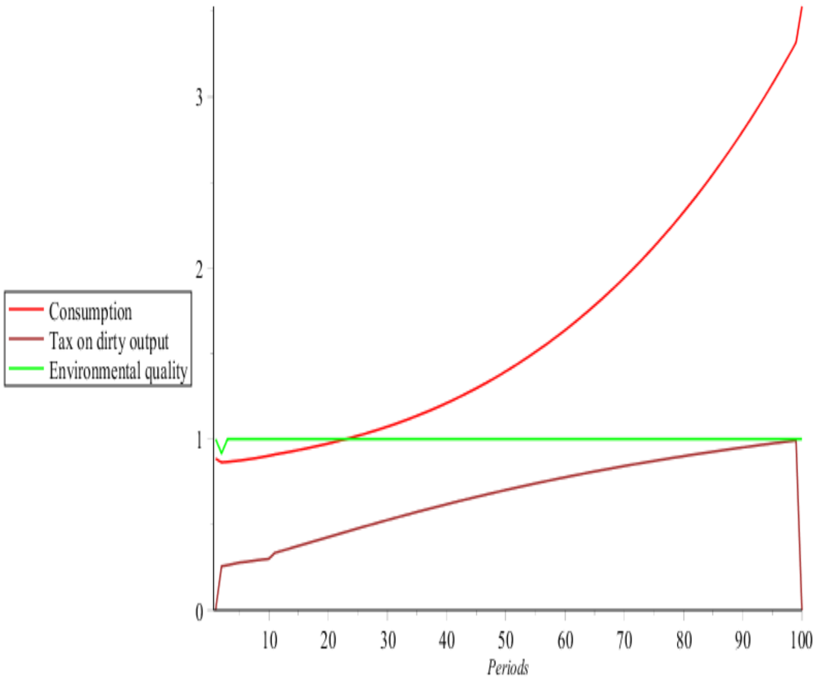
[43] To do this I ask Maple to use the procedure if: if the intermediate output is greater than $\frac{\delta S_{Nt}-\zeta_G Y_{Sdt}}{\zeta_G+\zeta_L}$ it must be fixed to this value, otherwise to be equal to $(\frac{\alpha p_{dt}}{\psi})^{\frac{1}{1-\alpha}} A_{Ndt} L_{Ndt}$. In this simulation almost all the values are below the net zero threshold, $\frac{\delta S_{Nt}-\zeta_G Y_{Sdt}}{\zeta_G+\zeta_L}$ , except the one in the transition from $t = 2$ to $t = 5$

the environment. The compensation can be used to subsidize the imitation process in the South allowing it to switch faster to clean production.

Figure 3.11: The green curve represents the environmental quality, the red one the consumption, the brown one the tax on the intermediate output in the North. There is transboundary pollution but no abatement technology



In the plot, figure(3.11), we can see that the tax reaches an initial level of 33% and after that increases steadily due to two factors. The first is the consumption in the North that increases its marginal value. The second is marginal utility of the environment in the South, that still depends by the southern consumption.[44] At period $t = 100$ the tax is equal to 0 and allows the economy to use more intensively the dirty output which increases the final output and consequently the consumption .[45]

In the plot(3.12) we can see how the labour force in the clean sector reacts to the tax that compensate the South for the pollution generated by the North. At period $t = 1$ the labour force is the 41% of the total work force, but after the tax is introduced in period $t = 2$ it reaches the 60% of it. Then it grows with a logarithmic path until period $t = 99$ with the value 98% implying that almost all the population is engaged in the clean production. At period $t = 100$ when the tax is eliminated the clean workforce reaches the level of 90%, which is its optimal equilibrium value given by the

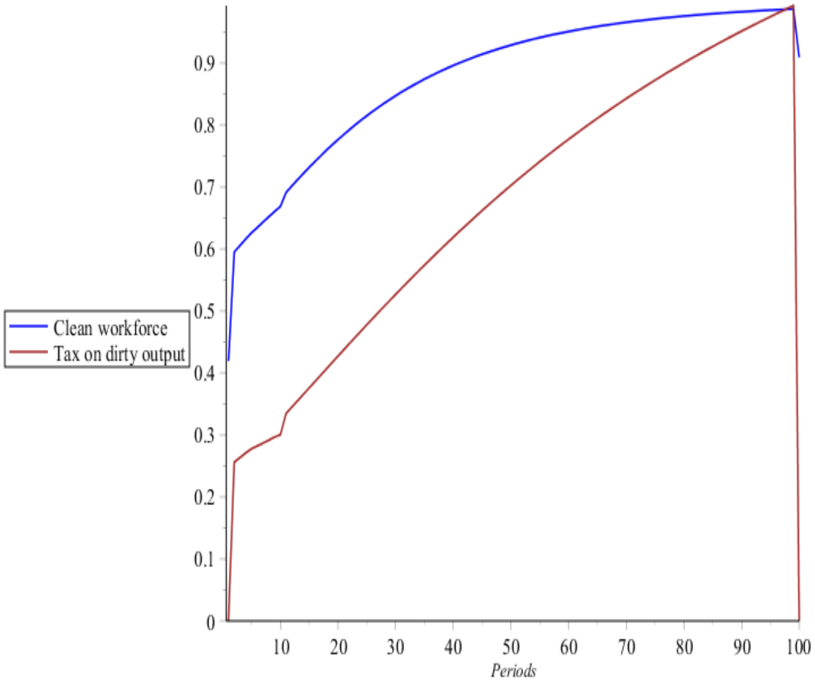
---

[44] We must remember that the tax is defined as $\tau_{Ot} = \frac{\omega_{St+1}\zeta_G}{p_{Ndt}\lambda_{Nt}}$ But the main factors are $\omega_{St+1}$ and $p_{Ndt}\lambda_{Nt}$ and their ratio establishes the value of the tax.

[45] To better understand this we must consider the marginal productivity of each factor and it will be more clear in the next plot. The main point is that the clean output is overproduced to balance the demand of final good meanwhile the dirty one is underused. Eliminating the tax bring the economy to have the marginal productivities equal to their prices and therefore obtaining the maximum amount of final good.

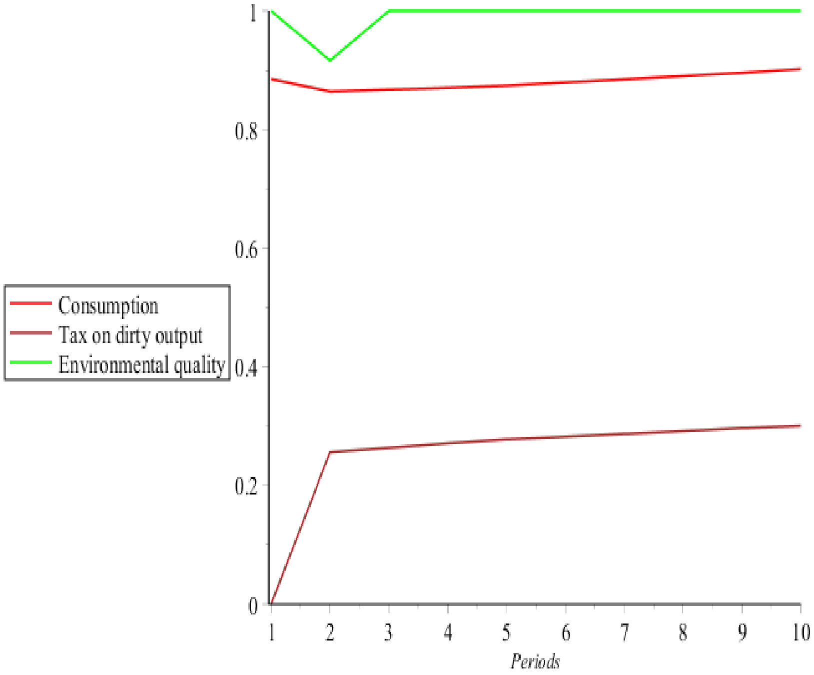
relationship $L_{Nct} = \frac{\frac{A_{Nct}}{A_{Ndt}}^{-\varphi}}{\frac{A_{Nct}}{A_{Ndt}}^{-\varphi}+1}$ that represents the demand of clean labour in terms of relative productivity.

Figure 3.12: The blue curve represents the percentage of workers employed in the clean sector in the North. The brown curve represents the tax applied to the intermediate dirty output. We have no abatement technology.



The next plots represents the transition period in the range from $t = 1$ to $t = 10$.
The first plot(3.13), shows that after the tax is introduced the consumption grows linearly according to the tax, which reflects the effect of the marginal utility on the policy tool.

Figure 3.13: The green curve represents the environmental quality, the red one the consumption, the brown one the tax on the intermediate output in the North. There is transboundary pollution but no abatement technology



In figure (18) the clean labour force in the first 10 periods perfectly mimics the behaviour of the tax growing at a similar rate[46].

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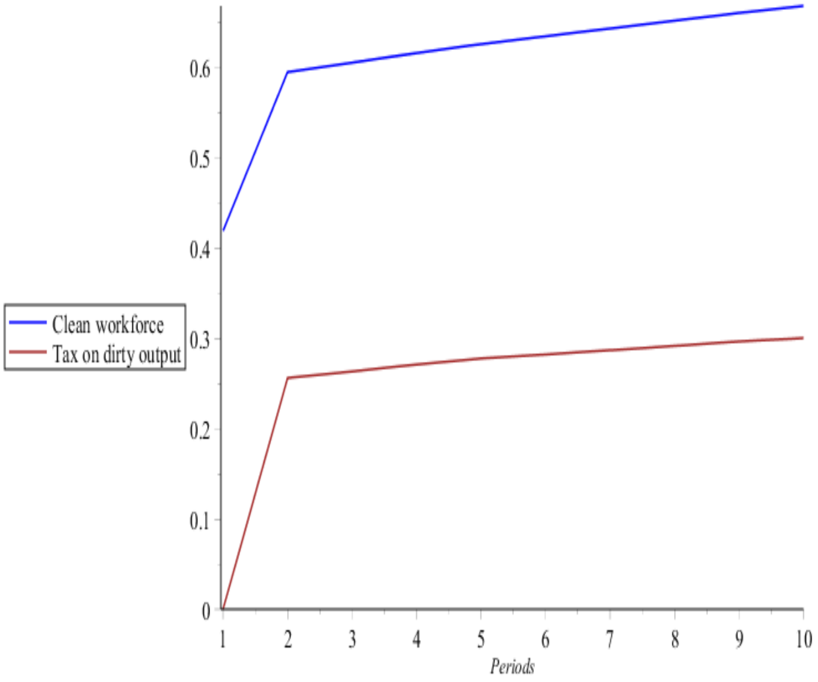
[46] The clean workforce grows faster due to the innovation in the clean sector that pushes the labour force.

Figure 3.14: The blue curve represents the percentage of workers employed in the clean sector in the North. The brown curve represents the tax applied to the intermediate dirty output. We have no abatement technology.



The main difference between this section and the one without net zero (Scenario III) is given by the transition period in the first 10 periods and the value of the tax.

## 3.9 Welfare Analysis

In this section I am investigating the welfare analysis to understand which policy better increases the utility of the representative household focusing on the North [47] concluding with the southern one. This analysis helps us understanding which is the optimum policy to be implemented by a social planner.
I use a CRRA utility function with the following specification

$$u = \sum_{t=1}^{100} \frac{1}{(1+\rho)^t} \frac{(C_{Nt}S_{Nt})^{1-\sigma} - 1}{1-\sigma} \tag{3.86}$$

where $\rho$ ha a value of 0.015, following the specification of Nordhaus (2007)[14], and $\sigma$ of 2. For each scenario we have a different value that we use to understand which policy better solve our problem. The analysis is done for the periods from $t = 1$ to $t = 100$. I start in order following the subsection of the numerical simulation.

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[47] The reason to do it only in the North is due to the fact that the South operates under lassaiz-faire having the same utility under all the different scenarios.

Before starting the analysis of the policy I introduce the lassaiz-faire utility in the North. We already know that at tiime $t = 7$ the environment is fully degraded with value $S_{Nt} = 0$. At this point the utility is represented as $\frac{C_{N7}0^{-1}}{-1}$ which gives $-\infty$.

The first scenario we analyze is the one with abatement technology. The analysis gives as a utility of 0.005. In fact, the utility in the first 29 periods. In the first period the utility is $-0.60$ decreasing in the second period to $-1.47$, growing to $-0.80$ in period 3. In the last period it has a value 0.15. The main reason the utility is so low is given by the abatement technology that is exogenous. In the second period the introduction of the tx, combined with the abatement technology strongly reduces the utility of the representative household. She reaches positive utility at period 29 when the consumption reaches the value of 1.

The next scenario is the one without abatement technology with transboundary pollution. Here the total utility in the whole horizon has the value 7.5. In the first period the utility is $-0.12$ to decrease in the next one to $-0.45$. In the period 3 the value is $-0.22$ then it steadily increase until period 25 where it get positive. Again, period $t = 25$ is the moment where the consumption reaches the value of 1. The final period the consumer has a utility of 0.16.

In the net zero scenario the welfare analysis gives an overall utility negative, with a value of $-28$. The initial value is $-0.60$ and in period 2 of $-2.5$. It can be easily seen in the numerical simulation when the consumption decrease of 49%. The utility reaches a positive level at period 52. This scenario is clearly unfeasible due to the total negative utility. In fact, to be realistic a policy should at least be positive.

To solve this problem I modeled the economy using a tax to prepare the net zero emission target decreasing the investment in abatement technology over time obtaining an utility of $-1.30$. We start with an utility of $-0.60$ at period 1, which decreases to $-1.53$ at period 2 to grow to $-0.80$ in period 3. It reaches a positive value at period 33. In the final period when the tax is eliminated reaches level 1.51. Diminishing the final output invested in abatement technology over time strongly increases the utility of the representative household but, unlikely it is not enough.

It is clear that the abatement technology does not improve the welfare when it is exogenous. To solve this problem I create an economy that enters net zero without abatement technology. Its total utility is 8.44. At period 1 is $-0.12$, decreasing to $-0.25$ in period 2 to reach a positive value at period 23. It grows steadily until it reaches the value of 0.16 in the final period. This can be considered the best policy compared to the other we have seen. The reason is given by the fact that the tax is embodied inside the net zero target, enhancing the demand of labour only with the compensation diminishing the shock on the economy at the initial periods.

The more reliable simulations are the one without abatement technology given the high welfare. Given this reason I start analyzing and explaining in detail why net zero has an higher value compared to the classical scenario refering respectively to the scenario with tranboundary pollution and to net zero without abatement technology. In period 1 they have the same values for labour force, 41%, consumption, 0.88, and final production, 1.32. At period 2, the tax is higher in the first scenario, with a value of 0.51, meanwhile in the second 0.25. At this time the clean labour force is 72% while the net zero scenario has 59% and this differences are reflected in the consumption levels, 0.74 against 0.86. In period 3 the tax is 0.44 compared to the other scenario with .26, the clean workforce is the 69%, which is still higher than the 60% in the best scenario which influence the consumption to the values 0.80 and 0.86. To conclude, in the final period when the tax the consumption has the value of 3.52 and 3.52 which is the same. From this values we understand that both policies reach the same values in the long run, but what is interesting is the transition where the net zero policy reaches higher consumption and sustains a better grow. From these data we understand that the best policy is the one that brings the workers to the less advanced sector more "naturally"[48].

In the South, the representative household has a negative utility to in all periods, which increases over time. Meanwhile it is negative, the marginal utility is still positive, meaning that the economy requires more goods to be consumed. The main reason is the low productivity that is not enough to satisfy a larger production. In fact in all simulations it goes from $-2.87$ at time $t = 1$to $-0.02$ at time $t = 100$ and a total welfare of $-92.7$.

## 3.10 Results

| Results Summary | | |
|---|---|---|
| Results | environmental constraint/target | Mechanism |
| I Proposition 3 | Transboundary pollution | Compensation |
| II Proposition 4 | Abatement technology | Carbon taxes and abatement subsidies |
| III Proposition 5 | Net zero emission | Quotas and cap |

To avoid an environmental disaster under unilateral policy there are two conditions. the first is described in Acemoglu et al. (2014) [2], where the elasticity of substitution must be sufficiently large. The second one is described in my first proposition and requires that the North is sufficiently more advanced than the south. Acemoglu et al.(2014) avoids the environmental disaster uses a combination of carbon taxes and subsidies. In my paper I add the use of compensation and quotas to avoid the ecological disaster.

One of the main effect of an environmental policy will be the change of employment. In fact the labour force must be redeployed in the clean sector to allow a decrease in the dirty production to avoid an environmental disaster. The direct productivity effect

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[48] In fact, it is understandable because before the clean sector is more advanced is less productive, meaning that each worker produce less in that sector compared to the dirty one, and consequentially this switch must be done steadily instead of immediately.

will push the demand of new technology to the most productive one and it needs to be counterbalanced by the other two, the market size and the price effect.

As we have seen in the first proposition, taking into consideration the transboundary pollution, a tax on intermediate dirty output to be optimal must embody the pollution received by the other country. This is a consequence of the nature of the pollution and the tax applied is considered as a compensation for the damage done to the other country. In fact the social planner corrects the externality that affects the South increasing the final tax on the intermediate output. To correct this externality the social planner requires the environmental shadow value of the South, because the pollution is decreasing its environmental quality.

In the section where I outline the optimality with abatement technology I show that the social optimum can be implemented through a combination of carbon taxes, and an abatement subsidy on the intermediate output. This is a consequence of the evolution of the abatement technology that depends by the amount of final good. Here the social planner wants to increase the marginal production to obtain an higher investment. Also, if the production is too high, the subsidy can become a tax reducing the production. This tax/subsidy is used by the social planner to balance the final production to obtain the optimal investment. Also, the shadow value of the abatement technology corrects the percentage of final output invested in removal technology endogenizing it. Given the fact that this shadow value converge to zero over time, it implies that the investment in abatement technology should be reduced over time.

In the net zero subsection we saw a different way to represent the environment. We can say that given net zero assumption, the social planner introduces a quota on the demand of dirty machines. In facts, the social planner do not want to degrade any more the environment and requires a limit to the production which is represented by the relative pollution. To apply this limit she introduces a quota on the demand of dirty machines reducing them, which automatically affects the expected profits of the scientists engaged in dirty research. This phenomenon can be seen also in the social planner solution where the quota is applied also on the shadow value of a dirty innovation in the North.

In the same section we find that given the net zero assumption, a sufficiently small relative pollution is enough strong to redirect the innovation without a subsidy on the clean research. Under this policy the quota is applied also to the social gain of an innovation in the dirty sector. The social planner allocates the scientists to the sector with the higher social gain which is the clean one without applying a subsidy. This is a consequence of the quota that affects directly the quantities , meanwhile the tax affects directly the prices.

The quota affects the price effect and the market size effect, meanwhile the carbon tax affects only the relative labour. Then to counter the direct productivity effect it is necessary a subsidy until the clean technology is greater than the dirty one. On the other hand, the South has not the same technological level as the North, which implies that it needs more time before reaching an environmental disaster. Also, before taking their decision on what technology imitate, the southern scientist want to know what sector has the higher productivity in the North. This means that it is possible to avoid an environmental disaster in the South with unilateral policy if and only if the North acts fast. The tax in the North is necessary for two reasons. The first is to finance the subsidy in the clean research, the second is to prevent people from working in the dirty sector. In fact since the dirty sector has a high productivity without the tax the demand of workers will be greater in the dirty sector until it is the less productive sector. It is curious how changing the environmental constraint moves the externality from one side of the firms to another. In fact in the standard model the externality was solved through a tax on the final sectors which it appears on the marginal products of the intermediate inputs. Using a net zero introduces a quota on the demand of machines used in the intermediate productions. The difference is that in the first scenario I apply a tax on the selling price of the intermediate producer charging the buyer of the cost of the pollution, meanwhile in the second scenario I apply the quota on the demand of machines, changing implicitly the selling price. In the net zero scenario the consumption is lower because the abatement technology has a low value, implying an higher quota, meanwhile in the tax scenario the labour force employed in the clean sector is a large part of the total population, but not as much as in the net zero scenario. This means that a net zero target should be used to avoid an environmental disaster from any level of environmental quality. After the first shock more the abatement technology grows less the new environmental law constraints the dirty production allowing more people to work in the dirty sector compared to the other scenarios.

When we apply the Net zero as a terminal condition the tax is useful to decrease the shock that the economy will face at the end of the transition period. To eliminate this shock the best solution is to reduce the abatement investment meanwhile we are closing that point. This implies that the investment must be done in the initial period to reach the optimal level in the long-run.

In all scenarios we can see a consumption reduction in the first two periods followed by a growing path. We have to remember that still, since we are trying to eliminate the externality the economy is switching the natural evolution of the technology developing the less advanced implying that a social cost is still necessary. The main point is which is the best social cost and the answer belongs to the scenario with abatement technology and trans-boundary pollution.

When we assume that there is no abatement technology a net zero is more beneficial to the economy because moves the workers more gradually in the clean sector allowing a technological switch with a lower shock. This phenomena is represented by a small reduction in the consumption which contradicts the previous results. The main reason , which is fully outlined in the previous section, is given by the composition of the tax, that embodies the local tax inside the dirty production cap. Then the labour force is enhanced by the compensation to the South. This implies that the tax does not depends by the shadow value of the northern environment but only by the southern one.

## 3.11 Conclusion

In this study I discussed the implications of different environmental constraints and targets in a North-South directed technical change model. The scenarios outlined are with trans-boundary pollution, with abatement technology and a net zero emission target. For each environmental problem the social planner introduce a policy to correct the externalities from a mix of taxes and subsidies to a quota on the production. The main effect of a tax on the intermediate output, $\tau_O$ is reflected on the market size effect, meanwhile a quota on the machines affects all the factors that characterize the technical change, the prices and the market size. The tax on the trans-boundary pollution should be used to subsidize the scientist engaged in clean research in the South. It could be difficult for an economy to satisfy a net zero emission target but it is possible to decrease the shock of the transition through taxes that help the clean sector. In other words the taxes and new technologies will allow the transition to net zero emission "less painful" for the representative household. This is the way decided by the UK and other countries aiming the Net zero emission target by banning or increasing the taxation on certain goods, such that their economy will be ready to sustain the transition. The net zero emissions is a target because it is supposed to be a terminal point where the emission going to the atmosphere are balanced by the one removed from it. You can always engineer the production to satisfy it but the real issue is the social cost of this decision, which is still a better decision than an environmental disaster. The best way to reach it is consistent with the policy adopted by the COP26 Glasgow, requiring steps to be reached before the terminal point. This has been decided to avoid the social losses that I show suddenly introducing the policy. Appling the policy only in the North allow us to see the development of the southern economy without shocks letting her enjoy the same growth that the most developed countries had during the second economic revolution. This could be a way to reduce the differences that we observe in the countries worldwide.

The point is what is the social cost that the economy is ready to suffer to reach this target, remembering that this is a long period target that allows us to avoid an environmental disaster on each path remembering that it is better to have welfare losses instead of an environmental catastrophe. Otherwise, the optimum solution requires to delay the Net zero target until the transition policy has modified the structure of the economy to the point where there is no economic damage. As we can see in the last simulation plots, reducing the amount of final output invested in the economy can eliminate the economic damage of the net zero emission target. Consequently, the economy must invest more at the beginning diminishing the percentage over time reaching the minimum amount entering the new policy.

The future research should be focused on monopolies that trade between countries, meaning that the intermediate firm can buy machines from both countries. This would require endogenous prices for the monopolist but allowing us to reduce the deadweight loss. Also, since the labour plays a ke role, endogenizing the salary would bring new result in this context because the tax affecting the demand of labour would affect the composition of the salaries. A second extension to this model would require endogenous abatement technology. Firstly, because would be automatically adjusted at every period giving us an understanding of the better allocation of the final output in the production of this technology. Secondly, because it reduces the final consumption of the representative household given its investment nature. An exogenous investment does not allow us to understand when it is optimal to allocate resources and offuscate the welfare analysis. Also, the abatement technology should be considered as a positive cost function in the environmental constraint to better reduce the environmental damage over time.

In conclusion it is possible to avoid an environmental disaster through unilateral policy if some conditions are satisfied. The first is that the South is less advanced in both sectors compared to the north such that the south can switch to clean imitation before creating an irreversible environmental disaster. The second one is that the elasticity of substitution is enough high.

## 3.12 Appendix

### 3.12.1 Proof of proposition 1

To prove it a cite Acemoglu et al 2012 demonstration where "Assumption 1 together with the characterization of equilibrium allocation of scientists above implies that, initially, innovation takes place in the dirty sector only ($s_{dt} = 1$ and$s_{ct} = 0$). From (3.12), this widens the gap between clean and dirty technologies and ensures that $s_{dt+1} = 1$ and $s_{ct+1} = 0$, and so on in subsequent periods. This shows that under Assumption 1, the equilibrium is uniquely defined under laissez-faire and involves $s_{dt} = 1$ and $s_{ct} = 0$ for all t."

### 3.12.2 Net zero production function

In this section I demonstrate that to achieve th net zero the social planner requires the maximum production to be

$$Y_{Ndt} = \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L_{ijt}^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{ijkt}^{\alpha} dk \tag{3.87}$$

We remember that a net zero implies that the environment of tomorrow cannot be smaller than the environment of today.

$$S_{t+1} \geq S_t \tag{3.88}$$

but to simplify the study we try to solve it only for the less constraining condition[49] which still satisfies the definition of net zero condition which is expressed as follow

$$S_{t+1} = S_t \tag{3.89}$$

which implies

$$(\zeta_G + \zeta_L) Y_{Nd} = \delta S_N + \zeta_{at} Y_{Nd} \tag{3.90}$$

Now the dirty production under net zero becomes

$$Y_{Nd} = \frac{\delta S_N}{(\zeta_G + \zeta_L)} + \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L_{ijt}^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{ijkt}^{\alpha} dk \tag{3.91}$$

The production under net zero emission must be equal to the relative regeneration plus the relative emissions produced by the production. This means that to satisfy our environmental condition the production should not exced the ability of the environment to heal itself plus the effective emissions that the production create. In fact if the

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[49] The constraint that we are applying is the first step decided by the Paris Agreement since, once you can satisfy it as an equality you have the tools to keep a constant pollution level avoiding an environmental disaster and restoring the environment in a longer period reducing the cost of the technological switch

emission that we can extract from the environment are the same that we emit, $\zeta_{at} = (\zeta_G + \zeta_L)$, we do not require any more quotas . To demonstrate the use of a restrictive production function let's assume that

$$Y^*_{Ndt} = \frac{\delta S_N}{(\zeta_G + \zeta_L)} + \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L^{1-\alpha}_{ijt} \int_0^1 A^{1-\alpha}_{jt} x^{\alpha}_{ijkt} dk \tag{3.92}$$

and that

$$Y_{Ndt} = \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L^{1-\alpha}_{ijt} \int_0^1 A^{1-\alpha}_{jt} x^{\alpha}_{ijkt} dk \tag{3.93}$$

Since

$$Y^*_{Ndt} > Y_{Ndt}$$

It implies that the emission generated are lower as it is shown

$$(\zeta_G + \zeta_L) Y^*_{Ndt} > (\zeta_G + \zeta_L) Y_{Ndt}$$

and given the same regeneration rate the environment increase its future, or

$$S^*_{Nt+1} < S_{Nt+1}$$

This means that solving the production function

$$Y_{Ndt} = \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L^{1-\alpha}_{ijt} \int_0^1 A^{1-\alpha}_{jt} x^{\alpha}_{ijkt} dk$$

assure me that the environment of tomorrow is always better than the present one.

$$S_{Nt+1} - S_{Nt} > 0$$

or in other words that the environment is improving.

### 3.12.3 Intermidiate dirty output

To obtain the dirty output we start from the dirty marginal product of the final producer

$$p_{dt} = Y_{dt}^{-\frac{1}{\epsilon}} [Y_{dt}^{\frac{\epsilon-1}{\epsilon}} + Y_{ct}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} \tag{3.94}$$

$$p_{dt} = [1 + (\frac{Y_{ct}}{Y_{dt}})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} \tag{3.95}$$

We remember that $(\frac{Y_{ct}}{Y_{dt}})^{\frac{-1}{\epsilon}} = \frac{p_{ct}}{p_{dt}} = (\frac{A_{ct}}{A_{dt}})^{-(1-\alpha)}$

$$Y_{dt} = p_{dt}^{\frac{\alpha}{1-\alpha}} A_{Ndt} L_{Ndt} \tag{3.96}$$

$$Y_{dt} = [(\frac{A_{ct}}{A_{dt}})^{(1-\alpha)(\epsilon-1)} + 1]^{\frac{1}{\epsilon-1}} A_{Ndt} L_{Ndt} \tag{3.97}$$

$$Y_{dt} = [(\frac{A_{ct}}{A_{dt}})^{(1-\alpha)(\epsilon-1)} + 1]^{\frac{1}{\epsilon-1}} A_{Ndt} L_{Ndt} \tag{3.98}$$

And since the amount of dirty labour is defined as $L_{Ndt} = 1 - L_{Nct}$, we can express it as $L_{Ndt} = \frac{1}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}+1}$. Meanwhile we define $\varphi$ as $(1-\alpha)(\epsilon-1) = \varphi$. [50] $(1-\alpha)\frac{\varphi}{\alpha-1} = -\varphi$

$$Y_{dt} = [(\frac{A_{ct}}{A_{dt}})^{(1-\alpha)(\epsilon-1)} + 1]^{\frac{1}{\epsilon-1}} A_{Ndt} \frac{1}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} + 1} \tag{3.99}$$

$$Y_{dt} = [(\frac{A_{ct}}{A_{dt}})^{-\varphi} + 1]^{\frac{-\alpha}{\varphi}} A_{Ndt} [(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} + 1]^{-1} \tag{3.100}$$

$$Y_{dt} = [(\frac{A_{ct}}{A_{dt}})^{-\varphi} + 1]^{\frac{-(\alpha+\varphi)}{\varphi}} A_{Ndt} \tag{3.101}$$

$$Y_{dt} = [(\frac{A_{dt}}{A_{ct}})^{\varphi} + 1]^{\frac{-(\alpha+\varphi)}{\varphi}} A_{Ndt} A_{Nct}^{\alpha+\varphi} A_{Nct}^{-(\alpha+\varphi)} \tag{3.102}$$

$$Y_{dt} = [(A_{dt})^{\varphi} + A_{ct}^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} A_{Ndt} A_{Nct}^{\alpha+\varphi} \tag{3.103}$$

Now e try to solve the equilibrium productions with the taxes introduced by the social planner. Here there are both on output and input. To find the desired result impose a tax equal to zero. We start again from the marginal productivity of the intermediate output, and since it is perfect competion is equal to the price times the tax

$$p_{dt}(1+\tau_O) = Y_{dt}^{-\frac{1}{\epsilon}} [Y_{dt}^{\frac{\epsilon-1}{\epsilon}} + Y_{ct}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} \tag{3.104}$$

$$p_{dt} = [1 + (\frac{Y_{ct}}{Y_{dt}})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} \frac{1}{1+\tau_O} \tag{3.105}$$

Then use the optimal production

$$Y_{Ndt}^{\star} = (\frac{p_{dt}}{1+\tau_I})^{\frac{\alpha}{1-\alpha}} A_{Ndt} L_{Ndt} \tag{3.106}$$

and we substitute the price inside the it rememberong that $(\frac{Y_{ct}}{Y_{dt}})^{\frac{-1}{\epsilon}} = \frac{p_{ct}}{p_{dt}} = (\frac{A_{ct}}{A_{dt}})^{-(1-\alpha)}$

$$Y_{Ndt} = (\frac{1}{1+\tau_O})^{\frac{\alpha}{1-\alpha}} (\frac{1}{1+\tau_I})^{\frac{\alpha}{1-\alpha}} [(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} (\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{-\frac{\alpha}{\varphi}} A_{dt} \frac{1}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} (1+\tau_O)^{\epsilon} (\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1} \tag{3.107}$$

Now applying linear algebra and powers' properties

$$Y_{Ndt} = (\frac{1}{1+\tau_O})^{\frac{\alpha}{1-\alpha}} (\frac{1}{1+\tau_I})^{\frac{\alpha}{1-\alpha}} [(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} (\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{-\frac{\alpha}{\varphi}} A_{dt} [(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} (1+\tau_O)^{\epsilon} (\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{-1} \tag{3.108}$$

$$Y_{Ndt} = (\frac{1}{1+\tau_O})^{\frac{\alpha}{1-\alpha}} (\frac{1}{1+\tau_I})^{\frac{\alpha}{1-\alpha}} [(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} (\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{-\frac{\alpha}{\varphi}} A_{dt} (1+\tau_O)^{-\epsilon} \tag{3.109}$$

$$[(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi} (\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + (1+\tau_O)^{-\epsilon}]^{-1}$$

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[50] It has been normalied by Acemoglu et al 2012

$$Y_{Ndt}^{-1} = (\frac{1}{1+\tau_O})^{\frac{-\alpha}{1-\alpha}}(\frac{1}{1+\tau_I})^{\frac{-\alpha}{1-\alpha}}[(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}+1]^{\frac{\alpha}{\varphi}}A_{dt}^{-1}(1+\tau_O)^{\epsilon}[(\frac{A_{Nct}}{A_{Ndt}})^{+\varphi}(\frac{1}{1+\tau_I}+1-1)^{\frac{\alpha\varphi}{1-\alpha}}]+$$
$$+(\frac{1}{1+\tau_O})^{\frac{-\alpha}{1-\alpha}}(\frac{1}{1+\tau_I})^{\frac{-\alpha}{1-\alpha}}[(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}+1]^{\frac{\alpha}{\varphi}}A_{dt}^{-1}(1+\tau_O)^{\epsilon}(1+\tau_O)^{-\epsilon}$$

Multiplying and regrouping we have

$$Y_{Ndt} = A_{dt}A_{ct}^{(\alpha+\varphi)}(1+\tau_O)^{\frac{-\alpha}{1-\alpha}}(1+\tau_I)^{\frac{-\alpha}{1-\alpha}}\{(A_{dt}^{\varphi}(1+\tau_I)^{\frac{-\alpha\varphi}{1-\alpha}}+A_{ct}^{\varphi})^{\frac{(\alpha+\varphi)}{\varphi}}-(A_{dt}^{\varphi}(1+\tau_I)^{-\frac{\alpha\varphi}{1-\alpha}}+A_{ct}^{\varphi})^{\frac{\alpha}{\varphi}}[(1+\tau_O)^{\epsilon}-1]\}^{-1} \tag{3.110}$$

### 3.12.4 Intermediate clean output

Again, we start from the clean price from the derivative of the final producer problem,

$$p_{ct} = Y_{ct}^{-\frac{1}{\epsilon}}[Y_{dt}^{\frac{\epsilon-1}{\epsilon}} + Y_{ct}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}} \tag{3.111}$$

$$p_{ct} = [(\frac{Y_{ct}}{Y_{dt}})^{-\frac{\epsilon-1}{\epsilon}} + 1]^{\frac{1}{\epsilon-1}} \tag{3.112}$$

Now we remember that the relationship between relative intermediate output, relative prices and relative productivity, $(\frac{Y_{ct}}{Y_{dt}})^{\frac{-1}{\epsilon}} = \frac{p_{ct}}{p_{dt}} = (\frac{A_{ct}}{A_{dt}})^{-(1-\alpha)}$, can be applyed to obtain

$$p_{ct} = [(\frac{A_{ct}}{A_{dt}})^{-(1-\alpha)(\epsilon-1)} + 1]^{\frac{1}{\epsilon-1}} \tag{3.113}$$

$$p_{ct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi} + 1]^{\frac{1}{\epsilon-1}} \tag{3.114}$$

Now substituting it in the optimal production gives us

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi} + 1]^{\frac{1}{\epsilon-1}}A_{Nct}L_{Nct} \tag{3.115}$$

and

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi} + 1]^{\frac{-\alpha}{\varphi}}A_{Nct}L_{Nct} \tag{3.116}$$

Now we know that the clean workforce is given by $L_{Nct} = 1-L_{Ndt}$ or $L_{Nct} = \frac{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}+1}$ which transform the dirty production in

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi} + 1]^{\frac{-\alpha}{\varphi}}A_{Nct}\frac{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}+1} \tag{3.117}$$

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi} + 1]^{\frac{-\alpha}{\varphi}}A_{Nct}[(\frac{A_{Nct}}{A_{Ndt}})^{\varphi} + 1]^{-1} \tag{3.118}$$

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi} + 1]^{\frac{-(\alpha+\varphi)}{\varphi}}A_{Nct}A_{Ndt}^{-(\alpha+\varphi)}A_{Ndt}^{\alpha+\varphi} \tag{3.119}$$

that help us finding the equilibrium production level

$$Y_{Nct} = [(A_{ct})^{\varphi} + (A_{dt})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} A_{Nct} A_{Ndt}^{\alpha+\varphi} \tag{3.120}$$

Now I apply the taxes that have been introduced by the social planner. I use the labour force from chapter 1 to find the equilibrium level

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{-\alpha}{\varphi}} A_{Nct}[\frac{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}}{(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1}]$$

rearranging the part introduced b the labour we obtain

$$Y_{Nct} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{-\alpha}{\varphi}} A_{Nct}[(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}}]$$
$$[(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{-1}$$

Now we eliminate the negative power at the end of the last equation obtaining the inverse of the intermediate production

$$Y_{Nct}^{-1} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1}[(\frac{A_{Nct}}{A_{Ndt}})^{\varphi}(1+\tau_O)^{-\epsilon}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}}]$$
$$[(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon}(\frac{1}{1+\tau_I})^{\frac{\alpha\varphi}{1-\alpha}} + 1]$$

then I multiply the last two squared parenthesis obtaining

$$Y_{Nct}^{-1} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1}[1 + (\frac{A_{Nct}}{A_{Ndt}})^{\varphi}(1+\tau_O)^{\epsilon}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}}]$$

then I regrouped

$$Y_{Nct}^{-1} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1}(1+\tau_O)^{-\epsilon}[(1+\tau_O)^{\epsilon} + (\frac{A_{Nct}}{A_{Ndt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}}]$$

$$Y_{Nct}^{-1} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1} +$$
$$+[(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1}(1+\tau_O)^{-\epsilon}[(\frac{A_{Nct}}{A_{Ndt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1 - 1]$$

$$Y_{Nct}^{-1} = [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1} +$$
$$+[(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha+\varphi}{\varphi}} A_{Nct}^{-1}(1+\tau_O)^{-\epsilon} - [(\frac{A_{ct}}{A_{dt}})^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + 1]^{\frac{\alpha}{\varphi}} A_{Nct}^{-1}(1+\tau_O)^{-\epsilon}$$

here I rise to the power of $-1$ both sides obtaining

$$Y_{Nct} = A_{Nct}A_{Ndt}^{(\alpha+\varphi)}\{[A_{ct}^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + A_{dt}{}^{\varphi}]^{\frac{\alpha}{\varphi}} + [A_{ct}^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + A_{dt}{}^{\varphi}]^{\frac{\alpha+\varphi}{\varphi}}(1+\tau_O)^{-\epsilon} +$$
$$-[A_{ct}^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}} + A_{dt}{}^{\varphi}]^{\frac{\alpha}{\varphi}}(1+\tau_O)^{-\epsilon}\}^{-1}$$

and finally we have the equilibrium production under all taxes.

$$Y_{Nct} = A_{Nct}A_{Ndt}^{(\alpha+\varphi)}\{(1+\tau_O)[A_{ct}^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}}+A_{dt}{}^{\varphi}]^{\frac{\alpha+\varphi}{\varphi}}(1+\tau_O)^{-\epsilon}-[A_{ct}^{\varphi}(1+\tau_I)^{\frac{\alpha\varphi}{1-\alpha}}+A_{dt}{}^{\varphi}]^{\frac{\alpha}{\varphi}}[(1+\tau_O)^{-\epsilon}-1]\}^{-1}$$

To find the desired result impose the value of the not desired tax equal to zero.

### 3.12.5 Social planner problem

**Abatement technology**

Here I show the Social planner problem with abatement technology and transboundary pollution. The social planner maximizes the utility of the representative household satisfying the following constraints To summarize the problem is

$$\max\sum_{t=0}^{\infty}\frac{1}{(1-\rho)^t}\frac{C_Nt^{1-\sigma}-1}{1-\sigma}S_{Nt}+\sum_{t=0}^{\infty}\frac{1}{(1-\rho)^t}\frac{C_St^{1-\sigma}}{1-\sigma}S_{St} \tag{3.121}$$

s.t.

$$A_{Nct}=\left(1+\gamma\eta_c s_{Nct}\right)A_{Nct-1} \tag{3.122}$$

$$A_{Ndt}=\left(1+\gamma\eta_d s_{Ndt}\right)A_{Ndt-1} \tag{3.123}$$

$$A_{Sct}=\kappa_c s_{Sct}A_{Nct}+(1-\kappa_c s_{Sct})A_{Sct-1} \tag{3.124}$$

$$A_{Sdt}=\kappa_d s_{Sdt}A_{Ndt}+(1-\kappa_d s_{Sdt})A_{Sdt-1} \tag{3.125}$$

$$\zeta_{at+1}=(1+\sigma Y_{Nt})\zeta_{at} \tag{3.126}$$

$$S_{Nt+1}=-[\zeta_G(Y_{Ndt}+Y_{Sdt})+\zeta_L Y_{Ndt}-Y_{Ndt}\zeta_{at}]+(1+\delta)S_{Nt} \tag{3.127}$$

$$S_{St+1}=-[\zeta_G(Y_{Ndt}+Y_{Sdt})+\zeta_L Y_{Sdt}]+(1+\delta)S_{St} \tag{3.128}$$

$$Y_{Nt}=(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}}+Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \tag{3.129}$$

$$Y_{St}=(Y_{Sct}^{\frac{\epsilon-1}{\epsilon}}+Y_{Sdt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \tag{3.130}$$

$$C_{Nt}+C_{St}=(1-\sigma)Y_{Nt}+Y_{St}-\psi\left(\int_0^1 x_{Nckt}dk+\int_0^1 x_{Ndkt}+\int_0^1 x_{Sckt}dk+\int_0^1 x_{Sdkt}dk\right) \tag{3.131}$$

$$Y_{ijt}=L_{ijt}^{1-\alpha}\int_0^1 A_{ijt}^{1-\alpha}x_{ijkt}^{\alpha}dk \tag{3.132}$$

$$L_{Nct}+L_{Ndt}=1 \text{ and } L_{Sct}+L_{Sdt}=1 \tag{3.133}$$

$$s_{Nct}+s_{Ndt}=1 \text{ and } s_{Sct}+s_{Sdt}=1 \tag{3.134}$$

$$A_{Nc0}>0 \text{ and } A_{Nd0}>0 \tag{3.135}$$

$$A_{Sc0}>0 \text{ and } A_{Sd0}>0 \tag{3.136}$$

the problem above lead us to the Lagrangian which is represented as

$$L=\sum_{t=0}^{\infty}\frac{1}{(1-\rho)^t}\frac{C_it^{1-\sigma}}{1-\sigma}S_{it}+\lambda_{it}[(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}}+Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}+(Y_{Sct}^{\frac{\epsilon-1}{\epsilon}}+Y_{Sdt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}-\psi\left(\int_0^1 x_{ickt}dk+\int_0^1 x_{idkt}dk\right) \tag{3.137}$$

$$-C_{Nt}-C_{St}]+\omega_{it}[-[\zeta_G(Y_{Ndt}+Y_{Sdt})+\zeta_L Y_{idt}]+(1+\delta)S_{it}-S_{it+1}]+\lambda_{jit}[L_{ijt}^{1-\alpha}\int_0^1 A_{ijt}^{1-\alpha}x_{ijkt}^{\alpha}dk-Y_{jit}]+$$

$$+\mu_{Sjt}[\kappa_j s_{Sjt}A_{Njt}+(1-\kappa_j s_{Sjt})A_{Sjt-1}-A_{Sjt}]+\mu_{Njt}[\left(1+\gamma\eta_j s_{Njt}\right)A_{Njt-1}-A_{Njt}]$$

where, $\lambda_{it}$ is the shadow value of consumption, $\omega_{it}$ the shadow value of the environmental quality, $\lambda_{jit}$ the shadow price of the intermediate output, $\mu_{ijt}$ the shadow value of an innovation.

Here we have in order, the derivative of consumption, of environmental quality, technology, abatement technology, clean input, dirty input, demand of machines, derivative of labour and the derivative of scientists. All of these are only in the North.

$$\begin{cases} \frac{1}{(1-\rho)^t} C_{it}^{-\sigma} S_{it} = \lambda_{it} \\ \omega_{it} = \frac{1}{(1-\rho)^t} \frac{C_{it}^{1-\sigma}}{1-\sigma} + (1+\delta)\omega_{it+1} \\ \mu_{Njt} = \lambda_{Njt}(1-\alpha)(L_{Njt}^{1-\alpha} A_{Njt}^{-\alpha} x_{Njt}^{\alpha}) + \mu_{Njt+1}(1+\gamma\eta_j s_{Njt+1}) + \mu_{Sjt}(\kappa_j s_{sjt}) \\ (1+\sigma Y_{Nt})\mu_{at+1} + \omega_{Nt+1} Y_{Ndt} = \mu_{at} \\ [Y_{ict}^{\frac{1}{\epsilon}}(Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}][1+\sigma(\frac{\mu_{at+1}\zeta_{at}}{\lambda_{Nt}} - 1)] = \hat{p}_{Nct} \\ [Y_{Ndt}^{\frac{-1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}][1+\sigma(\frac{\mu_{at+1}\zeta_{at}}{\lambda_{Nt}} - 1)] - \frac{[\zeta_G+\zeta_L-\zeta_{at}]\omega_{Nt+1}}{\lambda_{Nt}} - \frac{\zeta_G \omega_{St+1}}{\lambda_{Nt}} = \hat{p}_{Ndt} \\ x_{Njt} = \frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt} \\ (1-\alpha)(L_{Njt}^{-\alpha} A_{Njt}^{1-\alpha} x_{Njt}^{\alpha})\lambda_{Njt} = \chi_{NLj} \\ \mu_{Njt}\gamma\eta_j A_{Njt-1} = \chi_{Nsj} \end{cases}$$

To eliminate the monopoly distortion I use the first order condition with respect to the machines

$$\frac{\partial L}{\partial x_{ijt}} = (\alpha L_{ijt}^{1-\alpha} A_{ijt}^{1-\alpha} x_{ijt}^{\alpha-1})\lambda_{ijt} - \psi\lambda_{it} = 0 \tag{3.138}$$

And applying $\frac{\lambda_{ijt}}{\lambda_{it}} = \hat{p}_{ijt}$ we can rewrite it as

$$x_{ijt} = (\frac{\alpha \hat{p}_{ijt}}{\psi})^{\frac{1}{1-\alpha}} L_{ijt} A_{ijt} \tag{3.139}$$

Then, using the Foc with respect to labour we obtain

$$\frac{\partial L}{\partial L_{ijt}} = (1-\alpha)(L^{-\alpha} A_{ijt}^{1-\alpha} x_{ijt}^{\alpha})\lambda_{ijt} = 0 \tag{3.140}$$

Then, the number of machine in eq (3.139) is substituted inside the equation above which leads to

$$(1-\alpha)(\frac{\alpha \hat{p}_{ijt}}{\psi})^{\frac{\alpha}{1-\alpha}} A_{ijt}\lambda_{ijt} = 0 \tag{3.141}$$

Now, since in both sectors they are equal to zero, we derive the following arbitrage condition

$$\hat{p}_{ict}^{\frac{1}{1-\alpha}} A_{ict} = \hat{p}_{idt}^{\frac{1}{1-\alpha}} A_{idt} \tag{3.142}$$

### 3.12.6 Net zero emissions

To solve the problem with net zero emission we need to substitute the production function and the environmental constraint in the North with $Y_{Nd} = (\frac{\delta S_N - \zeta_G Y_{Sd}}{(\zeta_G+\zeta_L)} +$

$\frac{\zeta_{at}}{(\zeta_G+\zeta_L)} L_{Ndt}^{1-\alpha} \int_0^1 A_{Ndkt}^{1-\alpha} x_{Ndkt}^{\alpha} dk)$ obtaining in the Lagrangian the following equation

$$\lambda_{Ndt}(\frac{\delta S_N - \zeta_G Y_{Sd}}{(\zeta_G+\zeta_L)} + \frac{\zeta_{at}}{(\zeta_G+\zeta_L)} L_{Ndt}^{1-\alpha} \int_0^1 A_{Ndkt}^{1-\alpha} x_{Ndkt}^{\alpha} dk - Y_{Nd}) \tag{3.143}$$

The lagrangian we have seen before becomes

$$L = \sum_{t=0}^{\infty} \frac{1}{(1-\rho)^t} \frac{C_i t^{1-\sigma}}{1-\sigma} S_{it} + \lambda_{it}[(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} + (Y_{Sct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Sdt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - \psi \left( \int_0^1 x_{ickt} dk + \int_0^1 x_{idkt} dk \right) + \tag{3.144}$$

$$-C_{Nt} - C_{St}] + \omega_{it}[-[\zeta_G(Y_{Ndt}+Y_{Sdt}) + \zeta_L Y_{idt}] + (1+\delta)S_{it} - S_{it+1}] + \lambda_{jit}[L_{ijt}^{1-\alpha} \int_0^1 A_{ijt}^{1-\alpha} x_{ijkt}^{\alpha} dk - Y_{jit}] +$$

$$+\mu_{Njt}[(1+\gamma\eta_j s_{Njt})\, A_{Njt-1} - A_{Njt}] + \lambda_{Ndt}(\frac{\delta S_N - \zeta_G Y_{Sd}}{(\zeta_G+\zeta_L)} + \frac{\zeta_{at}}{(\zeta_G+\zeta_L)} L_{Ndt}^{1-\alpha} \int_0^1 A_{Ndkt}^{1-\alpha} x_{Ndkt}^{\alpha} dk - Y_{Nd}) +$$

$$+\mu_{Sjt}[\kappa_j s_{Sjt} A_{Njt} + (1-\kappa_j s_{Sjt}) A_{Sjt-1} - A_{Sjt}]$$

### 3.12.7 Demand of machines

hen the Social planner derives the demand of machines she obtains

$$\frac{\partial L}{\partial x_{Ndt}} = \alpha \lambda_{Ndt} \frac{\zeta_{at}}{(\zeta_G+\zeta_L)} L_{Ndt}^{1-\alpha} A_{Ndt}^{1-\alpha} x_{Ndt}^{\alpha-1} - \lambda_{Nt}\psi = 0$$

and since $\psi$ is equal to $\alpha^2$, and $\frac{\lambda_{Nt}}{\lambda_{Ndt}} = p_{\hat{N}dt}$ it becomes

$$x_{Ndt} = (\alpha p_{\hat{N}dt} \frac{\zeta_{at}}{\zeta_G+\zeta_L})^{\frac{1}{\alpha-1}} L_{Ndt} A_{Ndt}$$

### 3.12.8 Social gain of innovation

The social gain of an innovation in the dirty sector is different in the two scenarios. In the one with abatement technology it depends by

$$\frac{\partial L}{\partial A_{Ndt}} = \mu_{Ndt} = \lambda_{Ndt}(1-\alpha)(L_{Ndt}^{1-\alpha} A_{Ndt}^{-\alpha} x_{Ndt}^{\alpha}) + \mu_{Ndt+1}(1+\gamma\eta_d s_{Ndt+1}) + \mu_{Sdt}(\kappa_d s_{sdt}) \tag{3.145}$$

and substituting it with 3.139 we obtain

$$\mu_{Njt} = \lambda_{Nt}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} \hat{p}_{Njt}^{\frac{1}{1-\alpha}} L_{Njt} + \mu_{Njt+1}(1+\gamma\eta_j s_{Njt+1}) + \mu_{Sjt}(\kappa_j s_{sjt}) \tag{3.146}$$

Here I have exctracted $\hat{p}_{Njt}^{-1}$ from $\hat{p}_{Njt}^{\frac{\alpha}{1-\alpha}}$ and I have substituted it with $\frac{\lambda_{Nt}}{\lambda_{Njt}}$.

Meanwhile, with respect to the South

$$\frac{\partial L}{\partial A_{Sjt}} = \mu_{Sjt} = \lambda_{Sjt}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}}\hat{p}_{Sjt}^{\frac{1}{1-\alpha}}\frac{\lambda_{St}}{\lambda_{Sjt}}L_{Sjt} + \mu_{Sjt+1}(1-\kappa_j s_{Sjt+1}) \tag{3.147}$$

solving recursively and regroupping

$$A_{Nt}\mu_{Njt} = \sum_{v\geq t}\lambda_{Nt}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}}\hat{p}_{Njv}^{\frac{1}{1-\alpha}}L_{Njv}A_{Njv} + \mu_{Sjt}(\kappa_j s_{sjt})A_{Nt} \tag{3.148}$$

and doing the same in the south the following relationship is obtained

$$\mu_{Sjt} = \sum_{v\geq t}\lambda_{Sv}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}}\hat{p}_{Sjv}^{\frac{1}{1-\alpha}}L_{Sjv}(1-\kappa_j s_{Sjv}) \tag{3.149}$$

which becomes

$$\mu_{Njt}A_{Njt} = (1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}}[\sum_{v\geq t}\lambda_{Nv}\hat{p}_{Njv}^{\frac{1}{1-\alpha}}L_{Njv}A_{Njv} + \sum_{v\geq t}\lambda_{Sv}\hat{p}_{Sjv}^{\frac{1}{1-\alpha}}L_{Sjv}(\kappa_j s_{sjt})(1-\kappa_j s_{Sjv})A_{Nt}] \tag{3.150}$$

and applying the probability of a successful innovation I obtain

$$\mu_{Njt}A_{Njt-1}\gamma\eta_j = \gamma\eta_j(1+\gamma\eta_j s_{Njt})^{-1}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} \tag{3.151}$$

$$[\sum_{v\geq t}\lambda_{Nv}\hat{p}_{Njv}^{\frac{1}{1-\alpha}}L_{Njv}A_{Njv} + \sum_{v\geq t}\lambda_{Sv}\hat{p}_{Sjv}^{\frac{1}{1-\alpha}}L_{Sjv}(\kappa_j s_{sjt})(1-\kappa_j s_{Sjv})A_{Nt}]$$

where $\mu_{Njt}A_{jt-1}\gamma\eta_j$ is the social gain of a new innovation. Note that $(\kappa_j s_{sjt})(1-\kappa_j s_{Sjv})$ is composed by the probability of obtain a successful imitation times the probability of an unsuccessful imitation, representing the variance of a bernoulli distribution. To find the optimal allocation of scientists in the North we impose

$$\frac{(1+\gamma\eta_c s_{Nct})^{-1}[\sum_{v\geq t}\lambda_{Nv}\hat{p}_{Ncv}^{\frac{1}{1-\alpha}}L_{Ncv}A_{Njv} + \sum_{v\geq t}\lambda_{Sv}\hat{p}_{Scv}^{\frac{1}{1-\alpha}}L_{Scv}(\kappa_j s_{sct})(1-\kappa_j s_{Scv})A_{Nct}]}{(1+\gamma\eta_d s_{Ndt})^{-1}[\sum_{v\geq t}\lambda_{Nv}\hat{p}_{Ndv}^{\frac{1}{1-\alpha}}L_{Ndv}A_{Njv} + \sum_{v\geq t}\lambda_{Sv}\hat{p}_{Sdv}^{\frac{1}{1-\alpha}}L_{Sdv}(\kappa_j s_{sdt})(1-\kappa_j s_{Sdv})A_{Ndt}]} \tag{3.152}$$

From eq (3.139) with eq(4.101) we obtain

$$Y_{ijt} = (\frac{\alpha\hat{p}_{ijt}}{\psi})^{\frac{\alpha}{1-\alpha}}L_{ijt}A_{ijt} \tag{3.153}$$

Then using the FOC with respect to $Y_{ijt}$ and the relative prices we obtain

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{ct}}{A_{dt}})^{-\varphi}(1+\frac{[\zeta_G+\zeta_L]\omega_{Nt+1}}{\hat{p}_{idt}\lambda_{Nt}} + \frac{\zeta_G\omega_{St+1}}{\hat{p}_{idt}\lambda_{Nt}})^{\epsilon} \tag{3.154}$$

where $\frac{[\zeta_G+\zeta_L]\omega_{Nt+1}}{\hat{p}_{idt}\lambda_{Nt}} + \frac{\zeta_G\omega_{St+1}}{\hat{p}_{idt}\lambda_{Nt}}$ is our tax $\tau_O$. From the pre-tax profit we obtain the expected profits of the scientists

$$\frac{\Pi_{Nc}}{\Pi_{Nd}} = \frac{\eta_c}{\eta_d}[\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}]^{-1-\varphi}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{3.155}$$

Meanwhile in the net zero scenario the social gain of an innovation in the dirty sector is composed by the shadow value corresponding to the derivative of $A_{Ndt}$

$$\frac{\partial L}{\partial A_{Ndt}} = \mu_{Ndt} = \frac{\zeta_{At}}{\zeta_G + \zeta_L}\lambda_{Ndt}(1-\alpha)(L_{Ndt}^{1-\alpha}A_{Ndt}^{-\alpha}x_{Ndt}^{\alpha})+\mu_{Ndt+1}(1+\gamma\eta_d s_{Ndt+1})+\mu_{Sdt}(\kappa_d s_{sdt}) \tag{3.156}$$

It is easy to verify that for any $\zeta_{At} < \zeta_G + \zeta_L$ the shadow of a dirty innovation is lower under net zero compared to the other scenario.

### 3.12.9 Labour under net zero

To find the labour we insert the demand of machines

$$x_{idt} = (\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt} \tag{3.157}$$

into the net zero production function

$$Y_{Nd} = \frac{\delta S_N}{(\zeta_G + \zeta_L)} + \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L_{ijt}^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{ijkt}^{\alpha} dk \tag{3.158}$$

obtaining the optimal production

$$Y_{Nd} = \frac{\delta S_N}{(\zeta_G + \zeta_L)} + \frac{\zeta_{at}}{(\zeta_G + \zeta_L)} L_{ijt}^{1-\alpha} A_{jt}^{1-\alpha}((\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt})^{\alpha} \tag{3.159}$$

here I simplify it applying the power of $\alpha$

$$Y_{Nd} = \frac{\delta S_N}{(\zeta_G + \zeta_L)} + (\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt} \tag{3.160}$$

and I substitute it in the derivatitve

$$\frac{\partial L}{\partial Y_{Ndt}} = [Y_{Ndt}^{\frac{1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] = \hat{p}_{Ndt} \tag{3.161}$$

obtaining

$$[(\frac{\delta S_N}{(\zeta_G + \zeta_L)} + (\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt})^{\frac{1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] = \hat{p}_{Ndt} \tag{3.162}$$

Now, to simplify I require $(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}} = D$

$$[(\frac{\delta S_N}{(\zeta_G + \zeta_L)} + (\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt})^{\frac{1}{\epsilon}} D] = \hat{p}_{Ndt} \tag{3.163}$$

We remember that $(1-\alpha)(\epsilon - 1) = \varphi$ so

$$[(\frac{\delta S_N}{(\zeta_G + \zeta_L)} + (\frac{\zeta_{at}}{(\zeta_G + \zeta_L)}\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt})^{-\frac{1-\alpha}{1-\alpha-\varphi}} D] = \hat{p}_{Ndt} \tag{3.164}$$

Now i rearrange D and the left part

$$(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)}\frac{\alpha\hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}}L_{Ndt}A_{Ndt}=(\hat{p}_{Ndt}D^{-1})^{-\frac{1-\alpha-\varphi}{1-\alpha}}-\frac{\delta S_N}{(\zeta_G+\zeta_L)} \tag{3.165}$$

Now,

$$(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{1}{1-\alpha}}(\frac{\alpha\hat{p}_{Ndt}}{\psi})^{\frac{\alpha}{1-\alpha}}L_{Ndt}A_{Ndt}=\hat{p}_{Ndt}^{-\frac{1-\alpha-\varphi}{1-\alpha}}(1-\frac{\delta S_N}{(\zeta_G+\zeta_L)}(\frac{D}{p_{Ndt}})^{\epsilon})D^{\epsilon} \tag{3.166}$$

which lead to

$$L_{Ndt}=(1-\frac{\delta S_N}{(\zeta_G+\zeta_L)}(\frac{D}{p_{Ndt}})^{\epsilon})(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{-1}{1-\alpha}}(\frac{\alpha}{\psi})^{\frac{-1}{1-\alpha}}\hat{p}_{Ndt}^{\cdot\frac{\varphi-1}{1-\alpha}}A_{Ndt}^{-1}D^{\frac{1-\alpha-\varphi}{1-\alpha}} \tag{3.167}$$

which implies that the demand of labour depends b two forces, the first is the relative regeneration/damage that increases the demand of dirt workers raised to $\epsilon$, on the other hand the relative emissions that decreases the demand of dirty workers. The Clean labour depends by

$$L_{Nct}=(\frac{\alpha}{\psi})^{\frac{-1}{1-\alpha}}\hat{p}_{Nct}^{\frac{\varphi-1}{1-\alpha}}A_{Nct}^{-1}D^{\frac{1-\alpha-\varphi}{1-\alpha}} \tag{3.168}$$

The relative labour becomes

$$\frac{L_{Nct}}{L_{Ndt}}=(\frac{\hat{p}_{Nct}}{\hat{p}_{Nct}})^{\frac{\varphi-1}{1-\alpha}}(\frac{A_{Nct}}{A_{Ndt}})^{-1}(1-\frac{\delta S_N}{(\zeta_G+\zeta_L)}(\frac{D}{p_{Ndt}})^{\epsilon})(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{-1}{1-\alpha}} \tag{3.169}$$

Now I use $\frac{p_{Nct}}{p_{Ndt}}=(\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}(\frac{1}{1+\tau_I})^{\alpha}$

$$\frac{L_{Nct}}{L_{Ndt}}=(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1-\frac{\delta S_N}{(\zeta_G+\zeta_L)}(\frac{D}{p_{Ndt}})^{\epsilon})(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}} \tag{3.170}$$

where the value of $(\frac{D}{p_{Ndt}}$ is equal to the production dirty production we obtain

$$\frac{L_{Nct}}{L_{Ndt}}=(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1-\frac{\delta S_N}{(\zeta_G+\zeta_L)}(\frac{1}{Y_{Ndt}})^{\epsilon})(\frac{\zeta_{at}}{(\zeta_G+\zeta_L)})^{\frac{\alpha\varphi-\alpha-1}{1-\alpha}} \tag{3.171}$$

### 3.12.10 Demand of labour under Net zero without abatement technology

In this section I find the relative demand of workers in Net zero showing that applying this policy the demand of workers is the same as without the policy. Applying this definition of net zero

$$Y_{Ndt}=\frac{\delta S_{Nt}-\zeta_G Y_{Sdt}}{\zeta_G+\zeta_L} \tag{3.172}$$

we understand that the maximum amount of production must satisfy this equality. This implies that the optimum production can be described as

$$Y^*_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} = (\frac{\alpha p_{dt}}{\psi})^{\frac{-\alpha}{1-\alpha}} A_{Ndt} L_{Ndt} \tag{3.173}$$

which implies that the demand of dirty labour depends by the cap, the productivity and the price of the intermediate dirty good as follows

$$L_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} A^{-1}_{Ndt} (\frac{\alpha p_{Ndt}}{\psi})^{\frac{-\alpha}{1-\alpha}} \tag{3.174}$$

As we remember the clean labour depends by

$$L_{Nct} = (\frac{\alpha}{\psi})^{\frac{-\alpha}{1-\alpha}} \hat{p}_{Nct}^{\frac{\varphi-1}{1-\alpha}} A^{-1}_{Nct} D^{\frac{1-\alpha-\varphi}{1-\alpha}} \tag{3.175}$$

We remember that $D = (Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}$. Given these informations, the relative labour depends by

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{p_{Nct}^{\frac{\varphi}{1-\alpha}}}{p_{Ndt}} (\frac{p_{Nct}}{p_{Ndt}})^{\frac{-1}{1-\alpha}} (\frac{A_{Nct}}{A_{Ndt}})^{-1} (\frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L})^{-1} Y_{Ndt} ((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{3.176}$$

and as we said before $Y_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L}$. I also moltiply and divide $p_{Ndt}$ by $\frac{p_{Nct}}{p_{Nct}}$ and I substitute it with $(\frac{p_{Nct}}{p_{Ndt}}) = (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}$ which implies

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{p_{Nct}^{\frac{\varphi}{1-\alpha}}}{p_{Nct}(\frac{A_{Nct}}{A_{Ndt}})^{(1-\alpha)}} ((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{3.177}$$

now

$$\frac{L_{Nct}}{L_{Ndt}} = p_{Nct}^{-\epsilon} (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} ((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{3.178}$$

as we know, $\hat{p}_{Nct} = [Y_{Nct}^{\frac{-1}{\epsilon}} (Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}]$ and since it is raised to $-\epsilon$ it implies that $p_{Nct}^{-\epsilon} = [Y_{Nct}^{\frac{\epsilon}{\epsilon}} (Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{-\epsilon}{\epsilon-1}}]$ that can be transformed in

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{Y_{Nct}}{Y_{Ndt}} ((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{-\epsilon}{\epsilon-1}} (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} ((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{3.179}$$

so finally

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{Y_{Nct}}{Y_{Ndt}} (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} \tag{3.180}$$

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{p_{Nct}}{p_{Ndt}(1+\tau_O)})^{-\epsilon} (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} \tag{3.181}$$

and finally

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{\epsilon-\epsilon\alpha} (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} (1+\tau_O)^{\epsilon} \tag{3.182}$$

which leads to

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{\epsilon-\epsilon\alpha+\alpha-1}(1+\tau_O)^{\epsilon} \tag{3.183}$$

here we regroup the powers

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{(\epsilon-1)-\alpha(\epsilon-1)}(1+\tau_O)^{\epsilon} \tag{3.184}$$

and rearrangin the powers we obtain

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{(\epsilon-1)(1-\alpha))}(1+\tau_O)^{\epsilon} \tag{3.185}$$

In this model $(1-\epsilon)(1-\alpha) = \varphi$ so this implies that

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{3.186}$$

Which is the same solution of the social planner that corrects the environmental externalities.

### 3.12.11 Extra plots

**Net zero as a Terminal point**

We have seen in the previous subsections the effect of different policies and what happen when we introduce a Net zero policy without preparation. The main effect on the representative household is a welfare loss which might be too large to be politically acceptable. We should ask ourself if it is possible to reach net zero diminishing the welfare loss.

In this subsection I try to reach net zero as smoothly as possible. From the previous simulation I find that the optimal point is at period $t = 80$, where the consumption is at the level 1.774. So I decide first to solve with optimal taxation until the period $t = 79$ and then using the Net zero as a terminal condition seeing its effect from $t = 80$ to $t = 100$. Applying the net zero at that period we have a loss of welfare of $-10\%$. This welfare loss depends by the change of policy that is less tightening with regards to the dirty output[51].

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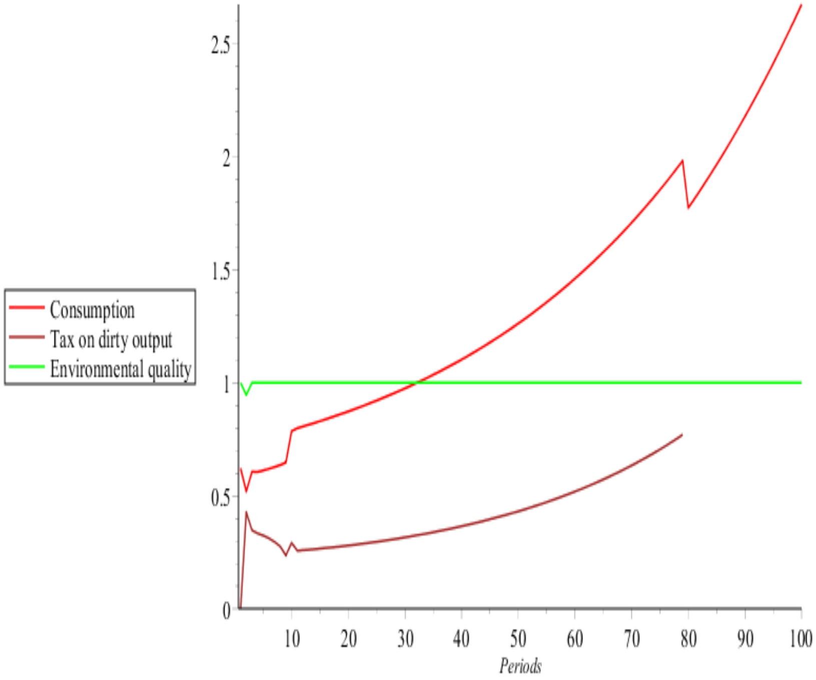
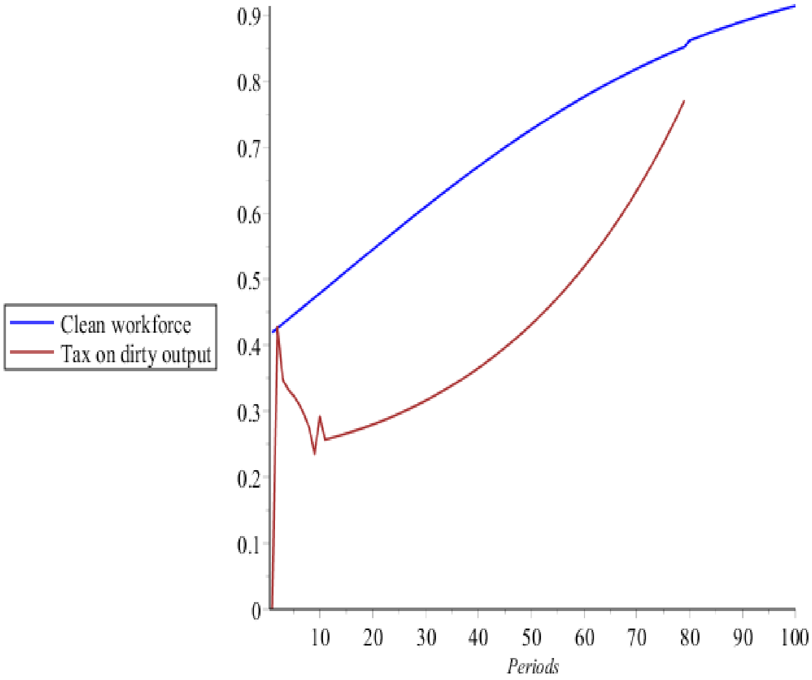
[51] It is less effective than the tax because the abatement technology is more advanced implying that the relative pollution $\frac{\zeta A_t}{\zeta}$ is close to 1. Also, the clean technology is not enough advanced to keep the relative labour force at the same level meaning that without an incentive like the tax on intermediate output the reorganization of the economy leads to a welfare loss.

Figure 3.15: The red curve represents the consumption. The green one the environmental quality and the brown one the tax on the dirt intermediate output. In the plot at time t= 80 the economy introduces the Net zero which is represented until t=100. As we can see the social planner keep the tax higher to introduce the new policy. At time t=80 the tax becomes the quota of intermediate dirty output required by the economy



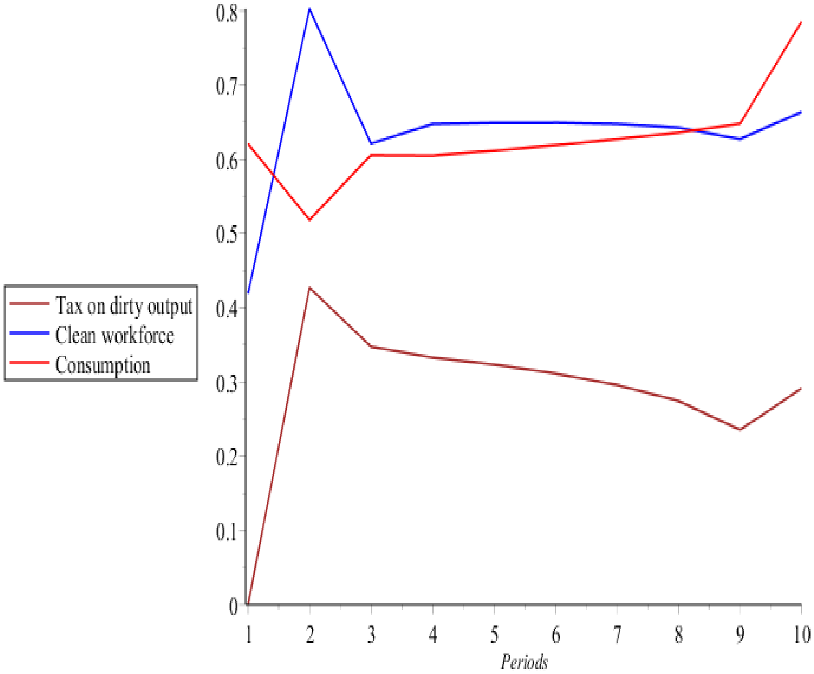
Comparing this scenario with the others we can see that the tax on intermediate dirty output is higher due to the social planner decision to prepare the economy to switch policy to Net zero. The labour in this scenario is represented by the next plot where it is sustained by the tax until period $t = 80$.

Figure 3.16: The blue line represents the workforce employed in the clean sector. The brown curve is the tax applied to the dirty sector. At time t=80 the tax becomes the quota of intermediate dirty output required by the economy.



At time t= 80 the abatement technology has reached its higher level and the demand of worker in the clean sector decreases of the 10% and as a consequence the consumption decreases due to the reorganization of the economy to grow again. Now the clean labour force grows with less shocks compared to figure (14) where it jumps from 99% to 53%.

Figure 3.17: The blue curve represents the clean workforce, the red one is the consumption level, and the brown one is the tax on the intermediate outpu. Note that the net zero start at time t=10 and the abatement technology has not reached the maximum level. Remember that at time t=10 the tax is substituted by the quota on the intermediate dirty production.

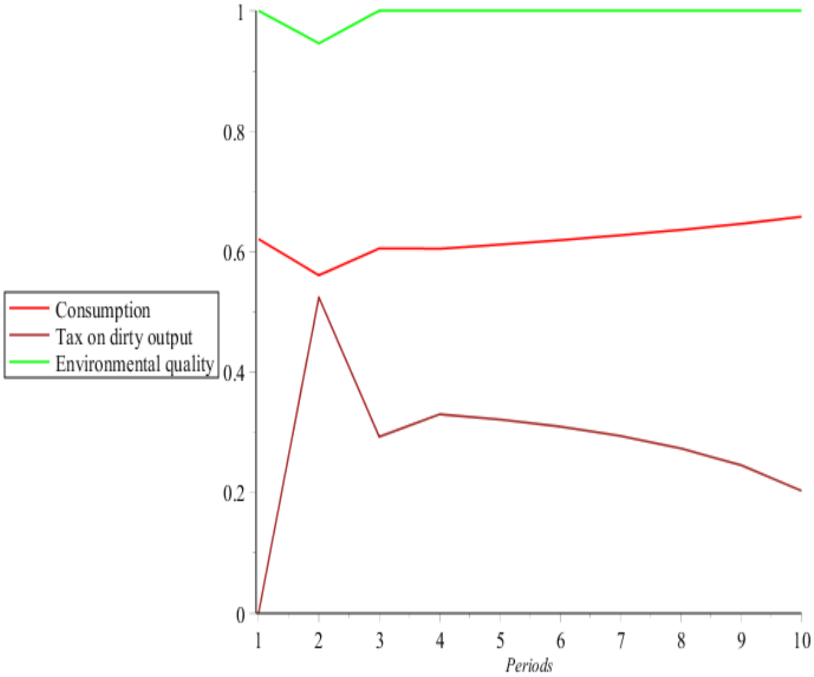


We will always have kinks due to the change of policy unless I diminish the investment in abatement technology to balance the consumption loss. Nonetheless it is evident that the policy needs preparation. In fact, all the countries that decided to adopt this policy have a transition period composed by taxes and bans on dirty goods to avoid the economic shocks that I have presented in the last plot. It is important to notice that at time $t = 79$ the tax is higher compared to the other scenario with abatement technology because the social planner needs to prepare the society to start a more restricting policy. This higher tax confirm the policy decided by the countries that agreed to reach Net zero by 2050, increasing the restriction on dirty goods over time. In the next subsection I represent the best solution to this problem applying the policy at period $t = 80$ with a lower investment rate. I have not presented the other attempts in different periods because they are similar to the plots in this subsection.

### Net zero smoothly

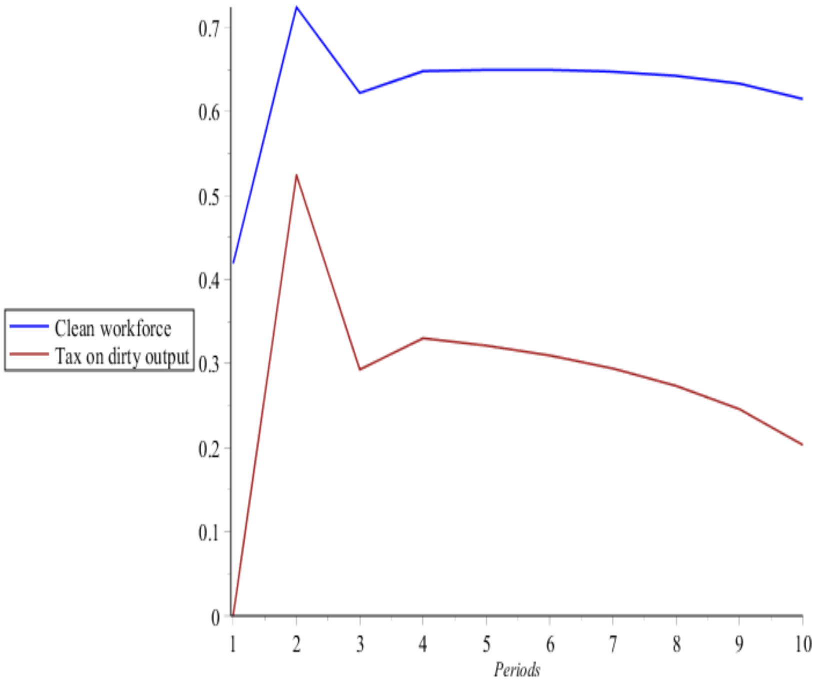
In this subsection I am diminishing the percentage of final product invested in abatement technology over time to avoid the welfare loss letting the policy be politically feasible. Here the percentage of investment in abatement technology decreases over time from 20% from $t = 1$ to $t = 9$, then from $t = 10$ to $t = 79$ the investment becomes the 10% of the final production to be fixed at 2.5% when the economy reaches the Net zero from $t = 80$. As we can see in the following plot, the first 80 periods are the same as in the scenario with abatement technology. The reason is that the previous policy is used to prepare the economy.

Figure 3.18: The red curve represents the consumption. The green one the environmental quality and the brown one the tax on the dirt intermediate output. Remember that at time t=80 the tax is substituted by the quota on the intermediate dirty production.



We can see that reducing the investment the plot becomes similar to the one with abatement technology. The jump that we see in the tax at period 80 is given b the quota that substitutes the tax. The next figure represents the effect of the tax on the labour force and is exactly the same as in the previous subsection but it is not reflected on the consumption level thanks to a different investment rate.

Figure 3.19: The blue curve represents the labour force engaged in the clean sector and the brown one the tax on the intermediate dirty output. Remember that at time t=80 the tax is substituted by the quota on the intermediate dirty production



In fact, the labour force reach an initial peak at time $t = 2$ to contrast the environmental degradation thanks to the tax on intermediate dirty output. Then, it follows the tax increasing with clean sector getting more productive over time.
As we can see diminishing the investment in abatement technology, the economy will have a welfare gain from $t = 79$ to $t = 80$ of 2% which is more than optimal comparing it with all the other scenarios.

Figure 3.20: The red curve represents the consumption. The green one the environmental quality and the brown one the tax on the dirt intermediate output. Remember that at time t=80 the tax is substituted by the quota on the intermediate dirty production.

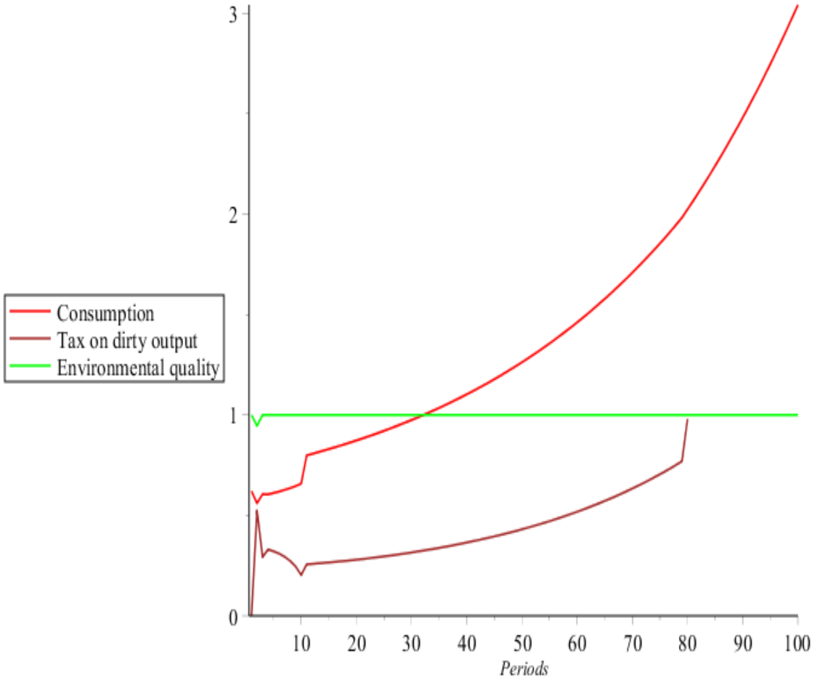
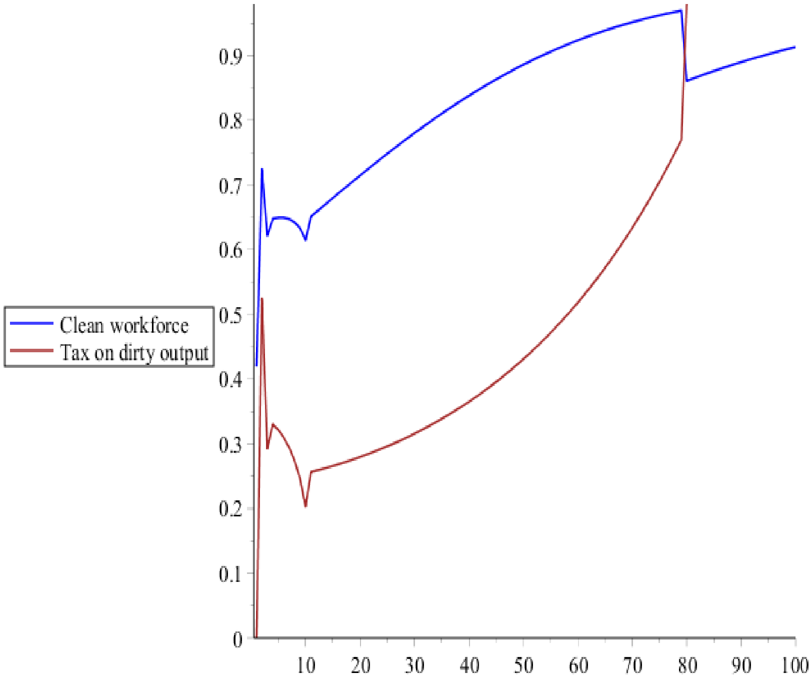


Figure 3.21: The blue curve represents the labour force engaged in the clean sector and the brown one the tax on the intermediate dirty output. Remember that at time t=80 the tax is substituted by the quota on the intermediate dirty production



We have the same situation we can see in the subsection before where the tax on intermediate output reaches higher levels compared to the abatement technology scenario where it is not higher than the 50%.
It is evident that the economy needs to decrease its relative investment over time to keep growing even entering the Net zero. Unlikely in this model the investment is exogenous so diminishing its quantity I am trying to make it more dynamic reducing its negative effects on the economy. In fact we have to consider the investment in abatement technology for its double effects. It is positive to reduce the externalities produced by the dirty production, but at the same time it is negative for the consumer welfare because reduces the final output that could be used by her.

### Net zero smoothly with trans-boundary pollution

Taking into account the previous results I introduce the net zero from the starting period with a tax on the intermediate output simultaneously. In this subsection we see the presence of transboundary pollution and the tax that compensate it.
From the plot we can see th compensation is small due to the lower dirty price[52]. The first two plots represent the economy in the short period during the transiction.

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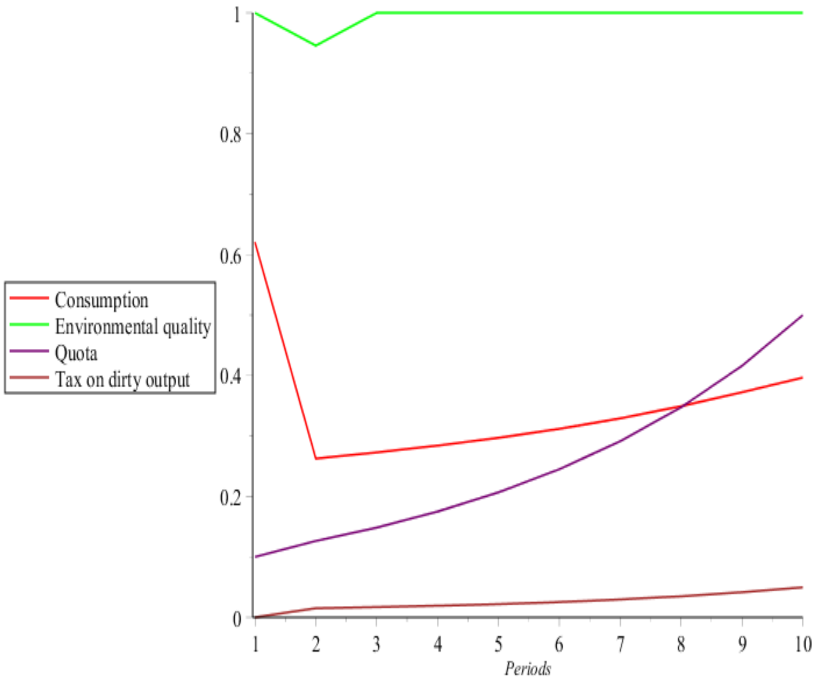
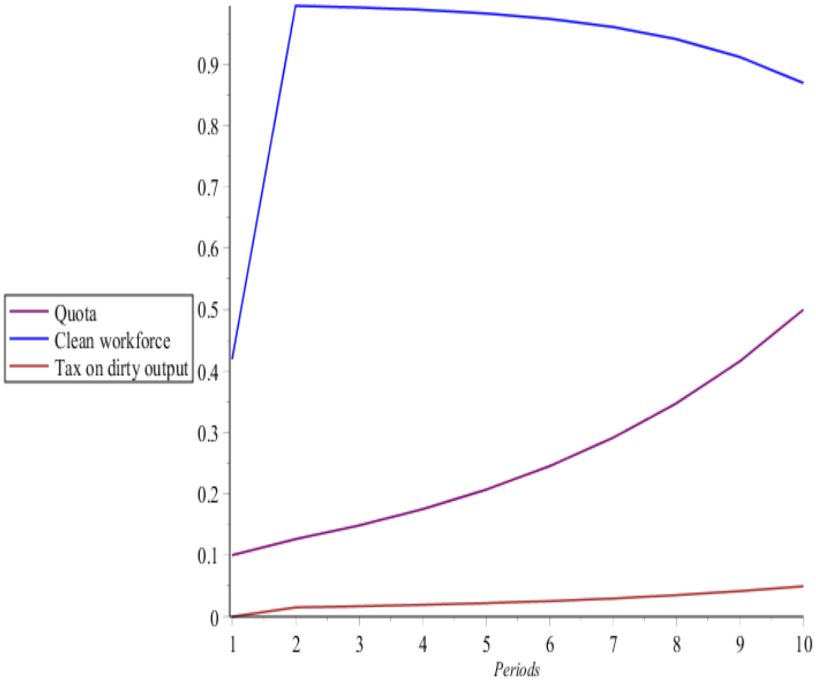
[52] We need to remember that the tax depends also by the dirty price which is represented as $p_{Ndt} = p_{Nct}(\frac{A_{Nct}}{A_{Ndt}})^{1-\alpha}(\frac{\zeta_G+\zeta_L}{\zeta_{At}})^{\alpha}$ which results in a decrease in the price thanks to the quota applied on the dirty production.

Figure 3.22: The green curve represents the environmental quality, the red one the consumption, the brown one the tax on the intermediate output in the North. Here I introduced the purple curve to outline the evolution of the quota over time. There is abatement technology



The plot above outlines the evolution of the consumption, that after the introduction of the net zero decreases to its lower level. After that moment, it grows slowly with the evolution of the abatement technology that relaxes the quota allowing more dirty production. The tax is lower than all the other scenarios because it is competing with the quota that is already reducing the environmental damage and it is incorporating the local tax.

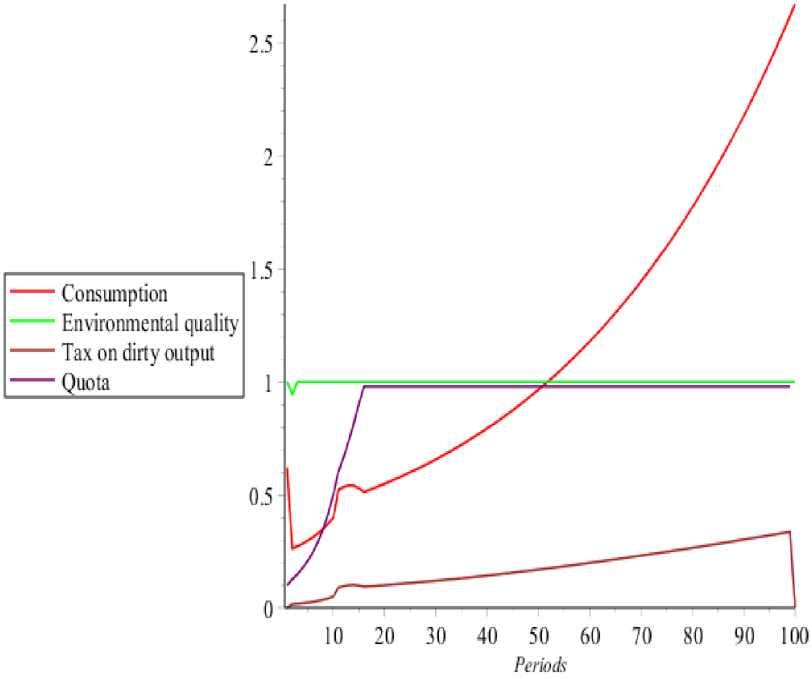
Figure 3.23: The blue curve represents the percentage of workers employed in the clean sector in the North. The brown curve represents the tax applied to the intermediate dirty output. The purple one is the quota applied to the dirty production. We have abatement technology.



We can see that the clean workforce is repeating the same behaviour I show in the previous sections with net zero. It reaches its local maximum at period 2 to decrease slowly while the quota allows more dirty production over time.

Meanwhile in the long period I show the evolution of the consumption with its kinks. There is a jump in the consumption thanks to new abatement technology that allows the economy to exploit again the most advanced sector, togheter with a lower investment in abatement technology, from 20% to 10%. Between period 15 and 20 the economy stagnates due to the decrease in the demand of clean workers after the quota is set at 90% of full capacity, where there is a switch to the dirt sector, that now is the less advanced.

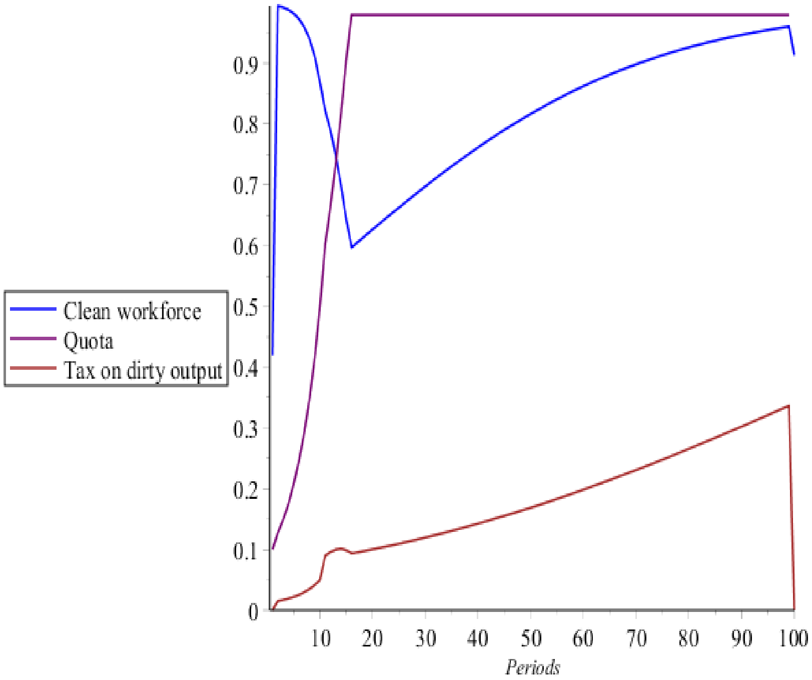
Figure 3.24: The green curve represents the environmental quality, the red one the consumption, the brown one the tax on the intermediate output in the North. Here I introduced the purple curve to outline the evolution of the quota over time. There is abatement technology



This behaviour is a consequence of the continuous shocks the economy is facing. In fact, to reduce the peaks we see in the previous sections, we are implicitly creating new inefficiency. The tax grows following the marginal consumption, and since the consumption is lower, it implies a lower tax.

The demand of clean labour is similar to the first scenario with net zero emissions. We see the initial peak were almost all the workforce is driven to the clean sector, that diminishes over time with the expansion of the abatement technology, where it falls to the lower level once the quota reaches its higher level, and there the tax and the technological change start influencing the demand.

Figure 3.25: The blue curve represents the percentage of workers employed in the clean sector in the North. The brown curve represents the tax applied to the intermediate dirty output. We have abatement technology.



We understand from this plot that the quota is stronger than the compensation, at least in the labour market.
In the next section I run the numerical simulation with net zero emissions without abatement technology. The reason is that the removal is exogenous and creates distortions that are evident.

# Bibliography

[1] Acemoglu D, 2009, *Introduction to Modern Economic Growth,* Princeton: Princeton University Press

[2] Acemoglu, D., Aghion, P., Bursztyn, L., Hémous, D. 2012a *'The Environment and Directed Technical Change',* American Economic Review, 102(1), 131–66.

[3] Acemoglu D.,Aghion P., Hémous D, 2014 *The environment and directed technical change in a North–South model,* Oxford Review of Economic Policy, Oxford University Press, Volume 30, Number 3 pp. 513–530

[4] Amigues J., Durmaz T., 2019 *A Two-Sector Model of Economic Growth with Endogenous Technical Change and Pollution Abatement,* Environmental Modelling & Assessment, Volume 24, pp 703-725

[5] Barro R. J., Sala-i-Martin X, 2003,*Economic Growth,* The MIT Press Cambridge, Massachusetts

[6] Bretschger L., Suphaphiphat N., 2014 *Effective climate policies in a dynamic North–South model,* European Economic Review, Volume 69, pp 59-77

[7] Bretschger L, Valente S, 2012 *Endogenous growth, asymmetric trade and resource dependence,* Journal of Environmental Economics and Management, Volume 64, pp 301-311

[8] Black, R., Cullen, K., Fay, B., Hale, T., Lang, J., Mahmood, S., Smith, S.M., 2021 *Taking Stock: A global assessment of net zero targets,* Energy & Climate Intelligence Unit and Oxford Net Zero

[9] Chackraborty P., Chatterjee C., 2017, *Does environmental regulation indirectly induce upstream innovation? New evidence from India* Research policy, Volume 46, pp 939-955

[10] Grossman G. M., Helpman E., Kircher P., 2017, *Matching, Sorting, and the Distributional Effects of International Trade* Journal of Political Economy, Volume 125, pp 224-264

[11] Harstad B., 2016, *The Dynamics of Climate Agreement* Journal of the European Economic Association, volume 14(3), pp 719-752

[12] Hassler J., Krusell P., Olovsson C., 2021, *Directed Technical Change as a Response to Natural Resource Scarcity* Journal of Political Economy, volume 129, number 11, November 2021 pp 3039-3072

[13] Hemous D., 2016, *The dynamic impact of unilateral environmental policies,* Journal of International Economics, Volume 103, pp 80-95

[14] Heomus D., Olsen M., 2021, *Directed Technical Change in Labor and Environmental Economics* Annual Review of Economics, Volume 13, pp 571–597

[15] IPCC, 2018: Annex I: Glossary [Matthews, J.B.R. (ed.)]. In: *Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*[Masson-Delmotte, V., P. Zhai, H.-O. Pörtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I. Gomis, E. Lonnoy, T. Maycock, M. Tignor, and T. Waterfield (eds.)]

[16] Khan A., 2015. *Unilateral Environmental Policy in a Two-Country World with Directed Technological Progress.* Thesis(M.Sc). Northwestern University

[17] Kolcava D., Nguyen Q., Bernauer 2019 *Does trade liberalization lead to environmental burden shifting in the global economy,* Ecological Economic, Volume 163, pp 98-112

[18] Kelly Levin K., Taryn Fransen T., Clea Schumer C, Davis C., 2017. *What Does "Net-Zero Emissions" Mean? 8 Common Questions, Answered.* Washington, DC. World Resource Institute. Available from: https://www.wri.org/insights/net-zero-ghg-emissions-questions-answered.

[19] Naqvi A, Stockhammer E., 2018, *Directed Technological Change in a Post-Keynesian Ecological Macromodel,* Ecological Economics, Volume 154, pp 168-188

[20] Naito T., 2017, *Growth and welfare effects of unilateral trade liberalization with heterogeneous firms and asymmetric countries,* Journal of International Economics, Volume 109, pp 167-173

[21] Nordhaus, William D., 2007 *A Review of the Stern Review on the Economics of Climate Change.* Journal of Economic Literature , 45(3): 686–702

[22] Papageorgiou C., Saam M., Schulte P., 2017, *Substitution between clean and dirty energy : A macroeconomic perspective,* The Review of Economics and Statistics, Volume 99(2), pp 281–290

[23] Peretto P., Valente S., 2011, *Resources, innovation and growth in the global economy,* Journal of Monetary Economics, Volume 58, pp 387-399

[24] Proops J., 2004 *The growth and distributional consequences of international trade in natural resources and capital goods: a neo-Austrian analysis,* Ecological economics, Volume 48, pp 83-91

[25] Zhang J., 2020 *International production fragmentation, trade in intermediate goods and environment,* Volume 87, pp 1-7

# Chapter 4

# International trade, environment and Directed technical change

## 4.1 Introduction

In the history of human race we have faced different challenges from the Black Death to the Cold war with the risk of nuclear annihilation. In the present days the risk of worldwide diseases threaten our life together with the uncertainty of artificial intelligence. The main challenge is the trade-off between economic growth and climate change.

In fact there is no doubt that the development of our economies increases the quality of our life decreasing the probability of starvation and diseases but this cannot be done through pollution that affects the next generations. It is clear that our way of production damages the environment we inhabit.

In this chapter I present two similar models with export of machines. In the first model the intermediate firms use machines produced locally and imported applying their local productivity. In the second model the productivity is linked with the machine used in the production, it implies that the one produced locally use the local technology meanwhile the imported one use the other country technology. These two models highlight the different kind of inputs that are traded in our economies. This methodology reflects the concept of property right and productivity. As an example, the same machine used in the UK needs to resist to wet weather, meanwhile if it is used in South Africa it needs to work in a drier environment. This implies that the productivity is related to the way a machine is adapted in a particular country. The second model represents the use of an engine or a software that needs to be applied with a particular technology, otherwise it could not work, or violate the copyright of the producer. This means that the concept of technology and productivity is stricter. In our world there is a variety of inputs that have different technological applications and constraints.

I show in my work how different type of inputs differently affect the demand of machines and the evolution of the world economy. In fact, under trade of inputs using local productivities, the demand of new technology is the same for the North and the

South. On th other hand, under free trade of machines that use their producer technology the expected profits of the two countries have a common trade component and a local one obtaining two different allocation of scientists.

## 4.2 Literature

Growth theory develops by addressing the question why there are cross-country income differences. It starts answering that research question analysing close economies and their characteristics. Now the focus is on trade and its effect on growth and first reason of cross-country differences are tariffs. Rivera-Batiz and Romer (1991)[16] is the first Directed Technical Change (DTC) model that use trade of inputs to understand the effect of tariffs on growth. In their model there are two regions, a North and a South that produce using imported and local inputs. The Rivera-Batiz and Romer model develops from the intuition of Krugman (1979)[9] and Krugman 1981[10] where the gains from trade are not comparative advantage but rather gains from variety expansion. In fact, in the 1981 paper similar economies create intra-industrial specialization instead of comparative advantages. The two multiple economies collaborate to the same projects creating collaborative industries instead of competitive markets. The classical example is the one of the automotive where different companies specialize on different components, which we call machines, to assemble the same final product. This paper follows the Krugman tradition applying the Rivera-Batiz and Romer model to understand its assumption on the Acemoglu et al. 2014[2] framework where there is trade of intermediate output. Naito (2021)[13] extends the Rivera-Batiz Romer model to find what is the optimal tariff for a growing economy finding that ”a unilateral tariff reduction by either country always increases the balanced growth rate” and that ”a zero tariff is locally optimal for a country under a mild condition, which is automatically satisfied at a symmetric balanced growth path with the zero tariff”.

After the first endogenous models it is clear in the literature the role between property rights and growth. Some economists tried to understand this topic from the relationship between trade and property rights. Gancia and Bonfiglioli (2008)[7] is a North South DTC model that addresses salaries and property rights with and without trade saying that ”protection of intellectual property is most beneficial in open countries”. Then, they analyse a cross-country panel that leads to the following statement: ”an increase in import penetration from low-wage, low-IPRs countries is followed by a sharp fall in R&D investment in a panel of US manufacturing sectors”.

In Benchekroun (2003)[5] the focus is on the effects of unilateral restrictions in a dynamic duopoly showing ”an increase of the asset’s stock and the long-run profits of the firm that is being imposed the production restriction”. His second result is important to our topic because it relates to one of the result of my model where using his words ”a unilateral decrease of the production of one firm can induce its rival to also decrease its production”. Lili Xu et al 2022 is a study on the ”strategic relations between emission tax and environmental corporate social responsibility in a Cournot–Bertrand comparison between a tax-then-ECSR (T game) and an ECSR-then-tax (E game)” where the authors argue that " the T game always yields higher emission tax than the E game irrespective of competition modes, but lower ECSR under Cournot competition and higher ECSR when the marginal damage is high under Bertrand competition". The main result is" that firms endogenously choose Cournot competition with the commitment of the E game irrespective of marginal damage and product substitutability."

The last oligopoly model is the one of Elsadany and Awad 2019 where the focus is on "the difference between price and quantity competition in a mixed duopoly game" with "Bertrand competition market with environmental taxes".
In their model Acemoglu et al (2014)[2] used a different evolution of technology in the South allowing it to grow through imitation. They discovered the conditions that allow to solve the model in two different ways, with global and local coordination. It depends by the elasticity of substitution that influences the effect of the tax on the economy. They outlined also when the South enters a pollution haven specializing in the dirty sector thanks to comparative advantages from trade. The idea of the imitation process and unilateral policy is a key point in my model and it is explored with different environmental constraints. There is a larger interest around unilateral policy, which is the scenario where the optimal policy is applied only to one country, usually the North, seeing how this decision influences the other country.

## 4.3 General framework

In this chapter I extend the previous work adding the international trade of machines. I do this using two similar model that differ only in the use of the productivities. In the first model, intermediate firm can only use local productivity. This implies that the technology cannot be transferred in different countries.[1]
In the last model every machine has its very own productivity that is released by the machine producer. The informatics sector is the best candidate to represent this concept where a machine is linked to a specific software that can be improved only by the producer .[2] Another example is the car sector where every vehicle has its own engine that cannot be changed without violating the copyright. These two models allow us to see how different types of machines affect the demands of new technologies avoiding an environmental disaster. In fact in the first model the demand of machines is the same for both regions meanwhile in the second model it is different. After that, I generalize the two models with infinite countries for future research.
This section describes a world divided in two economies, a North and a South. These two regions trade machines that will be used as input in the production of intermediate goods which will be assembled to create the final good. The North innovates while the South imitates in order to increase the productivity.
In the next sections I show the main characteristics of the model.

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[1] This represent the situation where the productivity coincide with the infrastructure level of a determined country.

[2] The simplest example is given by the main software used by Microsoft and Apple that are unique to that brands.

### 4.3.1 Preferences

The utility function takes the form of the one seen in the previous chapters

$$\sum_{t=1}^{\infty}(\frac{1}{1+\rho})^t u^i(C_t^i, S_t^i) \tag{4.1}$$

where $i = N, S$ with $N$ referring to the north and $S$ to the south. $C_t^i$ represents the consumption of the final good respectively in North and South at time $t$, $S_t^i$ denotes the quality of the environment at time t in the country $i$, and $\rho \geq 0$ is the discount rate. This utility function implies that the gain from consumption is complementary with the environmental quality. To have the maximum utility environment needs to not be damaged otherwise the representative household cannot fully enjoy the good she is consuming because the utility is directly proportional in consumption and environmental quality. In this work I consider the environment as a regional entity to highlight the idea that people care more about what they have closer compared to what they cannot see with their eyes[3].

As in Acemoglu et al (2012)[1] the environmental quality $S \in (0, \bar{S})$ where $\bar{S}$ is the quality of the environment without any pollution, while 0 implies that an environmental disaster occurs. In this chapter the local environment is affected by transboundary pollution generated by the production of the dirty sector. Having two different environments allows me to do a more detailed analysis of the intrinsic environmental characteristics and how they react to external stimulations.

Now in a more formal way, the instantaneous utility function $u^i(C_t^i, S_t^i)$ is increasing in $C_t^i$ and $S_t^i$, differentiable and concave. The following Inada-type conditions are verified in both countries

$$\lim_{C\to 0}\frac{\partial u(C,S)}{\partial C} = \infty \text{ and } \lim_{S\to 0}\frac{\partial u(C,S)}{\partial S} = \infty \tag{4.2}$$

$$\lim_{S\to 0} u(C,S) = -\infty \tag{4.3}$$

which implies that when the environmental quality reaches 0 a environmental disaster occurs and it is impossible to enjoy the final good implying the utility drops to zero[4].

### 4.3.2 Production technology

In this section I present the structure of the production. To summarize it, the world is divided in two perfectly symmetric economies that use machines imported and produced domestically. The machine producers create inputs that are traded between the countries following the setting of Rivera-Batiz and Romer (1991)[16]. The intermediate

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[3] This is the reason why we care more if there are waste close to our houses instead of the melting of the environmental disasters that occur in the other side of the world.

[4] The economic intuition is explained in the previous chapter. The main concept is that it does not matter how many goods you can consume in a world that cannot sustain life.

firms purchase the machines to produce competitively their intermediate output. The final producers assemble the intermediate goods in order to sell their final output to the consumers.

Now I explain in detail the structure of each production function. The intermediate firm is producing competitively using a Cobb-Douglas production function.

$$maxY_{jt}^N = p_{jt}^N (L_{jt}^N)^{1-\alpha} [\int_0^1 (A_{jkt}^N)^{1-\alpha} (d_{jkt}^N)^\alpha dk + \int_0^1 (A_{jkt}^N)^{1-\alpha} (m_{jkt}^S)^\alpha dk] - w_t^N L_{jt}^N + \tag{4.4}$$

$$- \int_0^1 p_{jkt}^N d_{jkt}^N dk - \int_0^1 p_{mjkt}^S m_{jkt}^S dk$$

The input share is represented as $\alpha \in (0,1)$. The firm uses a continuum of machines of type $k$, where $d_{jkt}^N$ is the one produced domestically to internal use, and $m_{jkt}^S$ is the number of machines imported from the South. The superscript $i = N, S$ represents the origin in this context and the subscript $j = c, d$ describes the sector ,clean or dirty, we are operating. I assume that the productivity $A_{jkt}^i$ is the know-how that allows a machine to work in a particular country[5]. This implies that a country can use the machines produced all around the world. $p_{jt}^N$ is the selling price at time $t$ of the intermediate good. $L_{jt}^N$ is the amount of people working in the sector $j$ at time $t$ and $w_t^N$ represents their salary. $p_{jkt}^N$ is the cost to buy a machine locally and $p_{mjkt}^S$ represents the price to purchase a machine from the South. Also, it is important to highlight that there is no accumulation of machine, instead they fully-depreciate at the end of each period.

Similarly in the South the intermediate production is represented as follows

$$maxY_{jt}^S = p_{jt}^S (L_{jt}^S)^{1-\alpha} [\int_0^1 (A_{jkt}^S)^{1-\alpha} (d_{jkt}^S)^\alpha dk + \int_0^1 (A_{jkt}^S)^{1-\alpha} (m_{jkt}^N)^\alpha dk] - w_t^S L_{jt}^S + \tag{4.5}$$

$$- \int_0^1 p_{jkt}^S d_{jkt}^S dk - \int_0^1 p_{mjkt}^N m_{jkt}^N dk$$

where $d_{jkt}^S$ is the number of machines locally produced in the South, and $m_{jkt}^N$ is the number of imported machines from the North. $p_{jkt}^S$ is the cost to produce the machine in the South, meanwhile $p_{mjkt}^N$ is the cost to import them from the North.

Now in both countries the machine producer is supplying the local economy and the foreign one, implying that it takes the form of

$$\pi_{jt}^i = \max_{d_{jt}^i, m_{jt}^i} p_{jkt}^i d_{jkt}^i + p_{mjt}^i m_{jt}^i - (d_{jkt}^i + m_{jt}^i)\psi \tag{4.6}$$

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[5] To comparison I pick two random countries which can easily represent the North and the South. When I use a machine in the UK I need a technology that allows me to use it in a country with high humidity meanwhile in a nation as Egypt I need a technology that allow me to use the machine in an arid environment. It can be applied for the roads. In fact, to build a road in the Uk and Egypt I need the same inputs that can be imported from a third country but the difference is given by the temperature of the road, which implies a different technology. In a cold country roads are not developed to sustain more than 40 degrees meanwhile in Egypt you need to sustain at least 50 degree. To summarize this concept, two countries can use the same input applying a different know-how to solve a local constraint which in this example is represented by the weather.

where here the machine producer is satisfying two demands simultaneously increasing its profits. This specification of inputs shows the characteristics of a globalized world with international trade. In fact, the most traded commodities by quantity are the inputs. For example, one of the most traded commodity by quantity is sand with a value close to 50 billion tonnes par year [17].

In each country there is a continuum of final producers that assemble intermediate goods $Y^i_{ct}$ and $Y^i_{dt}$ creating final output $Y^i_t$ according to the CES production function such that

$$Y^i_t = \{(Y^i_{ct})^{\frac{\epsilon-1}{\epsilon}} + (Y^i_{dt})^{\frac{\epsilon-1}{\epsilon}}\}^{\frac{\epsilon}{\epsilon-1}} \tag{4.7}$$

where the representative final producer sells the final output to the representative household at the numeraire price, $p_t = 1$. The intermediate dirty good cost $p^i_{dt}$ per unit, as well as the clean one that sustain the cost $p^i_{ct}$. I follow the intuition of Acemoglu et al 2012([1]) assuming the elasticity of substitution, $\epsilon$, to be larger than 1. This implies that the clean and dirty intermediate outputs are perfect substitutes, allowing corner solutions. Otherwise it would be impossible to move away from the dirty production[6].

### 4.3.3 Technology

In this model we have two types of technologies made using two different processes. The northern technology follows the innovation path on the shoulders of giants meanwhile the southern one evolves according to an imitation process. The North improves the productivity depending on the value of its past technologies meantime the South takes into account the northern technological progress before developing its own.

In the North there is a mass of scientists $s^N_t = 1$ which can engage research in the clean sector $s^N_{ct}$ or in the dirty sector $s^N_{dt}$ such that $s^N_{ct} + s^N_{dt} = s^N_t$. Similarly to the North there is a mass of scientists $s^S_t = 1$ such that $s^S_{ct} + s^S_{dt} = s^S_t$. Once a successful innovation occurs the scientists in the North obtain a one period patent becoming the monopolist of that type of machine. As in Acemoglu et al (2014) when a scientist in the South is successful "she is successful, she will have imitated the frontier machine in the North"[2]. Recalling my previous chapter the use of a technology to produce a machine of type k is protected by patents, but this property right do not cover the imitation in another part of the world and so, it is excludable only locally but not globally. This implies that the machine producers in the North cannot stop the imitation from the South. Also, the international trade of machines does not concern the input producers because they are still profiting from their production and do not have the power and the will to stop the trade which is increasing their profits. In fact they tolerate the import of imitated machines because they can export their own to the other region.[7]

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[6] The reason is simple. Under complementarity the economy needs both inputs to properly function, meanwhile with perfect substitution a sector can overwhelm the other eliminating possible externalities.

[7] In the globalized economy some firms tolerate the competition of copied good because they are considered of less quality compared to the original one. In fact branding has a major role in this part.

The evolution of the productivity $A_{Njt}$ is represented by

$$A_{jt}^{N} = (1 + \gamma\eta_j s_{jt}^{N}) A_{jt-1}^{N} \tag{4.8}$$

where $\eta_j \in (0, 1)$ is the probability of a successful innovation, $(1 + \gamma)$ is the increased amount of machine's quality with $\gamma > 0$ and $s_{jt}^{N}$ is the number of scientist in the North working in clean and dirty sectors with $j = c, d$.
In the South the productivity $A_{jt}^{S}$ is described as

$$A_{jt}^{S} = \kappa_j s_{jt}^{S} A_{jt}^{N} + (1 - \kappa_j s_{jt}^{S}) A_{jt-1}^{S} \tag{4.9}$$

The parameter $\kappa_j \in (0, 1)$ expresses the probability to copy an innovation in the North. The meaning of this equation is that the scientists in the South will imitate the technology from the North and if it is not possible they will keep using the one already existing in the South.

### 4.3.4 Environment

In each region the environment depends by the local and foreign dirty production. In fact I assume it as a public good that receive spillovers from the foreign country to better represent the effects of the generation of CO2. I decide to divide it in two parts because different regions respond differently from environmental degradation. For example, a desertic area does not react as a tropical one. Following my previous chapters the environment is defined as

$$S_{t+1}^{N} = -\{(\zeta(Y_{dt}^{N} + Y_{dt}^{S})\} + (1 + \delta) S_{t}^{N} \tag{4.10}$$

where $S_{it} \in (0, 1)$ is the environmental quality at time $t$,where $\zeta$, is emission rate. The one in the South is similar, and it takes the form of

$$S_{t+1}^{S} = -\{\zeta(Y_{dt}^{S} + Y_{dt}^{N})\} + (1 + \delta) S_{t}^{S} \tag{4.11}$$

where the variables have the same meaning of the equation before.
These two equations represent the environmental characteristic to regenerate faster when it is close to full capacity.[8] Once the pollution generated reaches a certain level, the environment reaches its lower level, 0 the condition $\lim_{S \to 0} u(C, S) = -\infty$ happens and an environmental disaster occurs implying the end of the economy.
To summarize, the environment is divided in two subsets[9] and then analysed. The environment has the same behaviour in both countries. When the value of $S_{t+1}^{i}$ reaches

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[8] The simplest example is given by the role of the oceans. They have the ability to absorb CO2 from the atmosphere, until they reach their limit.

[9] The two subsets represent the environment of each region. The reason is quite simple. We understand what we can see and care about what is happening in our backyard. Also, if the Environmental Kuznet curve is true, the environmental issues are linked with the income. This kind of model already proves that the environmental policy should be income related since it is increasing with consumption, or inversely proportional to the marginal utility of consumption.

0 there is an environmental disaster, while at 1 it has its higher value and it is fully restored. The environment is damaged at every period by the emissions, but it is also restored by its regenerative parameter. Our target is to decrease the dirty production before $\zeta(Y^S_{dt} + Y^N_{dt}) > (1 + \delta)S^i_t$ otherwise the economies lead to an environmental disaster.
In the next section I highlight the equilibrium variables and the direction of technical change in the lassaiz-faire.

## 4.4 Laissez-faire equilibrium

In this section I describe the lassaiz-faire where the two economies have no policy to address the environmental issue.

**Definition 5** *An equilibrium is given by sequences of wages* $(w_{it})$ *, prices for inputs* $(p_{jt})$*, prices for machines* $(p_{jkt})$*, prices of imported machines* $(p^i_{mjkt})$*, quantities of machines* $(d^i_{jkt})$*, quantities of machines imported* $(m^i_{jt})$*, quantities of inputs* $(Y^i_{jt})$*, quantities of labour* $(L^i_{jt})$ *by input producers* $j \in c, d$*, research allocations* $(s^i_{dt}, s^i_{ct})$*, and quality of environment* $(S_t)$ *such that, in each period t: (1)* $(p^i_{jkt}, x^i_{jkt})$ *and* $(p^i_{mjt}, m^i_{jt})$ *maximize profits by the producer of machine i in sector j; (2)* $L^i_{jt}$ *maximizes profits by producers of input j; (3)* $Y^i_{jt}$ *maximizes the profits of final good producers; (4)* $(s^i_{dt}, s^i_{ct})$ *maximizes the expected profit of a researcher at date t; (5) the wage* $(w^i_t)$ *and the prices* $p_{jt}$ *clear the labour and input markets respectively; and (6) the evolution of* $S^i_t$ *is given by (4.10) and (4.11).*

Such that the market clearing conditions of the workers are

$$L^N_{ct} + L^N_{dt} \leq 1 \text{ and } L^S_{ct} + L^S_{dt} \leq 1 \tag{4.12}$$

also, I normalize the total mass of scientists in each country $s_i$ to 1, denoting the total amount of researchers working on machines in sector $j \in c, d$ at time $t$ by $s^i_{jt}$. The market clearing conditions for scientists engaged in research are

$$s^N_{ct} + s^N_{dt} \leq 1 \text{ and } s^S_{ct} + s^S_{dt} \leq 1 \tag{4.13}$$

The market clearing conditions imply that the total amount of workers and researchers in each country needs to be equal to one[10]. The market clearing for the final good in the North implies

$$C^N_t = Y^N_t - \psi\left(\int_0^1 d^N_{ckt}dk + \int_0^1 d^N_{dkt}dk + \int_0^1 m^S_{ckt}dk + \int_0^1 m^S_{dkt}dk\right) \tag{4.14}$$

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[10] This allows us to study the allocation of the population using percentages. The main tipping point of this model requires to overtake the 51% of the population. In fact in the next sections I show how the workers increase their percentage following the growth of the productivity that reflect their sector.

and the one in the South

$$C_t^S = Y_t^S - \psi \left( \int_0^1 d_{ckt}^S dk + \int_0^1 d_{dkt}^S dk + \int_0^1 m_{ckt}^N dk + \int_0^1 m_{dkt}^N dk \right) \tag{4.15}$$

The market clearing for the final good represents the core of my framework. The representative household consumes final output sustaining the cost of the machines produced for local use and imported.

In this new framework, every firm is using two machines, one made locally and the other imported. This implies that every firm faces a national and a foreign demand. The main effect of the use of more machines is that the final output is larger compared to the closed economy version thanks to the greater supply of inputs.

The intermediate firms produce clean and dirty output competitively satisfying their demands which are represented by the demand of machines produced locally

$$d_{jt}^N = (\frac{p_{jkt}^N}{\alpha p_{jt}^N})^{\frac{1}{\alpha-1}} A_{jt}^N L_{jt}^N \tag{4.16}$$

and the one imported from the South is

$$m_{jt}^S = (\frac{p_{mjkt}^S}{\alpha p_{jt}^N})^{\frac{1}{\alpha-1}} A_{jt}^N L_{jt}^N \tag{4.17}$$

with the equation above we have defined the demand of machines in the North. Now since the cost to purchase a machines is the same independently from the origin, $p_{mjkt}^S = p_{jkt}^N = \alpha$ the demand of machine produced locally is equal to demand of the one imported. [11]

Similarly we want to represent the machine produced locally in the South as

$$d_{jt}^S = (\frac{p_{jkt}^S}{\alpha p_{kjt}^S})^{\frac{1}{\alpha-1}} A_{jt}^S L_{jt}^S \tag{4.18}$$

meanwhile the imported one

$$m_{jt}^N = (\frac{p_{mjkt}^N}{\alpha p_{jt}^S})^{\frac{1}{\alpha-1}} A_{jt}^S L_{jt}^S \tag{4.19}$$

The equation above describe the machine production The salaries in both countries depend by the following equation [12]

$$2(1-\alpha)(p_{jt}^N)^{\frac{1}{1-\alpha}} A_{jt}^N = w_t^N \tag{4.20}$$

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[11] The proof of the equality between the costs of the machine is in the appendix (A). This result should not surprise. The machine producers are colluding to obtain the maximum amount of profits from the other firms. A Social planner would impose a Bertrand oligopoly where the selling prices are equal to the marginal costs. It implies that in this model the collusion substitutes monopoly distortions.

[12] Here the salaries are doubled due to the use of two machines. The two inputs double the value of the output increasing the wages. In fact, most of the variables in this model are multiplied by two thanks to the two demands of machines.

Then dividing eq (4.20) by sector we obtain the relative prices of the intermediate goods which are defined as

$$\frac{p^i_{ct}}{p^i_{dt}} = (\frac{A^i_{ct}}{A^i_{dt}})^{-(1-\alpha)} \tag{4.21}$$

The relative prices have the same specification as in Acemoglu et al (2012)[1] implying that international trade does not affect the relative variables. Now given the fact that the machines producers create the inputs for intermediate firms in both countries, their profits depends by the sum of the two demands. The structure of the profits implies that the input producer considers the demand of machines from both countries simultaneously.

$$\pi^N_{jt} = \alpha(1-\alpha)[(p^N_{jt})^{\frac{1}{1-\alpha}} A^N_{jt} L^N_{jt} + (p^S_{jt})^{\frac{1}{1-\alpha}} A^S_{jt} L^S_{jt}] \tag{4.22}$$

The same happens in the South leading to

$$\pi^S_{jt} = \alpha(1-\alpha)[(p^N_{jt})^{\frac{1}{1-\alpha}} A^N_{jt} L^N_{jt} + (p^S_{jt})^{\frac{1}{1-\alpha}} A^S_{jt} L^S_{jt}] \tag{4.23}$$

Here they sell the machines at the price $\alpha$ sustaining the cost $\alpha^2$. This is the solution when the production cost of a machine is the same, which leads to the same profits in the North and the South where $\frac{\pi^N_{dt}}{\pi^S_{dt}} = 1$ representing the scenario where the machines producers perfectly share the global market. This is the collusion mechanism where the machine producers perfectly share the market. Otherwise, when the cost in the North is lower,$\lim_{t\to\infty} \frac{\pi^N_{Nt}}{\pi^S_{dt}} = \infty$ it implies that the North not only is obtaining the shares of the other country but is also leading the development of new technologies. When the cost to produce machines is lower in the South, it implies $\lim_{t\to\infty} \frac{\pi^N_{dt}}{\pi^S_{dt}} = 0$, or that the South is selling its machines in the North substituting the northern machine producers strongly influencing the allocation of scientists.[13]

The scientists engage in research in the sector that has the higher profitability to obtain patents. Since there are no trade barriers once they have the monopoly on a type of machine they can sell it in all the regions with no frictions. This mean that a scientists needs to take into account the expected profits of the demands of machines from the two regions. In fact once they develop a new machine for a foreign firm the buyers apply the technology they have. To better understand the expected profits of the scientists under free trade I start from the autarky ones in Acemoglu et al 2014 [2] where there is no competition in the production of machines but monopolists that supply the local intermediate firms. The expected profit in the North under autarky is represented by the equation below

$$\Pi^N_{jt} = \alpha(1-\alpha)(1+\gamma)\eta_j(p^N_{jt})^{\frac{1}{1-\alpha}} L^N_{jt} A^N_{jt-1} \tag{4.24}$$

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[13] The explanation of the limits is intuitive. The producer with the lower cost will increase its profits increasing its market share. In fact a lower cost imply a lower selling price affecting the trade equilibrium.
A different solution require a price war, which is not profitable for either firm. The result of this competition would be a firm acquiring all the markets.

where the profits of the machine producer depends by the probability to create a new technology $\eta_j$ and the augmented quality $(1+\gamma)$. In the South they are represented similarly as

$$\Pi^S_{jt} = \alpha(1-\alpha)\kappa_j (p^S_{jt})^{\frac{1}{1-\alpha}} L^S_{jt} A^N_{jt} \tag{4.25}$$

where the scientists in the South depends by the probability to imitate $\kappa_j$ a new technology from the North. This represent the expected profits from each demand of machines. Now the expected value of a sum of variables is equal to the sum of the expected values such that

$$\Pi^N_{ct} = \alpha(1-\alpha)[(1+\gamma)\eta_c (p^N_{ct})^{\frac{1}{1-\alpha}} L^N_{ct} A^N_{ct_{-1}} + \kappa_c (p^S_{ct})^{\frac{1}{1-\alpha}} L^S_{ct} A^N_{ct}]$$

The expected profit of a machine producer is equal to the sum of the individual expected earning of each country where it is operating[14]. This is a particular case where there is imitation, but with innovation in the South we would have the same overall results. The direction of technical change depends on

$$\frac{\Pi^N_{ct}}{\Pi^N_{dt}} = \frac{(1+\gamma)\eta_c (p^N_{ct})^{\frac{1}{1-\alpha}} L^N_{ct} A^N_{ct_{-1}} + \kappa_c (p^S_{ct})^{\frac{1}{1-\alpha}} L^S_{ct} A^N_{ct}}{(1+\gamma)\eta_d (p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}} + \kappa_d (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}} \tag{4.26}$$

which are represented in the canonical way

$$\begin{aligned}\frac{\Pi^N_{ct}}{\Pi^N_{dt}} &= \frac{\eta_c (p^N_{ct})^{\frac{1}{1-\alpha}} L^N_{ct} A^N_{ct_{-1}}}{\eta_d (p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}}}(1 + \frac{\kappa_d (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}}{(1+\gamma)\eta_d (p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}}})^{-1} + \\ &+ \frac{\kappa_c (p^S_{ct})^{\frac{1}{1-\alpha}} L^S_{ct} A^N_{ct}}{\kappa_d (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}}(1 + \frac{(1+\gamma)\eta_d (p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}}}{\kappa_d (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}})^{-1}\end{aligned} \tag{4.27}$$

where on the left side of the equation we have the relative expected profits of the scientists and on the right there are the three forces that move the direction of innovation[15]. We have respectively $(\frac{p^N_{ct}}{p^N_{dt}})^{\frac{1}{1-\alpha}}$, which is the price effect, $\frac{A^N_{ct-1}}{A^N_{dt-1}}$ the productivity effect and the market size effect $\frac{L^N_{ct}}{L^N_{dt}}$. At the same time we have the same values for the South, the price effect $(\frac{p^S_{ct}}{p^S_{dt}})^{\frac{1}{1-\alpha}}$,the market size effect $\frac{L^S_{ct}}{L^S_{dt}}$ and the productivity effect $\frac{A^N_{ct}}{A^N_{dt}}$. Note that in this framework the direction of technical change depends on the sum of the domestic factors weighted by the "trade effect"[16] composed by the price advantage

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[14] In fact, when a company is operating in different country, it should take into account the global earning before develop a ne technology, since it could be applied in every country they are operating. It is clear in the car industry where the decision to develop a new vehicle depends by the markets here it could be successful.

[15] All the passages to prove it are in the Appendix C.

[16] The trade effect can be represented by the value $(1 + \frac{\kappa_d (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}}{(1+\gamma)\eta_d (p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A_{dt^N_{-1}}})^{-1}$which it can be seen as the export size effect on the North.

$(\frac{p^S_{dt}}{p^N_{dt}})^{\frac{1}{1-\alpha}}$ , the market size advantage $\frac{L^S_{dt}}{L^N_{dt}}$ and the productivity advantage $\frac{A^N_{dt}}{A^N_{dt-1}}$. The price advantage implies that the country with the largest price influences more the direction of the technical change. The same can be said for the market advantage, where the country with the larger workforce has a greater effect on the scientists allocation. The productivity effect has a different meaning. With innovation in both countries we would have $\frac{A^S_{dt}}{A^N_{dt}}$ implying that the country with the largest productivity is influencing the scientists more. But here we have imitation and the scientists consider only the leading technology in their expectations. It implies that the scientists will weight more the southern demand when the technology grows over time implying that $\frac{A^N_{dt}}{A^N_{dt-1}} > 1$ is true. Otherwise when the relative productivity advantage is constant $A^N_{dt} = A^N_{dt-1}$ the scientists are not considering it because there is no possibility to imitate.
When the Southern values are larger than the Northern one, the scientists are weighting more the direction of technical change generated by the South, otherwise they will consider more the one of their country, the North.
Similarly for the South, the profits are represented by

$$\frac{\Pi^S_{ct}}{\Pi^S_{dt}} = \frac{\eta_c(p^N_{ct})^{\frac{1}{1-\alpha}} L^N_{ct} A^N_{ct_{-1}}}{\eta_d(p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}}}(1 + \frac{\kappa_d(p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}}{(1+\gamma)\eta_d(p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}}})^{-1} + \tag{4.28}$$

$$+\frac{\kappa_c(p^S_{ct})^{\frac{1}{1-\alpha}} L^S_{ct} A^N_{ct}}{\kappa_d(p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}}(1 + \frac{(1+\gamma)\eta_d(p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt_{-1}}}{\kappa_d(p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^N_{dt}})^{-1}$$

This implies that the expected profits in the North and in the South are equal as $\frac{\Pi^N_{ct}}{\Pi^N_{dt}} = \frac{\Pi^S_{ct}}{\Pi^S_{dt}}$ In a world where the economies are symmetric there is only a global direction of technical change. When the social planner influences the decision of an economy, e.g. a tax on the dirty intermediate output, it redirect the global economy. This means that it is possible to redirect the imitation process in the South using unilateral policy in the North. This implies our proposition

**Proposition 7** *It is possible that redirecting the profit of one country will automatically redirect the world profit, because* $\frac{\Pi^N_{ct}+\Pi^S_{ct}}{\Pi^N_{dt}+\Pi^S_{dt}} = \frac{\Pi^N_{ct}}{\Pi^N_{dt}} = \frac{\Pi^S_{ct}}{\Pi^S_{dt}}$

In fact, the relative expected world profits, are the same as the relative expected profits of each country. This implies that scientists, independently from their location, will take into consideration the development of the world economy before deciding to engage in research in a sector.
This result explains the global research system and its mechanics. Nowadays the scientists in one country study things happening in the other side of the world considering characteristics that happen worldwide. This phenomenon is typical especially in economics, where researchers need to find economic mechanisms that affect the whole world, trying to understand how they will influence their own nation and vice versa.
Now to find the equilibrium production level we start from the optimum levels that we describe inserting the demand of locally produced and imported machines into the

production function obtaining

$$(Y^i_{jt})^* = 2(p^i_{jt})^{\frac{\alpha}{1-\alpha}} L^i_{jt} A^i_{jt} \tag{4.29}$$

where the optimum production depends by the intermediate price, the labour force and the productivity level of the related sector. Substituting it inside the first order condition of the final producer from equation (4.7) we obtain the relative labour which is defined as

$$\frac{L^i_{ct}}{L^i_{dt}} = (\frac{A^i_{ct}}{A^i_{dt}})^{-\varphi} \tag{4.30}$$

which explain that under free trade of inputs the market size of each country depend by their relative productivity.[17]

Inserting the equilibrium price which is represented by $p^N_{ct} = [(\frac{A^N_{ct}}{A^N_{dt}})^{\varphi} + 1]^{\frac{1}{\epsilon-1}}$ inside the optimum production, the equilibrium production is defined by

$$Y^N_{ct} = 2[(A^N_{ct})^{\varphi} + (A^N_{dt})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} A^N_{ct} (A^N_{dt})^{\alpha+\varphi}$$

and the dirty one is represented

$$Y^N_{dt} = 2[(A^N_{ct})^{\varphi} + (A^N_{dt})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} A^N_{dt} (A^N_{ct})^{\alpha+\varphi}$$

The final good production in equilibrium terms is

$$Y^N_t = 2[(A^N_{ct})^{\varphi} + (A^N_{dt})^{\varphi}]^{\frac{1}{-\varphi}} A^N_{ct} A^N_{dt}$$

where $\varphi \equiv (\epsilon - 1)(\alpha - 1)$ with $\epsilon > 1$ representing the elasticity of substitution. The South has the same equations, expressed with southern subscript $S$. The final output is doubled by the use of two machines to create the intermediate inputs. Considering equation (4.14) it is clear that the cost of imported machines balances the larger output. Comparing the equilibrium production with the result of Acemoglu et al (2012)[1] we can see that trade allow the economies to grow faster producing more due to the high flow of inputs purchased by the intermediate firms.

Given the informations above I substitutes the price and market size effects in equation (4.140) obtaining

$$\frac{\Pi^N_{ct}}{\Pi^N_{dt}} = \frac{\eta_c (A^N_{ct})^{-1-\varphi} A^N_{ct_{-1}}}{\eta_d (A^N_{dt})^{1-\varphi} A^N_{dt_{-1}}} (1 + \frac{\kappa_d (1 + (\frac{A^S_{ct}}{A^S_{dt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A^N_{dt}}{(1+\gamma)\eta_d (1 + (\frac{A^N_{ct}}{A^N_{dt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A^N_{dt_{-1}}})^{-1} \tag{4.31}$$

$$+ \frac{\kappa_c (A^S_{ct})^{-1-\varphi} A^N_{ct}}{\kappa_d (A^S_{dt})^{-1-\varphi} A^N_{dt}} (1 + \frac{(1+\gamma)\eta_d (1 + (\frac{A^N_{ct}}{A^N_{dt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A^N_{dt_{-1}}}{\kappa_d (1 + (\frac{A^S_{ct}}{A^S_{dt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A^N_{dt}})^{-1}$$

The value $(1 + \frac{\kappa_d (1+(\frac{A^S_{ct}}{A^S_{dt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A^N_{dt}}{(1+\gamma)\eta_d (1+(\frac{A^N_{ct}}{A^N_{dt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A^N_{dt_{-1}}})^{-1}$ is the northern export size effect in equilibrium terms. Also, given my assumption it perfectly split the scientists interest in

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[17] This is the classical result of the DTC model developed after the ABHH model.

two equal parts.[18]. The most important part is that the innovation occur in the clean sector when the expected profits $\frac{\Pi^N_{ct}}{\Pi^N_{dt}}$ are larger than 1.

**Lemma 1** *When we allow trade of machines between different regions the expected profits of the scientists are the sum of the autarky ones weighted by the trade effect.*

**Proposition 8** *Under lassaiz-faire with symmetric economies and the elasticity of substitution $\epsilon$ is larger than 1, there is no comparative advantage, both countries will specialize in dirty machines and technologies.*

To prove this proposition we divide the expected profit of the clean sector in the North by the one in the South obtaining $\frac{\Pi^N_{ct}}{\Pi^S_{ct}} = 1$.
The reason of this outcome is due to the fact that researchers in both countries are developing the same sector simultaneously. The second reason is due to the hidden collusion mechanism that affect the machine producers. Meanwhile, complementarity gives ambiguous results which are difficult to interpret in terms of comparative advantage.[19]
Here the demand of the most advanced economy leads the profits of the follower. In the next section I explain the mechanisms that affect the allocation of scientists highlighting when an innovation occurs in the clean and dirty sector.

### 4.4.1 Scientists allocation

In the model of Acemoglu et al 2012, the allocation of scientists is determined by the expected profits. When the relative expected profits,$\frac{\Pi^i_{ct}}{\Pi^i_{dt}}$ , are larger than 1, all the scientists are engaged in clean research, otherwise, when the expected relative profits are smaller than 1 the opposite. Otherwise the researchers are perfectly split in both sectors. This behaviour is a consequence of the perfect substitution between the intermediate goods. In fact, the final producer can assemble the final good using clean or dirty intermediate outputs. When there is complementarity between the two sectors, the final firm needs a combination of the two inputs. To conclude the point, the scientists decide the technology to develop taking into consideration the elasticity of substitution. When there is perfect substitution they study the most advanced sector which leads to a corner solution, otherwise they will study both of them obtaining an interior solution. To describe the scientists allocation I approximate the trade effect to be equal to $\frac{1}{2}$[20]. To summarize, I approximate the function to obtain a more elegant equation that can be easily understood. I do it because the scientists are considering the two countries similarly and it is a direct consequence of the two first propositions where

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[18] When I simulate it for different parameters and productivities the result is always close to$\frac{1}{2}$. It implies that the scientists have no preference but will decide the next research considering all the factors affecting the countries without any bias.

[19] In this model complementarity between clean and dirty sectors allows interior solution implying different results in different periods affecting the trade side of the equation.

[20] I show the reason of the approximations in the appendix.

the expected profits of the two countries are equal and that there is no comparative advantage between North and South. Thanks to my approximation this threshold is now equal to 2. It implies that an innovation occur in the clean sector when the expected profits satisfy the following relationship

$$\frac{\eta_c}{\eta_d}(\frac{A^N_{ct}}{A_d t}^{N})^{-1-\varphi}\frac{A^N_{ct-1}}{A^N_{dt-1}} + \frac{\kappa_c}{\kappa_d}(\frac{A^S_{ct}}{A^S_{dt}})^{-1-\varphi}\frac{A^N_{ct}}{A^N_{dt}} > 2 \tag{4.32}$$

The scientists will engage the dirty sector when the sum of the expected profits are

$$\frac{\eta_c}{\eta_d}(\frac{A^N_{ct}}{A_d t}^{N})^{-1-\varphi}\frac{A^N_{ct-1}}{A^N_{dt-1}} + \frac{\kappa_c}{\kappa_d}(\frac{A^S_{ct}}{A^S_{dt}})^{-1-\varphi}\frac{A^N_{ct}}{A^N_{dt}} < 2 \tag{4.33}$$

Otherwise they will engage both sectors simultaneously which can be expressed in a more formal way as

$$\frac{\eta_c}{\eta_d}(\frac{A^N_{ct}}{A^N_{dt}})^{-1-\varphi}\frac{A^N_{ct-1}}{A^N_{dt-1}} + \frac{\kappa_c}{\kappa_d}(\frac{A^S_{ct}}{A^S_{dt}})^{-1-\varphi}\frac{A^N_{ct}}{A^N_{dt}} = 2 \tag{4.34}$$

It is noteworthy to cite that the sum of the expected profit of each country under autarky is equal to the expected profits under free trade. It is plausible to assume that the global allocation of scientists is the sum of every contribution.[21]

**Lemma 2** *Under lassaiz-faire it is an equilibrium for an innovation at time t to occur i the clean sector only when* $\eta_c(A^N_{ct-1})^{-\varphi} > \frac{2\eta_d(A^N_{dt-1})^{-\varphi}(1+\gamma\eta_c)^{1+\varphi}}{1+\frac{\kappa_c\eta_d}{\kappa_d\eta_c}\phi_A(1+\gamma\eta_c)}$*,in the dirty sector only when* $\eta_d(A^N_{dt-1})^{-\varphi} > \frac{\eta_c}{2}(A^N_{ct-1})^{-\varphi}(1+\gamma\eta_d)^{1+\varphi}[1+\frac{\kappa_c\eta_d}{\kappa_d\eta_c}\phi_A(1+\gamma\eta_d)^{-1}]$ *and in both sectors when* $2\eta_d(A^N_{dt-1})^{-\varphi}(1+\gamma\eta_c s^N_{ct})^{1+\varphi} = \eta_c(A^N_{ct-1})^{-\varphi}(1+\gamma\eta_d s^N_{dt})^{1+\varphi}[1+\frac{\kappa_c\eta_d}{\kappa_d\eta_c}\phi_A\frac{(1+\gamma\eta_c s^N_{ct})}{(1+\gamma\eta_d s^N_{dt})}]$

where $\phi_A = (\frac{A^S_{ct}A^N_{dt}}{A^S_{dt}A^N_{ct}})^{-1-\varphi}$ is the relative comparative advantage .[22] As in in the Lemma of Acemoglu et al (2012)[1] the conclusion is that the scientists develop the most advanced sector.

## 4.5 Optimal policy

In the last section we see the equilibrium solution of the model where the scientists choose to continue researching the most advanced sector which is the dirty one going

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[21] In this paper I outline that the allocation of scientists is a global mechanism composed by different parts. This equation shows this behaviour properly. In the next section I show how this effect depends by the diffusion and use of foreign productivity.

[22] Note that $\phi_A$ has no time. It is due to the fact that it is constant over time because it is equal to $(\frac{A^S_{ct}A^N_{dt}}{A^S_{dt}A^N_{ct}})^{-1-\varphi}$. When the scientists innovate in the clean sector it happens simultaneously in both countries balancing the equation. The same argument can be made for the dirty sector and for the condition when they are growing together.

toward an environmental disaster.
As I did in the previous chapter, the Social planner is applying a policy only in the North to correct the externalities in both countries. In this framework I consider an optimal policy that generates locally optimal allocations. This is a consequence of the principle "who pollutes pay" which has been established by the Kyoto protocol where the first polluters should address the externalities generated at the beginning of the 20th century that are still in the atmosphere. Nonetheless, to correct the externality the Social planner needs some informations from the South given the transboundary nature of the CO2 gas such as her evolution of the environmental quality. The main reason why the optimal policy is only local is given by the fact that a global one would slow the growth of the South making it less political acceptable. In the paper of Acemoglu et al. 2014 the authors refer to optimal policy when there are globally optimal allocations but that definition would go against the aim and questions of my research where only a country is willing to solve the environmental issues.
In this section I correct the externalities of the model using the optimal policy chosen by a social planner that control the two economies. She maximizes the utility of the representative household in the North and the South equation(4.1) subject to the two intermediate productions equation (4.4) and (4.5), the final producers equation (4.7), the evolution of technologies in the North equation (4.8) and in the South equation (4.9), the environmental qualities equations (4.10) and (4.16), the market clearing conditions for the final goods equations (4.14) and (4.15) and the one for labour (4.12) and scientists (4.13).[23] As we discuss in section (4) the machine producers collude to keep the same price and cost. The social planner corrects the collusion of the machine producers bringing them to perfect competition.[24] Then she corrects the researchers' knowledge spillovers subsidizing the social optimum technology. Also, she taxes the intermediate dirty production to eliminate the environmental externalities. The marginal productivity of the intermediate dirty output is

$$[(Y^N_{dt})^{\frac{-1}{\epsilon}}((Y^N_{ct})^{\frac{\epsilon-1}{\epsilon}} + (Y^N_{dt})^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] - \frac{\zeta\omega^N_{t+1}}{\lambda^N_t} - \frac{\zeta\omega^S_{t+1}}{\lambda^N_t} = \hat{p}^N_{dt} \tag{4.35}$$

where $\zeta$ is our emission rate, $\omega^N_{t+1}$ is the shadow value of the environment of the next period in the North, $\lambda^N_t$ the shadow value of the consumption in the North today, meanwhile $\omega^S_{t+1}$ is the shadow value of the environment of the next period in the South. $\hat{p}^N_{dt}$ is the price of th intermediate dirty good which is represented by the ratio between the Lagrange multiplier of the intermediate dirty production of the North and the shadow value of the final good $\frac{\lambda^N_{dt}}{\lambda^N_t}$.

Here the Social planner takes into consideration the difference between an additional unit of dirty output with an additional unit of pollution in the North and the South. To do this she is comparing the ratio between marginal utility of environment and consumption adjusted by the emission rate. In fact, a small emission rate automatically reduces the need of a tax. Meanwhile the need for growth reduces the demand for a

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[23] The full problem is written in the Appendix (G).

[24] The social planner solves the Bertrand problem asking the machine producers to sell with the price at marginal cost

tax[25]. It implies that there exists a tax applied to the dirty intermediate output $\tau_O = \frac{\zeta(\omega^N_{t+1}+\omega^S_{t+1})}{\lambda^N_t \hat{p}^N_{dt}}$ that corrects the environmental externality. As Chapter 3 argued, the policy takes the form of a carbon tax on the damage sustained by the North $\frac{\zeta\omega^N_{t+1}}{\lambda^N_t \hat{p}^N_{dt}}$ and a compensation for the externality that affects the South $\frac{\zeta\omega^S_{t+1}}{\lambda^N_t \hat{p}^N_{dt}}$.
To reach the optimality we need to correct the machine producers collusion as

$$d^N_{jt} = (\frac{\hat{p}^N_{jt}}{\alpha})^{\frac{1}{1-\alpha}} L^N_{jt} A^N_{jt} \tag{4.36}$$

and for the imported good

$$m^S_{jt} = (\frac{\hat{p}^N_{jt}}{\alpha})^{\frac{1}{1-\alpha}} L^N_{jt} A^N_{jt} \tag{4.37}$$

Comparing this with equations (4.16) and (4.17) we can see that the collusion prices are not applied any more, instead the social planner applies competitive prices eliminating the dead weight loss. These new equations show how the production of machine reaches an higher level improving the efficiency of the economies. In fact, a higher level of efficiency implies more input for the intermediate firms increasing the production of final output and subsequentially the welfare of the representative household[26].

The last externality that needs to be corrected is the knowledge spillovers. The social planner solves the problem with respect to the productivities ignoring the accumulation of profits. Instead she moves the researchers to the sector with an higher social gain of an innovation. The shadow values of an innovation in the North and the South are defined as

$$\mu^N_{jt} = \lambda^N_{jt}(1-\alpha)[L^N_{jt}(2(\frac{\alpha\hat{p}^N_{jt}}{\psi})^{\frac{\alpha}{1-\alpha}})] + \mu^N_{jt+1}(1+\gamma\eta_j s^N_{jt+1}) + \mu^S_{jt}(\kappa_j s^S_{jt}) \tag{4.38}$$

and

$$\mu^S_{jt} = \lambda^S_{jt}(1-\alpha)[L^S_{jt}(2(\frac{\alpha\hat{p}^S_{jt}}{\psi})^{\frac{\alpha}{1-\alpha}})] + \mu^S_{jt+1}(1-\kappa_j s^S_{jt+1}) \tag{4.39}$$

The social planner recognizes that the innovation in the North has an effect in the South due to the imitation process. Now applying the probability of a successful innovation I obtain

$$\mu^N_{jt} A^N_{jt-1}\gamma\eta_j = 2\gamma\eta_j(1+\gamma\eta_j s^N_{jt})^{-1}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} \tag{4.40}$$

$$[\sum_{v\geq t}\lambda^N_v(\hat{p}^N_{jv})^{\frac{1}{1-\alpha}} L^N_{jv} A^N_{jv} + \sum_{v\geq t}\lambda^S_v(\hat{p}^S_{jv})^{\frac{1}{1-\alpha}} L^S_{jv}(\kappa_j s^S_{jt})(1-\kappa_j s^S_{jv}) A^N_t]$$

where $\mu^N_{jt} A^N_{jt-1}\gamma\eta_j$ is the social gain of a new innovation. Note that $(\kappa_j s^S_{jt})(1-\kappa_j s^S_{jv})$ is composed by the probability of obtain a successful imitation times the probability of

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[25] This is the mathematical explanation why a policy intervention in a not enough developed South is not welcomed. It could decrease the consumption rate using a less productive input without changing the environment.

[26] To explain it in a more economic way, banning the collusion between the machine producers, the social planner eliminates the dead weight loss allowing perfect competition.

an unsuccessful imitation, representing the variance of a Bernoulli distribution. Also, having two different machines that satisfy the same demand imply the presence of the number 2 at the beginning of the equation. To find the optimal allocation of scientists in the North needs to satisfy the following condition

$$\frac{(1+\gamma\eta_c s_{ct}^N)^{-1}[\sum_{v\geq t}\lambda_v^N(\hat{p}_{cv}^N)^{\frac{1}{1-\alpha}}L_{cv}^N A_{jv}^N+\sum_{v\geq t}\lambda_v^S(\hat{p}_{cv}^S)^{\frac{1}{1-\alpha}}L_{cv}^S(\kappa_j s_{ct}^S)(1-\kappa_j s_{cv}^S)A_{ct}^N]}{(1+\gamma\eta_d s_{dt}^N)^{-1}[\sum_{v\geq t}\lambda_v^N(\hat{p}_{dv}^N)^{\frac{1}{1-\alpha}}L_{dv}^N A_{jv}^N+\sum_{v\geq t}\lambda_v^S(\hat{p}_{dv}^S)^{\frac{1}{1-\alpha}}L_{dv}^S(\kappa_j s_{dt}^S)(1-\kappa_j s_{dv}^S)A_{dt}^N]} \tag{4.41}$$

where it represents the direction of technical change in optimum terms. When the ratio is larger than 1, the clean sector has a larger social value, when is smaller than 1 it is the opposite and when it is equal to 1 they have the same social value and the scientists are engaging both sectors simultaneously. This result is a consequence of the imitation process, but it is also similar to my decentralized equilibrium.

In the next section I show the decentralized economy applying the global coordination solution, which implies a tax applied on both country. We will see the implication of the same tax, different taxes and to conclude unilateral policy only on the North.

In this section we see how the social planner corrects the externalities . In the next section I introduce the implementation of the carbon tax in the lassaiz-faire economies.

## 4.6 Tax applied to the final output

In this section I show the effect of two different policies, a global coordination and an unilateral policy. The first implies that the two countries are applying the carbon tax on the dirty intermediate prices and the subsidy on research that the social planner suggests to avoid the environmental problem. The second represents the scenario where a country decides to introduce a policy alone. This is a hybrid scenario where the North uses a policy to address its environmental degradation meanwhile the other operates in lassaiz-faire. The reason behind this policy is that the South is less developed and is destroying its environment slower than the North implying that the correction of the environmental externality could happen to the imitation process instead of the carbon tax.

### 4.6.1 Global coordination

In this section I apply the carbon tax on both countries to understand the effect on the demand of machines and the direction of technical change. I assume the two regions agree to do the same carbon tax after an environmental international agreement in order to eliminate the externality. This is also the simpler case that allow us to deeply see the effect of an identical tax on the expected profits of the scientists. From Acemoglu et al (2012) [1] ”optimal environmental regulation, or even simple suboptimal policies just using carbon taxes or profit taxes/research subsidies, would be sufficient to redirect technical change and avoid an environmental disaster”. I am using the tax $\tau_O$ to finance a subsidy $q$ on the research of clean technologies that can increase its demand.

Applying the tax $\tau_O^i$ where $i = N, S$ to the intermediate dirty output such that the new final production maximization problem takes the form of

$$\max Y_t^i = ((Y_{ct}^i)^{\frac{\epsilon-1}{\epsilon}} + (Y_{dt}^i)^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - p_{ct}^i Y_{ct}^i - p_{dt}^i(1+\tau_O^i)Y_{dt}^i \tag{4.42}$$

where the final firms sell the final good produced assembling the intermediate outputs $Y_{ct}^i$ and $Y_{dt}^i$ sustaining their cost $p_{ct}^i$ and the new one for the dirty output $(1+\tau_O^i)p_{dt}^i$. Since the new price is higher, the dirty intermediate output is less convenient than the clean one. The maximizing problem above leads to the new relative demand of labour

$$\frac{L_{ct}^i}{L_{dt}^i} = (\frac{A_{ct}^i}{A_{dt}^i})^{-\varphi}(1+\tau_O^i)^{\epsilon} \tag{4.43}$$

which now is influenced by the tax as in Acemoglu et al 2012 [1]. Given the assumption of perfect substitution, the relative labour increases with a carbon tax implying that the clean workforce is larger with a carbon tax than without. The equilibrium prices are represented by

$$p_{ct}^i = (1 + (\frac{A_{ct}^i}{A_{dt}^i})^{\varphi}(1+\tau_O^i)^{\epsilon-1})^{\frac{1}{\epsilon-1}} \tag{4.44}$$

in the clean sector and

$$p_{dt}^i = (1 + (\frac{A_{ct}^i}{A_{dt}^i})^{-\varphi}(1+\tau_O^i)^{\epsilon-1})^{\frac{1}{\epsilon-1}}(1+\tau_O^i)^{-1} \tag{4.45}$$

in the dirty sector. Given this informations, substituting the equation above into the expected profits, the direction of technical change can be rewritten as

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}(\frac{A_{ct}^N}{A_{dt}^N})^{-1-\varphi}\frac{A_{ct-1}^N}{A_{dt-1}^N}(1+\tau_O^N)^{\epsilon}[1+\frac{\kappa_d(1+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi}(1+\tau_O^S)^{\epsilon})^{-1}((1+\tau_O^S)^{\epsilon-1}+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi})^{\frac{1}{-\varphi}}}{(1+\gamma)\eta_d(1+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi}(1+\tau_O^N)^{\epsilon})^{-1}((1+\tau_O^N)^{\epsilon-1}+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi})^{\frac{1}{-\varphi}}}]^{-1} \tag{4.46}$$

$$+\frac{\kappa_c}{\kappa_d}(\frac{A_{ct}^S}{A_{dt}^S})^{-1-\varphi}\frac{A_{ct}^N}{A_{dt}^N}(1+\tau_O^S)^{\epsilon}[1+\frac{(1+\gamma)\eta_d(1+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi}(1+\tau_O^N)^{\epsilon})^{-1}((1+\tau_O^N)^{\epsilon-1}+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi})^{\frac{1}{-\varphi}}}{\kappa_d(1+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi}(1+\tau_O^S)^{\epsilon})^{-1}((1+\tau_O^S)^{\epsilon-1}+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi})^{\frac{1}{-\varphi}}}]^{-1}$$

When the tax is the same for both countries, $\tau_O^N = \tau_O^S$ the fraction that refers to the North $[1+\frac{\kappa_d(1+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi}(1+\tau_O^S)^{\epsilon})^{-1}((1+\tau_O^S)^{\epsilon-1}+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi})^{\frac{1}{-\varphi}}}{(1+\gamma)\eta_d(1+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi}(1+\tau_O^N)^{\epsilon})^{-1}((1+\tau_O^N)^{\epsilon-1}+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi})^{\frac{1}{-\varphi}}}]^{-1}$ and the one to the South $[1+\frac{(1+\gamma)\eta_d(1+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi}(1+\tau_O^N)^{\epsilon})^{-1}((1+\tau_O^N)^{\epsilon-1}+(\frac{A_{ct}^N}{A_{dt}^N})^{-\varphi})^{\frac{1}{-\varphi}}}{\kappa_d(1+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi}(1+\tau_O^S)^{\epsilon})^{-1}((1+\tau_O^S)^{\epsilon-1}+(\frac{A_{ct}^S}{A_{dt}^S})^{-\varphi})^{\frac{1}{-\varphi}}}]^{-1}$ can be approximated to $\frac{1}{2}$ because the tax is the same and is not influencing the approximation we see in the scientists allocation at the end of the Equilibrium in section (4). This lead us to a solution that take this form

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{2\eta_d}(\frac{A_{ct}^N}{A_{dt}^N})^{-1-\varphi}\frac{A_{ct-1}^N}{A_{dt-1}^N}(1+\tau_O^N)^{\epsilon} + \frac{\kappa_c}{2\kappa_d}(\frac{A_{ct}^S}{A_{dt}^S})^{-1-\varphi}\frac{A_{ct}^N}{A_{dt}^N}(1+\tau_O^S)^{\epsilon}$$

where the scientists prefer to engage research in the clean sector thank to the carbon taxes. Also, this equation implies that there is not a preference between the North or the South given by the approximation. But this is a special case, when $\tau_O^N \neq \tau_O^S$ the result of the previous equation is difficult to determine. With different taxes, the scientists consider the fact that a country is less restricted and could continue to study new dirty technologies because there exists at least a market where it is profitable leading to a comparative advantage for the country with the lower tax. In fact, when a country has a sufficient low tax, the scientists ignore the effect of the tax generating a pollution haven.
The optimal tax depends by the marginal consumptions and assuming the South is poorer, it implies an higher shadow value and consequently a lower tax. This implies that a tax applied only to one country could create a pollution haven effect that avoid the global economy to move in the direction of clean research.

### 4.6.2 Implications unilateral tax

In this section I show the implication of an unilateral policy on the world expected profits. Solving the optimal problem we see before using an unilateral policy, eq(4.46) becomes

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{A_{ct}^N}{A_{dt}^N}\right)^{-1-\varphi}\frac{A_{ct-1}^N}{A_{dt-1}^N}(1+\tau_O^N)^{\epsilon}\left[1+\frac{\kappa_d\left(1+\left(\frac{A_{ct}^S}{A_{dt}^S}\right)^{-\varphi}\right)^{\frac{1+\varphi}{-\varphi}}}{(1+\gamma)\eta_d\left(1+\left(\frac{A_{ct}^N}{A_{dt}^N}\right)^{-\varphi}(1+\tau_O^N)^{\epsilon}\right)^{-1}\left((1+\tau_O^N)^{\epsilon-1}+\left(\frac{A_{ct}^N}{A_{dt}^N}\right)^{-\varphi}\right)^{\frac{1}{-\varphi}}}\right]^{-1}+ \tag{4.47}$$

$$+\frac{\kappa_c}{\kappa_d}\left(\frac{A_{ct}^S}{A_{dt}^S}\right)^{-1-\varphi}\frac{A_{ct}^N}{A_{dt}^N}\left[1+\frac{(1+\gamma)\eta_d\left(1+\left(\frac{A_{ct}^N}{A_{dt}^N}\right)^{-\varphi}(1+\tau_O^N)^{\epsilon}\right)^{-1}\left((1+\tau_O^N)^{\epsilon-1}+\left(\frac{A_{ct}^N}{A_{dt}^N}\right)^{-\varphi}\right)^{\frac{1}{-\varphi}}}{\kappa_d\left(1+\left(\frac{A_{ct}^S}{A_{dt}^S}\right)^{-\varphi}\right)^{\frac{1+\varphi}{-\varphi}}}\right]^{-1}$$

The direction of technical change is influenced by the unilateral policy. Given a large tax, the scientists reduce their interest in the northern mechanisms weighting more the southern ones. The researchers recognize that the tax is affecting the demand of machines of the North switching their focus on the southern demand. This is possible because the innovators are satisfying two demands simultaneously[27]. Solving the problem with $\frac{A_{ct-1}^N}{A_{dt-1}^N} = 0.8$, $\frac{A_{ct-1}^S}{A_{dt-1}^S} = 0.75$ and a tax applied only in the North $\tau_O^N = 0.3$ such that $(1+\tau_O^N) = 1.3$ we obtain as a result $\frac{\Pi_{ct}^N}{\Pi_{dt}^N} = 1.07$ which is larger than 1, implying that every scientist will engage in clean research. With a larger tax, the expected profits will increase. It means that it exists a tax that can unilaterally change the direction of technical change. It is possible to redirect the direction of technical change unilaterally applying a tax only in the North. In order to do this the scientists require a larger tax compared to the global coordination scenario. Nonetheless, it can be used to finance a subsidy on clean research. Comparing it with the previous scenario we can see that the tax clearly change the equilibrium of the scientists allocation. The

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[27] The tax interferes with the prices changing the structure of the problem

absence of a tax in the South move the scientists focus to this market implicitly decreasing the demand of dirty machines in the North throughout the effects on the dirty workforce. Nonetheless, with an enough small tax the scientists could still generate dirty technologies in a market distorted by the carbon policy.
To understand the aforementioned results we need to split the expected profits in two parts. The first refers to the impact of the tax on the local component and the second on trade. In the closed economy the tax has a positive effect on the demand of clean technology when

$$\tau > (\frac{\eta_d}{\eta_c})^{\frac{1}{\epsilon}}(\frac{A_{ct}}{A_{dt}})^{\frac{1+\varphi}{\epsilon}}(\frac{A_{ct_{-1}}}{A_{dt_{-1}}})^{\frac{\varphi}{\epsilon}} - 1 \tag{4.48}$$

it satisfies the relationship above.
From eq 4.28 we know that the trade can be represented as

$$\frac{(p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt}}{(p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} A^N_{dt} + (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt} A^S_{dt}} \tag{4.49}$$

Now I will call $(p^N_{dt})^{\frac{1}{1-\alpha}} = (p^N_{dt})^e$ to represent the dirty component of the price effect. Then the effect of the tax on the trade component depends by

$$\frac{(p^N_{dt})^{e'}}{(p^N_{dt})^e} > -\frac{(L^N_{dt})'}{L^N_{dt}} \tag{4.50}$$

This last equation introduces the next proposition.

**Proposition 9** *Under unilateral policy, if equation(4.50) holds the demand of clean technology is positively affected by trade; if equation(4.50) is violated the demand of clean technology is negatively affected by trade.*

In fact when the tax has a larger effect on the demand of labour compared to the price the demand of clean technology from the North reduces. Meanwhile, when the opposite happens, the demand increases. This strongly depends by the elasticity of substitution between clean and dirty. In fact, when the goods are highly substitutes dirty labour is more sensitive to a tax compared to price. This proposition implies that trade has a counter-intuitive effect on the direction of technical change that depends by which effect dominates it. Nonetheless, the result of a unilateral tax is overall positive since the northern economy is larger than the southern by assumption. The interpretation of this result depends by the fact that trade allow the scientists to substitute one country with the other as a consequence of perfect substitution between clean and dirty sector. It is possible that when goods are complement these characteristic translates to the competing economies.

### 4.6.3 Comparative statics

In this part I show the effect of the northern unilateral tax on the direction of change of the two regions

$$\frac{\partial \frac{\Pi^N_{ct}}{\Pi^N_{dt}}}{\partial \tau^N_O} > 0 \tag{4.51}$$

This means that the expected profits of the two regions are monotonically growing with a positive variation of the tax in the North. This implies that the scientists in the two regions recognize the effect of the tax and decide to allocate in the clean sector because it will be the most profitable.

$$-\frac{L^N_{dt}\frac{\partial (p^N_{dt})^e}{\partial \tau} + (p^N_{dt})^e \frac{\partial L^N_{dt}}{\partial \tau}}{L^N_{dt}\frac{\partial (p^N_{dt})^e}{\partial A^N_{ct}} + (p^N_{dt})^e \frac{\partial L^N_{dt}}{\partial A^N_{ct}}} \tag{4.52}$$

The theorem says that a positive change in the tax in the tax substitutes the effect of an increase in the clean productivity depending by the difference between the variation of the cost of dirty output and the variation in the dirty workforce relatively to an increase in the tax against a change in the clean productivity.

## 4.7 Net zero

As we explain in the past chapters th net zero is a cap applied to the production of dirty machines. First I define it as follows.

IPCC[8]

**Definition 6** *Net zero emissions are achieved when anthropogenic emissions of greenhouse gases to the atmosphere are balanced by anthropogenic removals over a specified period. Where multiple greenhouse gases are involved, the quantification of net zero emissions depends on the climate metric chosen to compare emissions of different gases (such as global warming potential, global temperature change potential, and others, as well as the chosen time horizon).*

To better represent the anthropogenic removals we need an abatement technology that can remove the emissions, which we introduced in the last section. It is noteworthy that I define inside the net zero emission all types of pollution, from CO2 to production waste, meaning that the economy must remove all of them to reach this condition. Under Net zero our problem becomes

$$\zeta(Y^N_{dt} + Y^S_{dt}) \leq \delta S^N_t \tag{4.53}$$

where the global emissions need to be equal to the environmental regeneration. This is the best way to represent the balance represented in the definition above. It implies that the optimal production needs to satisfy the following condition

$$(Y^N_{dt})^* \leq \frac{\delta S^N_t - \zeta Y^S_{dt}}{\zeta} = 2(\frac{\alpha}{\psi} p^N_{dt})^{\frac{\alpha}{1-\alpha}} L^N_{dt} A^N_{dt} \tag{4.54}$$

which means that when the optimal production is lower than the net zero emission target the production follows the optimal market path. Otherwise, when the market production could be above the target a cap is applied. Also, as in the previous chapter, under net zero the relative labour depends by

$$\frac{L^N_{ct}}{L^N_{dt}} = (\frac{A^N_{ct}}{A^N_{dt}})^{-\varphi} \tag{4.55}$$

When the carbon emission from the dirty production exceed the cap, a carbon tax on the cost of dirty machines can be introduced such that the demand of dirty machines is

$$x^N_{dkt} = (\frac{p^N_{dt}}{1+\tau_I})^{\frac{1}{1-\alpha}} A^N_{dkt} L^N_{dt} \tag{4.56}$$

It implies that the tax on the input reduces the use of machines decreasing the intermediate dirty output satisfying the cap established by the emission target.

## 4.8 Different technology(Trade of property rights)

In the previous sections I present a model where the imported machines work with the local productivity[28]. In this part of the paper I show an economy that needs the patent related to the machine to work. The typical example is the use of a software. Once you buy it, it is linked with the developer patent and and they are not separable legally. The user of the software needs upgrade from the producer to increase the productivity. The reason I add this extension is to cover different types of inputs that are traded in the world economy to complete the analysis. In fact, it helps me explaining mechanisms that the previous model is not able to describe highlighting different answers to my research questions. Using a new Cobb-Douglas production function, where the machine is imported with the producer technology we have for the North

$$Y^N_{jt} = p^N_{jt}(L^N_{jt})^{1-\alpha}[\int_0^1 (A^N_{jkt})^{1-\alpha}(d^N_{jkt})^{\alpha} + (A^S_{jkt})^{1-\alpha}(m^S_{jkt})^{\alpha} dk] - w^N_t L^N_{jt} - \int_0^1 (d^N_{jkt} + m^S_{jkt})\alpha dk \tag{4.57}$$

and for the South

$$Y^S_{jt} = p^S_{jt}(L^S_{jt})^{1-\alpha}[\int_0^1 (A^S_{jkt})^{1-\alpha}(d^S_{jkt})^{\alpha} + (A^N_{jkt})^{1-\alpha}(m^N_{jkt})^{\alpha} dk] - w^S_t L^S_{jt} - \int_0^1 (d^S_{jkt} + m^N_{jkt})\alpha dk \tag{4.58}$$

The value of the intermediate output $Y^i_{jt}$, depends by the intermediate price $p^N_{jt}$ given to the product of the workers ($L^i_{jt}$) that produce using a combination of machines

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[28] The example I use is the same input used in different countries. The road are made of the same materials worldwide but need to sustain different environments to perfectly operate.

produced locally ($d^i_{jkt}$ and imported from the other country $m^S_{jkt}$ which are combined with the respective productivity $A^N_{jkt}$ and ($A^S_{jkt}$. All of this given the cost of labour $w^i_t$ and the one of the machines. In this new model every imported machines increase the intermediate output depending by the productivity of the importer country.
The northern production function leads to the following demand of machines

$$d^N_{jt} = (p^N_{jt})^{\frac{1}{1-\alpha}} L^N_{jt} A^N_{jt} \tag{4.59}$$

and

$$m^S_{jt} = (p^N_{jt})^{\frac{1}{1-\alpha}} L^N_{jt} A^S_{jt} \tag{4.60}$$

The demand of machines for the South have the same form with the opposite superscript. Now the demand of imported machines depends by the productivity of the exporter. In fact, using this specification we start highlighting a new mechanism that influences the demand of machines[29].
The relative prices in the North are represented as

$$\frac{p^N_{ct}}{p^N_{dt}} = (\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{-(1-\alpha)} \tag{4.61}$$

Now the relative prices are influenced by the sum of the clean sectors productivities engaged in the trade relatively to the dirty ones. The relative prices in the South are shown in the same way as in the North

$$\frac{p^S_{ct}}{p^S_{dt}} = (\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{-(1-\alpha)} \tag{4.62}$$

The open economies using free trade are integrated to the point where they have the same relative prices.
The optimum production

$$Y^N_{jt} = (p^N_{jt})^{\frac{\alpha}{1-\alpha}} L^N_{jt} (A^N_{jt} + A^N_{jt}) \tag{4.63}$$

The relative labour by

$$\frac{L^N_{ct}}{L^N_{dt}} = (\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{-\varphi} \tag{4.64}$$

The reason behind the sum of productivities is due to the strong connection between the economies. Since the foreign productivities influence the output of the local firms, it also change the structure of the workforce[30]. This equation shows that to understand the effects on the labour market we need to understand the changes in the global

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[29] It is reasonable that a firm is interested by the productivity of the machines used in the production. Especially when you need to import the improvements spares from another country because you are unable to produce them due to property rights or technological boundaries.

[30] When a factory applies a new way to produce increasing the productivity they will hire more people to increase the output enjoying scale effects.

productivity by sector.
The South is following the same path

$$\frac{L^S_{ct}}{L^S_{dt}} = (\frac{A_{Nct} + A_{Sct}}{A_{Sct} + A_{Sdt}})^{-\varphi} \tag{4.65}$$

The machine producer profits are

$$\pi^N_{jt} = \alpha(1-\alpha)((p^N_{jt})^{\frac{1}{1-\alpha}} L^N_{jt} A^N_{jt} + (p^S_{jt})^{\frac{1}{1-\alpha}} L^S_{jt} A^N_{jt})$$

which can be simplified in

$$\pi^N_{jt} = \alpha(1-\alpha) A^N_{jt}((p^N_{jt})^{\frac{1}{1-\alpha}} L^N_{jt} + (p^S_{jt})^{\frac{1}{1-\alpha}} L^S_{jt})$$

and for the South

$$\pi^S_{jt} = \alpha(1-\alpha) A^S_{jt}((p^N_{jt})^{\frac{1}{1-\alpha}} L^N_{jt} + (p^S_{jt})^{\frac{1}{1-\alpha}} L^S_{jt})$$

The machines producers satisfy the machine demands of the North and the South simultaneously. The expected profits are

$$\frac{\Pi^N_{ct}}{\Pi^N_{dt}} = \frac{\eta_c}{\eta_d} \frac{A^N_{ct-1}}{A^N_{dt-1}} (\frac{(p^N_{ct})^{\frac{1}{1-\alpha}} L^N_{ct} + (p^S_{ct})^{\frac{1}{1-\alpha}} L^S_{ct}}{(p^N_{dt})^{\frac{1}{1-\alpha}} L^N_{dt} + (p^S_{dt})^{\frac{1}{1-\alpha}} L^S_{dt}}) \tag{4.66}$$

now substituting the relative prices and the market size effects we obtain the new expected profits for the North which are represented by

$$\frac{\Pi^N_{ct}}{\Pi^N_{dt}} = \frac{\eta_c}{\eta_d} \frac{A^N_{ct-1}}{A^N_{dt-1}} (\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{-\varphi-1}$$

and for the South we have

$$\frac{\Pi^S_{ct}}{\Pi^S_{dt}} = \frac{\kappa_c}{\kappa_d} \frac{A^N_{ct}}{A^N_{dt}} (\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{-\varphi-1}$$

The first main difference between the previous section is that the expected profits are slowly different between the North and the South. In equilibrium the machine producers have a common part that represents the trade effect, similar in all countries $(\frac{A^N_{ct}+A^S_{ct}}{A^N_{dt}+A^S_{dt}})^{-\varphi-1}$ and a specific part that define the local characteristics. This attributes that are typical of every country are enhanced by the world trade evolution. The second difference is in the trade effect value. It substitute the relative productivity $(\frac{A^i_{ct}}{A^i_{dt}})^{-1-\varphi}$. The equilibrium levels are defined as

$$p^N_{ct} = [(\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{\varphi} + 1]^{\frac{1}{\epsilon-1}} \tag{4.67}$$

and for the production

$$Y^N_{ct} = [(\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{\varphi} + 1]^{\frac{-\alpha}{\varphi}} [(\frac{A^N_{ct} + A^S_{ct}}{A^N_{dt} + A^S_{dt}})^{\varphi} + 1]^{-1} (A^N_{ct} + A^S_{ct}) \tag{4.68}$$

$$Y_{ct}^{N} = [(\frac{A_{ct}^{N} + A_{ct}^{S}}{A_{dt}^{N} + A_{dt}^{S}})^{\varphi} + 1]^{\frac{-(\alpha+\varphi)}{\varphi}} (A_{ct}^{N} + A_{ct}^{S}) \tag{4.69}$$

The equilibrium clean production can be described as

$$Y_{ct}^{N} = [(A_{ct}^{N} + A_{ct}^{S})^{\varphi} + (A_{dt}^{N} + A_{dt}^{S})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}} (A_{ct}^{N} + A_{ct}^{S})(A_{dt}^{N} + A_{dt}^{S})^{\alpha+\varphi} \tag{4.70}$$

This section outlines a scenario where the import of machines linked with their productivities links the economies to the same values. In fact the relative prices and the market size are the same in both countries. In fact the relative labour is represented as $\frac{L_{ct}^{N}}{L_{dt}^{N}} = \frac{L_{ct}^{S}}{L_{dt}^{S}} = (\frac{A_{ct}^{N}+A_{ct}^{S}}{A_{dt}^{N}+A_{dt}^{S}})^{-\varphi}$. Meanwhile the relative prices take the form of $\frac{p_{ct}^{N}}{p_{dt}^{N}} = \frac{p_{ct}^{S}}{p_{dt}^{S}} = (\frac{A_{ct}^{N}+A_{ct}^{S}}{A_{dt}^{N}+A_{dt}^{S}})^{-(1-\alpha)}$

This result is different from the one of the previous model. In fact, the expected profits are now different between the to regions meanwhile before they are the same implying the scientists can research different technologies. This means the scientists in the North can research clean technologies meanwhile the one in the South the dirty one generating pollution havens. Also, the equilibrium production variables are the same between the North and the South meaning that the representative household is consuming more.

### 4.8.1 Optimal policy

Like in the previous sections after the description of the model I describe the social planner solution. To obtain optimality it is necessary a carbon tax that corrects the environmental externality, a subsidy on the machine production and a policy on the knowledge spillovers. For convenience I am showing only the results for the North, but they are applied also in the South. The tax takes the form of the previous chapter where the marginal productivity of the intermediate dirty output is

$$[(Y_{dt}^{N})^{\frac{-1}{\epsilon}}((Y_{ct}^{N})^{\frac{\epsilon-1}{\epsilon}} + (Y_{dt}^{N})^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] - \frac{[\zeta]\omega_{t+1}^{N}}{\lambda_{t}^{N}} - \frac{\zeta\omega_{t+1}^{S}}{\lambda_{t}^{N}} = \hat{p}_{dt}^{N} \tag{4.71}$$

which again, it implies that there is a tax applied to the dirty intermediate output $\tau_{O}^{N} = \frac{\zeta(\omega_{t+1}^{N}+\omega_{t+1}^{S})}{\lambda_{t}^{N}\hat{p}_{dt}^{N}}$

To reach the optimality we need to correct the machine producers externalities as

$$d_{jt}^{N} = (\frac{\hat{p}_{jt}^{N}}{\alpha})^{\frac{1}{1-\alpha}} L_{jt}^{N} A_{jt}^{N} \tag{4.72}$$

and for the imported good

$$m_{jt}^{S} = (\frac{\hat{p}_{jt}^{N}}{\alpha})^{\frac{1}{1-\alpha}} L_{jt}^{N} A_{jt}^{S} \tag{4.73}$$

The Social planner allocates the researchers to the clean sector when it has the higher social value, or when

$$\frac{\mu_{ct}^{N} A_{ct-1}^{N} \eta_{c} \gamma}{\mu_{dt}^{N} A_{dt-1}^{N} \eta_{d} \gamma} > 1$$

and it implies that

$$\frac{\eta_c(1+\gamma\eta_j s_{ct}^N)^{-1}\sum_{v\geq t}[\lambda_v^N L_{cv}^N(\hat{p}_{ct}^N)^{\frac{1}{1-\alpha}}+\lambda_v^S L_{cv}^S(p_{ct}^S)^{\frac{1}{1-\alpha}}]A_{cv}^N(1-(k_c s_{cv}^S(1-\kappa_j s_{cv}^S)))}{\eta_d(1+\gamma\eta_d s_{dt}^N)^{-1}\sum_{v\geq t}[\lambda_v^N L_{dv}^N(\hat{p}_{dt}^N)^{\frac{1}{1-\alpha}}+\lambda_v^S L_{dv}^S(p_{dt}^S)^{\frac{1}{1-\alpha}}]A_{dv}^N(1-(k_d s_{dv}^S(1-\kappa_j s_{dv}^S)))}>1 \tag{4.74}$$

Comparing this equation with equation 4.66 we can see same similarity due to the structure of the problem. In fact, the social planner is comparing the sum of the prices adjusted by the clean labour force in the world with the dirty ones. They are the main drivers adjusted by the relative productivities.

## 4.9 Generalization of the model

Now,given the conclusions of th previous sections I try to extend the results to a generalized framework with $n$ different countries. I start from eq(4.140) where it is clear that the expected profits are the sum of the autarky profits. Given $n$ countries the expected relative profits of the country $i$ under free trade takes the form

$$\frac{\Pi_{ict}^{Trade}}{\Pi_{idt}^{Trade}}=\frac{\Pi_{1ct}^{Autarky}+\Pi_{2ct}^{Autarky}+...\Pi_{nct}^{Autarky}}{\Pi_{1dt}^{Autarky}+\Pi_{2dt}^{Autarky}+...\Pi_{ndt}^{Autarky}}$$

which in an easier way is

$$\frac{\Pi_{ict}^{Trade}}{\Pi_{idt}^{Trade}}=\frac{\sum_{i=1}^{n}\Pi_{ict}^{Autarky}}{\sum_{i=1}^{n}\Pi_{idt}^{Autarky}}$$

and the scientists will engage the clean sectors only if

$$\sum_{i=1}^{n}\frac{\Pi_{ict}^{Autarky}}{\Pi_{idt}^{Autarky}}>n$$

It is clear that with multiple countries the effect on the expected profits is the same where the scientists take into consideration the profits of every firm engage in trade before developing new technology. Once the expected profits are defined, the scientists study the same technologies simultaneously across the world. Under this framework the equilibrium production takes the form of

$$Y_{ct}^N=n[A_{Nct}^{\varphi}+A_{Ndt}^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}}A_{Nct}A_{Ndt}^{\alpha+\varphi}$$

and

$$Y_{dt}^N=n[A_{Nct}^{\varphi}+A_{Ndt}^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}}A_{Ndt}A_{Nct}^{\alpha+\varphi}$$

to conclude with the equilibrium final good

$$Y_t=n[A_{Nct}^{\varphi}+A_{Ndt}^{\varphi}]^{\frac{1}{-\varphi}}A_{Nct}A_{Ndt}$$

This result is just a preliminary study to extend the international trade literature joining it wit the growth theory models.

A similar generalization applies for the extension of my model where when the country innovate it takes the form of

$$\frac{\Pi_{ict}^{Trade}}{\Pi_{idt}^{Trade}} = \frac{\eta_c^i A_{ct-1}^i (\sum_{i=1}^n A_{ct}^i)^{-\varphi-1}}{\eta_d^i A_{dt-1}^i (\sum_{i=1}^n A_{dt}^i)^{-\varphi-1}}$$

where adding countries in the trade enlarge the expectations of the scientists due to the amount of potential markets for new machines. In the next part I am showing the generalization of the second model

## 4.10 Numerical simulations

In this section there is the numerical simulation of the two models. I am running the decentralized model applying the policies I discuss in the previous parts of the chapter.

In fact, when I simulate the value it is close to 1 implying that no country is specialized in one specific sector. When it diverge from 1 it implies that the comparative advantage exists.

### 4.10.1 Methodology

As in Acemoglu et al (2012) [1] and my previous chapter each period corresponds to 5 years and the other variables are defined as: the quality of the innovation $\gamma = 1$, the share of inputs $\alpha = 0.33$, the probability of innovate in each sector $\eta_c = \eta_d = 0.02$,the risk aversion $\sigma = 2$, the elasticity of substitution $\epsilon = 3$ and the cost of machines $\psi = \alpha^2$. Following Nordhaus assumption, the discount rate of every period is $\rho = 0.015$ following Nordhaus (2007)[14] specification.

Meanwhile as in the previous chapter, I decided that the probability to imitate in the South is $\kappa_c = \kappa_d = 0.04$[31]. The initial value of the technologies are represented as follow: $A_{Nd0} = 0.4, A_{Nc0} = 0.3, A_{Sd0} = 0.3, A_{Sc0} = 0.2$[32]. The values of the average technologies depends by the fact that the dirty sector must be more advanced than the clean one, otherwise the scientists innovate only in the clean sector avoiding the pollutive production. Also, since there is no capital in this model, I impose that the North is more advanced than the south to represent the difference between a developed country and a developing one.
The initial environmental quality is at full capacity, $S_{it} = 1$ and $S_{i0} = 1$. Also, in this section I substitute the upper value of the environmental quality, $\bar{S}$ with 1 implying that $S \in (0, 1)$. This simplification satisfies the Inada-conditions at the beginning of the model. Also, the simulation runs for 100 periods, but I am showing only the first 20 to simplify the analysis. The emission rates is respectively $\zeta = 0.3$ meanwhile on

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[31] This decision depends by the fact that it is easier to copy and adapt an existing technology instead of creating a new one.

[32] They are lower compared to the previous chapter because the economies develop too fast under free trade and therefore an environmental disaster occurs before it can be avoided.

the other side the regeneration rate is $\delta = 0.25$. I used the software Maple generating two loops to solve the system of equation that describe the economy. The first loop runs the first period where the social plan maximizes the economy without the tax. The second loop last 99 periods where the tax is in place.

In the next subsections I describe the plots from three different scenarios for the two models. In order the scenarios are: 1)with Lassaiz-faire ;2) with global coordination and same tax ; 3) with global coordination but different taxes 4) with unilateral policy; and 4) under net zero. They are shown for the first twenty periods, $t = 1...20$ to show the short run transition.

I decided to have an arbitrary taxation that is enough strong to avoid an environmental disaster. In fact it is not optimal in the sense that it has been decided by a social planner but in a general sense it is enough to eliminate the environmental externality correcting the market failure. The importance of this simulation is the fact that numerically it is possible to avoid an environmental disaster with trade under unilateral policy contradicting the conclusion of Acemoglu et al. 2014. If it is possible to do so with an arbitrary taxation is also possible to obtain the same result with an optimal policy.

### 4.10.2 Scenario I: Lassaiz-faire

In this section I show the results of the numerical simulation in lassaiz-faire.

Both economies are heading towards an environmental disaster which is happening at period $t = 12$. From the figure 4.1 we can see the consumption starting at 1.37 is growing slowly to 1.58 meanwhile the clean workforce is decreasing from 0.39 to .33 over time[33].

Figure 4.1: This plot shows the lassaiz faire equilibrium for the North

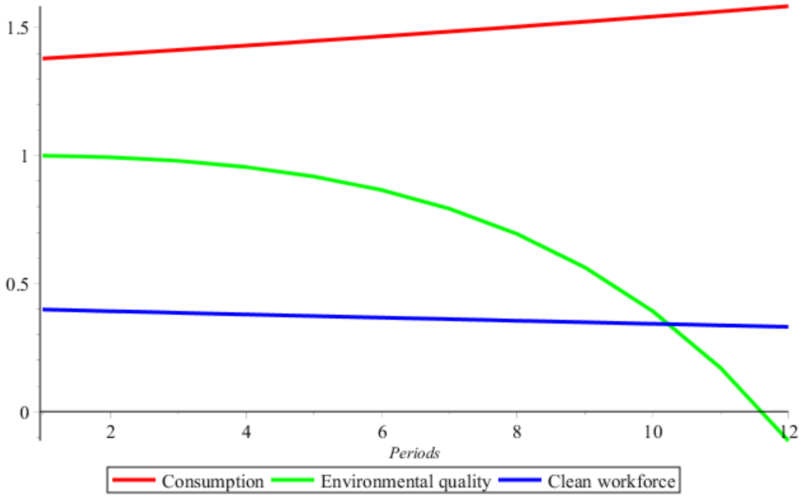

Figure 4.2 shows the lassaiz-faire equilibrium in the South

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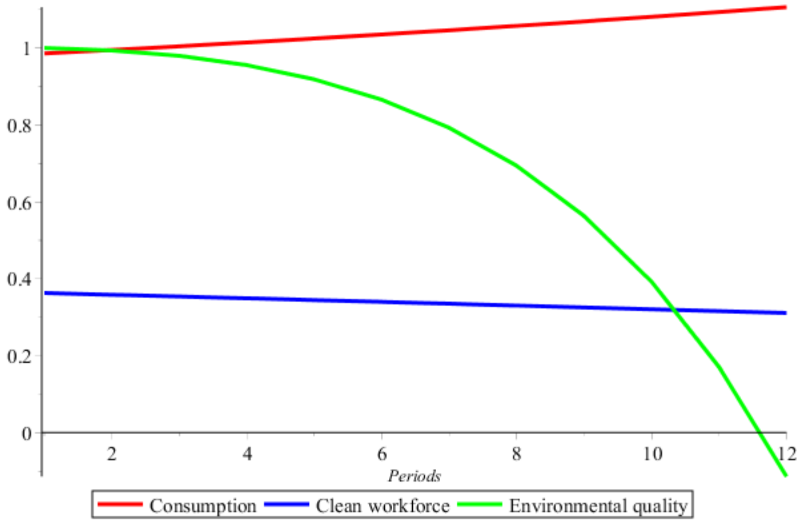
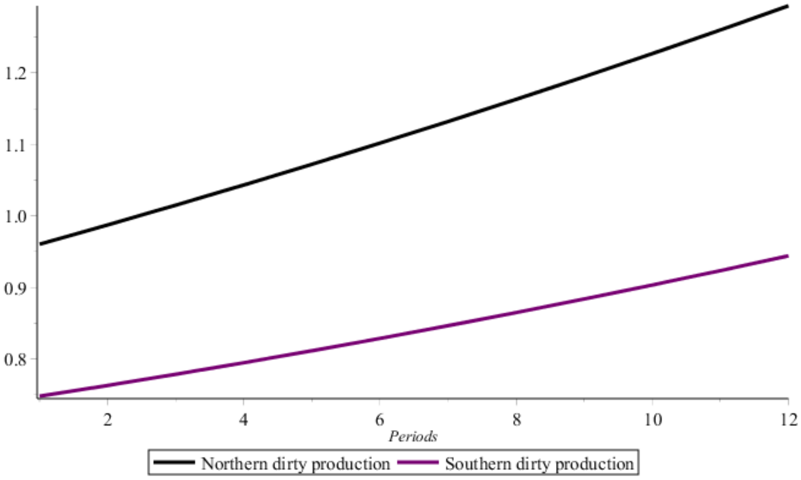
[33] It is difficult to see from the plot because the simulation end too quickly before curve starts the exponential behaviour.

Figure 4.2: The plot shows the south



where the consumption is growing from 0.98 to 1.10 and the clean workforce is decreasing from 0.36 to .31.

Figure 4.3:

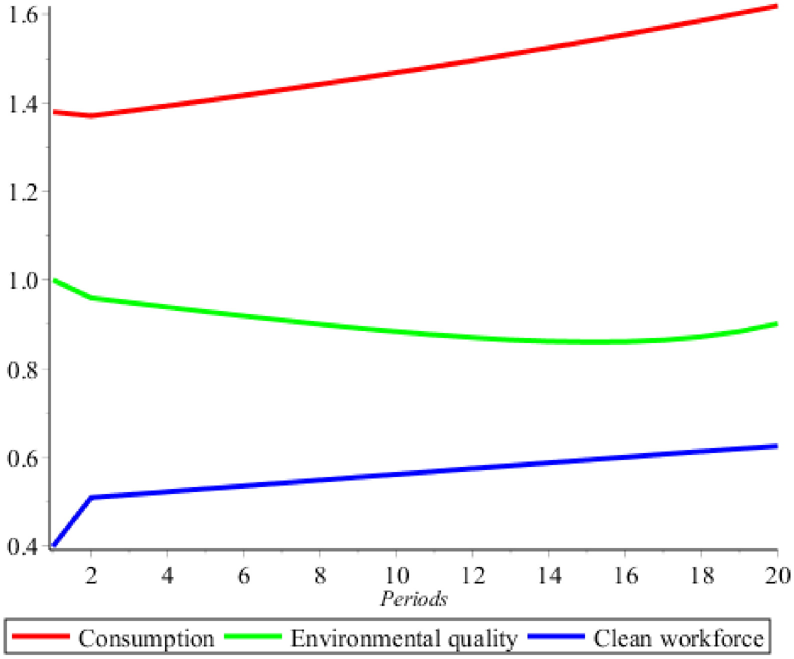


It is evident that the economies will reach an environmental disaster simultaneously due to the trans-boundary pollution that is affecting them. It is important to highlight that the clean workforce is inversely proportional to the dirty productivity. It implies that when a new technology is developed more people are attracted to work with it. As a consequence the people that are moving to the more prominent sector are abandoning the other one. This behaviour is a consequence of the growth of the dirty technology. In the model we see that the dirty market is larger than the clean one, implying the clean workforce shrinks over time. The dirty output increases in both countries, as shown in figure 3, reducing the environmental quality that reaches zero.

### 4.10.3 Scenario II: Same taxes implemented in both countries

Here I present the global coordination. It means that the two countries are trying to solve the environmental issue using policies in contrast with an unilateral policy as it is introduced in Acemoglu et al (2014) [2]. In this section I show the result of the numerical simulation when the same tax is applied in both countries. To solve the environmental problem I am using an exogenous tax $\tau_O = 0.15$[34]. The first plot, figure 4, represents the North in the first 20 periods.

Figure 4.4: The figure shows the short run in the north



We can see that the consumption decreases from 1.379 at period $t = 1$ to 1.370 in the second period after the tax is introduced. The clean workforce increases from 0.39 to 0.50. We can see that the environment is changing trajectory regenerating itself. At period $t = 20$ the environment is almost fully restored. We can see that the consumption is growing following the evolution of the clean productivity. The decrease in consumption is a consequence of the extensive use of a less productive input.
In figure 5 I show the same interval for the South. Here the tax is the same as before.

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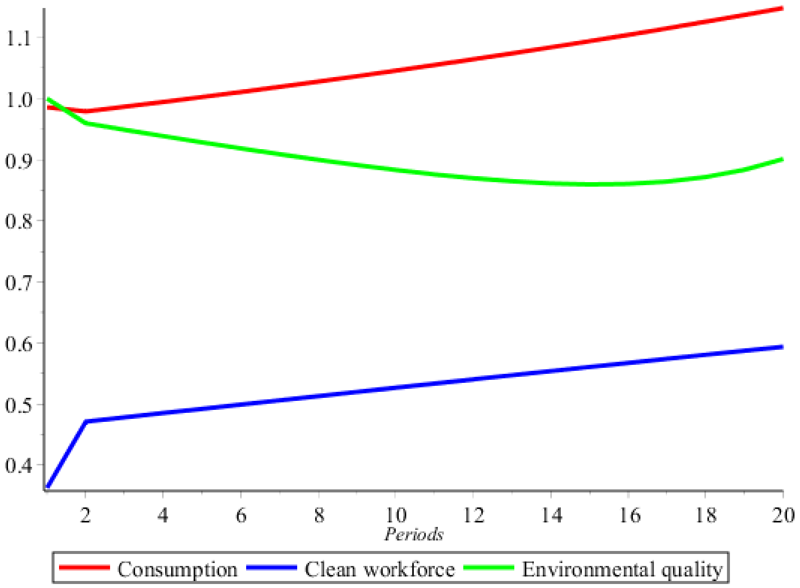
[34] It is very large, but the only one that allow me to correct the environmental externality before the environmental disaster occurs. This framework reacts very fast due to the large flow of machines.

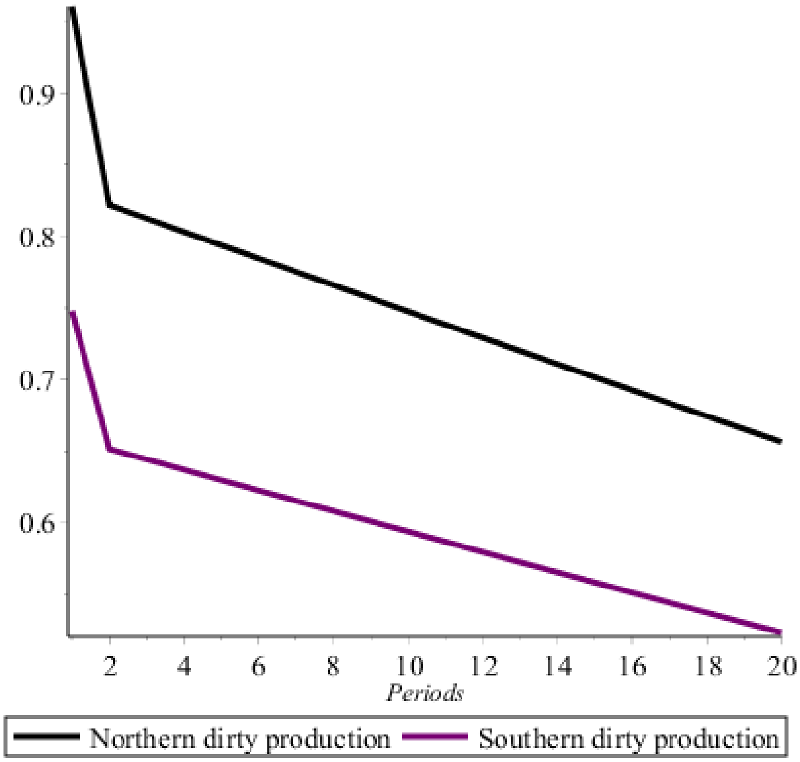
Figure 4.5:



After the introduction of the tax consumption decreases from 0.98 to 0.97. At the same time, the clean workforce goes from 0.36 to 0.47. The environment follows the same path we have seen in the previous plot. Due to the environmental damage it is moving from full capacity to 0.73 then, thanks to the tax it increases its value until it is fully recovered.

In the last plot, figure 4.6, I show the evolution of the dirty productions in both countries.

Figure 4.6:



The amount of intermediate output produced in the countries drops after the introduction of the tax. The policy increases the cost to purchase these inputs reducing the use in the final production. For the North, the value jumps from 0.96 to 0.82 at time $t = 2$. The same happens for the South where it decreases from 0.74 to 0.65. They continue to decrease over time due to the development of clean technologies that affect the composition of the workforce and consequently the production of dirty output.
The next section shows the effect of different taxes on the two economies.

### 4.10.4 Scenario III: Different taxes

The next plots represents the economies with two different taxes. The reason is given by the fact that the two countries have different shadow values of the environment. It implies that they are considering their environment differently[35]. The northern tax is $\tau_O^N = 0.2$ and the southern one $\tau_O^S = 0.1$. This is one of the combinations that avoid an environmental disaster. A lower tax in the South is possible only when we raise the tax in the North.
In the first plot. figure 4.7, we can see that the consumption strongly decreases after the tax is introduced. It grows slowly after period $t = 2$ due to the growth of the clean productivity moving from 0.86 to 1.1. The tax is enough strong to boost the clean workforce from 0.39 to 0.89. The environment is fully restored at period 17.

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[35] It means that the South is more willing to grow compared to the North.

Figure 4.7: Consumption short run

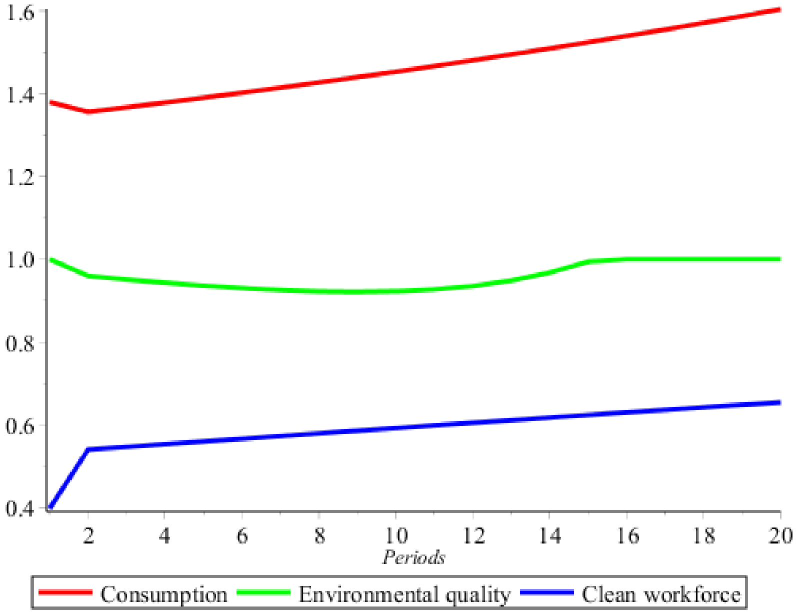
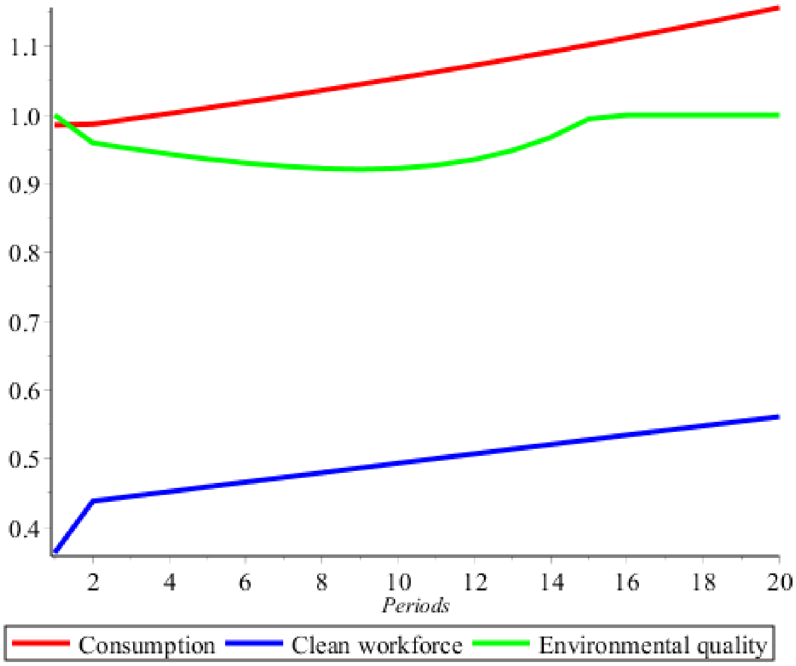
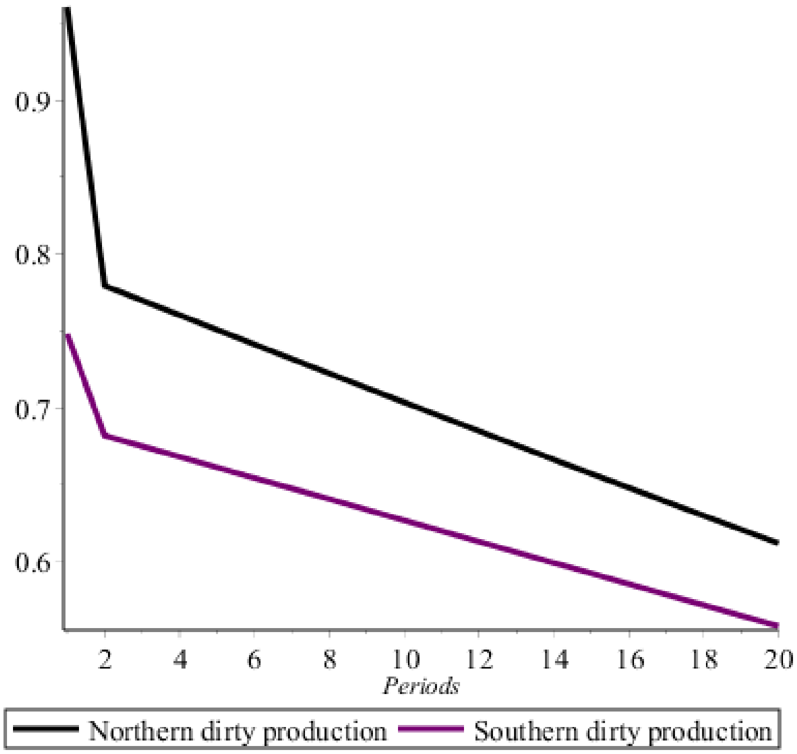


Figure 4.8: south-consumption-short run



In the South, figure 4.8, we are seeing a similar path. The consumption falls from 0.98 to 0.65 in period $t = 2$. After that it steadily grows over time to 0.85 at period twenty. The environment is again restored at time $t = 17$. the dirty productivities

Figure 4.9: dirty production

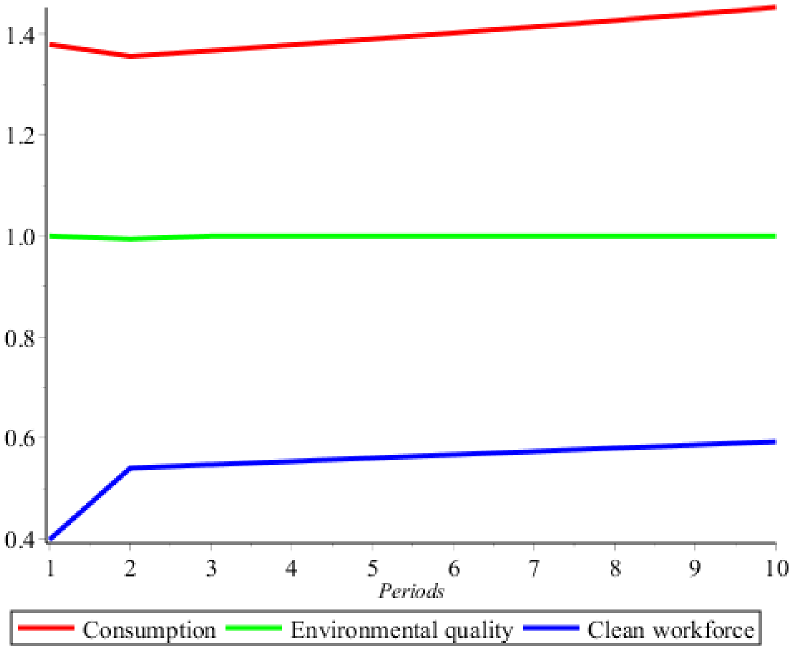


Analysing the dirty production in figure 4.9 we see that the North is falling from 0.96 to 0.33 and after that it is slowly decreasing to 0.22 in period $t = 20$. The same happens for the South where it goes from 0.74 to .31 ending at .21. With these plots I conclude the part with two taxes applied in both countries. In the next part I introduce unilateral policy.

### 4.10.5 Scenario IV: Unilateral Policy

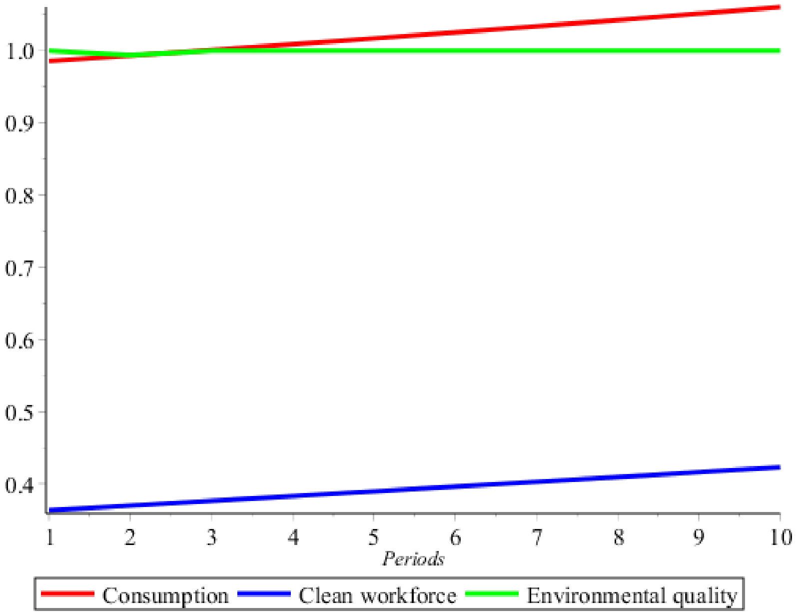
Here I show the effects of the unilateral policy applying a tax $\tau_O$ only in the North with value 0.25.

Figure 4.10:



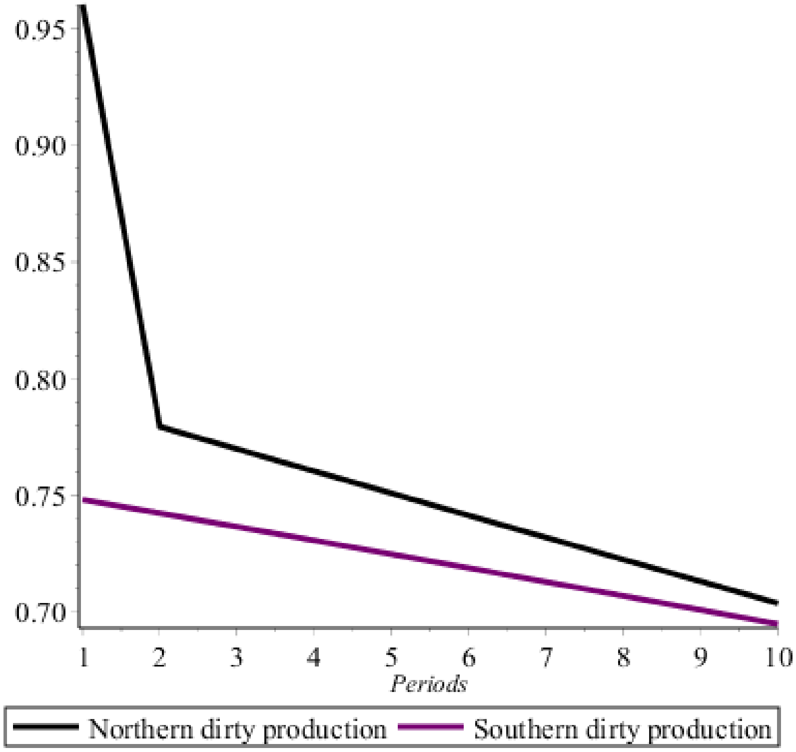
In this scenario the tax is sufficient to avoid an environmental disaster. The tax is enough large to allow a movement of workers to the clean sector. The consumption is slowly decreasing to increase from period 3.

Figure 4.11:



The South is growing as in lassaiz faire except the workers that are slowly changing sector due to the growth of the clean technology. The effect of the unilateral policy on the productivities is represented by the next plot

Figure 4.12:



where the northern production drastically change due to the taxation meanwhile the southern one decreases naturally thanks to the imitation process. The tax needs to be larger compared to the previous section, therefore the environmental issue is solved faster, in fact it is difficult to see the change on the plots.

## 4.11 Welfare analysis

In this section I am doing the welfare analysis to understand which policy better increases the utility of the representative household focusing on the North [36] concluding with the southern one. This analysis helps us understanding which is the optimum policy to be implemented by a social planner.
I use a CRRA utility function with the following specification

$$u = \sum_{t=1}^{100} \frac{1}{(1+\rho)^t} \frac{(C_{Nt}S_{Nt})^{1-\sigma} - 1}{1-\sigma} \tag{4.75}$$

---

[36] The reason to do it only in the North is due to the fact that the South operates under lassaiz-faire having the same utility under all the different scenarios.

where $\rho$ ha a value of 0.015, following the specification of Nordhaus (2007)[14], and $\sigma$ of 2. For each scenario we have a different value that we use to understand which policy better solve our problem. The analysis is done for the periods from $t = 1$ to $t = 100$. I start in order following the subsection of the numerical simulation.

In the lassaiz-faire scenario we have impossible as a result. It happens because the environmental disaster violates the Inada-conditions.

In the scenario with the same tax for both countries, we have 14.1 for the North and $-3.0$ for the South with 11.0 as a global result. In the part with different taxes the North has the value 11.8, the South $-2.7$ with a total value of 9.1. In the net zero scenario 10.8, for the South $-4.0$ and globally 6.8.

The next part I analyse is the second model where the productivity is imported with the machines. The lassaiz-faire has th same result as before, impossible. Then I move to the scenario with the same tax applied for both countries where their welfare is 5.77 with a global value of 11.5. In the scenario with different taxes, $\tau_O^N = 095$ and $\tau_O^S = 0.85$ we have respectively 4.12 for the North and 6.85 for the South with a global result of 10.9. To conclude I analyse the Net zero welfare where the North is at 7.48, and the South at 5.48 with the total welfare at 12.9.

## 4.12 Discussion of the model

<table>
<tr><th colspan="3">Results Summary</th></tr>
<tr><th>Results</th><th>Degree of cordination</th><th>Mechanism</th></tr>
<tr><td>I Proposition 8</td><td>Lassaiz-faire</td><td>No comparative advantage</td></tr>
<tr><td>II Proposition 9</td><td>Lassaiz-faire</td><td>Unilaterally changing the profit of a country change the world profits</td></tr>
<tr><td>III Proposition 10</td><td>Unilateral policy</td><td>Necessary conditions to affect the world profits under free trade and unilateral taxation</td></tr>
</table>

In this model the machine producers collude to maximize their profits perfectly sharing the market. In fact, the machine producers in the North take into account their own demand combined with the one from the South. Since the countries are symmetric, the South has the same expected profits like the one of the North.

In the decentralized equilibrium the environmental disaster is the only solution. To avoid it is necessary a policy enough strong to redirect the allocation of the scientists to the clean sector. When a tax is applied on both countries it influence the importance of the individual demand of machines inside the expected profits. It implies that the trade part is strongly influenced by the tax. The country with the larger tax on intermediate dirty output will affect the scientists allocation. In fact, when the northern tax is larger than the other, the scientists will be more focused on the demand of machines from the South. Nonetheless there exist different taxes combinations $(\tau_O^N, \tau_O^S)$ such that an environmental disaster is avoidable.

When it is applied only on a country implying unilateral policy we obtain ambiguous results. This last relationship is a particular case where $\tau_O^S$ is equal to 0. In this scenario the scientists in the North will evaluate more the southern demand of machine because it is not distorted by any mechanism. Nonetheless there exists a tax $\tau_O^N$ enough large such that the economies switch to clean technologies. This is a consequence of the initial conditions. First of all the North is more advanced than the South, implying that the scientists cannot ignore the northern demand of machines because the clean productivity in the North is not too far from the dirty productivity in the South. This behaviour is amplified by a tax enough large.

From the social planner solution we see that the trade mechanism is incorporated requiring the shadow value to double its original value which is a result of the fact that the expected profits now depend from two demands of machines.

I expect a solution where both technologies need to grow at the same rate because otherwise, the South would specialize in the dirty sector leading to its environmental disaster, meanwhile the North gets cleaner. A solution could be a tariff on the import of dirty machines, which will indirectly affects the demand of machines from the South.

## 4.13 Conclusion

In this paper I present a directed technical change model with trade of machines. The trade shows collusions between the machine producers that substitute the monopolist prices. To solve this distortion the social planner applies competitive prices that eliminate the dead weight losses. Trade increases the flow of machines used in the production of intermediate output resulting in the production of more consumption goods compared to my previous chapters. This increase of output negatively affects the environment, degrading it faster than before. On the other hand, the exchange of machines create a link between the scientists implying they are taking the same decisions thanks to a new markets beyond their countries. In fact the decision on which technology needs to be developed is taken by the whole scientific community simultaneously allowing more possibilities to tackle the environmental issues.

In the extension of the model, this global decision is represented by the trade component. However, the main difference is in the intermediate and final production where the two regions have the same equilibrium output implying that an exchange of productivity can increased the welfare of a developing country reducing the GDP differences in the world. This could also explain why some countries grow faster than others. The reason could be that they are learning how to combine their productivity with the one

of other nations.
This paper contradicts some of the finding of the previous literature adding new points of view on trade mechanisms. In fact my paper says that an unilateral policy has a positive effect on the demand of machines for both countries contrary to the results of Acemoglu et al. 2014. The main novelty is the necessary condition shown in proposition 9 that explain the complexity of the elasticity of substitution with trade. When clean and dirty goods are complementary the unilateral policy amplifies the technological spillovers of the imitation process. Meanwhile when they are substitute the unilateral policy is decreasing the global effect of the taxation enhancing pollution havens effects.
In this work we see the strong correlation between trade and endogenous growth. Future research should continue to study the effect of technologies on trade adding more complexities. The first extension should consider trade between all sectors to understand the effect on the global economy as well as on the demand of new technologies. A second and more ambitious extension should start from close economy to analyse the role of investment and research to the development of an economy to conclude with free trade. An example could include the role of investment in infrastructures on the direction of technical change to study if it is stronger than the discovery of new technologies.

## 4.14 Appendix

### 4.14.1 Normalization of the monopolists cost

Now the machine producer in the North maximizes

$$\pi_{Ndt} = max_{d_{Ndt},m_{Ndt}} p_{Ndkt} d_{Ndkt} + p_{mNdt} m_{Ndt} - (d_{Ndkt} + m_{Ndt})\psi \tag{4.76}$$

where the firm generates revenue from the machines it sells to the intermediate firms at the price $p_{Ndkt}$ and $p_{mNdkt}$ in the North and in the South sustaining the cost to produce them $\psi$.
To find the value of the prices I maximize the firm with respect to the machines locally produced

$$\frac{\partial \pi_{Ndt}}{\partial d_{Ndkt}} = \alpha^2 p_{Ndt} L_{Ndt}^{1-\alpha} A_{Ndkt}^{1-\alpha} (d_{Ndkt})^{\alpha-1} = \psi$$

obtaining

$$d_{Ndkt} = (\frac{\psi}{\alpha^2 p_{Ndt}})^{\frac{1}{\alpha-1}} L_{Ndt} A_{Ndkt}$$

where $\psi = \alpha^2$. Substituting the supply of machine from the producer inside the inverse demand

$$\alpha p_{Ndt} L_{Ndt}^{1-\alpha} A_{Ndkt}^{1-\alpha} (d_{Ndkt})^{\alpha-1} = p_{Ndkt}$$

which leads to

$$p_{Ndkt} = \alpha$$

and respectively for the imported good I am doing the same so

$$\frac{\partial \pi_{Ndt}}{\partial m_{Ndkt}} = \alpha^2 p_{Ndt} L_{Ndt}^{1-\alpha} A_{Ndkt}^{1-\alpha} (m_{Ndkt})^{\alpha-1} = \psi$$

which leads to

$$p_{Sdkt} = \alpha$$

### 4.14.2 Proof of proposition I

The expected profits in the North are given by

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{(1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}$$

and in the South

$$\frac{\Pi_{Sct}}{\Pi_{Sdt}} = \frac{(1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}$$

The world profits are represented as $\frac{\Pi_{ct}^{World}}{\Pi_{dt}^{World}} = \frac{\Pi_{Nct}+\Pi_{Sct}}{\Pi_{Ndt}+\Pi_{Sdt}}$that can be shown as

$$\frac{\Pi_{ct}^{World}}{\Pi_{dt}^{World}} = \frac{(1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct} + (1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt} + (1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}$$

or

$$\frac{\Pi_{ct}^{World}}{\Pi_{dt}^{World}} = \frac{2[(1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}]}{2[(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}]}$$

which implies

$$\frac{\Pi_{ct}^{World}}{\Pi_{dt}^{World}} = \frac{(1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}$$

such that

$$\frac{\Pi_{ct}^{World}}{\Pi_{dt}^{World}} = \frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\Pi_{Sct}}{\Pi_{Sdt}}$$

### 4.14.3 Expected profits

The direction of technical change depends by

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{(1+\gamma)\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}} + \kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}} \tag{4.77}$$

To find the direction of technical change we need to separate the fraction

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{(1+\gamma)\eta_c [p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}}]}{(1+\gamma)\eta_d [p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + \kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}]} + \frac{\kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{(1+\gamma)\eta_d [p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}} + p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}]}$$

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c [p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}}]}{\eta_d [p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}}](1 + \frac{\kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}{(1+\gamma)\eta_c p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{dt_{-1}}})} + \frac{\kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{\kappa_d [p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}](1 + \frac{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}}}{\kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}})}$$

which are represented in the canonical way

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct_{-1}}}{\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}}} (1 + \frac{\kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}}})^{-1} + \tag{4.78}$$

$$+ \frac{\kappa_c p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct} A_{Nct}}{\kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}} (1 + \frac{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}}}{\kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}})^{-1}$$

### 4.14.4 Trade specification

(You can discuss about comparative advantage of the ratios, use the proper terms of the literature as market size, price effect and so on.) From the direction of technical change we understand that the profits of the scientists engaged in the North depends by the local one combined with the exported machines wheighted by

$$(1+\frac{\kappa_d p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Ndt}}{(1+\gamma)\eta_d p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt_{-1}}})^{-1} \tag{4.79}$$

Now, we know that the equilibrium dirty price of the South is represented as

$$p_{Sdt} = [1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1}{\epsilon-1}}$$

and for the North

$$p_{Ndt} = [1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1}{\epsilon-1}}$$

combining them we can describe the price advantage as

$$\frac{p_{Sdt}}{p_{Ndt}} = [\frac{1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}}{1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}}]^{\frac{1}{\epsilon-1}}$$

Now raising to the power of $\frac{1}{1-\alpha}$

$$(\frac{p_{Sdt}}{p_{Ndt}})^{\frac{1}{1-\alpha}} = [\frac{1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}}{1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}}]^{\frac{1}{(\epsilon-1)(1-\alpha)}}$$

where $(\epsilon-1)(1-\alpha) = -\varphi$ which implies that

$$(\frac{p_{Sdt}}{p_{Ndt}})^{\frac{1}{1-\alpha}} = (\frac{1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}}{1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}})^{\frac{1}{-\varphi}}$$

Now we need to define the market size advantage $\frac{L_{Sdt}}{L_{Ndt}}$ which is represented as

$$\frac{L_{Sdt}}{L_{Ndt}} = (\frac{1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}}{1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}})^{-1}$$

Now substituting it in equation (4.79) we obtain

$$(1+\frac{\kappa_d(1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A_{Ndt}}{(1+\gamma)\eta_d(1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A_{Ndt_{-1}}})^{-1} \tag{4.80}$$

### 4.14.5 Scientists allocation

To allocate the scientists we need first to describe the market mechanisms. The scientists are decided simultaneously in the north as well in the south. It implies that the number of clean scientists in the south as well in the North is represented as $s_{Nct} =_{Sct}$ and $s_{Ndt} = s_{Sdt}$. To solve it I focus on the clean corner solution where $s_{Nct} = 1$ and $s_{Ndt} = 0$. We first substitute the expected relative profits $\frac{\Pi_{Nct}}{\Pi_{Ndt}}$ with $f(s)$ which is a function that represent the allocation of the scientists, than we impose it greater than 1. This is the condition when all the scientist are engaged in clean research. The first consequence is that the expected profits will simplify. Also, since $A_{Ndt} = (1 + \gamma\eta_d s_{Ndt})A_{Ndt-1}$, when all the scientists are engaged in the clean sector it implies that $A_{Ndt} = A_{Ndt-1}$.

$$f(s) = \frac{\eta_c A_{Nct}^{-1-\varphi} A_{Nct_{-1}}}{\eta_d A_{Ndt}^{1-\varphi} A_{Ndt_{-1}}}(1 + \frac{\kappa_d(1 + (\frac{A_{Sct}}{A_{Sdt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A_{Ndt}}{(1+\gamma)\eta_d(1 + (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A_{Ndt_{-1}}})^{-1}$$

$$+\frac{\kappa_c A_{Sct}^{-1-\varphi} A_{Nct}}{\kappa_d A_{Sdt}^{-1-\varphi} A_{Ndt}}((1 + \frac{(1+\gamma)\eta_d(1 + (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A_{Ndt_{-1}}}{\kappa_d(1 + (\frac{A_{Sct}}{A_{Sdt}})^{-\varphi})^{\frac{1+\varphi}{-\varphi}} A_{Ndt}})^{-1} > 1$$

Summing and regrouping we obtain

$$(1+\gamma)\eta_d[1 + (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}} + \kappa_d[1 + (\frac{A_{Sct}}{A_S dt})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}} <$$

$$(1+\gamma)\eta_c(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}}[1 + (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}} + \kappa_c(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}}[1 + (\frac{A_{Sct}}{A_S dt})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}$$

which leads to

$$(1+\gamma)\eta_d[1 + (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}} - (1+\gamma)\eta_c(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}}[1 + (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}} <$$

$$\kappa_c(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}}[1 + (\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}} - \kappa_d[1 + (\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}$$

from this point we obtain

$$[1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}[(1+\gamma)\eta_d - (1+\gamma)\eta_c(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}}] < [1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}[\kappa_c(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}} - \kappa_d] \tag{4.81}$$

To solve it I approximate the value of $\frac{[1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}}{[1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}} \approx 1$.[37] This gives me

$$(1+\gamma)\eta_d - (1+\gamma)\eta_c(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}} < \kappa_c(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}} - \kappa_d \tag{4.82}$$

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[37] In this case $\frac{{Adt_{-1}}^N}{A_{dt}^N}$ is eliminated because $A_{dt}^N$ is equal at every period. In the case where scientists are allocated in the dirty sector that fraction, $\frac{{Adt_{-1}}^N}{A_{dt}^N}$ is inside the approximation otherwise it is impossible to obtain an analitical result.

To conclude I reorganize the equation as

$$(1+\gamma)\eta_d + \kappa_d < \kappa_c(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}} + (1+\gamma)\eta_c(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}} \tag{4.83}$$

and

$$(1+\gamma)\eta_d + \kappa_d < (\frac{A_{Nct}}{A_N dt})^{-1-\varphi}[(1+\gamma)\eta_c\frac{A_{Nct-1}}{A_{Ndt-1}} + \kappa_c(\frac{\frac{A_{Sct}}{A_{Sdt}}}{\frac{A_{Nct}}{A_{Sdt}}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}}]$$

where $(\frac{\frac{A_{Sct}}{A_{Sdt}}}{\frac{A_{Nct}}{A_{Sdt}}})^{-1-\varphi} = \phi(A_t)$, a function that mesuares the ratio of the relative productivities. regrouping I obtain

$$(1+\gamma)\eta_d + \kappa_d < (\frac{A_{Nct}}{A_N dt})^{-1-\varphi}[(1+\gamma)\eta_c\frac{A_{Nct-1}}{A_{Ndt-1}} + \kappa_c\phi(A_t)\frac{(1+\gamma\eta_c)A_{Nct-1}}{A_{Ndt-1}}]$$

and

$$(1+\gamma)\eta_d + \kappa_d < (1+\gamma\eta_c)^{-1-\varphi}(\frac{A_{Nct-1}}{A_{Ndt-1}})^{-\varphi}[(1+\gamma)\eta_c + \kappa_c\phi(A_t)(1+\gamma\eta_c)]$$

which leads to

$$\eta_c A_{Nct-1}^{-\varphi} > \frac{\eta_c[(1+\gamma)\eta_d + \kappa_d]A_{Ndt-1}^{-\varphi}}{[(1+\gamma)\eta_c + \kappa_c\phi(A_t)(1+\gamma\eta_c)](1+\gamma\eta_c)^{-1-\varphi}} \tag{4.84}$$

This implies that an innovation in the clean sector occurs when $\eta_c A_{Nct-1}^{-\varphi} > \frac{\eta_c[(1+\gamma)\eta_d+\kappa_d]A_{Ndt-1}^{-\varphi}}{[(1+\gamma)\eta_c+\kappa_c\phi(A_t)(1+\gamma\eta_c)](1+\gamma\eta_c)^{-1-\varphi}}$ is satisfied.

### 4.14.6 Approximation of scientists allocation and probability of imitation

I reorganize equation (4.81) such that

$$(1+\gamma)\eta_d[1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}[1-\frac{\eta_c}{\eta_d}(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}}] < \kappa_d[1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}[\frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}}-1] \tag{4.85}$$

To solve it I approximate the value of $\frac{(1+\gamma)\eta_d[1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}}{\kappa_d[1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}} \approx 1$. In this simplification I am assuminng that $\kappa_d = 2\eta_d$. This gives me

$$\frac{\eta_c}{\eta_d}(\frac{A_{Nct}}{A_N dt})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}} + \frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}} > 2 \tag{4.86}$$

This leads to

$$\eta_c A_{Nct-1}^{-\varphi} > \frac{2\eta_d A_{Ndt-1}^{-\varphi}(1+\gamma\eta_c)^{1+\varphi}}{1 + \frac{\kappa_c\eta_d}{\kappa_d\eta_c}(\frac{A_{Sct}A_{Ndt}}{A_{Sdt}A_{Nct}})^{-1-\varphi}(1+\gamma\eta_c)} \tag{4.87}$$

The same happens for the different scenarios, except that I am using a stronger approximation where $\frac{(1+\gamma)\eta_d[1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}A_{Ndt-1}}{\kappa_d[1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}]^{\frac{1+\varphi}{-\varphi}}A_{Ndt}} \approx 1$, Now for $f(s) < 1$ we have

$$\frac{\eta_c}{\eta_d}(\frac{A_{Nct}}{A_{Ndt}})^{-1-\varphi}\frac{A_{Nct-1}}{A_{Ndt-1}} + \frac{\kappa_c}{\kappa_d}(\frac{A_{Sct}}{A_{Sdt}})^{-1-\varphi}\frac{A_{Nct}}{A_{Ndt}} < 2 \tag{4.88}$$

which implies that an innovation occurs in the dirty sector when

$$\eta_d A_{Ndt-1}^{-\varphi} > \frac{\eta_c}{2}(A_{Nct-1})^{-\varphi}(1+\gamma\eta_d)^{1+\varphi}[1+\frac{\kappa_c\eta_d}{\kappa_d\eta_c}(\frac{A_{Sct}A_{Ndt}}{A_{Sdt}A_{Nct}})^{-1-\varphi}(1+\gamma\eta_d)^{-1}] \tag{4.89}$$

### 4.14.7 Social planner problem

To summarize the problem is

$$\max\sum_{t=0}^{\infty}\frac{1}{(1-\rho)^t}\frac{C_N t^{1-\sigma}-1}{1-\sigma}S_{Nt} + \sum_{t=0}^{\infty}\frac{1}{(1-\rho)^t}\frac{C_S t^{1-\sigma}-1}{1-\sigma}S_{St} \tag{4.90}$$

s.t.

$$A_{Nct} = (1+\gamma\eta_c s_{Nct})\,A_{Nct-1} \tag{4.91}$$

$$A_{Ndt} = (1+\gamma\eta_d s_{Ndt})\,A_{Ndt-1} \tag{4.92}$$

$$A_{Sct} = \kappa_c s_{Sct}A_{Nct} + (1-\kappa_c s_{Sct})A_{Sct-1} \tag{4.93}$$

$$A_{Sdt} = \kappa_d s_{Sdt}A_{Ndt} + (1-\kappa_d s_{Sdt})A_{Sdt-1} \tag{4.94}$$

$$S_{Nt+1} = -[\zeta(Y_{Ndt}+Y_{Sdt})] + (1+\delta)S_{Nt} \tag{4.95}$$

$$S_{St+1} = -[\zeta(Y_{Ndt}+Y_{Sdt})] + (1+\delta)S_{St} \tag{4.96}$$

$$Y_{Nt} = (Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \tag{4.97}$$

$$Y_{St} = (Y_{Sct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Sdt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \tag{4.98}$$

$$C_{Nt} = Y_{Nt} - \psi\left(\int_0^1 d_{Nckt}dk + \int_0^1 d_{Ndkt}dk + \int_0^1 m_{Sckt}dk + \int_0^1 m_{Sdkt}dk\right) \tag{4.99}$$

$$C_{St} = Y_{St} - \psi\left(\int_0^1 d_{Sckt}dk + \int_0^1 d_{Sdkt}dk + \int_0^1 m_{Nckt}dk + \int_0^1 m_{Ndkt}dk\right) \tag{4.100}$$

$$Y_{ijt} = L_{ijt}^{1-\alpha}\int_0^1 A_{ijt}^{1-\alpha}(d_{ijkt}^{\alpha} + m_{ijkt}^{\alpha})dk \tag{4.101}$$

$$L_{Nct} + L_{Ndt} = 1 \text{ and } L_{Sct} + L_{Sdt} = 1 \tag{4.102}$$

$$s_{Nct} + s_{Ndt} = 1 \text{ and } s_{Sct} + s_{Sdt} = 1 \tag{4.103}$$

$$A_{Nc0} > 0 \text{ and } A_{Nd0} > 0 \tag{4.104}$$

$$A_{Sc0} > 0 \text{ and } A_{Sd0} > 0 \tag{4.105}$$

the problem above lead us to the Lagrangian which is represented as

$$L = \sum_{t=0}^{\infty} \frac{1}{(1-\rho)^t} \frac{C_i t^{1-\sigma} - 1}{1-\sigma} S_{it} + \lambda_{it}[(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} + (Y_{Sct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Sdt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - \psi \left( \int_0^1 x_{ickt} dk + \int_0^1 x_{idkt} dk \right) \tag{4.106}$$

$$-C_{Nt} - C_{St}] + \omega_{it}[-[\zeta_G(Y_{Ndt} + Y_{Sdt}) + \zeta_L Y_{idt}] + (1+\delta)S_{it} - S_{it+1}] + \lambda_{jit}[L_{ijt}^{1-\alpha} \int_0^1 A_{ijt}^{1-\alpha}(d_{ijkt}^{\alpha} + m_{ijkt}^{\alpha}) dk - Y_{jit}] +$$

$$+\mu_{Sjt}[\kappa_j s_{Sjt} A_{Njt} + (1 - \kappa_j s_{Sjt}) A_{Sjt-1} - A_{Sjt}] + \mu_{Njt}[(1 + \gamma\eta_j s_{Njt}) A_{Njt-1} - A_{Njt}]$$

Here we have in order, the derivative of consumption, of environmental quality, technology, abatement technology, clean input, dirty input, demand of machines, derivative of labour and the derivative of scientists. All of these are only in the North.

$$\begin{cases} \frac{1}{(1-\rho)^t} C_{it}^{-\sigma} S_{it} = \lambda_{it} \\ \omega_{it} = \frac{1}{(1-\rho)^t} \frac{C_{it}^{1-\sigma}}{1-\sigma} + (1+\delta)\omega_{it+1} \\ \mu_{Njt} = \lambda_{Njt}(1-\alpha)[L_{Njt}^{1-\alpha} A_{Njt}^{-\alpha}(d_{Njt}^{\alpha} + m_{Sjt}^{\alpha})] + \mu_{Njt+1}(1 + \gamma\eta_j s_{Njt+1}) + \mu_{Sjt}(\kappa_j s_{sjt}) \\ [Y_{Nct}^{\frac{1}{\epsilon}} (Y_{ict}^{\frac{\epsilon-1}{\epsilon}} + Y_{idt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] = \hat{p}_{Nct} \\ [Y_{Ndt}^{\frac{-1}{\epsilon}} (Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}] - \frac{[\zeta_G + \zeta_L]\omega_{Nt+1}}{\lambda_{Nt}} - \frac{\zeta_G \omega_{St+1}}{\lambda_{Nt}} = \hat{p}_{Ndt} \\ d_{Njt} = (\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt} \\ m_{Sjt} = (\frac{\alpha \hat{p}_{Ndt}}{\psi})^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt} \\ (1-\alpha)(L_{Njt}^{-\alpha} A_{Njt}^{1-\alpha} x_{Njt}^{\alpha})\lambda_{Njt} = \chi_{NLj} \\ \mu_{Njt}\gamma\eta_j A_{Njt-1} = \chi_{Nsj} \end{cases}$$

### 4.14.8 Social value of an innovation

Now we rewrite the social gain of an innovation such that

$$\mu_{Njt} = \lambda_{Njt}(1-\alpha)[L_{Njt}^{1-\alpha} A_{Njt}^{-\alpha}(d_{Njt}^{\alpha} + m_{Sjt}^{\alpha})] + \mu_{Njt+1}(1 + \gamma\eta_j s_{Njt+1}) + \mu_{Sjt}(\kappa_j s_{sjt}) \tag{4.107}$$

substituting the demands of machines

$$\mu_{Njt} = \lambda_{Njt}(1-\alpha)[L_{Njt}(2(\frac{\alpha \hat{p}_{Njt}}{\psi})^{\frac{\alpha}{1-\alpha}})] + \mu_{Njt+1}(1 + \gamma\eta_j s_{Njt+1}) + \mu_{Sjt}(\kappa_j s_{sjt}) \tag{4.108}$$

as well as for the South

$$\mu_{Sjt} = \lambda_{Sjt}(1-\alpha)[L_{Sjt}(2(\frac{\alpha \hat{p}_{Sjt}}{\psi})^{\frac{\alpha}{1-\alpha}})] + \mu_{sjt+1}(1 - \kappa_j s_{sjt+1}) \tag{4.109}$$

From this point I am using the appendix of chapter 2.
Here I have exctracted $\hat{p}_{Njt}^{-1}$ from $\hat{p}_{Njt}^{\frac{\alpha}{1-\alpha}}$ and I have substituted it with $\frac{\lambda_{Nt}}{\lambda_{Njt}}$. solving recursively and regroupping

$$A_{Nt}\mu_{Njt} = \sum_{v \geq t} 2\lambda_{Nt}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} \hat{p}_{Njv}^{\frac{1}{1-\alpha}} L_{Njv} A_{Njv} + \mu_{Sjt}(\kappa_j s_{sjt}) A_{Nt} \tag{4.110}$$

and doing the same in the south the following relationship is obtained

$$\mu_{Sjt} = \sum_{v \geq t} 2\lambda_{Sv}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} \hat{p}_{Sjv}^{\frac{1}{1-\alpha}} L_{Sjv}(1-\kappa_j s_{Sjv}) \tag{4.111}$$

which becomes

$$\mu_{Njt}A_{Njt} = 2(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}}[\sum_{v \geq t} \lambda_{Nv}\hat{p}_{Njv}^{\frac{1}{1-\alpha}} L_{Njv}A_{Njv} + \sum_{v \geq t} \lambda_{Sv}\hat{p}_{Sjv}^{\frac{1}{1-\alpha}} L_{Sjv}(\kappa_j s_{sjt})(1-\kappa_j s_{Sjv})A_{Nt}] \tag{4.112}$$

and applying the probability of a succesful innovation I obtain

$$\mu_{Njt}A_{Njt-1}\gamma\eta_j = 2\gamma\eta_j(1+\gamma\eta_j s_{Njt})^{-1}(1-\alpha)(\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} \tag{4.113}$$

$$[\sum_{v \geq t} \lambda_{Nv}\hat{p}_{Njv}^{\frac{1}{1-\alpha}} L_{Njv}A_{Njv} + \sum_{v \geq t} \lambda_{Sv}\hat{p}_{Sjv}^{\frac{1}{1-\alpha}} L_{Sjv}(\kappa_j s_{sjt})(1-\kappa_j s_{Sjv})A_{Nt}]$$

where $\mu_{Njt}A_{jt-1}\gamma\eta_j$ is the social gain of a new innovation. Note that $(\kappa_j s_{sjt})(1-\kappa_j s_{Sjv})$ is composed by the probability of obtain a succesful imitation times the probability of an unsuccesful imitation, representing the variance of a bernoulli distribution. To find the optimal allocation of scientists in the North we impose

$$\frac{(1+\gamma\eta_c s_{Nct})^{-1}[\sum_{v \geq t} \lambda_{Nv}\hat{p}_{Ncv}^{\frac{1}{1-\alpha}} L_{Ncv}A_{Njv} + \sum_{v \geq t} \lambda_{Sv}\hat{p}_{Scv}^{\frac{1}{1-\alpha}} L_{Scv}(\kappa_j s_{sct})(1-\kappa_j s_{Scv})A_{Nct}]}{(1+\gamma\eta_d s_{Ndt})^{-1}[\sum_{v \geq t} \lambda_{Nv}\hat{p}_{Ndv}^{\frac{1}{1-\alpha}} L_{Ndv}A_{Njv} + \sum_{v \geq t} \lambda_{Sv}\hat{p}_{Sdv}^{\frac{1}{1-\alpha}} L_{Sdv}(\kappa_j s_{sdt})(1-\kappa_j s_{Sdv})A_{Ndt}]} \tag{4.114}$$

### 4.14.9 Net zero 1st model

In this section I find the relative demand of workers in Net zero showing that applying this policy the demand of workers is the same as without the policy. Applying this definition of net zero

$$Y_{Ndt} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} \tag{4.115}$$

we understand that the maximum amount of production must satisfy this equality. This implies that the optimum production can be described as

$$Y_{Ndt}^{*} = \frac{\delta S_{Nt} - \zeta_G Y_{Sdt}}{\zeta_G + \zeta_L} = 2(\frac{\alpha p_{dt}}{\psi})^{\frac{\alpha}{1-\alpha}} A_{Ndt}L_{Ndt} \tag{4.116}$$

which implies that the demand of dirty labour depends by the cap, the productivity and the price of the intermediate dirty good as follows

$$L_{Ndt} = \frac{1}{2}\frac{\delta S_{Nt} - \zeta Y_{Sdt}}{\zeta} A_{Ndt}^{-1}(\frac{\alpha p_{Ndt}}{\psi})^{\frac{-\alpha}{1-\alpha}} \tag{4.117}$$

As we remember the clean labour depends by

$$L_{Nct} = \frac{1}{2}(\frac{\alpha}{\psi})^{\frac{-\alpha}{1-\alpha}} \hat{p}_{Nct}^{\frac{\varphi-1}{1-\alpha}} A_{Nct}^{-1} D^{\frac{1-\alpha-\varphi}{1-\alpha}} \tag{4.118}$$

We remember that $D = (Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}$. Given these informations, the relative labour depends by

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{p_{Nct}^{\frac{\varphi}{1-\alpha}}}{p_{Ndt}}(\frac{p_{Nct}}{p_{Ndt}})^{\frac{-1}{1-\alpha}}(\frac{A_{Nct}}{A_{Ndt}})^{-1}(\frac{\delta S_{Nt} - \zeta Y_{Sdt}}{\zeta})^{-1} Y_{Ndt}((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{4.119}$$

and as we said before $Y_{Ndt} = \frac{\delta S_{Nt} - \zeta Y_{Sdt}}{\zeta}$. I also moltiply and divide $p_{Ndt}$ by $\frac{p_{Nct}}{p_{Nct}}$ and I substitute it with $(\frac{p_{Nct}}{p_{Ndt}}) = (\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}$ which implies

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{p_{Nct}^{\frac{\varphi}{1-\alpha}}}{p_{Nct}(\frac{A_{Nct}}{A_{Ndt}})^{(1-\alpha)}}((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{4.120}$$

now

$$\frac{L_{Nct}}{L_{Ndt}} = p_{Nct}^{-\epsilon}(\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{4.121}$$

as we know, $\hat{p}_{Nct} = [Y_{Nct}^{\frac{-1}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon-1}}]$ and since it is raised to $-\epsilon$ it implies that $p_{Nct}^{-\epsilon} = [Y_{Nct}^{\frac{\epsilon}{\epsilon}}(Y_{Nct}^{\frac{\epsilon-1}{\epsilon}} + Y_{Ndt}^{\frac{\epsilon-1}{\epsilon}})^{\frac{-\epsilon}{\epsilon-1}}]$ that can be transformed in

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{Y_{Nct}}{Y_{Ndt}}((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{-\epsilon}{\epsilon-1}}(\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}((\frac{Y_{Nct}}{Y_{Ndt}})^{\frac{\epsilon-1}{\epsilon}} + 1)^{\frac{\epsilon}{\epsilon-1}} \tag{4.122}$$

so finally

$$\frac{L_{Nct}}{L_{Ndt}} = \frac{Y_{Nct}}{Y_{Ndt}}(\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} \tag{4.123}$$

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{p_{Nct}}{p_{Ndt}(1+\tau_O)})^{-\epsilon}(\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)} \tag{4.124}$$

and finally

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{\epsilon-\epsilon\alpha}(\frac{A_{Nct}}{A_{Ndt}})^{-(1-\alpha)}(1+\tau_O)^{\epsilon} \tag{4.125}$$

which leads to

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{\epsilon-\epsilon\alpha+\alpha-1}(1+\tau_O)^{\epsilon} \tag{4.126}$$

here we regroup the powers

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{(\epsilon-1)-\alpha(\epsilon-1)}(1+\tau_O)^{\epsilon} \tag{4.127}$$

and rearrangin the powers we obtain

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{(\epsilon-1)(1-\alpha))}(1+\tau_O)^{\epsilon} \tag{4.128}$$

In this model $(1-\epsilon)(1-\alpha)=\varphi$ so this implies that

$$\frac{L_{Nct}}{L_{Ndt}} = (\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O)^{\epsilon} \tag{4.129}$$

Which is the same solution of the social planner that corrects the environmental externalities.

### 4.14.10 Expected profits 2nd model

The expected profits are

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}\frac{A_{Nct-1}}{A_{Ndt-1}}(\frac{p_{Nct}^{\frac{1}{1-\alpha}}L_{Nct}+p_{Sct}^{\frac{1}{1-\alpha}}L_{Sct}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}+p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}) \tag{4.130}$$

now rearranging the equation we obtain $(\frac{A_{Nct}+A_{Sct}}{A_{Ndt}+A_{Sdt}})^{-\varphi-1}$

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}\frac{A_{Nct-1}}{A_{Ndt-1}}(\frac{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}+p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}+\frac{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}+p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}})$$

Regrouping the parenthesis we have

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}\frac{A_{Nct-1}}{A_{Ndt-1}}(\frac{p_{Nct}^{\frac{1}{1-\alpha}}L_{Nct}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}\frac{1}{1+\frac{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}}+\frac{p_{Sct}^{\frac{1}{1-\alpha}}L_{Sct}}{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}\frac{1}{1+\frac{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}})$$

Now, since $\frac{p^N_{ct}}{p^N_{dt}}=\frac{p^S_{ct}}{p^S_{dt}}=(\frac{A^N_{ct}+A^S_{ct}}{A^N_{dt}+A^S_{dt}})^{-(1-\alpha)}$ and for the same reason $\frac{L^N_{ct}}{L^N_{dt}}=\frac{L^S_{ct}}{L^S_{dt}}=(\frac{A^N_{ct}+A^S_{ct}}{A^N_{dt}+A^S_{dt}})^{-\varphi}$ we can substitute them with $(\frac{A_{Nct}+A_{Sct}}{A_{Ndt}+A_{Sdt}})^{-\varphi-1}$ obtaining

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}\frac{A_{Nct-1}}{A_{Ndt-1}}(\frac{A_{Nct}+A_{Sct}}{A_{Ndt}+A_{Sdt}})^{-\varphi-1}(\frac{1}{1+\frac{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}}+\frac{1}{1+\frac{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}})$$

but since the $(\frac{1}{1+\frac{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}}+\frac{1}{1+\frac{p_{Ndt}^{\frac{1}{1-\alpha}}L_{Ndt}}{p_{Sdt}^{\frac{1}{1-\alpha}}L_{Sdt}}})=1$ we obtain

$$\frac{\Pi_{Nct}}{\Pi_{Ndt}} = \frac{\eta_c}{\eta_d}\frac{A_{Nct-1}}{A_{Ndt-1}}(\frac{A_{Nct}+A_{Sct}}{A_{Sct}+A_{Sdt}})^{-\varphi-1}$$

and for the South
$$\frac{\Pi_{Sct}}{\Pi_{Sdt}} = \frac{\kappa_c}{\kappa_d}\frac{A_{Nct}}{A_{Ndt}}(\frac{A_{Nct}+A_{Sct}}{A_{Sct}+A_{Sdt}})^{-\varphi-1}$$

### 4.14.11 Equilibrium production 2nd model

To obtain the equilibrium production we start from the optimum production

$$Y_{Nct} = p_{Nct}^{\frac{\alpha}{1-\alpha}} L_{Nct}(A_{Nct} + A_{Sct})$$

$$Y_{Nct} = [(\frac{A_{Nct} + A_{Sct}}{A_{Ndt} + A_{Sdt}})^{\varphi} + 1]^{\frac{-\alpha}{\varphi}} [(\frac{A_{Nct} + A_{Sct}}{A_{Ndt} + A_{Sdt}})^{\varphi} + 1]^{-1}(A_{Nct} + A_{Sct}) \tag{4.131}$$

$$Y_{Nct} = [(\frac{A_{Nct} + A_{Sct}}{A_{Ndt} + A_{Sdt}})^{\varphi} + 1]^{\frac{-(\alpha+\varphi)}{\varphi}}(A_{Nct} + A_{Sct}) \tag{4.132}$$

The equilibrium clean production can be described as

$$Y_{Nct} = [(A_{Nct} + A_{Sct})^{\varphi} + (A_{Ndt} + A_{Sdt})^{\varphi}]^{\frac{-(\alpha+\varphi)}{\varphi}}(A_{Nct} + A_{Sct})(A_{Ndt} + A_{Sdt})^{\alpha+\varphi} \tag{4.133}$$

### 4.14.12 Social gain of an innovation 2nd model

From the F.O.C. of the technology we obtain

$$\mu_{jt}^{N} = \lambda_{jt}^{N}(1-\alpha)(L_{jt}^{N})^{1-\alpha}(A_{jt}^{N})^{-\alpha}(d_{jt}^{N})^{\alpha} + \mu_{jt+1}^{N}(1 + \gamma\eta_j s_{jt+1}^{N}) + \mu_{jt}^{S}(\kappa_j s_{jt}^{S}) + \tag{4.134}$$

$$+\lambda_{jt}^{S}(1-\alpha)(L_{jt}^{S})^{1-\alpha}(A_{jt}^{N})^{-\alpha}(m_{jt}^{N})^{\alpha}$$

For the south we have

$$\mu_{jt}^{S} = \lambda_{jt}^{S}(1-\alpha)(L_{jt}^{S})^{1-\alpha}(A_{jt}^{S})^{-\alpha}(d_{jt}^{S})^{\alpha} + \mu_{jt+1}^{S}(1-\kappa_j s_{jt}^{S}) + \lambda_{jt}^{N}(1-\alpha)(L_{jt}^{N})^{1-\alpha}(A_{jt}^{S})^{-\alpha}(m_{jt}^{S})^{\alpha}$$

Now substituting the demand of machines inside we obtain

$$\mu_{jt}^{N} = \lambda_{jt}^{N}(1-\alpha)L_{jt}^{N}(\frac{\alpha\hat{p}_{jt}^{N}}{\psi})^{\frac{\alpha}{1-\alpha}} + \mu_{jt+1}^{N}(1+\gamma\eta_j s_{jt+1}^{N}) + \mu_{jt}^{S}(\kappa_j s_{jt}^{S}) + + \lambda_{jt}^{S}(1-\alpha)L_{jt}^{S}(\frac{\alpha\hat{p}_{jt}^{N}}{\psi})^{\frac{\alpha}{1-\alpha}}$$

solving we obtain the social gain of an innnovation, $\mu_{jt+1}^{N} A_{jt-1}^{N} \gamma\eta_j$ depends by

$$\gamma\eta_j(1+\gamma\eta_j s_{jt}^{N})^{-1}(1-\alpha)(\frac{\alpha}{\varphi})^{\frac{\alpha}{1-\alpha}} \sum_{v\geq t}[\lambda_v^{N} L_{jv}^{N}(\hat{p}_{jt}^{N})^{\frac{1}{1-\alpha}} + \lambda_v^{S} L_{jv}^{S}(p_{jt}^{S})^{\frac{1}{1-\alpha}}]A_{jv}^{N}(1-(k_j s_{jv}^{S}(1-\kappa_j s_{jv}^{S})))$$

this implies that the social planner allocates the scientists to the sector that satisfies the following condition

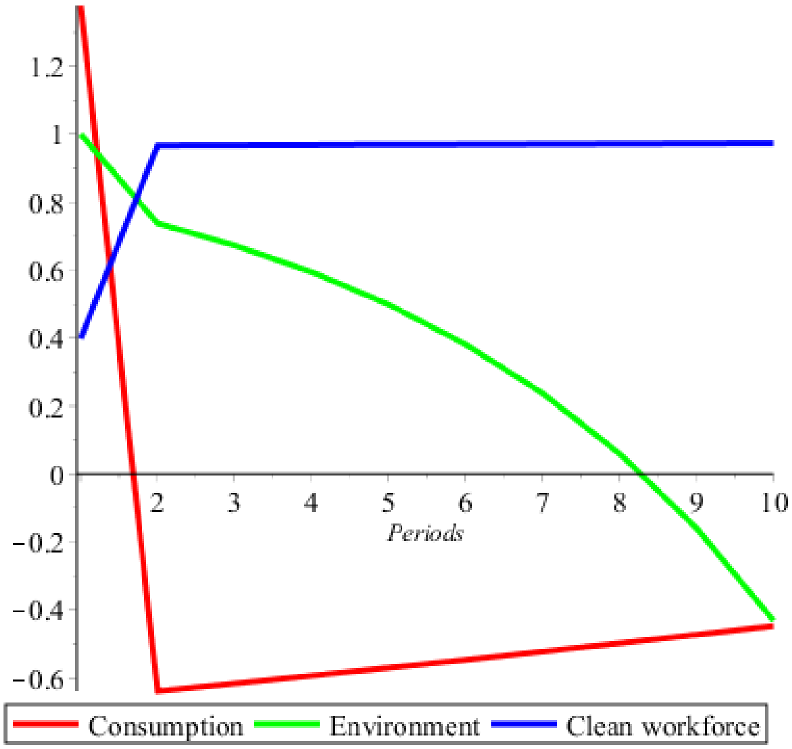
$$\frac{\eta_c(1 + \gamma\eta_j s_{ct}^{N})^{-1} \sum_{v\geq t}[\lambda_v^{N} L_{cv}^{N}(\hat{p}_{ct}^{N})^{\frac{1}{1-\alpha}} + \lambda_v^{S} L_{cv}^{S}(p_{ct}^{S})^{\frac{1}{1-\alpha}}]A_{cv}^{N}(1 - (k_c s_{cv}^{S}(1 - \kappa_j s_{cv}^{S})))}{\eta_d(1 + \gamma\eta_d s_{dt}^{N})^{-1} \sum_{v\geq t}[\lambda_v^{N} L_{dv}^{N}(\hat{p}_{dt}^{N})^{\frac{1}{1-\alpha}} + \lambda_v^{S} L_{dv}^{S}(p_{dt}^{S})^{\frac{1}{1-\alpha}}]A_{dv}^{N}(1 - (k_d s_{dv}^{S}(1 - \kappa_j s_{dv}^{S})))} \tag{4.135}$$

### 4.14.13 Additional plots

**Unilateral Policy**

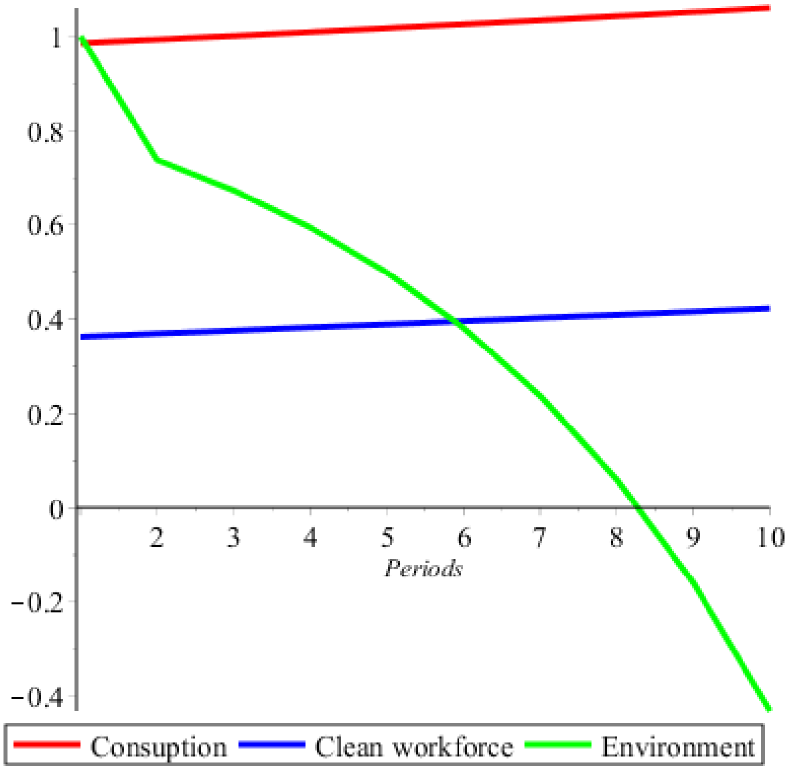
Here I show the effects of the unilateral policy applying a tax $\tau_O$ only in the North with value 2.5.

Figure 4.13:



In this scenario the tax is not sufficient to avoid an environmental disaster. The tax is enough large to obtain negative values of consumptions, which are absurds. I bring this exaple to show ho difficult it is to choose a combination of variables that is enough strong to restore the environment. The meaning of this plot is that under free trade and my initial values, an environmental disaster is not avoidable even with negative consumption.

Figure 4.14:

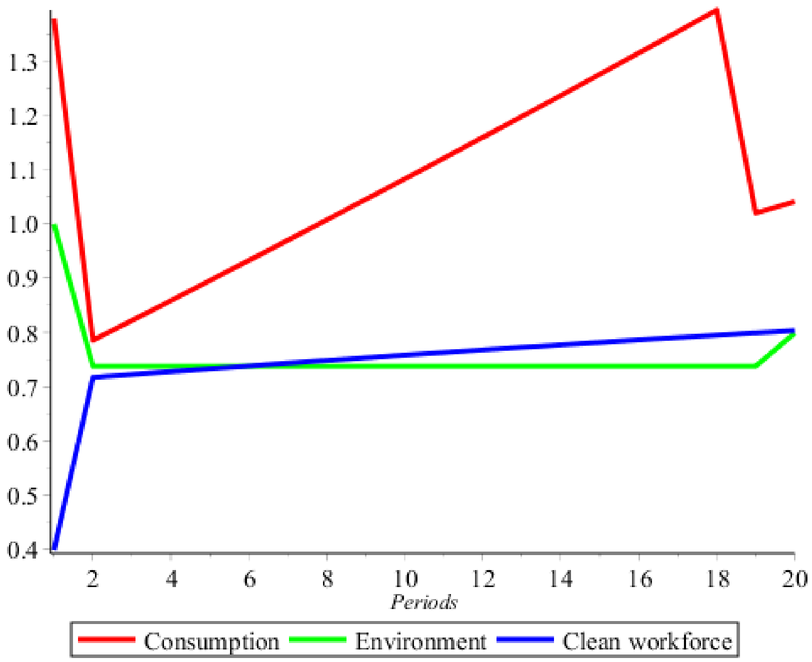


The South is growing over time reaching an environmental disaster at period $t = 8$.

### Net zero

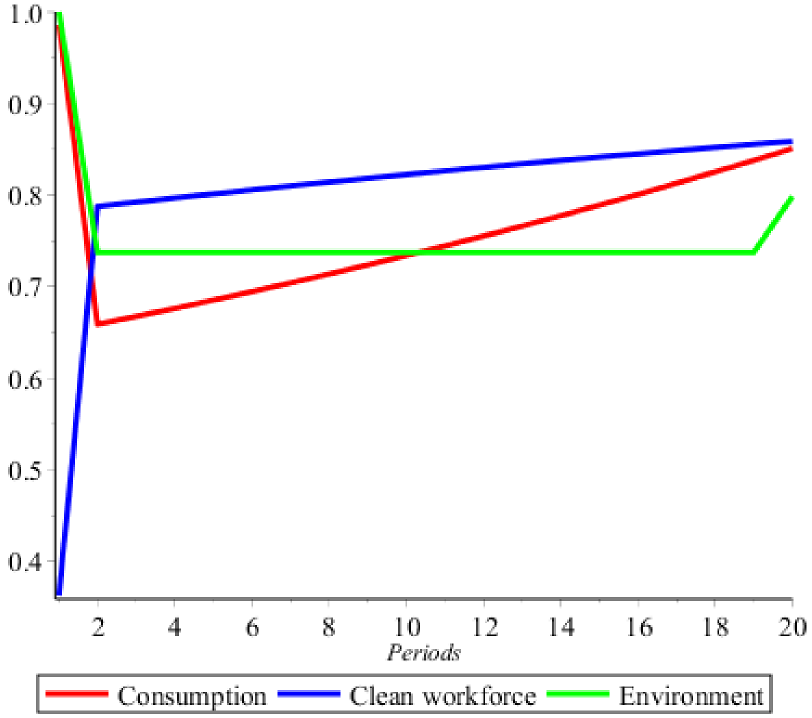
In this section I show the effect of the Net zero emission target on the economies. The policy is represented by a cap on the production such that, when it is too large the cap beyond the acceptable levels the cap is activated. Also, a small tax, $\tau_O^N = 0.55$ is applied to the North to compensate for the pollution that is affecting the South.

Figure 4.15: Consumption north short



Here we have the consumption in the North that goes from 1.37 to 0.78 after the introduction of the cap. Immediately after we can see that the environmental quality is constant at 0.73 due to the fact that the net zero fix the value. The consumption is increasing due to the fact that the dirty production in the South is decreasing thanks to the tax on the dirty output. At the period $t = 19$.
In the next plot we have the South applying the tax on intermediate dirty output $\tau_O^S = 0.85$.

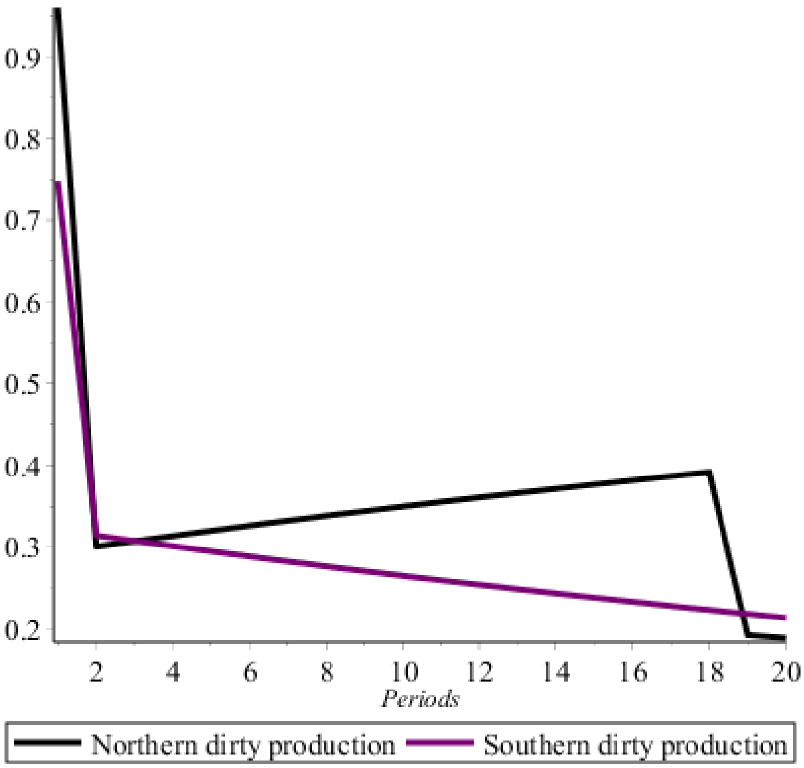
Figure 4.16:



The consumption moves from 0.98 to 0.65 due to the introduction of the tax. Then it is growing alongside with the clean workforce that jumps to 0.78 from its initial value of 0.36. At peropd $t = 20$ the consumption reaches the level 0.85, the workforce is at 0.85.

Then we have the dirty production

Figure 4.17: Dirty product

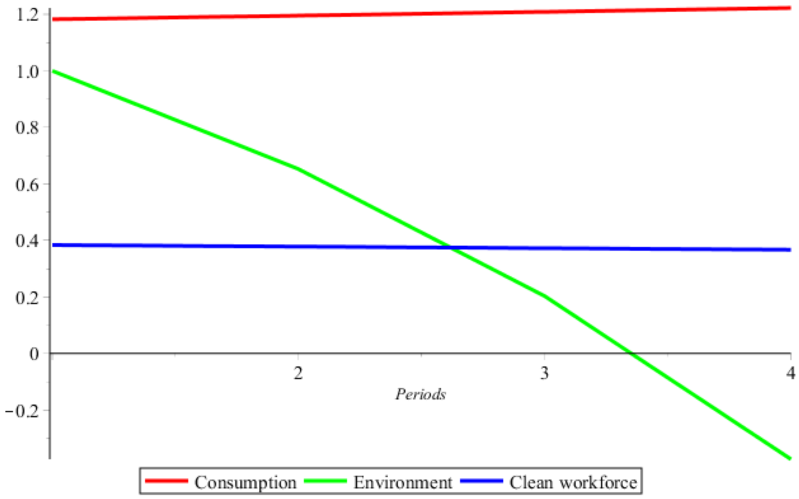


The dirty production has two different behaviours depending on the policy applied. The southern one declines to 0.31 thanks to the tax. It is slowly decreasing over tim due to the technological progress in the clean sector which affects the demand of workers. The northern production after the first fall, it is increasing due to the decraese in the production from the South. In fact, the cap is established as $Y_{dt}^N = \frac{\delta S_t^N - \zeta Y_{dt}^S}{\zeta}$. At period 19 the clean workforce is so large that the dirty production falls to 0.19.
In the next figure I represent the long run in the north. We can see the consumption growing to the value of 5.0. I finish the section with the South where the consumption reaches 3.49.

### Lassaiz faire under property rights trade

In this part the are the plots of the second model under lassaiz-faire. Here the consumption is slightly lower because the firms use a combination of productivity.

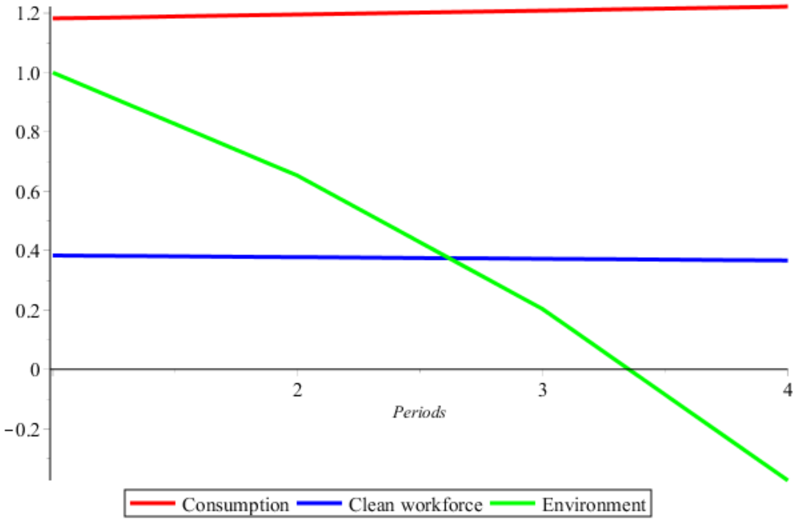
Figure 4.18: Northern variables



The consumption starts at 1.18 ending in period 4 at 1.22. The clean workforce is moving from 0.38 to 0.36. The environment degrades slower compared to the previous model reaching the disaster by period 4[38].

The next plot represents the same variables for the South.

Figure 4.19: Southern variables



Given the structure of the problem, the two countries are identical in the equilibrium thanks to the use of a productivity mix.

Finally the dirty production of the two countries.

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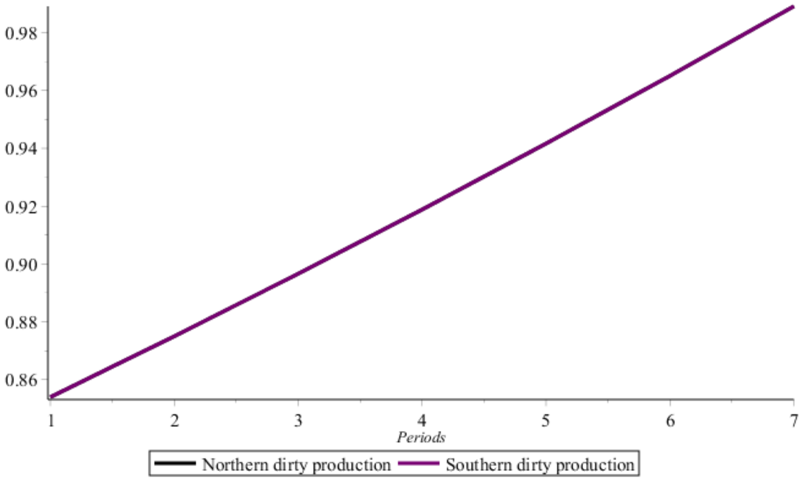
[38] In the plot it is happening before, between periods $t = 3$ and $t = 4$, but since it is a discrete time model, I cannot consider it happening in the middle.

Figure 4.20: Dirty prod



The two dirty production overlap in all periods. This is due to the fact that the two economies are linked to their productivities implying that they have the same value for every variable.

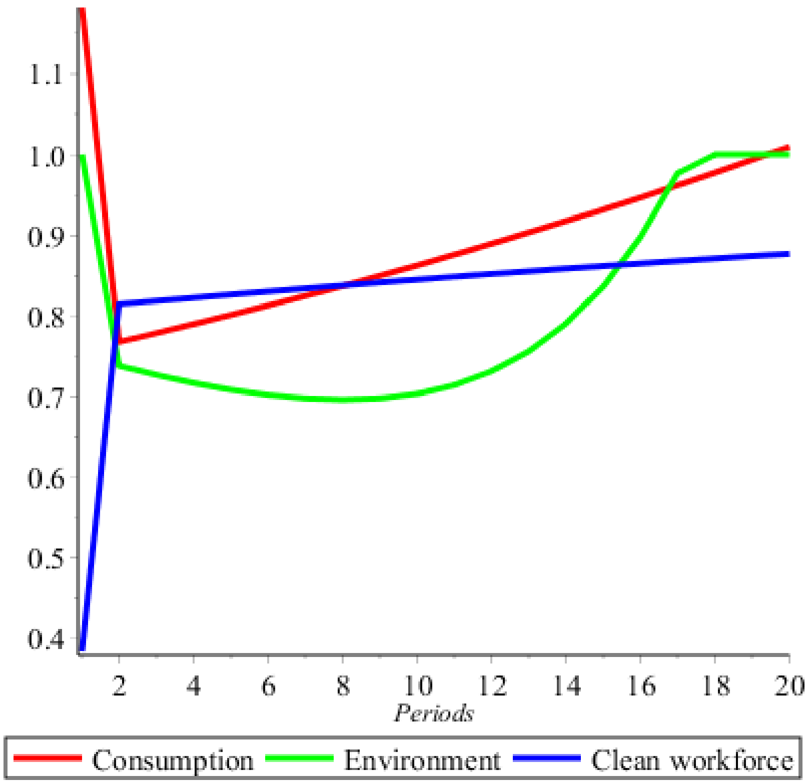
### Global coordination with same tax implemented with trade of property rights

We reached the point where the same tax is applied to both countries. Given the previous plot where the prodution of intermediate inputs has the same value for every country, this is the natural solution of the model[39]. The economies have a tax on intermediate dirty output with value $\tau_O = 0.9$.

---

[39] This conclusion is given by the fact that having the same variables we are expecting to address the environmental issue using the same tax in both countries. In fact, the marginal productivity are the same, as well as the marginal consumption implying the same tax

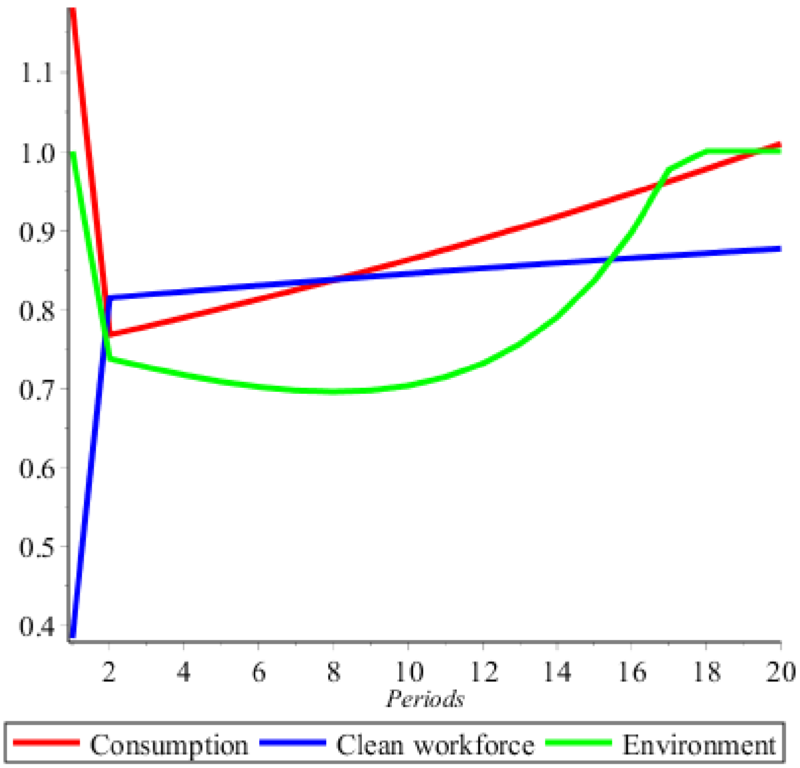
Figure 4.21: Consumption north short



We can see that the consumption goes from 1.18 to 0.76 and after that, the consumption starts growing steadily until it reaches 1 at period $t = 20$. The environment is recovering from period 2 at a level of 0.73 recovering its full value at period 17.

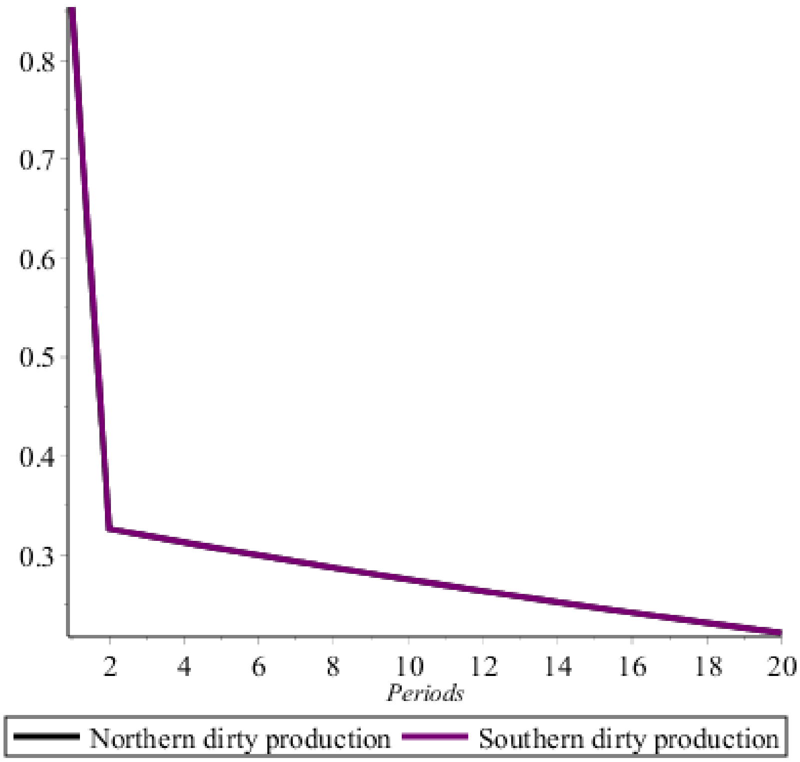
The south is following the same path

Figure 4.22: Consumption in the south



where all the variables have the same behaviour as in the North.
The next one represents the dirty production in both countries,

Figure 4.23: Dirty production



which again the dirty productions coincide given the use of the same productivities and tax. The value of the production goes from 0.85 to 0.32 at period 2 decreasing over time reaching 0.22 at period $t = 20$.

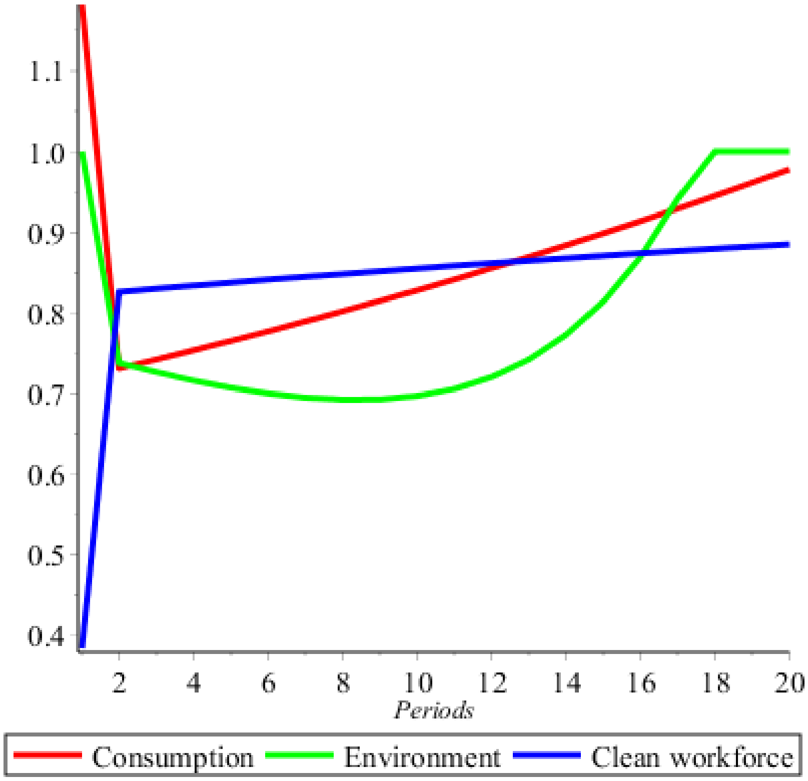
The next figures show the evolution of the economies in the long run. Where the consumption grows exponentially reaching the value 4.29.

In the south we have the same consumption path. In the next section I am showing the effect of different taxes.

### Global coordination with different taxes

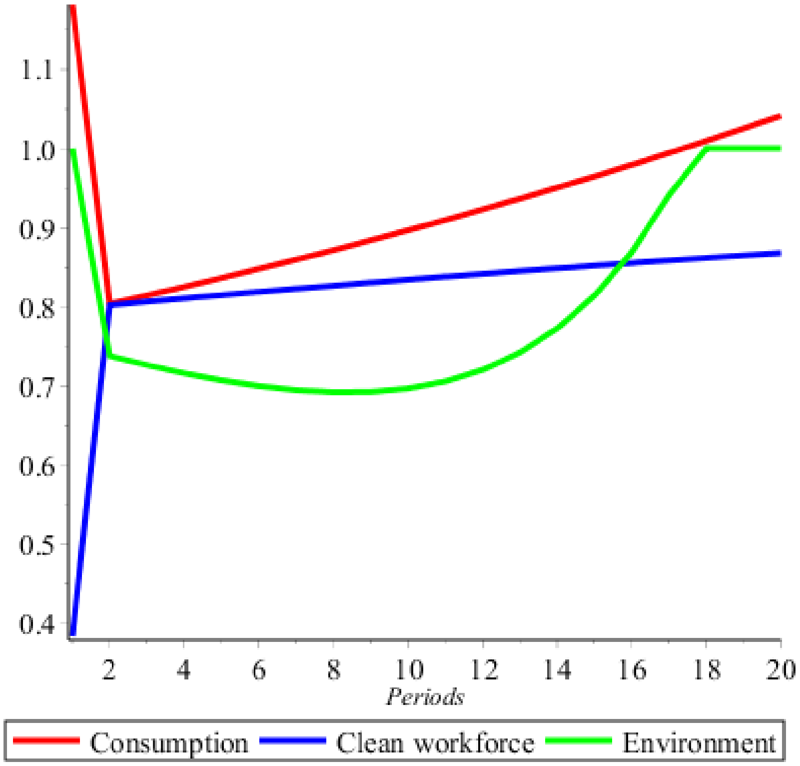
In this section I am applying two different taxes on the intermediate output of the two countries. In the North $\tau_O^N = 0.95$ meanwhile in the South $\tau_O^S = 0.85$.

Figure 4.24: Consumption in the north



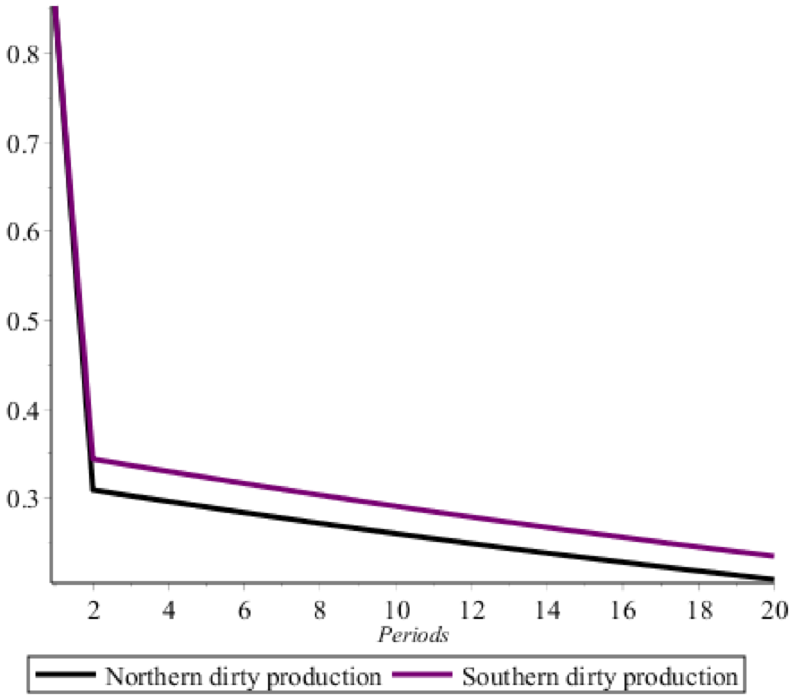
The consumption moves from 1.18 to 0.73 at period $t = 2$. At the same time, the clean workforce goes from 0.38 to 0.82. The environment after the introduction of the tax slows it fall and start recovering reaching its full capacity at period $t = 17$. The consumption is at level 0.97 at the end in period 20.
The next one is the short run in the South

Figure 4.25: Consumption south



where now, thanks to a new tax the consumption takes a different path. It starts from the value of 1.18 falling to 0.80 and after that at period $t = 20$ it raises to 1.14. The workforce goes from 0.38 to 0.80. the dirty production

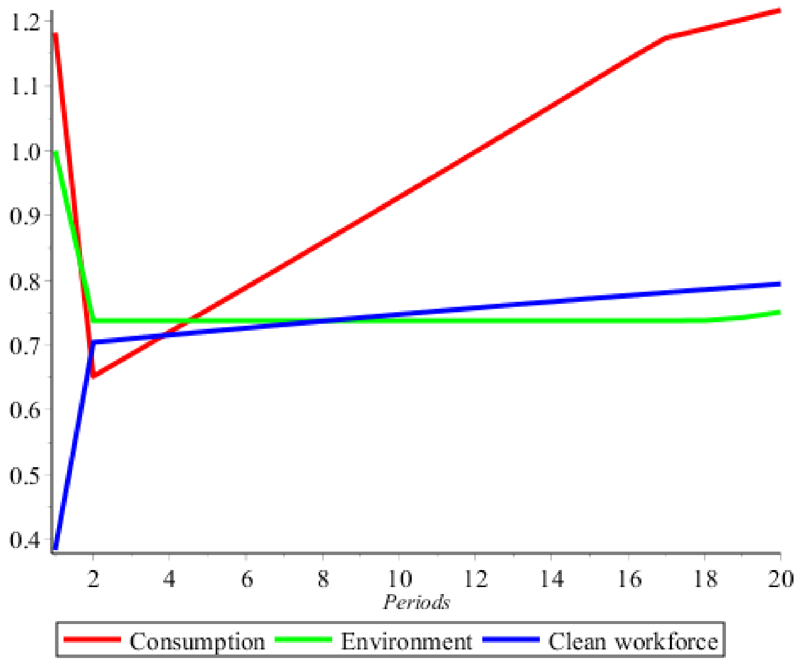
Figure 4.26: Dirty production



### Net zero

In this part I show the plots where the North engages the Net zero emission policy trying to reach a sustainable growth. The North is sustaining on top of the sustainable policy a carbon tax on the emiisions that affects the South of value $\tau_O^N = 0.55$ and one in the South $\tau_O^S = 0.85$.

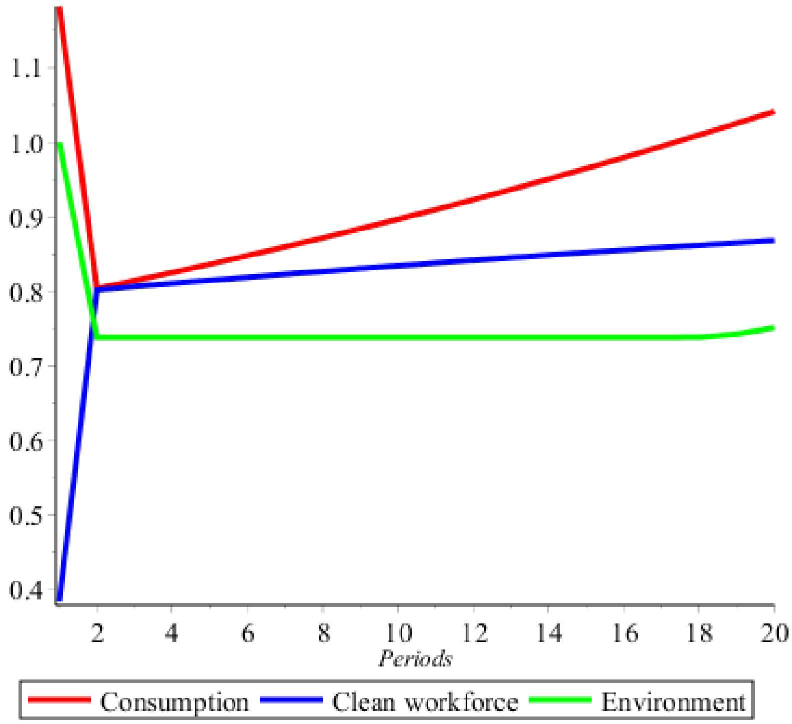
Figure 4.27: Short run north



In the North consumption moves from 1.18 to .65 increasing to 1.21 at period $t = 20$. The clean workforce is jumping to 0.70 from its initial value 0.38. The environment is stationary due to the introduction of the sustainable policy until period $t = 19$ where the environment starts regenerating.

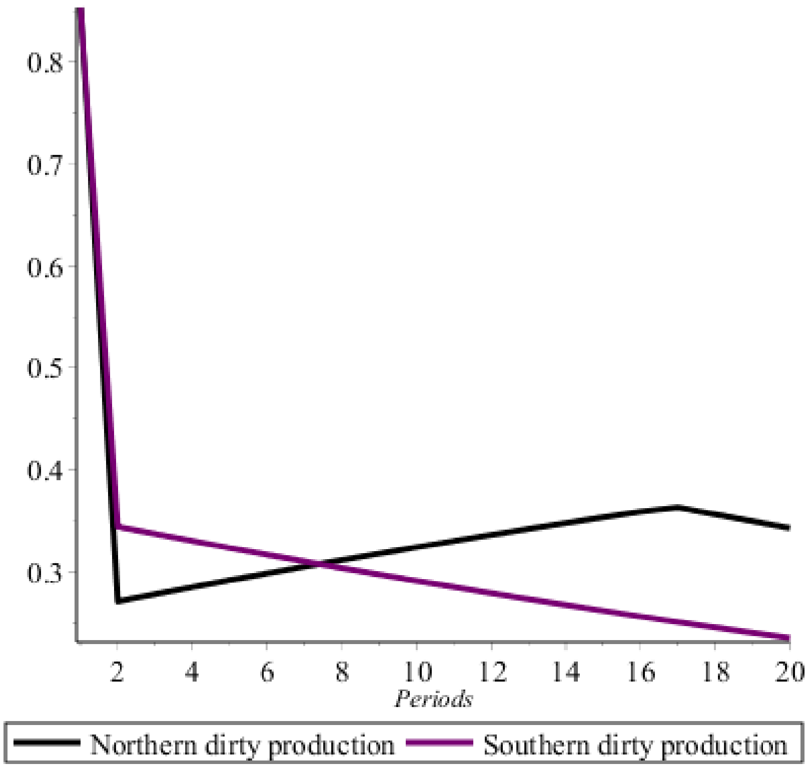
The next plot shows the evolution of the southern economy

Figure 4.28: Short run



where the consumption moves from 1.18 to 0.80 after the introduction of the tax and then it grows slowly to 1.04 at period $t = 20$. The clean workdorce is larger than in the North, going to 0.80 at time $t = 2$ then the dirty productions

Figure 4.29: The dirty production



### 4.14.14 Analysis of the trade component

I start from the closed economy

$$\frac{\pi_c}{\pi_d} = (\frac{A_c}{A_d})^{-1-\varphi}(1+\tau)^{\epsilon} \tag{4.136}$$

Now, we are looking for the scenario when the profits are larger than 1, then

$$(\frac{A_c}{A_d})^{-1-\varphi}(1+\tau)^{\epsilon} > 1 \tag{4.137}$$

it happens only when

$$\tau > (\frac{A_c}{A_d})^{\frac{1+\varphi}{\epsilon}} - 1 \tag{4.138}$$

Now looking at the open economy, we need to solve the trade component. We start from

$$\frac{\pi^N_{ct}}{\pi^N_{dt}} = \frac{(p^N_{ct})^{\frac{1}{1-\alpha}} A^N_{ct} L^N_{ct} + (p^S_{ct})^{\frac{1}{1-\alpha}} A^S_{ct} L^S_{ct}}{(p^N_{dt})^{\frac{1}{1-\alpha}} A^N_{dt} L^N_{dt} + (p^S_{dt})^{\frac{1}{1-\alpha}} A^S_{dt} L^S_{dt}} \tag{4.139}$$

$$\frac{\pi_{Nct}}{\pi_{Ndt}} = \frac{p_{Nct}^{\frac{1}{1-\alpha}} L_{Nct} A_{Nct}}{p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt}}(1 + \frac{p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt} A_{Sdt}}{p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt} A_{Ndt}})^{-1} + \tag{4.140}$$

$$+\frac{p_{Sct}^{\frac{1}{1-\alpha}} L_{Sct}A_{Sct}}{p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt}A_{Sdt}}(1+\frac{p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt}}{p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt}A_{Sdt}})^{-1}$$

The first trade component is

$$\frac{p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt}}{p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt}+p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt}A_{Sdt}} \tag{4.141}$$

we know that the dirty price depends by

$$p_{Ndt}=(1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O^i)^{\epsilon-1})^{\frac{1}{\epsilon-1}}(1+\tau_O^i)^{-1} \tag{4.142}$$

and the dirty workforce by

$$L_{Ndt}=\frac{1}{(1+(\frac{A_{Nct}}{A_{Ndt}})^{-\varphi}(1+\tau_O^N)^{\epsilon})} \tag{4.143}$$

The one of the south by

$$p_{Sdt}=(1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi})^{\frac{1}{\epsilon-1}} \tag{4.144}$$

and

$$L_{Sdt}=\frac{1}{(1+(\frac{A_{Sct}}{A_{Sdt}})^{-\varphi}} \tag{4.145}$$

The tax reduces the amount of labour forces and it slightly increases the price of the dirty input, such that the value of the numerator is decreasing.
In the second fraction we have

$$\frac{p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt}A_{Sdt}}{p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt}+p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt}A_{Sdt}} \tag{4.146}$$

where the tax decreases the value of the denominator increasing the value of the fraction, therefore, the first fraction is smaller than the second, because $p_{Ndt}^{\frac{1}{1-\alpha}} L_{Ndt}A_{Ndt} < p_{Sdt}^{\frac{1}{1-\alpha}} L_{Sdt}A_{Sdt}$ thanks to the tax

### 4.14.15 Implicit function

To simplify I substitute $\frac{A_{Nct}}{A_{Ndt}}$ with $x$. This is the derivative of the numerator,

$$(-(1+x^{-\varphi}\epsilon(1+\tau)^{\epsilon-1})^{-2}((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}+(1+x^{-\varphi}(1+\tau)^{\epsilon})^{-1}(-\frac{1}{\varphi})((\epsilon-1)(1+\tau)^{\epsilon-2}+x^{-\varphi})^{\frac{(1+\varphi)}{-\varphi}}) \tag{4.147}$$

$$(1+x^{-\varphi}(1+\tau)^{\epsilon})^{-1}((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}$$

In solving the problem using the chain rule I subtract the numerator times the derivative of the denominator.

$$(-(1+x^{-\varphi}\epsilon(1+\tau)^{\epsilon-1})^{-2}(((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}+(1+y^{-\varphi})^{\frac{1+\varphi}{(-\varphi)}})+(1+x^{(-\varphi)}(1+\tau)^{\epsilon})^{-}1(-\frac{1}{\varphi}(\epsilon-1)(1+\tau)^{\epsilon-2}+x^{-\varphi}) \tag{4.148}$$

$$(1+x^{-\varphi}(1+\tau)^{\epsilon})((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}(1+x^{(-\varphi)}(1+\tau)^{\epsilon})$$

Then I have the denominator

$$(((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}(1+x^{-\varphi}(1+\tau)^{\epsilon})^{-1}+(1+y^{-\varphi})^{\frac{1+\varphi}{(-\varphi)}})^{2} \tag{4.149}$$

It can be summarized by

$$\{-(1+x^{-\varphi}\epsilon(1+\tau)^{\epsilon-1})^{-2}((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}+(1+x^{-\varphi}(1+\tau)^{\epsilon})^{-1} \tag{4.150}$$

$$\frac{1}{-\varphi}((\epsilon-1)(1+\tau)^{\epsilon-2}+x^{-\varphi})^{\frac{1+\varphi}{-\varphi}}\}(1+y^{-\varphi})^{\frac{1+\varphi}{-\varphi}}$$

Now, the denominator in equation (14) is always positive. Equation (15) is positive because $-\frac{1}{\varphi}$ is larger than zero since $\varphi < 0$. One of the conditions that need to be satisfied are

$$\frac{\frac{1}{-\varphi}((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1+\varphi}{-\varphi}}}{(1+x^{-\varphi}(1+\tau)^{\epsilon})}(\epsilon-1)(1+\tau)^{\epsilon-2} > \epsilon(1+\tau)^{\epsilon-1}\frac{((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}}{(1+x^{-\varphi}(1+\tau)^{\epsilon})^{2}} \tag{4.151}$$

It can be represented as

$$L_{Ndt}p_{Ndt}^{'} > L_{Ndt}^{'}p_{Ndt} \tag{4.152}$$

It implies that the marginal price of the dirty sector per worker employed in that sector must be larger than the marginal worker of the dirty sector adjusted by the price. In other words, the variation of the cost of dirty output due to the tax times the dirty workforce needs to be larger than the variation of the dirty workforce time the cost of the intermediate dirty output. Now I am doing the derivative of $x$ for the numerator

$$\{-(1-\varphi x^{-\varphi-1}(1+\tau)^{\epsilon})^{-2}((1+\tau)^{\epsilon-1}x^{-\varphi})^{\frac{1}{-\varphi}}+ \tag{4.153}$$

$$+(1+x^{-\varphi}(1+\tau)^{\epsilon})^{-1}\frac{1}{-\varphi}((1+\tau)^{\epsilon-1}-\varphi x^{-\varphi-1})\}(1+y^{-\varphi})^{\frac{1+\varphi}{-\varphi}}$$

It is larger than zero when

$$\frac{((1+\tau)^{\epsilon-1}-\varphi x^{-\varphi-1})}{(1+x^{-\varphi}(1+\tau)^{\epsilon})} > \frac{((1+\tau)^{\epsilon-1}x^{-\varphi})^{\frac{1}{-\varphi}}}{(1-\varphi x^{-\varphi-1}(1+\tau)^{\epsilon})^{2}} \tag{4.154}$$

or

$$L_{Ndt}p_{Ndt}^{'} > L_{Ndt}^{'}p_{Ndt} \tag{4.155}$$

The values of the denominator is the same to the one found before. The implicit function theorem requires

$$-\frac{\frac{\frac{1}{-\varphi}((\epsilon-1)(1+\tau)^{\epsilon-2}+x^{-\varphi})^{\frac{1+\varphi}{-\varphi}}}{(1+x^{-\varphi}(1+\tau)^{\epsilon})}-\frac{((1+\tau)^{\epsilon-1}+x^{-\varphi})^{\frac{1}{-\varphi}}}{(1+x^{-\varphi}\epsilon(1+\tau)^{\epsilon-1})^{2}}}{\frac{((1+\tau)^{\epsilon-1}-\varphi x^{-\varphi-1})}{(1+x^{-\varphi}(1+\tau)^{\epsilon})}-\frac{((1+\tau)^{\epsilon-1}x^{-\varphi})^{\frac{1}{-\varphi}}}{(1-\varphi x^{-\varphi-1}(1+\tau)^{\epsilon})}} \tag{4.156}$$

This can be represented as

$$-\frac{L_{Ndt}\frac{\partial p_{Ndt}}{\partial\tau}-p_{Ndt}\frac{\partial L_{Ndt}}{\partial\tau}}{L_{Ndt}\frac{\partial p_{Ndt}}{\partial A_{Nct}}-p_{Ndt}\frac{\partial L_{Ndt}}{\partial A_{Nct}}} \tag{4.157}$$

The theorem says that a positive change in the tax in the tax substitutes the effect of an increase in the clean productivity depending by the difference between the variation of the cost of dirty output and the variation in the dirty workorce relatively to an increase in the tax against a change in the clean productivity.

# Bibliography

[1] Acemoglu D, 2009, *Introduction to Modern Economic Growth,* Princeton: Princeton University Press

[2] Acemoglu D., Aghion P., Bursztyn L., Hémous D. 2012a *'The Environment and Directed Technical Change',* American Economic Review, 102(1), 131–66.

[3] Acemoglu D.,Aghion P., Hémous D., 2014 *The environment and directed technical change in a North–South model,* Oxford Review of Economic Policy, Oxford University Press, Volume 30, Number 3 pp. 513–530

[4] Amigues J., Durmaz T., 2019 *A Two-Sector Model of Economic Growth with Endogenous Technical Change and Pollution Abatement,* Environmental Modelling & Assessment, Volume 24, pp 703-725

[5] Benchekroun H., 2003 *Unilateral production restrictions in a dynamic duopoly,* Journal of Economic Theory, Volume 111 pp. 214–239

[6] Bijgaart I. 2017 *The unilateral implementation of a sustainable growth path with directed technical change,* European Economic Review, Volume 91, January 2017, Pages 305-327

[7] Gancia G., Bonfiglioli A. 2008 *North–South trade and directed technical change,* Journal of International Economics Volume 76, pp. 276–295

[8] IPCC, 2018: Annex I: Glossary [Matthews, J.B.R. (ed.)]. In: *Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*[Masson-Delmotte, V., P. Zhai, H.-O. Pörtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I. Gomis, E. Lonnoy, T. Maycock, M. Tignor, and T. Waterfield (eds.)]

[9] Krugman P., 1979 *Increasing returns, monopolistic competition, and international trade,* Journal of International Economics, Volume 9, Issue 4, November 1979, Pages 469-479

[10] Krugman P., 1981, *Intraindustry Specialization and the Gains from Trade,* Journal of Political Economy, Vol. 89, No. 5 (Oct., 1981), pp. 959-973

[11] Leite D., Afonso O., Silva S. 2019 *A tale of two countries: Directed technical change, trade and migratory movements,* Economic Modelling, Volume 83, December 2019, Pages 173-194

[12] Naito T., 2019 *A lab-equipment model of growth with heterogeneous firms and asymmetric countries,* Economics Letters Volume 181, August 2019, Pages 33-36

[13] Naito T. 2021 *CAN THE OPTIMAL TARIFF BE ZERO FOR A GROWING LARGE COUNTRY?,* INTERNATIONAL ECONOMIC REVIEW Vol. 62, No. 3, August 2021 pp. 1237-1280

[14] Nordhaus, William D., 2007 *A Review of the Stern Review on the Economics of Climate Change.* Journal of Economic Literature , 45(3): 686–702

[15] Papageorgiou C., Saam M., Schulte P., 2017, *Substitution between clean and dirty energy : A macroeconomic perspective,* The Review of Economics and Statistics, Volume 99(2), pp 281–290

[16] Rivera-Batiz L., Romer P. 1991 *International trade with endogenous technological change,* European Economic Review Vol. 35 pp. 971-1004

[17] UNEP 2019. *Sand and sustainability: Finding new solutions for environmental governance of global sand resources.* GRID-Geneva, United Nations Environment Programme, Geneva, Switzerland

# Chapter 5

# Conclusions

## 5.1 Summary of Conclusions

In this thesis I continue the work of Acemoglu et al. 2012[1] showing that, when the dirty sector is the most advanced the economies will lead to an environmental disaster. Then, extending the paper of Acemoglu et al. 2014[2] I show how to avoid an environmental disasters with unilateral policies. The elasticity of substitution has a central role in the allocation of scientists and on the effect of taxation. In fact, the elasticity of substitution altogether with the imitation process allows the southern economy to transition to clean technologies.

In the second chapter I made the model baseline and the literature results showing that a correct taxation activates an immediate regime switch, going from an intensive carbon economy to a clean one. The new technologies will be adopted by the rest of the world in a short time. Also, a taxation on the use of intermediate dirty goods has a stronger effect compared to one on dirty machines.

In the third chapter the definition of pollution plays a key role to establish the targets and policy tools. In fact, when we consider trans-boundary pollution, the compensation has a key role. The North needs to internalize into its taxation the damage generated to the South which can be used to subsidize the allocation of scientists engaged in the clean imitation. The existence of an abatement technology changes the allocation of final output generating a trade-off between consumption and investment. Nonetheless, in this paper it is and exogenous variable that generates distortions.

A Net zero emissions target is considered as a target that, in the presence of abatement technology imposes a quota on the use of dirty machines. Also, the optimum dirty production is affected by a cap that fixes it to the regeneration of the environment. In fact, this policy stops the environmental degradation from the moment of its application.

When we allow trade of machines the unilateral policy has ambiguous effects depending on the elasticity of substitution. In the fourth chapter I prove it is a sufficient condition where a tax on dirty inputs with complementary goods increases the demand of clean technologies. The fact that the two economies are symmetric eliminates the comparative advantages allowing them to specialize in the same sector simultaneously.

The total effect of the unilateral policy can be divided in two components, the national one and the trade one. Its effect on the local economy is always stronger compared to trade and overcome it. Contrarily to Acemoglu et al. 2014[2] I prove it is possible to avoid an environmental disaster with unilateral policy and free trade.

### 5.1.1 Policy implications

In this thesis I show different solutions to solve environmental issues. The first step should be the identification of the problem and its characteristics. Then the policy makers can choose the correct and faster answer to address it.
Moving from a carbon based to a clean system can increase the size of the economy but the transition period will have a price that can be called the cost of delay. This is due to the accumulation of environmental damage that increases the externality and subsequentially the effort to reduce it.

The tax on trans-boundary pollution should be used to subsidize the scientist engaged in clean research in the South. It can also be used to cover the damages of climate change. In fact, the atolls could disappear and constantly ask for money to mitigate the sea level rise. This compensation is a justification for programs already existing such as can fund programs such as REDD+ or Foreign direct investments (FDI). This mechanism is also already existing with the environmental trade scheme to finance companies that pollute less.
A Net zero emissions target can have a double effect: it can be used as a terminal point or as an extreme solution to avoid an environmental disaster. Nonetheless, it needs to be reached. It could be difficult for an economy to satisfy a net zero emission target but it is possible to decrease the shock of the transition through taxes that help the clean sector. In other words the taxes and new technologies will allow the transition to net zero emission "less painful" for the representative household. This is the way decided by the UK and other countries aiming the Net zero emission target by banning or increasing the taxation on certain goods, such that their economy will be ready to sustain the transition.

### 5.1.2 Further study

This is a model with a lot of potential that can develop new characteristics. Its complexity could show hidden properties that are not visible. This is something typical of complex systems which is referred as emergence. The first step should focus on endogenous abatement technologies and endogenous investment.
Future research should continue to study the effect of technologies on trade adding more complexities. The first extension should consider trade between all sectors to understand the effect on the global economy as well as on the demand of new technologies. A second and more ambitious extension should start from close economy to analyse the role of investment and research to the development of an economy to conclude with free trade. An example could include the role of investment in infrastructures on

the direction of technical change to study if it is stronger than the discovery of new technologies.
Another aspect that is new in the field that deserve to be studied is the effect of finance on the development of new technologies developing a discussion on the role of finance on growth.

# Bibliography

[1] Acemoglu, D., Aghion, P., Bursztyn, L., Hémous, D. 2012a *'The Environment and Directed Technical Change',* American Economic Review, 102(1), 131–66.

[2] Acemoglu D.,Aghion P., Hémous D, 2014 *The environment and directed technical change in a North–South model,* Oxford Review of Economic Policy, Oxford University Press, Volume 30, Number 3 pp. 513–530