Abstract
The “sub-prime” crisis, which led to major turbulence in global financial markets beginning in mid-2007, has posed major challenges for monetary policymakers. We analyse the impact on monetary policy of the widening differential between policy rates and the 3-month Libor rate, the benchmark for private sector interest rates. We show that the optimal monetary policy rule should include the determinants of this differential, adding an extra layer of complexity to the problems facing policymakers. Our estimates reveal significant effects of risk and liquidity measures, suggesting the widening differential between base rates and Libor was largely driven by a sharp increase in unsecured lending risk. We calculate that the crisis increased Libor by up to 60 basis points; in response base rates fell further and quicker than would otherwise have happened as policymakers sought to offset some of the contractionary effects of the sub-prime crisis.

Keywords: optimal monetary policy; sub-prime crisis

JEL Classification: C51, C52, E52, E58

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1) Introduction
The “sub-prime” crisis, which led to major turbulence in global financial markets beginning in mid-2007, has posed major challenges for monetary policymakers. Most prominence has been given to the attempts by policymakers to avoid systemic failures in financial institutions by means of liquidity injections and proposed regulatory reforms. But the crisis also posed new problems for policymakers in setting interest rates in order to steer the economy towards stable inflation and output levels. One of the main symptoms of the sub-prime crisis has been the widening differential between medium-term interest rates such as the 3-month Libor rate and the short-term base rate set by policymakers. This differential is important since aggregate demand is more responsive to the Libor rate than to the base rate as it is the benchmark interest rate that influences the interest rate at which the private sector, both corporate and personal, can borrow. A changing relationship between base and Libor rates implies that a given base rate implies a different level of aggregate demand and hence different levels of inflation and output. This added an extra layer of complexity to the problems facing policymakers.

This paper analyses the effects of this changing relationship on the behaviour of monetary policymakers in the UK. We begin by extending a prominent model of optimal monetary policy to introduce the distinction between the interest rate set by the policymaker and the interest rate that affects aggregate demand, something that is neglected by existing models. Doing so, we obtain an optimal monetary policy rule for the base rate that includes not just inflation and the output gap but also the determinants of the differential between the base rate and the rate at which the private sector can
borrow. The emerging literature on the sub-prime crisis has identified risk and
liquidity factors as being central to the changing relationship between Libor
and the base rate; our model implies that these factors should also be
components of the optimal policy rule. The augmented policy rule suggests
two new insights into monetary policy; first, a rising differential implies that the
level of aggregate demand can contract even if the base rate is constant or
even falling, a situation that arguably occurred in the UK in early 2008 (e.g.
Lomax, 2007). This also implies that in the medium-term a higher differential
between Libor and base rates will imply lower base rates on average (Smith,
2007). Second, any factor that affects the slope of the relationship between
the base rate and Libor will affect the optimal response of interest rates to
inflation and output. For example, we shall argue below that a deterioration in
market liquidity after mid-2007 made base rates more responsive to inflation
and output.

We estimate an empirical version of the monetary policy rule,
augmented by an equation for the yield curve relationship between base rates
and Libor. We use monthly data as the interest rate-setting Monetary Policy
Committee meets monthly. For inflation, we use the rate targeted by the
MPC, the annual change in the Retail Price Index excluding mortgage interest
payments (RPIX) until December 2003 and the annual change in the
Consumer Price Index (CPI) thereafter. Correspondingly, the inflation target
is 2.5% until December 2003 and 2% thereafter. For output we use monthly
GDP data. Following the literature on the sub-prime crisis (e.g. Michaud and
Upper, 2008, and Taylor and Williams, 2008), we use the difference between
rates on secured and unsecured borrowing in the inter-bank market as our
main measure of risk in the relationship between the 3-month Libor rate and the base rate. This has been used to capture the perceived risk in lending between banks when there is concern that the counter-party may default, a prominent issue during the crisis. For liquidity, we use the composite index published in the bi-annual Financial Stability Report. This index reflects bid-ask spreads, return-to-volume ratios and liquidity premia using data for the US, Eurozone and the UK. For further details, see Bank of England (2007).

We find considerable empirical support for our model; the exclusion of risk and liquidity measures from the policy rule is rejected as is the assumption of a constant response of interest rates to inflation and output. Unsecured lending risk and liquidity are significant determinants of the differential between base and Libor rates; however the increase in the differential since mid-2007 is largely driven by increases in unsecured lending risk. Our evidence therefore further supports the argument that the sub-prime crisis was largely the result of the unwillingness of banks to enter the inter-bank market because of uncertainty of the value of assets on offer and, at times, because of fears of the solvency of their counter-parties.

The remainder of the paper is structured as follows. The theoretical model is developed in section 2). Section 3) describes the variables used in our empirical model. Section 4) contains our estimates. Section 5) concludes.

2) Theory

We use a slightly-modified version of the canonical model of Svensson (1997) in which a standard Taylor rule emerges as the result of a theoretical
framework in which policymakers adjust interest rates in order to pursue an inflation target. The model is

\[ \pi_{t+1} = \pi_t + \alpha_y y_t + \nu_{t+1} \]  

Equation (1) is a Phillips curve in which inflation depend on inflation and the output gap in the previous period and on a supply shock \( \nu \). Equation (2) is an aggregate demand relationship in which the output gap \( y \) depends on the lagged output gap, the real interest rate at which the private sector can borrow relative to its' equilibrium value and on a demand shock \( \eta \). This differs from the standard formulation in using the nominal interest rate at which the private sector can borrow \( i_{\text{borrow}} \) rather than the base rate \( i_{\text{base}} \). Since the borrowing rate is closely linked to medium-term interest rates, most prominently the 3-month Libor rate, equation (3) relates the nominal borrowing rate to the base rate using a yield-curve relationship. We allow both the intercept and slope of this relationship to vary over time, to reflect the pronounced movements in the differential between medium-term rates and base rates that have been observed since mid-2007.

The model is a simple extension of Svensson (1997), which is obtained if \( i_{\text{borrow}} = i_{\text{base}} \). To solve for the optimal policy rule we follow the approach
Svensson. We assume that at time \( t \) policymakers choose current and future base rates to minimise the loss function

\[
L_t = E_t \sum_{i=0}^{\infty} \delta^i \left\{ \frac{1}{2} (\pi_{t+i} - \pi^T)^2 \right\}
\]

Equation (4) specifies the policymakers’ loss function as the discounted sum of expected quadratic deviations of inflation \( \pi \) from the inflation target \( \pi^T \), where \( \delta \) is the discount factor. We assume policymakers know the value of \( \epsilon_i \) but not the other errors, which become apparent at the end of the period. Since the base rate chosen at time \( t \) affects the inflation rate in only period, that two periods ahead, the policymakers’ problem is equivalent to minimising

\[
L_t = \delta^2 E_t \frac{1}{2} (\pi_{t+2} - \pi^T)^2. \quad \text{The first-order condition is}^1
\]

\[
E_{t-1} \pi_{t+2} = \pi^T
\]

where we assume that policymakers choose interest rates at the start of period \( t \) based on information available up to the end of period \( t-1 \). Using (1)-(3), we can express the optimal policy rule as

\[
i_t^{\text{base}} = \bar{r} + \pi^T - \omega_{\eta y} + \frac{1 + \alpha_x \alpha_y}{\omega_h \alpha_x} (E_t \pi_t - \pi^T) + \frac{1 + \beta_y + \alpha_x}{\omega_h \alpha_x} E_t y_t
\]

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1 This argument closely follows Svensson (1997); see equation 2.8) of that paper and the preceding discussion for a more extensive analysis.
This extends the familiar Taylor rule by including the yield curve factors, $\omega_0t$ and $\omega_1t$. Intuitively, because aggregate demand and inflation depend on the borrowing rate, the optimal base rate will be a function of factors that affect the relationship between the base rate and the borrowing rate. The inclusion of these factors is consistent with evidence that inclusion of yield curve determinants improves the performance of Taylor rules in the US (e.g. Piazzesi, 2005).

3) Empirical Specification

In this section we develop an estimable model consisting of empirical versions of the yield curve relationship in (3) and the optimal policy rule in (6). Beginning with the yield curve relationship, the recent literature on the “sub-prime crisis” of 2007-2008 has focused on the widening differential between overnight and medium-term rates. Using daily data, Michaud and Upper (2008) consider the differential between the overnight indexed swap (OIS) rate and the 3-month LIBOR rate and investigate the role of risk and liquidity factors in explaining the widening of this differential from mid-2007. They consider two measures of risk, the spread between secured and unsecured inter-bank rates and premia on credit default swaps and argue that movements in the secured-unsecured spread are more closely related to movements in the OIS-Libor differential. Liquidity is measured using indicators of trading volume, bid-ask spreads and the impact of trades on prices, derived using data from the e-MID electronic trading platform. These measures have no clear relationship with movements in the Libor-base rate spread.
Taylor and Williams (2008) mainly consider the US and also use daily data. They focus on the role of risk factors in explaining the spread between 1- and 3-month interbank rates and OIS rates. As with Michaud and Upper (2008), risk is measured credit default swap premia and the spread between secured and unsecured inter-bank rates, although the former measure is preferred as a less noisy measure of risk. The effects of liquidity are confined to allowing for effects of the Term Auction Facility (TAF) introduced in December 2007. Both measures of risk and the effects of the TAF are significant in regressions explaining both the 1- and 3-month spreads.

Our econometric model will further test some of these ideas and integrate them into a model that also allows us to analyse the response of monetary policymakers to the "sub-prime crisis". We use monthly data since the Monetary Policy Committee meets monthly. This forces us to use somewhat different explanatory variables than in the recent literature. We use the base rate rather than the overnight indexed swap rate as this is the rate directly set by the monetary policy committee and since the OIS rate is only available for a relatively short period of time. In practice this has little effect; for the period for which they are available monthly OIS rates are highly correlated with the base rate. We are also unable to use credit default swap premia as data on these are only available from mid-2004 and so use the spread between secured and unsecured inter-bank rates as our main measure of risk.

To capture liquidity effects\(^2\) we use the index of liquidity calculated by the Bank of England; unlike data from the e-MID platform, this is available on

\(^2\) For more detailed analyses of the impact of liquidity, see papers by Goodhart, Crocker and Tirole in Banque de France (2008).
a monthly basis since 1992. This index reflects three factors: bid-ask spreads (for Gilt Repos, the FTSE100 and major currencies) as a measure of the “tightness” of markets (Kyle, 1985); the return-to-volume ratio (for Gilts, the FTSE100 and equity options) as a measure of the impact of volumes of prices (Amihud, 2002); and liquidity premia, measured as the spread between corporate bonds and a credit spread and between bond and Libor rates in the US, Eurozone and the UK. For further details, see Bank of England (2007)

On the basis of this discussion the empirical version of the yield curve equation in (3) is

\[
(7) \ i_{t}^{lib,3} = \omega_{00} + \omega_{01} \left( i_{t}^{lib,1} - i_{t}^{repo,1} \right) + \omega_{02} \left( i_{t}^{repo,3} - i_{t}^{repo,1} \right) + \omega_{03} \text{liq}_{t}^{FSR} + \left( \omega_{11} + \omega_{12} \text{liq}_{t}^{FSR} \right) \text{base} + \epsilon_{t}
\]

Comparing (7) with (3)\(^3\), \( \omega_{00} = \omega_{00} + \omega_{01} \left( i_{t}^{lib,1} - i_{t}^{repo,1} \right) + \omega_{02} \left( i_{t}^{repo,3} - i_{t}^{repo,1} \right) + \omega_{03} \text{liq}_{t}^{FSR} \)

and \( \omega_{1t} = \omega_{11} + \omega_{12} \text{liq}_{t}^{FSR} \). \( i_{t}^{lib,3} \) is the average 3-month Libor rate. \( i_{t}^{lib,1} - i_{t}^{repo,1} \) is the differential between 1-month Libor and Gilt-Repo rates. This measures the differential between unsecured and secured lending rates\(^4\). We also include the differential between the 3-month Libor and 1-month Gilt Repo rates to capture term structure effects. \( \text{liq}_{t}^{FSR} \) is the liquidity index published by the Bank of England in the Financial Stability Report. We include an interaction term, whereby the effect of liquidity on \( i_{t}^{lib,3} - i_{t}^{base} \) varies with the base rate, to allow for changes in the slope of the yield curve.

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\(^3\) The interest rates in (7) are monthly averages of daily observations, obtained from the Bank of England website.

\(^4\) Borrowing on the LIBOR market is unsecured while borrowing on the Gilt Repo rate market is secured.
For the optimal policy rule in (6), the main modelling choices are how to interpret the timing of events and which measures of inflation and the output gap to use. In our preferred specification, policymakers respond to forecasts of inflation and the output gap over the coming three months. Although this may appear to conflict with the specification of the optimal policy rule in (6), it has been argued that policymakers in effect review decisions every three months as forecasts of inflation, output and the time profile of interest rates are updated in the quarterly Inflation Report. Using (7) to express $\omega_{0t}$ and $\omega_{1t}$ in terms of risk and liquidity factors, our empirical policy rule is then

$$i_{t}^{\text{base}} = \bar{r} + \pi^{T} - \omega_{00} - \omega_{01}(i_{t}^{hth} - i_{t}^{repo,1}) - \omega_{02}(i_{t}^{repo,2} - i_{t}^{repo,1}) - \omega_{03}\bar{q}_{t}^{FSR}$$

$$+ \frac{\phi_{\pi}}{\omega_{11} + \omega_{12}\bar{q}_{t}^{FSR}} \sum_{k=0}^{2} (\pi_{t+k} - \pi_{t}^{T}) + \frac{\phi_{y}}{\omega_{11} + \omega_{12}\bar{q}_{t}^{FSR}} \sum_{k=0}^{2} y_{t+k} + \zeta_{t},$$

where $\phi_{\pi} = \frac{1 + \alpha_{\pi} \alpha_{y}}{\alpha_{\pi} \alpha_{y}}$, $\phi_{y} = \frac{1 + \beta_{y} + \alpha_{\pi} \alpha_{y}}{\alpha_{y}}$ and the error term reflects the errors induced by replacing expected values of inflation and the output gap with the realised ex-post values. We use the inflation rate targeted by monetary policy, namely the annual change in the RPIX price index until December 2003 and the annual change in the CPI thereafter. Correspondingly, the inflation target is 2.5% until December 2003 and 2% thereafter. For output we use monthly GDP data (kindly provided by the National Institute of Economics and Social Research) and derive the output gap as the proportional difference between GDP and its' Hodrick-Prescott trend. Finally, we allow for the
effects of interest rate smoothing\(^5\) by expressing the observed base rate as a weighted average of the current optimal and previous base rates:

\[
\hat{i}_t^{\text{base}} = \rho \hat{i}_{t-1}^{\text{base}} + (1 - \rho) \hat{i}_t^{\text{base}}
\]

where \(\hat{i}_t^{\text{base}}\) is the observed base rate and the optimal base rate, \(\hat{i}_t^{\text{base}}\), is given by (8). Combining (8) and (9),

\[
i_t^{\text{base}} = \rho i_{t-1}^{\text{base}} + (1 - \rho) \left\{ F + \pi_T - \omega_{01}(i_t^{\text{lib}1} - i_t^{\text{repo}1}) - \omega_{02}(i_t^{\text{repo}3} - i_t^{\text{repo}1}) - \omega_{03}l_i^{\text{FSR}} \right\} \frac{\omega_{11} + \omega_{12}l_i^{\text{FSR}}}{\omega_{11} + \omega_{12}l_i^{\text{FSR}}} + \phi_{\pi} \left( \sum_{k=0}^{2} (\pi_{t+k} - \pi_T) \right) \frac{\omega_{11} + \omega_{12}l_i^{\text{FSR}}}{\omega_{11} + \omega_{12}l_i^{\text{FSR}}} + \phi_{y} \left( \sum_{k=0}^{2} y_{t+k} + \zeta_t \right)
\]

Our empirical model comprises equations (7) and (10).

There are two sets of testable restrictions that simplify our model to models estimated elsewhere in the literature. First, the yield curve model in (7) simplifies to models estimated in the existing literature if \(\omega_{11} = 1\) and \(\omega_{12} = 0\), in which case

\[
i_t^{\text{lib}3} - i_t^{\text{base}} = \omega_{00} + \omega_{01}(i_t^{\text{lib}1} - i_t^{\text{repo}1}) + \omega_{02}(i_t^{\text{repo}3} - i_t^{\text{repo}1}) + \omega_{03}l_i^{\text{FSR}} + \epsilon_t
\]

This simplified model for the Libor-base rate differential is similar to the model in Michaud and Upper (2008) for the Libor-OIS differential, where, as they

\(^5\) Interest rate smoothing is difficult to model in the context of the Svensson approach to deriving the optimal monetary policy rule. As an alternative approach, we could derive the optimal policy rule from an amended model in which the loss function has quadratic terms in inflation, output and interest rate changes. The resultant rule would be similar to (6) above but with an interest rate smoothing term.
discuss, variations in liquidity between financial institutions and market microstructure effects are captured in the error term. Although the theoretical framework in Taylor and Williams (2008), based on the Ang and Piazzesi (2003) model of the term structure, is rather different, the model they estimate is consistent with (11). With these restrictions, (10) simplifies to

\[
\frac{\rho}{1 - \rho} \left[ T + \pi^T - \omega_i \right] - \omega_i (i_{lib,1} - \hat{i}_{repo}) - \omega_i (i_{repo,3} - \hat{i}_{repo})
\]

\[
-\omega_3 \rho \pi^T + \phi_\delta \sum_{k=0}^{2} (\pi_{T+k} - \pi^T) + \phi_\delta \sum_{k=0}^{2} \pi_{T+k} + \pi^T
\]

In this policy rule, risk and liquidity continue to affect base rates but the optimal response to inflation and output is now independent of these factors.

A second simplified model is obtained if \( \omega_0 = \omega_2 = \omega_3 = \omega_2 = 0 \), in which case our model simplifies to

\[
\frac{i_{lib,3} - i_{base}}{\omega_i} = \omega_i + \omega_i
\]

and

\[
\frac{i_{base}}{\omega_i} = \rho \left[ T + \pi^T - \omega_i \right] - \omega_i (i_{lib,1} - \hat{i}_{repo}) - \omega_i (i_{repo,3} - \hat{i}_{repo})
\]

In this case there is a fixed proportional relationship between the Libor and base rates and our policy rule simplifies to the familiar Taylor rule.

Our empirical strategy will be to compare estimates of the system comprising (7) and (10) with the simplified models in (11)-(12) and (13)-(14). We also note that the model in (7) and (10) implies that estimates of each of
(11)-(14) will suffer from parameter instability. We will use this as an additional test of the specification of our model.

The main features of our data are depicted in figure 1). Figure a) shows the differential between the 3-month Libor and base rates; this exhibits a sharp increase in mid 2007, followed by fluctuations around an elevated level until the end of our sample. Figure 1b) shows \((i_t^{lib,3} - i_t^{repo,1})\), where we see a sharp jump in mid 2007 that matches the sharp rise in \((i_t^{lib,3} - i_t^{base})\). Movements in \((i_t^{repo,3} - i_t^{repo,1})\), depicted in figure 1c), are less dramatic and are less correlated with \((i_t^{lib,3} - i_t^{base})\). Movements in this liquidity index are depicted in figure 1d), where it is apparent that liquidity fell sharply in mid-2007 having increased steadily over the previous five years\(^6\). Figures 1e) and 1f) depict the inflation rate relative to the target and the output gap.

4) Estimates

Our main estimates are presented in table 1). We estimated our system using GMM. We treat all variables as endogenous, using the first four lags of each as instruments. Column (i) presents estimates of (7) and (10), column (ii) has estimates of (11)-(12) while column (iii) has estimates of (13)-(14). There is considerable support for our model in equations (7) and (10). We reject \(\omega_h = \omega_{h2} = \omega_{h3} = \omega_{h4} = 0\), which simplify the model to that in column (ii) and reject \(\omega_{h1} = 1\) and \(\omega_{h2} = 0\), which simplify the model to that in column (iii). We

\(^6\) The index is expressed in standardised form, relative to the mean value of the mid-1990s and where the vertical scale measures deviations in terms of standard deviations; data on the liquidity index are available since 1992, whereas data on the Libor and Repo rates are only available since 1996.
also find parameter instability on each of (11)-(14), which further supports our preferred specification.

The estimated parameters of our preferred model are all significant and of the expected sign (this is not true of the simplified model in (11)-(12)). In the policy rule, the responses of interest rates to inflation and output are comparable to those estimated in the existing literature, although the estimated equilibrium real interest rate is perhaps a little low. In the Libor equation, increases in unsecured lending risk and medium term risk are associated with a larger differential. Liquidity has two, off-setting, effects on the differential between Libor and the base rate; an increase in liquidity reduces the differential, via the negative estimate on $\omega_3$, but also increases it via the positive estimate $\omega_2$.

Our estimates imply a more complex policy rule than usually considered, including measures of the risk and liquidity that affect the relationship between the base rate set by policymakers and rate at which the private sector can borrow. To illustrate this, we conducted a simple counterfactual analysis, by calculating the implied predicted value of the base rate assuming that our risk and liquidity measures were fixed at their 2007Q1 values for the remainder of our sample\(^7\). As figure 2) shows, base rates would have risen by 50 basis points in response to rising inflation in the spring in 2007 before falling by 25 basis points late in the year. With risk and liquidity factors unchanged, movements in the 3-month libor rate would have mirrored these changes. The impact of increasing risk and deteriorating market liquidity is apparent in the divergence between the actual and

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\(^7\) We use realised values of inflation and output; this can of course only be justified in the very short run and in the context of a limited illustration of the implications of our results.
counterfactual Libor rates, leading to an additional increase of up to 60 basis points in the Libor rate, representing a sharp tightening in monetary policy in the summer and autumn of 2007. The impact of risk and liquidity on the base rate is also apparent; the base rate was cut by 25 basis points in December 2007 compared to a counterfactual cut in January 2008, and the further 25 basis point reduction in February 2008 is not predicted by this counterfactual experiment. This illustrates the impact of the sub-prime crisis on monetary policy, both in terms of increasing the Libor rate relative to the base rate and in terms of lowering the base rate itself. The impact of the sub-prime crisis on the response of interest rates to inflation and output is depicted in figure 3; this shows the expansion in liquidity in the early year of this century reduced these responses, which then returned to the higher levels of the late 1990s following the sharp deterioration in liquidity in 2007-8.

Our estimates suggest that the key indicator of the “sub-prime” crisis, the rise in the differential between Libor and base rates, is largely explained by increases in unsecured lending risk. Figure 4) shows the results of using our model to decompose the rise in this differential since July 2007 (this is also discussed in the Appendix). The offsetting effects of liquidity largely cancel themselves out, while, as expected, the yield curve effects of the differential between the 3- and 1-month Gilt repo rates are small. The effect of the increase in the differential between 1-month Libor and Gilt Repo rates is dominant. These estimates support the view that the rise in the perceived risk of unsecured lending on the inter-bank market was the main driving force behind the sub-prime crisis.
The remainder of the paper considers the robustness of our findings. First, we use the differential between corporate bonds and 10-government bonds, both to assess this alternative measure of risk and to assess the effects of a longer sample, since we have data on this from 1992. We amend our model to be:

(15) \[ i_t^{lb,3} = \omega_{00} + \left( \omega_{03} liq_t^{FSR} + \omega_{04} (i_t^{corp} - i_t^{gb}) + (\omega_{11} + \omega_{12} liq_t^{FSR}) \right) i_t^{base} + \varepsilon_t \]

(16) \[ i_t^{base} = \rho i_t^{base-1} + (1 - \rho) \left\{ \bar{T} + \pi^T - \omega_{00} - \omega_{03} liq_t^{FSR} - \omega_{04} (i_t^{corp} - i_t^{gb}) \right\} + \frac{\phi_\pi}{\omega_{11} + \omega_{12} liq_t^{FSR}} \sum_{k=0}^{2} (\pi_{t+k} - \pi^T) + \frac{\phi_\pi}{\omega_{11} + \omega_{12} liq_t^{FSR}} \sum_{k=0}^{2} y_{t+k} + \varepsilon_t \}

where \( i_t^{corp} \) is the corporate bond rate and \( i_t^{gb} \) is the 10-year government bond rate. Estimates of this model, presented in column (iv) of table 1) are similar to those in column (i). The restrictions that would simplify this model to (11)-(12) or (13)-(14) are again rejected. Second, we use the overnight interest rate swap rate in place of the base rate. The model in this case is

(17) \[ i_t^{lb,3} = \omega_{00} + \omega_{03} liq_t^{FSR} + \omega_{04} (i_t^{repo,3} - i_t^{repo,1}) + (\omega_{11} + \omega_{12} liq_t^{FSR}) \mu_t^{ols} + \varepsilon_t \]

(18) \[ i_t^{ols} = \rho i_t^{ols-1} + (1 - \rho) \left\{ \bar{T} + \pi^T - \omega_{00} - \omega_{03} liq_t^{FSR} - \omega_{04} (i_t^{repo,3} - i_t^{repo,1}) - \omega_{04} liq_t^{FSR} \right\} + \frac{\phi\\bar{\pi}}{\omega_{11} + \omega_{12} liq_t^{FSR}} \sum_{k=0}^{2} (\pi_{t+k} - \pi^T) + \frac{\phi_\pi}{\omega_{11} + \omega_{12} liq_t^{FSR}} \sum_{k=0}^{2} y_{t+k} + \varepsilon_t \}
where $i^{\text{OIS}}$ is the monthly average overnight interest rate swap rate. We are limited by lack of data here, since data on OIS rates are only available from late 2000. This seems to have particularly affected the estimates on the inflation term in the policy rule and on medium term risk in the Libor equation. This aside, the main features of our estimates are unchanged and we are unable to simplify to either (11)-(12) or (13)-(14).

We also tried estimating (7)-(10) using the inflation forecasts provided by the Bank’s *Inflation Report* and using real time data on output. These experiments are hampered by the fact that inflation forecasts and real time data are only available on a quarterly frequency; to overcome this we assumed a constant inflation forecast for each month within the same quarter and a constant growth rate of real-time output data within a quarter. Doing so makes no qualitative difference to our empirical results, although estimates of the policy rule become less reliable. Estimates of models which used alternative filters to de-trend output data were similar to those reported in table 1).

Finally, we note that our estimates suffer from some residual ARCH effects, which seem to be associated with the liquidity measure, which is highly volatile towards the end of the sample. Estimating (7) and (10) with a correction for these ARCH effects has little effect on the estimates. Alternatively, we obtain a model free of ARCH effects if the liquidity variable is dropped from the model, estimates of the other parameters being relatively unaffected. We prefer not to omit liquidity effects as these are important factors in the sub-prime crisis.
5) Conclusions

We have analysed the impact of the sub-prime crisis which began in mid-2007 on the interest rate-setting behaviour of UK monetary policymakers. Our focus is on the widening differential between base rates and the 3-month Libor rate, the latter being a key determinant of aggregate demand as it is the baseline against which many interest rates relevant to the private sector are set. In order to do so, we extend a familiar model of optimal monetary policy, due to Svensson (1997) to allow for the distinction between the interest rate set by the Central Bank and the interest rate relevant to private sector expenditure decisions. We show that the resulting optimal policy rule includes the determinants of the differential between the two interest rates and that factors which affect the slope of the relationship between the interest rates affect the optimal response of interest rates to inflation and output.

We estimate our model using UK data using the 3-month Libor rate to measure the interest rate relevant to aggregate demand and following the literature on the “sub-prime” crisis in using measures of risk and liquidity as determinants of the differential between this and the base rate. Our estimates support our model. We find strong effects from both unsecured lending risk, measured by the difference between the 1-month Libor and Gilt repo rates, and liquidity; exclusion of these factors from the policy rule is strongly rejected by the data. We use our model to investigate the effects of the sub-prime crisis. We calculate that the effects of the sub-prime crisis increased the 3-month libor rate by up to 60 basis points in the summer and autumn of 2007, representing a significant tightening in monetary policy. They also affected the base rate, which fell further and quicker than would otherwise have
happened as policymakers sought to offset some of the contractionary effects of the sub-prime crisis on aggregate demand. We also establish that the rise in the differential between the 3-month libor rate and base rates was largely driven by unsecured lending risk, supporting the view that the perceived risk of unsecured lending on the inter-bank market was the main driving force behind the sub-prime crisis, perhaps because of the unwillingness of banks to enter the inter-bank market in view of the uncertain value of the assets on offer and fears of the solvency of counter-parties.

Although suggestive, our work is necessarily preliminary as the sub-prime crisis is still ongoing at the time of writing, in summer 2008. A more definitive analysis must wait for the end of the crisis and for its macroeconomics effects to have unwound. We intend to return to this in future work.
Appendix: Decomposing changes in the relationship between the borrowing rate and the base rate

The relationship between the borrowing rate and the base rate is

\[ i_t^{\text{borrow}} = \omega_0 + \omega_1 i_t^{\text{base}} \]  \hspace{1cm} (A1)

So the differential between the borrowing rate and base rate is

\[ \text{diff}_t = i_t^{\text{borrow}} - i_t^{\text{base}} = \omega_0 + \omega_1 i_t^{\text{base}} - i_t^{\text{base}} \]  \hspace{1cm} (A2)

where \( \omega_0 = \omega_0 + \omega_{01}(i_t^{\text{dbh,1}} - i_t^{\text{rep,1}}) + \omega_{02}(i_t^{\text{rep,3}} - i_t^{\text{rep,3}}) + \omega_{03} \text{liq}_t^{FSR} \) and \( \omega_1 = \omega_1 + \omega_{12} \text{liq}_t^{FSR} \).

The impact of a change in a variable \( x \) on the differential is then

\[ \frac{\partial \text{diff}_t}{\partial x_t} = i_t^{\text{borrow}} - i_t^{\text{base}} = \frac{\partial \omega_0}{\partial x_t} + \frac{\partial \omega_1}{\partial x_t} i_t^{\text{base}} - \frac{\partial i_t^{\text{base}}}{\partial x_t} \]  \hspace{1cm} (A3)

Since

\[ \omega_{1t} i_t^{\text{base}} = \rho i_{t-1} + (1 - \rho) \{ F + \pi^T - \omega_{0t} + \phi_x \sum_{k=0}^{2} (\pi_{t+k} - \pi^T) + \phi_y \sum_{k=0}^{2} y_{t+k} \} \]  \hspace{1cm} (A4)

then

\[ \frac{\partial \omega_{1t} i_t^{\text{base}}}{\partial x_t} = -(1 - \rho) \frac{\partial \omega_{0t}}{\partial x_t} \]  \hspace{1cm} (A5)

Also since

\[ i_t^{\text{base}} = \frac{\rho}{\omega_{1t}} i_{t-1} + (1 - \rho) \{ F + \pi^T - \omega_{0t} + \phi_x \sum_{k=0}^{2} (\pi_{t+k} - \pi^T) + \phi_y \sum_{k=0}^{2} y_{t+k} \} \]

then
\[ \frac{\partial i_{\text{base}}}{\partial x_i} = -(1 - \rho) \left( \frac{1}{\omega_{lt}} \frac{\partial \omega_{lt}}{\partial x_i} - \frac{\omega_{lt}^2}{\omega_{lt}^2} \frac{\partial \omega_{lt}}{\partial x_i} \right) - \frac{\rho}{\omega_{lt}^2} \frac{\partial \omega_{lt}}{\partial x_i} i_{t-1} \]

\[-(1 - \rho)(\bar{r} + \pi^t)(\frac{1}{\omega_{lt}^2} \frac{\partial \omega_{lt}}{\partial x_i}) - (1 - \rho) \left( \frac{1}{\omega_{lt}^2} \frac{\partial \omega_{lt}}{\partial x_i} \right) \phi_k \sum_{k=0}^{2} (\pi_{t+k} - \pi^T) \]

\[-(1 - \rho)(\frac{1}{\omega_{lt}^2} \frac{\partial \omega_{lt}}{\partial x_i}) \phi_k \sum_{k=0}^{2} y_{t+k} \]  

which simplifies to

\[ \frac{\partial i_{\text{base}}}{\partial x_i} = -(1 - \rho) \frac{1}{\omega_{lt}} \frac{\partial \omega_{lt}}{\partial x_i} - i_{t-1} \]  

(A6)

Combining (A3)-(A6),

\[ \frac{\partial \text{diff}}{\partial x_i} = \left( \rho + \frac{1 - \rho}{\omega_{lt}} \right) \frac{\partial \omega_{lt}}{\partial x_i} + i_{t-1} \frac{\partial \omega_{lt}}{\partial x_i} \]  

(A7)

Next we calculate individual effects.

1. Unsecured lending risk \((i_{t}^{lib,1} - i_{t}^{repo,1})\)

\[ \frac{\partial \omega_{lt}}{\partial x_i} = 1.688 \text{ and } \frac{\partial \omega_{lt}}{\partial x_i} = 0 \]

Therefore

\[ \frac{\partial \text{diff}}{\partial (i_{t}^{lib,1} - i_{t}^{repo,1})} = 1.688 \left( \rho + \frac{1 - \rho}{\omega_{lt}} \right) \]

so change due to changes in unsecured lending risk is

\[ 1.688 \left( \rho + \frac{1 - \rho}{\omega_{lt}} \right) \{ (i_{t}^{lib,1} - i_{t}^{repo,1}) - (i_{t}^{lib,1} - i_{t}^{repo,1}) \} \]

2. Medium term lending risk \((i_{t}^{repo,3} - i_{t}^{repo,1})\)

\[ \frac{\partial \omega_{lt}}{\partial x_i} = 1.196 \text{ and } \frac{\partial \omega_{lt}}{\partial x_i} = 0 \]

so

\[ \frac{\partial \text{diff}}{\partial (i_{t}^{repo,3} - i_{t}^{repo,1})} = 1.196 \left( \rho + \frac{1 - \rho}{\omega_{lt}} \right) \]
so change due to changes in medium term lending risk is

\[ 1.196\left(\rho + \frac{1-\rho}{\omega_t}\right)\{r_{repo,3}^t - r_{repo,1}^t\} - (r_{july,2007}^{repo,3} - r_{july,2007}^{repo,1}) \]

3. Liquidity \( (liq_{t}^{FSR}) \)

\[ \frac{\partial\omega_{lt}}{\partial x_{t}} = -0.804 \quad \text{and} \quad \frac{\partial\omega_{lt}}{\partial x_{t}} = 0.171 \]

so

\[ \frac{\partial\text{diff}_t}{\partial liq_{t}^{FSR}} = -0.804(\rho + \frac{1-\rho}{\omega_{lt}}) + 0.171\frac{\text{base}_{t}}{\omega_{lt}} \]

so change due to changes in liquidity is

\[ \{-0.804(\rho + \frac{1-\rho}{\omega_{lt}}) + 0.171\frac{\text{base}_{t}}{\omega_{lt}}\}\{liq_{t}^{FSR} - liq_{july,2007}^{FSR}\} \]

The two effects of liquidity in A7) go in opposite directions as the reduction in liquidity will:

- increase \( \omega_{lt} \) and thus widen the differential
- reduce \( \omega_{lt} \) and thus reduce the differential
Table 1)
Main Estimates

<table>
<thead>
<tr>
<th>Eqns estimated</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7) and (10)</td>
<td>(11)-(12)</td>
<td>(13)-(14)</td>
<td>(15)-(16)</td>
<td>(17)-(18)</td>
</tr>
<tr>
<td>Policy rule</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.900 (0.040)</td>
<td>0.894 (0.030)</td>
<td>0.911 (0.042)</td>
<td>0.891 (0.041)</td>
<td>0.901 (0.012)</td>
</tr>
<tr>
<td>( \bar{r} )</td>
<td>1.109 (0.174)</td>
<td>3.076 (0.056)</td>
<td>3.053 (0.057)</td>
<td>2.466 (0.161)</td>
<td>-0.672 (0.129)</td>
</tr>
<tr>
<td>( \phi_x ) (inflation)</td>
<td>1.130 (0.161)</td>
<td>1.572 (0.173)</td>
<td>1.628 (0.150)</td>
<td>1.537 (0.128)</td>
<td>0.567 (0.085)</td>
</tr>
<tr>
<td>( \phi_y ) (output gap)</td>
<td>2.276 (0.136)</td>
<td>2.780 (0.173)</td>
<td>2.524 (0.137)</td>
<td>1.721 (0.116)</td>
<td>1.942 (0.107)</td>
</tr>
<tr>
<td>Regression standard error</td>
<td>0.15</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
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<tr>
<td>Parameter stability (p-value)</td>
<td>0.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.28</td>
<td>0.11</td>
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<tr>
<td>Libor equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \omega_{00} )</td>
<td>0.159 (0.024)</td>
<td>-0.150 (0.011)</td>
<td>0.120 (0.009)</td>
<td>0.080 (0.039)</td>
<td>0.321 (0.016)</td>
</tr>
<tr>
<td>( \omega_{01} ) (unsecured lending risk)</td>
<td>1.688 (0.087)</td>
<td>1.689 (0.063)</td>
<td>1.677 (0.045)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td>p-value</td>
<td>Value</td>
<td>p-value</td>
<td>Value</td>
</tr>
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<td>---------</td>
<td>---------</td>
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</tr>
<tr>
<td>$\omega_{02}$ (medium term risk)</td>
<td>1.196 (0.080)</td>
<td>-0.535 (0.067)</td>
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<tr>
<td>$\omega_{03}$ (liquidity)</td>
<td>-0.804 (0.103)</td>
<td>-1.162 (0.139)</td>
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<tr>
<td>$\omega_{04}$ (corporate risk)</td>
<td>0.221 (0.039)</td>
<td>0.206 (0.012)</td>
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</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>0.950 (0.005)</td>
<td>0.988 (0.005)</td>
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<tr>
<td>$\alpha_{12}$ (liquidity)</td>
<td>0.171 (0.020)</td>
<td>0.236 (0.027)</td>
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</tr>
<tr>
<td>Regression standard error</td>
<td>0.14</td>
<td>0.18</td>
<td>0.19</td>
<td>0.18</td>
<td>0.09</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Parameter stability (p-value)</td>
<td>0.17</td>
<td>0.00</td>
<td>0.00</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>Simplify to (13)-(14) (p-value)</td>
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<tr>
<td>Simplify to (11)-(12) (p-value)</td>
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<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
(i) Numbers in parentheses are standard errors. Parameter stability is an F test of parameter stability (see Lin and Teräsvirta, 1994).
(ii) “Simplify to (13)-(14) reports the p values from tests of the hypotheses $H_0 : \omega_{01} = \omega_{02} = \omega_{03} = \omega_{12} = 0$ for columns (i) and (v) and $H_0 : \omega_{01} = \omega_{04} = \omega_{12} = 0$ for column (iv).
(iii) “Simplify to (11)-(12) reports the p values from tests of the hypotheses $H_0 : \alpha_{11} = 1; \alpha_{12} = 0$.
References


Figure 1: Interest rate spreads, liquidity, inflation and output gap in the UK

(a) 3-month LIBOR rate minus Base rate

(b) 1-month LIBOR rate minus 1-month REPO rate

(c) 3-month REPO rate minus 1-month REPO rate

(d) Liquidity index

(e) Inflation target

(f) Output gap
Figure 2: Counterfactual Experiment
Figure 3: Time-varying inflation and output gap effects

Note: Figure 3 plots the time-varying inflation effect \( \frac{\phi_x}{\omega_{l1} + \omega_{l2} liq^{FSR}} \) and the time-varying output gap effect \( \frac{\phi_y}{\omega_{l1} + \omega_{l2} liq^{FSR}} \) using estimates of equation (10) reported in Table 1.
Figure 4: Decomposing the differential between the 3-month Libor rate and base rate

Note: The decomposition is based on the estimates of equations (7) and (10) reported in Table 1.