F2008SC032

THE DYNAMICS OF A THREE-WHEELED NARROW-TRACK TILTING VEHICLE

[1]Berote, Johan*, [1]van Poelgeest, Auguste, [1]Darling, Jos, [1]Edge, Kevin, [1]Plummer, Andrew
[1]University of Bath, United Kingdom

KEYWORDS – dynamics, hydraulics, modelling, three-wheeler, control systems

ABSTRACT

Three-wheeled tilting vehicles have many potential advantages. They combine the comfort and safety of a car with the fuel economy, size and manoeuvrability of a motorcycle. One major challenge is to fully understand and account for the unusual dynamics of this type of vehicle. This paper sets out the equations of motion for a vehicle with single front and twin rear wheels and compares them with those of a car and a motorcycle. The mathematical model shows that it is necessary to introduce a tilt-dependent rear wheel steer term to obtain balanced handling characteristics. A vehicle with a hydraulically actuated tilting system is considered as a case study. As the tilt actuators act against the non-tilting engine module, this has significant effects on the cornering dynamics. Finally, the difference between the lateral acceleration and the hydraulic response time is shown to be a key issue.

INTRODUCTION

In today's society, the need for more environmentally friendly transportation is constantly increasing. In particular the demand for safe, clean and individual transportation is growing. The CLEVER car was developed as part of an EU funded consortium at the University of Bath between 2002 and 2005 to fill this gap in the market. The design objectives were to design an environmentally friendly, safe mode of transport that would take up as little space as possible on the road. The designers opted for a narrow track three-wheeled vehicle that would seat a driver and one passenger in tandem. This configuration significantly reduced the weight and frontal area of the vehicle, resulting in a road footprint similar to that of a motorcycle. The narrow wheelbase, however, meant that the roll-over stability of the vehicle was very limited. This problem was overcome by introducing a hydraulic tilt mechanism that would lean the driver cabin into a corner like a motorcycle.

The combination of three-wheels and tilting ability meant that the vehicle showed very different dynamic characteristics from those of a motorcycle or a car. The difference in dynamics between a car and a three-wheeled vehicle have been analysed by Huston et al. (1). However, this publication did not cover tilting three-wheelers. On the topic of tilting narrow track vehicles a number of papers have been published. Karnopp and Fang (2) were the first to suggest that, to prevent roll-over, the vehicle should lean into the turn in the same way a motorcyclist leans. Subsequently, Karnopp and Hibbard (3) discussed the optimum lean angle required when tilting a vehicle. Karnopp and Hibbard continued in this area of research and described the dynamics of a narrow tilting vehicle (4) and the methods to achieve the tilting position (5). The two methods described were Direct Tilt Control (DTC) and Steer Tilt Control (STC). The former consisted of a control loop which calculated the optimum tilt angle required for the speed and steer angle, and an actuator that pushed the vehicle to this tilting position. STC contained a similar control loop, but it estimated the countersteer required to achieve the optimum tilt angle, and steered the vehicle accordingly. With regards to safety, the CLEVER designers preferred DTC to STC, the latter requiring a steer-by-wire system and therefore cutting the direct link between the driver and the directional control. It should be noted that the authors of papers (2) to (5) only considered the tilt control. None of the publications considered the actuation in detail, and in particular, the effect the actuation system would have on the assembly it was acting against. The hydraulic actuators on the CLEVER vehicle are mounted on the non-tilting rear unit, so when they are driving the cabin to the balanced position, they are also acting against the suspension of the rear unit causing it to roll as well. If the torque from the actuators becomes large enough, the rear unit could roll over.

This paper discusses the various anomalies of the dynamics. It starts with a comparison of the equations of motion of a car, a motorcycle and a three-wheeled tilting vehicle. It then looks closer at one particular dynamic effect, namely rear steer. Following on is a discussion of the effect the hydraulics have on the system, in particular the effect of the actuators on the rear suspension and also the response time of the hydraulic actuation system.

NOMENCLATURE

m	vehicle mass
m_f	front axle mass
m_r	rear axle mass
I_z	vehicle yaw inertia
a_y	lateral acceleration
V_x	forward velocity
V_y	lateral velocity
ψ	yaw angle
a	c.g to front tyre
b	c.g. to rear tyre
l	vehicle length
R	turn radius
F_{yf}	front wheel lateral force
F_{yr}	rear wheel lateral force
$C_{\alpha f}$	front tyre stiffness
$C_{\gamma f}$	front tyre camber stiffness
$C_{\alpha r}$	rear tyre stiffness
$K_{\delta r}$	rear steer gain
α_f	front tyre slip angle
α_r	rear tyre slip angle
δ	vehicle steer angle
δ_f	front steer angle
δ_r	rear steer angle
γ_f	front tyre camber angle
g	gravity
θ	tilt angle

EQUATIONS OF MOTION

Car

The equations of motion of a car are generally derived from the bicycle model shown in Figure 1. This model assumes that the dynamic behaviour of the two front and rear wheels can be modelled by one single wheel producing twice the lateral force. Using this model the equations of motion can be derived as shown in equations [1]and [2]. The side forces F_{yf} and F_{yr} are caused by the tyre slip angles α_f and α_r. The resulting force is dependent on the tyre stiffness C_α, which varies per tyre, with the driving conditions, and many more factors. This relationship is shown in equation [3]. The slip angles are a result of the difference in the direction the tyre is pointing and the direction the vehicle is going. The equations that describe this phenomenon are shown in equation [4].

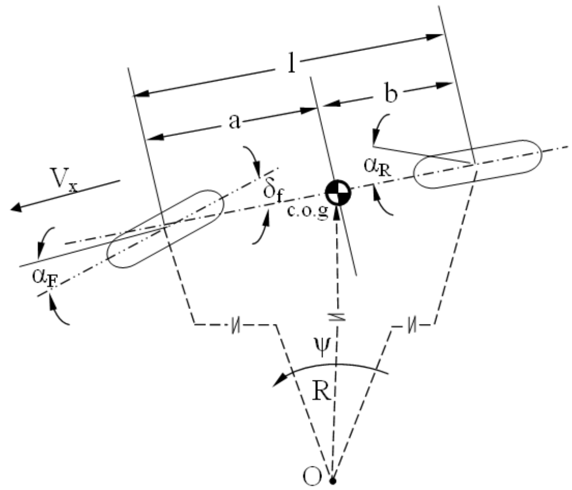

Figure 1: Bicycle model

$$m\left(a_y + V_x\dot{\psi}\right) = F_{yf} + F_{yr} \qquad [1]$$

$$I_z\ddot{\psi} = aF_{yf} - bF_{yr} \qquad [2]$$

$$F_{yf} = C_{\alpha f}\alpha_f \qquad F_{yr} = C_{\alpha r}\alpha_r \qquad [3]$$

$$\alpha_f = \delta_f - \arctan\left(\frac{V_y + a\dot{\psi}}{V_x}\right) \qquad \alpha_r = -\arctan\left(\frac{V_y - b\dot{\psi}}{V_x}\right) \qquad [4]$$

Motorcycle

Because the equations of motion of a car are usually derived from a bicycle model as shown in the section above, the motorcycle equations are not too different from those of a car. The main difference is the effect the roll motion of a motorcycle has on its dynamics. This can first be seen in the side force equation [5]. The side force is also dependent on the camber angle γ, also noted as the tilt angle θ, and the related tyre stiffness. The slip angles α_f and α_r are generated as in equation [4]. A motorcycle rider must lean the vehicle into a corner in order to balance the leaning moment caused by lateral acceleration and the moment caused by the gravitational force. This roll angle is described in equation [6].

$$F_{yf} = C_{\alpha f}\alpha_f + C_{\gamma f}\gamma_f \qquad F_{yr} = C_{\alpha r}\alpha_r + C_{\gamma r}\gamma_r \qquad [5]$$

$$\theta = \arctan\left(\frac{a_y}{g}\right) \approx \frac{a_y}{g} \qquad [6]$$

Three wheeled tilting vehicle

The equations of motion of a three-wheeled tilting vehicle are a combination of those of a car and a motorcycle. The tilting front tyre yields side forces from both the sideslip and the camber, whereas the rear tyres only yield side forces from their respective slip angles as shown in equation [7]. The generation of the slip angles is similar to equation [4] for the front tyre. However, the rear tyres of a three-wheeled tilting vehicle can steer too and an additional steer term is added to the second part of equation [8]. This rear steer term is derived in the next section.

$$F_{yf} = C_{\alpha f}\alpha_f + C_{\gamma f}\gamma_f \qquad F_{yr} = 2 \cdot C_{\alpha r}\alpha_r \qquad [7]$$

$$\alpha_f = \delta_f - \arctan\left(\frac{V_y + a\dot{\psi}}{V_x}\right) \qquad \alpha_r = \delta_r - \arctan\left(\frac{V_y - b\dot{\psi}}{V_x}\right) \qquad [8]$$

HANDLING CHARACTERISTICS OF A THREE-WHEELED TILTING VEHICLE

Steady State Characteristics

Using equations [5] and [6] from the bicycle model described previously and restricting steer and slip angles to fairly small values, the steer angle required to negotiate a curve with radius R is given by equation [9]. At low speed, α_f and α_r cancel each other out and the steer angle δ is equal to l/R. This is referred to as the Ackermann angle. At higher speeds, tyre slip increases and the steering angle deviates from this idealised condition. The slip angles are written with respect to the lateral acceleration as shown in equation [10]. Equation [10] is substituted into equation [8], the 1/R term is replaced with a_y/V^2, and the camber angle is replaced by the tilt angle from equation [6]. This then yields the new equation for a steady state steering radius, equation [11].

$$\delta = \frac{1}{R} + \left(\alpha_f - \alpha_r\right) \qquad [9]$$

$$\alpha_f = \frac{m_f \cdot a_y - C_{\gamma f} \cdot \theta}{C_{\alpha f}} \qquad \alpha_r = \frac{m_r \cdot a_y}{2 \cdot C_{\alpha r}} \qquad [10]$$

$$\delta = \frac{l \cdot a_y}{V^2} + \left(\frac{m_f \cdot g \cdot a_y - C_{\gamma f} \cdot a_y}{C_{\alpha f} \cdot g} - \frac{m_r \cdot a_y}{2 \cdot C_{\alpha r}}\right) \qquad [11]$$

Using typical tyre data for a 120/70R17 front motorcycle tyre and 195/65R15 rear car tyres (7), graphs of steer and slip angles vs. lateral acceleration can be plotted for constant radius turns, Figure 2. These show that the vehicle would considerably oversteer. This is due to the front tyre generating the majority of the cornering force through camber. The front slip, rear slip and camber angles remain the same for all turn radii because the equations are linearised. The linearization means that the angle approximations become less accurate as the lateral acceleration increases. Similarly, the camber angle is higher than 45^o at 1g cornering because the tan function in equation [6] is linearised.

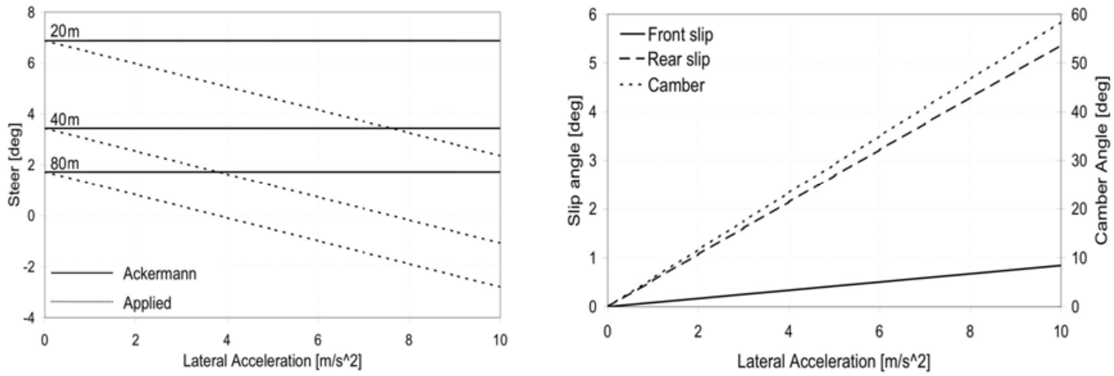

Figure 2: Cornering characteristics (7)

To compensate for the significant oversteer, the rear wheels would have to steer into the corner as the cabin leans. This rear steer is represented as a gain of the tilt angle [12]. Substituting the latter into equation [9] then yields the equation for the steer angle for a given turn [13]. Substituting in the slip angles from [10] and differentiating with respect to the lateral acceleration yields what is known as the oversteer estimation [14]. From this oversteer estimation, the rear steer gain $K_{\delta r}$ can be calculated [15].

$$\delta_r = K_{\delta r}\theta \qquad [12]$$

$$\delta = \frac{1}{R} + (\alpha_f - \alpha_r) + \delta_r \qquad [13]$$

$$\frac{d\delta}{da_y} = \frac{1}{V^2} + \frac{m_f \cdot g - C_{\gamma f}}{C_{\alpha f} \cdot g} - \frac{m_r}{2 \cdot C_{\alpha r}} + \frac{K_{\delta r}}{g} \qquad [14]$$

$$K_{\delta r} = \frac{m_r \cdot g}{2 \cdot C_{\alpha r}} - \frac{m_f \cdot g - C_{\gamma f}}{C_{\alpha f}} \qquad [15]$$

Rear Steer and Tilt Axis Inclination

In the previous section it was shown that rear wheels need to steer as the vehicle rolls in order to achieve neutral handling characteristics. This can be achieved through the inclination of the tilt axis. When considering the inertia tensors of both the front and the rear unit, they will both take the form of equation [16]. In this case the pitch inertia I_2 is also the inertia about the principle axis. Now it can be seen that a roll acceleration of the cabin will yield a yaw moment.

$$\begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_2 & 0 \\ I_{zx} & 0 & I_{zz} \end{bmatrix} \qquad [16]$$

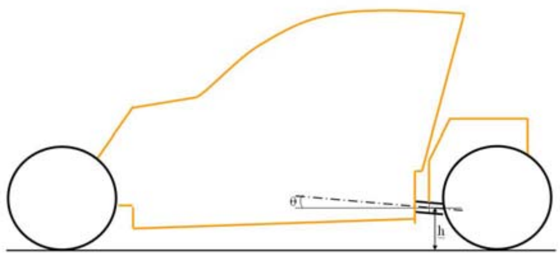

Figure 3: CLEVER and its tilt axis

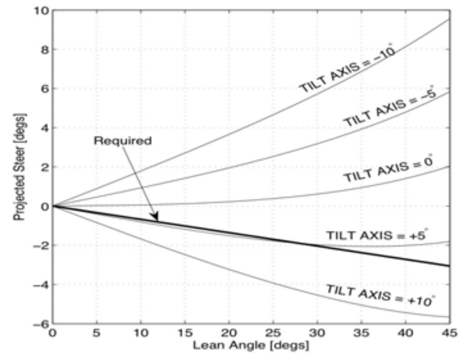

Figure 4: Effect of tilt axis inclination (7)

If the tilt axis is parallel to the ground, the yaw moment will be directly transferred to the rear unit and they will yaw together in the same direction and with the same acceleration. Now, if the tilt axis is inclined, the moment transfer will not be direct and the rear will yaw either more or less than the front assembly depending on the inclination. Figure 3 shows a positive tilt axis inclination. This results in a yaw rate of the rear which is greater than the yaw rate of the front, so the rear will steer into the turn. If the tilt axis is negative, the rear will steer away from the turn. Figure 4 shows the effect of various tilt axis inclination angles; it shows how much the rear wheels will steer for a given tilt angle. The bold line shows the necessary inclination to satisfy equation [15].

CONTROL

In order to stabilise the vehicle during cornering, CLEVER has an active direct tilt control system. The hydraulic circuit was designed to control the position of the tilting part of the vehicle with two single acting linear hydraulic actuators. When pressurised, these cylinders control the lean angle of the tilting cabin by rotating it with respect to the upright rear module. A proportional directional control valve with a closed centre position modulates the flow to the actuators, controlling their position and locking the cylinders when no command is given.

Having a neutral steering response allows a reasonably accurate estimate of the lateral acceleration to be made using vehicle speed and steer angle. The controller currently implemented is a simple Proportional controller with closed loop feedback from a transducer measuring actuator displacement.

SIMULATION

A non-linear model of CLEVER was created using the MATLAB/SIMULINK package in order to investigate the unique dynamics of the vehicle. One of the key areas of concern relating to the stability of the vehicle is the link between the static rear module and the tilting cabin. Due to the DTC method of controlling the lean angle, additional forces act on the rear module as a result of the actuators. The force acting on the rear module creates a roll moment that acts in such a way as to give the inner wheel the tendency to lift off the ground. This situation arises when a rapid steering input is made which results in a high actuator torque demand. The relatively small inertia of the rear module compared to that of the tilting cabin can lead to the lift off of the inner wheel and loss of control of the vehicle.

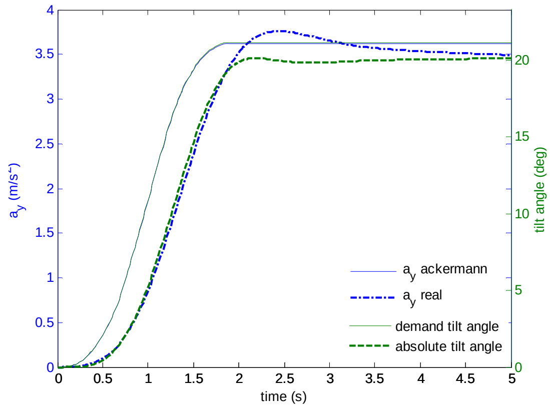

Figure 5: Simulated lateral acceleration and tilt angle

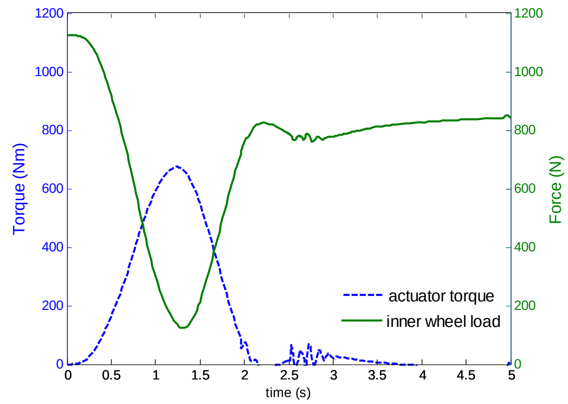

Figure 6: Simulated actuator torque and inner wheel load

Figure 5 and 6 illustrate the vehicle's response to a ramp-type input in the steering angle while driving at a constant velocity of 30kph. Figure 5 shows the calculated (Ackermann) lateral acceleration and the equivalent tilt angle required to bring the front module of the vehicle in equilibrium. The delayed vehicle's lateral acceleration response due to the generation of slip and camber in the tyres can also be seen. Furthermore, the tilt angle response of the front module as a result of the actuator torque is shown. Figure 6 illustrates the required actuator torque for the manoeuvre and the load on the inner wheel. It can be seen that actuator torque results in a significant drop in the load on the inner wheel. Once the load on the inner wheel reaches zero, the vehicle will roll over, showing that this particular manoeuvre is close to the vehicle's capabilities in terms of dynamic response. Figure 5 illustrates the favoured controller response, where the tilt angle required to balance the front module is achieved at the same time as the lateral acceleration is generated. By slowing down the response of the controller, the actuator torque demand is reduced along with the risk of roll-over. It is however important that the vehicle reaches the required tilt angle before the lateral acceleration builds up, because at higher lateral accelerations this would result in increased weight being transferred to the outer wheel and ultimately the rolling of the vehicle. Also, the response of the vehicle would feel sluggish, which could result in the driver compensating by additional steering demand. Notable is also the fact that the absolute tilt angle does not reach the demand tilt angle. This is due to the measurement of the cabin tilt angle being relative to the rear module, which rolls out of the corner with the increase in lateral acceleration.

DISCUSSION

The simulation has shown that a combination of speed and rapid steering inputs can cause the vehicle to roll over as a result of the large torque requirement for the manoeuvre. The actuator applied such a large force to the rear unit that the resulting roll acceleration was large enough to roll the unit out of the bend.

The simulation also revealed the minimum required response speed of the hydraulic actuation system. The Ackermann lateral acceleration, its resulting desired tilt angle, the non-linear lateral acceleration estimate, and the resulting tilt angle from the actuation system were compared. The time delay between the calculated Ackermann lateral acceleration and the real lateral acceleration estimate derived from the non-linear model allowed the hydraulic actuation system to respond before the vehicle's lateral acceleration built up, reducing the actuator torque demand and the risk of the rear module rolling over. The controller gain had to be set so that the actuation system response fell between the Ackermann lateral dynamics tilt demand and the actual tilt angle required to balance the vehicle according to the non-linear lateral acceleration model. This meant that the vehicle was in fact 'overleaning' during transient manoeuvres. This was found to be a positive response, as the gravitational component of the front cabin would help it reach the desired angle, reducing the torque required and hence the likelihood of roll-over.

CONCLUSIONS

An investigation of the dynamics of three-wheeled tilting vehicles has been presented. This investigation showed that the equations of motion of a three-wheeled tilting vehicle are a combination of those of a car and a motorcycle. The most important difference between the dynamics of a three-wheeled tilting vehicle and that of a car or motorcycle was the steering action from the non-tilting unit. It was shown that the rear wheels had to steer into the corner in order to achieve neutral steering dynamics. Also, the rear wheel steer had to be related to the amount of lean. To achieve this task, it was revealed that the tilt axis had to be inclined with respect to the road. In addition the characteristics and problems of the hydraulic tilt control system were highlighted. The actuation system could cause the rear unit to roll out of the bend when applying large torques in response to fast steering inputs. The controller set up was such that the cabin would start leaning into the corner just before the lateral acceleration built up. It was critical to slow down the response time of the actuators to avoid large torques, while at the same time preventing the lateral acceleration of the front cabin from building up.

ACKNOWLEDGEMENTS

The authors would also like to thank Dr M Barker and Dr B Drew for the work they carried out on the subject preceding this study.

REFERENCES

(1) Huston, J.C., Graves, B.J., Johnson, D.B., "Three wheeled vehicle dynamics", Society of Automotive Engineers, 820137, 1982
(2) Karnopp, D., Fang, C., "Simple model of steering-controlled banking vehicles", ASME Dynamics Systems and Control Division (DSC), 44, pp. 15-28, 1992
(3) Karnopp, D., Hibbard, R., "Optimum roll angle behavior for tilting ground vehicles", ASME Dynamics Systems and Control Division (DSC), 44, pp. 29-37, 1992
(4) Hibbard, R., Karnopp, D., "The dynamics of small, relatively tall and narrow tilting ground vehicles", ASME Dynamics Systems and Control Division (DSC), 52, pp. 397-417, 1993
(5) Hibbard, R., Karnopp, D., "Methods of controlling the lean angle of tilting vehicles", ASME Dynamics Systems and Control Division (DSC), 52, pp. 311-320, 1993
(6) Pacejka, H.B., Bakker, E., "Magic formula tyre model", Vehicle System Dynamics, 21 (suppl), pp 1-18, 1993
(7) Barker, M , "Chassis design and dynamics of a tilting three wheeled vehicle", PhD thesis, University of Bath, Bath, UK, 2006