Assessment of Conservativeness in Design of FRP-based Structural Strengthening Systems

Submitted by

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of the

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Kunal D Kansara
I do believe in God.
My research is an attempt to write this fact mathematically.
Acknowledgements

I feel greatly pleased in writing this acknowledgement not only for the sense of relief on seeing the ‘symbolic end’ of my thesis writing phase, but also for the fact that all the positive and negative sentiments I have gone through during my PhD tenure are floating in front of my eyes. Working for this PhD has been a great learning exercise. While the research phase of this PhD has given me stimulated pleasure of learning many new things, the writing up phase has given me opportunities to learn to be patient - a survival skill for persisting though immense psychological pressures. I am very confident in stating that completion of my PhD could have been far too difficult without the support of my family, friends and professional colleagues.

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My mother and father have been a source of inspiration and courage to me, and I thank them for the warmth and opportunities to excel that they have provided to me. I also thank my little niece Saumya whose innocence has given me reasons to smile when I was feeling down.

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I thank all whom I could not fit-in on this A4 size paper!

Kunal Kansara
January 15, 2014
Synopsis

Conservativeness (or conservatism), in general, is a measure of the lack of confidence in any activity that we do in the spheres of life. It is a reflection of our apprehension for the consequences of failure, and hence we instinctively tend to be conservative in order to be safe. Engineering design involves incorporation of various physical characteristics of the materials and systems through various mathematical models and design criteria. These models and criteria are developed based on empirically observed, experimentally measured, logically anticipated, analytically testified and/or hybrid behaviours and responses of materials and systems. All these processes invariably involve uncertainties arising from the deficiency in terms of knowledge, data and time-testimony. Uncertainties also arise from the lack of precision in expressing a phenomenon or mechanics, or from the inherent variability associated with them. All these factors lead to lack of confidence in the use of materials and systems, and compels our designs to be conservative. Comprehending uncertainties in engineering design, and refining their design treatment can pay big dividends.

Design of fibre reinforced polymer (FRP) based structural strengthening systems involves an interesting interplay of uncertainties between those inherent in the existing structure being strengthened and those arising from the lack of complete knowledge and time-testimony of using FRP composites for structural strengthening. Most strengthening design guidelines tend to be more conservative than the conventional structural design norms in order to meet the safety targets. A popular approach for achieving this requirement is through prescribing a set of safety factors within the strengthening design, which are substantially higher compared to those used in structural design using conventional materials like concrete and steel. However, FRP composites in general, and their use as externally bonded reinforcement in particular, involve considerable peculiarities compared to the conventional structural materials. Also, the type and form of post-strengthening failure modes exhibit substantial qualitative distinctions compared to the pre-strengthening failure modes. Therefore, the design processes for strength (for new constructions) and additional strength (for strengthening existing structures) can have conflicting design requirements and objectives. A strategy of prescribing quantitatively higher safety factors, under this condition, could be ineffective in producing required conservativeness for some design scenarios, and can instigate undesirable side-effects.

This thesis aims at assessing performance of flexural and shear strengthening design processes under the identified contradictory and contrasting features of the safety format used in strengthening design. It also provides a deeper interpretation of conservativeness in strengthening design by identifying implications of the means employed for producing conservativeness on the course of strengthening design process and on the quality of the resultant strengthening design solutions.

An exhaustive review of literature, spanning over past three decades, on the design for structural strengthening using externally bonded FRP reinforcement has been carried out. This review has identified various sources of uncertainties, gap in knowledge and design conflicts associated with the mechanics of FRP-based structural strengthening systems. Detailed taxonomies of uncertainties and safety parameters
Concerning FRP-based structural strengthening systems have been proposed. The uncertainties are classified into constitutive and behavioural uncertainties. The former are attributed to the variability in constitutive material properties of FRP, while the latter are due deviations between the ideally expected or real behaviours of FRP composites and that predicted within the design process.

A comprehensive mapping of the identified uncertainties and safety parameters is developed, which presents a framework that enables analytical treatment of conservativeness within strengthening design. This mapping indicates four distinct levels at which safety parameters in various forms are incorporated within the design process with an intention to produce conservativeness while estimating the design value of post-strengthening resistance. The first two of these four levels within the strengthening design process include various safety parameters are prescribed on the mean and characteristic values of FRP material properties to arrive at their design values. This approach aims at setting ‘reserved strength’ while estimating the design post-strengthening resistance by under estimating the load carrying capacity of the FRP-based structural strengthening system. This reserved strength accounts for the constitutive uncertainties, and the conservativeness produced in design post-strengthening resistance due to safety parameters prescribed on FRP material properties is called the material conservativeness. The last two of the four levels within the strengthening design process include various safety parameters are prescribed on the nominal values of resistance contribution of FRP reinforcement and strengthened member to arrive at the design post-strengthening resistance. This approach aims at setting ‘over strength’ while estimating the design post-strengthening resistance by under estimating the load carrying capacity of the FRP-based structural strengthening system. This over strength accounts for the behavioural uncertainties, and the conservativeness produced in design post-strengthening resistance due to safety parameters prescribed on resistance is called the resistance conservativeness. The aggregative effect of the material and resistance conservativeness comprises the total conservativeness, which can be segregated from the design post-strengthening resistance of a strengthening design solution. This process, when performed for a range of flexural and shear strengthening design solutions, enables assessing the effectiveness of various safety formats in producing conservativeness under different design scenarios. Conservativeness in estimation of the post-strengthening resistance, being a direct indicator of the global safety targets (e.g., probability of failure and reliability index), presents a very useful insight for a designer as well as for a calibrator of strengthening design guidelines.

In addition to enabling the analytical treatment of conservativeness, the mapping of uncertainties and safety parameters also reveals some interesting facts, which by-and-large remain hidden within the different formats of prescribing design criteria. Firstly, it reveals existence of ‘failure mode switchers’ within the strengthening design process. The strategic locations of these switchers set bifurcations within the strengthening design process by manipulating the design predictions for the modes of failure for an externally bonded FRP reinforcement (e.g., rupture and debonding). It is shown in this thesis that different modes of failure of FRP are differently sensitive to the safety parameters prescribed on FRP material properties. It is also shown that certain formats of the failure mode switcher (e.g., switcher for flexural strengthening according to ACI440) can quantitatively inflate the prescribed material safety parameters, without projecting any increase in the safety factors on FRP material properties. Thus, understanding of the working mechanism of failure mode switchers in manipulating modes of failure of
FRP can be a means of controlling conservativeness. Switchers in flexural and shear strengthening design processes are identified, and the mathematical criteria depicting design predictions for modes of failure of FRP are presented.

Secondly, the contention raised by the fact that the material and resistance safety parameters account for the constitutive and behavioural uncertainties respectively sets a ground to contest the strategy, adopted by many strengthening design guidelines, of not prescribing safety factors on post-strengthening structural resistance merely on the basis of the differences arising from the limit state design (LSD) and load and resistance factor design (LRFD) philosophies. In light of the confirmed differential sensitivity of various failure modes to the material safety parameters, it is demonstrated that such a strategy not only suggests ignorance towards accounting behavioural uncertainties in strengthening design, but also results into substantial reduction in conservativeness in estimated design post-strengthening resistance for strengthening design solutions governed by certain types of failure modes.

Assessment methodologies for flexural and shear strengthening are developed, which provide a common platform for comprehending flexural and shear strengthening design processes irrespective of all the philosophical and operational distinctions associated with various strengthening design guidelines. Both methodologies are in non-dimensional format, which can be calibrated against any existing strengthening design guidelines. For illustration, these methodologies are calibrated against ACI440 and TR55 design specifications. The assessment methodology for flexural strengthening design is based on ductility-based definitions of the post-strengthening failure modes. These definitions take the strain in tension steel reinforcement as a prime parameter that intuitively classifies the possible post-strengthening failure modes into concrete- and FRP-controlled failure modes covering all the possible variants based on sectional ductility and possibility of debonding or rupture of FRP. The assessment methodology for shear strengthening design presents the shear resistance contribution of FRP reinforcement comprising of an effective failure strain in FRP with modifications accounting for variations in orientation of principal fibres and wrapping configurations. These design solutions are primarily clustered into productive and unproductive design solutions. The former and the latter involve a positive and a non-positive value of shear resistance contribution of FRP under given conditions respectively. These methodologies present a complete picture of the internal architecture of flexural and shear strengthening design processes, and enable tracking propagation of conservativeness within them.

Both methodologies can produce a wide range of possible strengthening design solutions under given conditions, which can be clustered according to their qualitative characteristics and governing failure modes. The influence of differences in qualitative and quantitative prescriptions for various safety parameters, and different formats of FRP debonding limits, FRP bond length models, failure mode switchers and different approaches for accounting bond reduction (for FRP shear reinforcements only) on strengthening design processes are captured through a set of parametric and sensitivity analyses.

Based on this study it is concluded that the quantitative prescription of the material safety parameters can influence the design predictions for the modes of failure of FRP and the governing post-strengthening failure mode for the strengthening design solutions. This in turn, influences the course of strengthening design process and quality of strengthening design solutions considerably. Mathematical expressions
depicting various post-strengthening failure modes and their design predictions within flexural strengthening design are provided. Mathematical criteria for avoiding undesirable failure modes and promoting optimal design solutions are also provided. For the strengthening design guidelines not prescribing resistance safety parameters (e.g. TR55), suitable strength reduction factors to compensate for the reduced in conservativeness for the design solutions governed by concrete-controlled and FRP-controlled involving debonding of FRP are recommended. It is also recommended to include design criteria for mechanically anchored FRP reinforcement allowing an increase in the permissible strain in FRP at debonding towards better utilising the higher rupture strain capacity FRP materials. It is concluded that a certain format of debonding strain limit (e.g., according to ACI440) can produce significantly higher permissible value of strain in FRP at debonding, especially for low modulus high rupture strain capacity FRP materials. An upper limit, in form of a pre-set numerical constant strain value, is suggested for such circumstances. An apparent oversight in the numerical values for bond reduction coefficients for FRP shear reinforcement prescribed by ACI440 is identified. It is demonstrated that this values of bond reduction coefficients are thrice as high as compared to those prescribed by TR55, and results into a substantial increase in the extent of unproductive shear strengthening design solutions. A modification of these values is recommended.

It is suggested that the utility of this study can further be increased by developing an expert system based on the directions and knowledge-based presented within this study. The applicability of this study can be expanded by converting it into probability-based reliability format that can inform us on the fragility and risk profiling. It is also shown that the concept of conservativeness can also be extended to cover issues related to structural robustness and resiliency.
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<td>Cross-sectional area of an FRP shear reinforcement</td>
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<td>Width of an FRP shear reinforcement</td>
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<td>$b$</td>
<td>Width of an RC flexural member</td>
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<td>$C_R$</td>
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<td>$d_f$</td>
<td>Effective depth of an FRP shear reinforcement</td>
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<tr>
<td>$d$</td>
<td>Effective depth of an RC flexural member</td>
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<td>$D$</td>
<td>Overall depth of RC flexural member</td>
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<td>$E_{fd}$</td>
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<td>$E_s$</td>
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<td>$[J]$</td>
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<td>$k_1, k_2$</td>
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Assessment of conservativeness in design of FRP-based structural strengthening systems
Kunal D. Kansara (2014)

$K$ Ratio of effective to overall depth for a RC flexural member

$L_e$ Effective bond length (or active bond length) of externally bonded FRP

$M_{\text{concrete-design}}$ Design flexural resistance contribution of compression concrete of a strengthened or an unstrengthened RC flexural member

$M_{\text{d-calculated}}$ Calculated design flexural resistance of a strengthened RC flexural member

$M_{\text{d-required}}$ Required design flexural resistance of a strengthened RC flexural member

$M_d$ Actual design flexural resistance of a strengthened RC flexural member

$M_{\text{design}}$ Design flexural resistance contribution of externally bonded FRP reinforcement in tension for a strengthened or an unstrengthened RC flexural member

$M_{\text{FRP-design}}$ Design flexural resistance contribution of externally bonded FRP reinforcement in compression for a strengthened or an unstrengthened RC flexural member

$M_{\text{nominal}}$ Nominal flexural resistance for a strengthened or an unstrengthened RC flexural member

$M_{\text{sc-design}}$ Design flexural resistance contribution of internal compression steel reinforcement for a strengthened or an unstrengthened RC flexural member

$M_{\text{st-design}}$ Design flexural resistance contribution of internal tension steel reinforcement for a strengthened or an unstrengthened RC flexural member

$m, n, q$ Powers of $[S_E], [S_F]$ and $[F]$ respectively

$p_{\text{design}}$ Design value of an output for an engineered model

$p_{\text{e-max}}$ Threshold value of the probability of exceedence

$p_e$ Probability of exceedence

$p_{\text{nominal}}$ Nominal value of an output for an engineered model

$p_f$ Coefficient of variation for an arbitrary FRP material property

$R_{\text{FRP,RC (design)}}$ Design post-strengthening resistance of a strengthened RC member

$R_{\text{FRP,RC (nominal)}}$ Nominal post-strengthening resistance of a strengthened RC member

$R_{\text{FRP (design)}}$ Design resistance contribution of FRP (i.e. excluding RC section resistance contribution)

$R_{\text{FRP (nominal)}}$ Nominal resistance contribution of FRP (i.e. excluding RC section resistance contribution)

$R_{\text{RC (design)}}$ Design resistance contribution of RC section (i.e. excluding FRP resistance contribution)

$R_{\text{RC (nominal)}}$ Nominal resistance contribution of RC section (i.e. excluding FRP resistance contribution)

$R$ Mean axial rigidity per unit width for FRP (product $n_f t_f E_F$)

$R_{\text{RCI}}$ Residual conservativeness index

$R_{\text{RCI}}$ Relative residual conservativeness index

$[S_F]$ Condensed material safety parameter for an arbitrary FRP material property

$[S_E]$ Condensed material safety parameter for modulus of elasticity of FRP

$[S_F]$ Condensed material safety parameter for rupture strain capacity of FRP

$S_f$ Longitudinal centre-to-centre spacing of FRP shear reinforcement

$S$ Ratio of effective cover to effective death of an RC flexural member

$t_1$ Ratio $d_f / d$

$t_2$ Ratio $b / d_f$

$t_f$ Effective thickness of FRP ply
**Parameters**:

- $v_{\text{composite}}$: Total volume of FRP composite (i.e. including fibres and matrix both)
- $v_{\text{fibre}}$: Volume of fibres comprising an FRP composite
- $v_{\text{matrix}}$: Volume of matrix comprising an FRP composite
- $V_{\text{conc-design}}$: Design shear resistance contribution of concrete for a strengthened or an unstrengthened RC flexural member
- $V_{\text{design}}$: Design shear resistance of a strengthened or an unstrengthened RC flexural member
- $V_{\text{FRP-design}}$: Design shear resistance contribution of an FRP reinforcement
- $V_{\text{FRP,design}}$: Design shear resistance contribution of FRP reinforcement
- $V_{\text{FRP,nominal}}$: Nominal shear resistance contribution of FRP reinforcement
- $V_{\text{st-design}}$: Design shear resistance contribution of internal steel sirrups for a strengthened or an unstrengthened RC flexural member
- $x_1, x_2$: Parameters dictating spacing of FRP shear reinforcements
- $x_{\text{ut}}$: Depth of neutral axis for a RC flexural member
- $Y_{\text{(composite)}}$: An arbitrary material property value for an FRP composite (i.e. comprising of fibres and matrix both)
- $Y_{\text{(fibre)}}$: An arbitrary material property value of fibres comprising an FRP composite
- $Y_{\text{(matrix)}}$: An arbitrary material property value of matrix comprising an FRP composite
- $Y_0$: Initial (pre-deterioration) value for an arbitrary FRP material property
- $Y_1$: Final (post-deterioration) value for an arbitrary FRP material property
- $Y_c$: Characteristic value of an arbitrary FRP material property
- $Y_f$: Design value of an arbitrary FRP material property
- $\bar{\gamma}$: Statistical mean value of an arbitrary FRP material property
- $\bar{\varepsilon}_f$: Mean value of rupture strain capacity of FRP
- $\gamma$: Partial safety factor on rupture strain capacity of FRP according to TR55-2004 notations
- $\gamma_{\text{APR}}$: Application process-reliant safety parameter for FRP
- $\gamma_{\text{CMP}}$: Compensatory safety parameter for FRP
- $\gamma_{\text{E}}$: Partial safety factor on modulus of elasticity of FRP according to TR55-2004 notations
- $\gamma_{\text{EDT}}$: Environmental deterioration based safety parameter
- $\gamma_{\text{m-st}}$: Partial safety factor on material properties of structural steel used as a reinforcement in RC
- $\gamma_{\text{mc}}$: Partial safety factor on compressive strength of concrete
- $\gamma_{\text{MDG}}$: Mechanical degradation based safety parameter
- $\gamma_{\text{mm}}$: Additional partial safety factor for FRP according to TR55-2004 notations
- $\gamma_{\text{ms}}$: Partial safety factor on steel reinforcement
- $\gamma_{\text{PNT}}$: Punitive safety parameter for FRP
- $\gamma_{\text{PQR}}$: Production quality-reliant safety parameter
- $\gamma_{\text{SUP}}$: Supplementary safety parameter for FRP
- $\delta_{\text{fe}}$: Multiplier to the design rupture strain capacity of FRP in flexural switch
- $\delta_{\text{se}}$: Multiplier to the design rupture strain capacity of FRP in shear switch
- $\varepsilon_c$: Strain in outmost layer in compression concrete
- $\varepsilon_{\text{cu}}$: Limiting failure strain for compression concrete
- $\varepsilon_f$: Initial soffit strain at the time of installing FRP
- $\varepsilon_{\text{f,debond}}$: Design value of the debonding strain limit for externally bonded FRP
- $\varepsilon_{\text{f,disintegration}}$: Concrete integrity strain limit for externally bonded FRP shear reinforcement (beyond which concrete’s integrity through aggregate interlocking gets...
vanished)

\( \varepsilon_{fd, \text{fracture}} \) Average rupture strain limit for externally bonded FRP stirrups

\( \varepsilon_{fd, \text{rupture}} \) Design value of the rupture strain capacity of FRP

\( \varepsilon_{fd} \) (Governing or effective) Design rupture strain capacity of FRP

\( \varepsilon_{fe} \) Effective strain at tension face of the concrete substrate at which FRP is externally bonded

\( \varepsilon_{fl} \) Strain in FRP due to imposed load

\( \varepsilon_{fse} \) Effective failure strain in FRP stirrup

\( \varepsilon_{sc} \) Strain in internal compression steel reinforcement in a RC flexural member

\( \varepsilon_{st} \) Value of adequately yielding strain for internal tension steel reinforcement (or adequate ductility content) for a strengthened RC flexural member

\( \varepsilon_{st, \text{adequate}} \) Strain in internal tension steel reinforcement (or ductility content) for unstrengthened or strengthened RC flexural member

\( \varepsilon_{sy} \) Yield strain for steel reinforcement

\( \eta_1 \) Fibre orientation factor

\( \eta_2 \) Wrapping effectiveness factor

\( \kappa_\nu \) Bond reduction coefficient for externally bonded FRP stirrups according to ACI440

\( \kappa_{\nu s} \) Bond reduction coefficient for externally bonded FRP stirrups in shear

\( \rho_{\text{FRP}} \) FRP-content

\( \rho_{sc} \) Compression reinforcement content

\( \rho_{st} \) Tension reinforcement content

\( \sigma_Y \) Standard deviation for an arbitrary FRP material property

\( \phi_1 \) Related to supplementary safety parameter

\( \psi_f \) Strength reduction factor on resistance contribution of FRP according to ACI440-2008 notations

\( \alpha, \beta \) Principle direction of fibres relative to the longitudinal

\( \zeta \) Over-strength factor for flexural strengthening, expressed as a ratio of the required design flexural resistance to its actual design flexural resistance for a strengthened RC member

\( \lambda \) Equalising parameter

\( \xi \) Extent of a cluster for shear strengthening design solutions

\( \phi \) Strength reduction factor (resistance factor) for a strengthened or an unstrengthened RC flexural member according to ACI318-2008 and ACI440-2008 notations
## Schedule of acronyms

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>FRP</td>
<td>Fibre reinforced polymer (or plastic) composite</td>
</tr>
<tr>
<td>RC</td>
<td>Reinforced concrete (using steel reinforcement unless specified otherwise)</td>
</tr>
<tr>
<td>SM</td>
<td>Surface mounted technique for externally bonded FRP-based structural strengthening</td>
</tr>
<tr>
<td>NSM</td>
<td>Near surface mounted technique for externally bonded FRP-based structural strengthening</td>
</tr>
<tr>
<td>LSD</td>
<td>Limit state design philosophy</td>
</tr>
<tr>
<td>LRFD</td>
<td>Load and resistance factor design philosophy</td>
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<tr>
<td>PQ Class</td>
<td>Production quality class for clustering safety parameter $\gamma_{PQR}$</td>
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<td>EE Class</td>
<td>Environmental exposure class for clustering safety parameter $\gamma_{ENT}$</td>
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<tr>
<td>M &amp; I Class</td>
<td>Manufacturing and installation class for clustering safety parameter $\gamma_{APR}$</td>
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<td>CMP Class</td>
<td>Compensation class for clustering safety parameter $\gamma_{CMP}$</td>
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<td>PNT Class</td>
<td>Penalty class for clustering safety parameter $\gamma_{PNT}$</td>
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<tr>
<td>SUP Class</td>
<td>Supplement class for clustering safety parameter $\gamma_{SUP}$</td>
</tr>
<tr>
<td>RDS</td>
<td>Rupture-debonding switch (in flexural strengthening)</td>
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<tr>
<td>NRDS</td>
<td>Near rupture-debonding switch (in flexural strengthening)</td>
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<tr>
<td>FDDS</td>
<td>Fracture-debonding-disintegration switch (in shear strengthening)</td>
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<tr>
<td>DDS</td>
<td>Debonding-disintegration switch (in shear strengthening)</td>
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### Flexural Strengthening

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<td>Cluster A</td>
<td>Flexural strengthening design solutions governed by concrete-controlled failure mode</td>
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<td>Cluster B</td>
<td>Flexural strengthening design solutions governed by FRP-controlled failure mode involving rupture/near-rupture of FRP</td>
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<td>Cluster C</td>
<td>Flexural strengthening design solutions governed by FRP-controlled failure mode involving debonding of FRP</td>
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### Shear Strengthening

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<td>Unproductive shear strengthening design solutions</td>
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<tr>
<td>Cluster B</td>
<td>Productive shear strengthening design solutions</td>
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<td>Cluster C</td>
<td>Productive shear strengthening design solutions with controlled productivity</td>
</tr>
<tr>
<td>Cluster D</td>
<td>Productive or unproductive shear strengthening design solutions, which violate the minimum spacing requirements for FRP shear reinforcement</td>
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<table>
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<tr>
<th>Class</th>
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<tbody>
<tr>
<td>MLC</td>
<td>Most liberal combination of material safety parameters</td>
</tr>
<tr>
<td>MSC</td>
<td>Most stringent combination of material safety parameters</td>
</tr>
<tr>
<td>PDCC</td>
<td>Point of dual contra characteristics</td>
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<tr>
<td>CPP</td>
<td>Conservativeness propagation path</td>
</tr>
</tbody>
</table>
Introduction
1.1 Background

The perception of safety has been of prime importance to mankind from the dawn of civilisation. In fact, the recognition of engineering as a specialist discipline is a result of our need to have a streamlined notion of safety while converting theoretical concepts into reality. In structural engineering practice, this is attained by carrying out a ‘structural design’ based on the ‘structural analysis’ of a ‘structural system’ – generally in light of the ‘design standards’. While analysis aims to predict a realistic behaviour of a simulated virtual structural system under the probable actions that are likely to influence the actual structure, design aims to idealise a virtual structural system that ensures that the response of the physical structure when constructed will be within permissible limits. It is to be noted that analysis and design do involve considerable extents of assumptions, simplifications, omissions, ignorance and errors. Consequently, neither ‘as analysed’ nor ‘as designed’ behaviours of a virtual structural system exactly match with the ‘real’ behaviour of the physical structure. Since it is impossible to capture the absolute real behaviour of a physical structure through what-so-ever sophisticated analysis, design and monitoring approaches, the meaning of ‘real behaviour’ is diluted to refer to the ‘realistic behaviour’ as an analysis goal, and the ‘permissible behaviour’ as a design objective. It is for the engineer to ensure that the deviations between these behaviours remain as low as possible, and do not lead to a catastrophe.

The design standards include a set of established norms, which are calibrated based on analytical analogies, numerical interpretations, computational simulations, experimental observations, logical intuitions and empirical judgements. These standards, in a sense, are submissive to the fact that the failure of a structure is inevitable, and thus they appear to advocate the supremacy and irrepressibility of Nature over the human abilities. This is evident from the fact, for example, that we can at the most design our structures to be ‘earthquake resistant’, but essentially not to be ‘earthquake proof’. However, this factual limitation is by no means an allegation of inferiority for the design standards. In fact, in spite of having submitted their incapability to overrule the power of Nature, the design standards suggest fantastic mathematical tactics to manipulate the type, form and time of failure. For example, the concrete design standards specify partial factors of safety on concrete such that the shear strength of the concrete remains substantially lower than its flexural strength. This, in turn, suppresses the possibility of an undesirable failure mode, such as shear failure of an RC beam, to govern the design solution. With such an approach, we are able to retain at least a certain control over the structures that we build within a setup that is ruled by Nature.
with infinite powers. The sheer beauty lying within this philosophy appears to have inspired from the theosophical wisdoms professed by the religions that if we cannot avoid death, let us at least live the life in a manner so as to have a respectful death! Under the light of this, a conscientiously written design standard, in the view of the author, is a technical equivalent of the religious holy books (e.g., Bhagavad Gita [Edwin (1994)] or the Holy Bible [Collins (2012)]). The author is aware of the possibility that a reader might find the above description, in spite of a few metaphoric technical examples, a bit too philosophical to be a part of an engineering thesis. However, the author, in his defence, wishes to clarify that one of the major objectives of this thesis is to investigate if the format and content of a design standard can influence the design predictions for the failure modes of structures. The above philosophical narrative, therefore, fits well within the context of this thesis.

1.2 Safety and conservativeness

In the context of structural design, a design philosophy provides a fabric to conceptually incorporate safety within the design process, whereas a safety format woven within this fabric provides an analytical framework to ensure that the set safety goals are achieved at the end of a design process. These safety goals can be prescribed through different qualitative and quantitative means, which makes the incorporation of safety within engineering design a subjective science – probably more art than science. For engineers, however, it is essential to have this art written mathematically. This is due to the fact that the requirements of safety and economy run hand-in-hand. For each of these requirements, the other is a constraint, and the very essence of engineering design, in fact, is to have a justifiable balance between the two. Mathematical representation of safety enables us to achieve this by rationalising and optimising various means employed to incorporate safety in engineering design. This, in turn, facilitates us to achieve an efficacious design process and an efficient use of materials and systems, which could actually be seen as a form of bringing sustainability into the design practice. The maturing of structural engineering design philosophies through evolution – from the primitive judgement-based empirical design to a more thoughtful working stress, limit state and load and resistance factor designs – clearly explicates this fact. While this technical evolution has facilitated us incorporating safety more judiciously, there is at least one characteristic that has been incredibly successful in surviving through the entire course of this evolution – conservativeness.

The design standards have retained the flavour of conservativeness in prescribing the design criteria, irrespective of the employed design philosophy. While this fact
momentarily appears as a downside, there are justifiable reasons for its existence. Foremost amongst these reasons is analogous to the way life is preserved on the planet. By having an ample buffer to tolerate the disturbances to the existence of life, Mother Nature herself appears to advocate conservativeness as a hereditary survival instinct, and not a downside. Therefore, it is not surprising that we prefer to be instinctively conservative while dealing with the situations involving a lack of surety (i.e. uncertainties) in order to be safe. This equally applies to structural design, which is invariably performed under a considerable amount of uncertainties. However, by-and-large we ignore associating any idealisation, qualification and quantification to the term ‘conservativeness’. Having a clearer taxonomy of conservativeness with duly considering its implications on the design process not only provides a better control over retaining safety under uncertainties, but also facilitates accommodating newer and less time-tested materials, methods and techniques within the existing practice without jeopardising safety. This forms a primary motive behind this thesis’s inclination towards assessing conservativeness associated with structural design processes.

A critic of this study might expect a justification for the choice of word ‘conservativeness’ in the title of this thesis. A linguist might even contest the legitimacy of the term ‘conservativeness’ as a noun derived from the adjective ‘conservative’. In fact, the Cambridge Dictionary (http://dictionary.cambridge.org) does not find this term within it, and offer ‘conservatism’ as the closest alternative choice. The Oxford English Dictionary (http://www.oed.com), on the other hand, does approve ‘conservativeness’ as an authentic English word with sufficient etymologic details in support. This settles the linguistic legitimacy issue in the mind of the critic. However, the technical legitimacy behind choosing the noun ‘conservativeness’ instead of ‘safety’ can still be questioned. While ‘conservativeness’ is popularly believed as a synonym of ‘safety’, there is a clear distinction between the two terms, which is notionally represented through Fig. 1.1 in the context of structural design. It can be seen that safety is a more holistic term, and needs ‘resistance estimation’ and ‘load-effect estimation’ processes to be considered together. The resistance and the load-effect estimation processes individually need to be ‘conservative’ in order to achieve safety. Thus, conservativeness is a subset of safety. It is shown later in this chapter that this thesis deals with the process of resistance estimation for FRP-based structural strengthening systems only, while the load-effect estimation process is not within the scope. Therefore, the term conservativeness is used in lieu of safety within the title of this thesis. A more detailed discussion on conservativeness is presented in Chapter 3.
Figure 1.1
Indicative ‘conservativeness’ and ‘safety-content’ in structural design

1.3 Structural engineering design processes

The structural design processes primarily focus on deriving safety based on structural strength (also called structural resistance or capacity). The following are important strength-based design processes in contemporary structural engineering practice:

- Design for strength (for new constructions)
- Design for additional strength (for rehabilitating existing structures)

While ‘strength’ is a primary design objective in these design processes, they invariably involve checking that the structure at hand also meets the serviceability requirements. In fact, many existing structures are rehabilitated in order to improve their deficiency in meeting the serviceability requirements. Having an added strength, in such cases, forms a by-product. Conversely, it is also imperative to ensure that the means employed for rehabilitating an existing structure, primarily for improving its strength deficiency, do not impair its serviceability.

This thesis explicitly deals with the design for additional strength towards rehabilitating existing concrete structures, in particular using fibre reinforced polymer (FRP) based structural strengthening systems. FRP composites are a set of contemporary structural materials, which are relatively new for the construction industry but are prolifically used in other engineering industries since long. This thesis aims to broaden the understanding of the safety protocols used in structural rehabilitation designs by studying the implications of conservativeness involved in the design of FRP-based structural strengthening systems. While this thesis is written with a central focus on the use of
FRP composites in strengthening design, the philosophy and concepts proposed within this thesis can be extended to any engineering design process.

1.4 Structural rehabilitation

1.4.1 Purpose

The process of enhancing structural performance of an existing under-performing structure (or a structural member) is called structural rehabilitation. Fig. 1.2 presents a summary of the situations demanding structural rehabilitation including structural under-performance due to distress, deficiency or defect [Kansara et al. (2007)]. An existing structure is deemed distressed if it has undergone considerable deterioration, degradation or damage.

![Figure 1.2](image)

Design situations demanding structural rehabilitation

Of these, deterioration and degradation involve gradual loss of strength (over a relatively longer time period) under environmental and mechanical influences respectively. Damage, on the other hand, involves a sudden and accidental loss of strength. Rehabilitating a distressed structure includes restoration and enhancement of the lost strength or reduced serviceability. An existing structure is deemed to be deficient when its available strength or serviceability fails to meet the required demands. This may be caused by a change in function, and/or change in the live and seismic loads (arising from the revision of design code specifications, especially for old structures). Rehabilitating a deficient structure includes enhancement of strength occasionally with modifications. Structural defects include inherent construction, design or detailing errors. Rehabilitating a defective structure includes rectification occasionally with enhancement of strength. Based on the purpose, structural
rehabilitation can be classified into \textit{structural repair}, \textit{structural strengthening} and \textit{structural retrofitting} [Fig. 1.3].

**Figure 1.3**

Classification of structural rehabilitation

\textit{Structural repair} aims at \textit{restoring} the original (or near original) performance of an existing distressed structure, whereas \textit{structural strengthening} aims at \textit{raising} the performance of an existing distressed or an un-distressed structure to meet increased structural demands. \textit{Structural retrofitting}, on the other hand, aims at \textit{rectifying} and/or \textit{modifying} an existing defective or deficient structure to make it more adaptive in withstanding loads and conditions that were not considered in the original design. In literature, structural retrofitting is often synonymously referred to as seismic retrofitting.

1.4.2 Importance

An ever increasing deficit between the required and the deployable economic resources for the construction, maintenance and replacement of civil infrastructure worldwide has led to a situation, popularly referred to as the \textit{global infrastructure crisis} [Fig. 1.4]. Moreover, a significant increase in the carbon footprints of manmade activities has already sounded alarms towards reaching the ‘tipping-point’, risking life on the globe through extinction [Lynas (2007)]. Together, both these situations compel us to ensure that any improvement brought within existing engineering practices contributes towards economic and environmental sustainability. This is of fundamental importance if we are to avoid producing solutions to the problems that are problems themselves that remain to be solved later.
Assessment of conservativeness in design of FRP-based structural strengthening systems
Kunal D. Kansara (2014)

Figure 1.4
Global infrastructure crisis


In this context, structural strengthening or retrofitting of an under-performing structure offers many benefits. Firstly, structural strengthening and retrofitting are means to make existing infrastructure adaptable to changes [Kansara et al. (2007)], which in itself is one of the forms of resilience [Park et al. (2013)]. Secondly, imparting additional strength to an under-performing structure not only refurbishes its structural performance strength-wise, but it also renewes its service life. This extended service life effectively delays the need of a complete replacement of the structure, which in turn, instantly brings the exigency of the problem down from chronic to at least acute or moderate state. Thus, continued (social, economic, strategic and/or otherwise) revenues can be ensured at a low capital investment, without jeopardising the safety [Kansara et al. (2007)]. Thirdly, an extended service life not only provides a buffer period to build the required economic resources for a complete replacement (which is inevitable in most cases), but also facilitates setting a priority-based allocation of the limited economic resources to the needy existing structures [Kansara et al. (2007)]. Fourthly, the material, time, energy and resource requirements and the waste generation in a typical structural
rehabilitation exercise are significantly less than constructing a new substitute structure. Thus, it has a positive impact on the environmental sustainability front, when grossly compared with the complete substitution of an under-performing structure. It is, therefore, not surprising that the rehabilitation of constructed facilities comprises about 50% of all construction activities [Presidencia Espanola (2010)]. It can be appreciated that any improvement brought into structural rehabilitation practice is of substantial benefit towards mitigating the global infrastructure crisis. Scientific approaches for infrastructure engineering and management, therefore, form an important mitigating strategy for governmental and non-governmental agencies worldwide.

1.5 FRP composites in construction

FRP composites can be used for a wide range of applications, both for new constructions and existing structures, as summarised in Fig. 1.5, in which the applications within the direct scope of this study are highlighted.

![Diagram of FRP composites in construction](image)

Figure 1.5
Summary of applications of FRP composites in construction (Highlighted applications fall within the direct scope of the present study)

It can be seen that for new construction, FRP composites can be used either to produce fully polymeric structures (or structural components) or as an alternative to conventional steel reinforcement in reinforced concrete (RC) structures and a prestressing tendon in prestressed concrete (PSC) structures. FRP composites are also used as formwork systems for new concrete construction, which can either be
participatory, also called stay-in-place formwork system [Gai et al. (2012)], or non-participatory, also called flexible or reusable formwork system [Veenendaal et al. (2011)]. FRP composites are also used for structural rehabilitation of existing concrete, steel, masonry and timber structures.

For strengthening RC flexural members, FRP composites can be used as an externally bonded flexural tension reinforcement system (for flexural strengthening) and as an externally bonded ‘stirrup’ (for shear strengthening). It can also be used for axial strengthening and ductility enhancement applications – both mainly through confinement, but essentially not as a direct compression reinforcement – for RC compression members. The externally bonded tension and shear reinforcements can be installed through either surface mounted (SM) or near surface mounted (NSM) techniques. The former involves direct bonding of the FRP reinforcement (in the form of a strip or a plate) onto a surface of the concrete element, whereas the latter involves installing the FRP reinforcement (in the form of a square, rectangular or circular bar) into slots cut into the concrete subgrade [Parera et al. (2009)]. FRP shear reinforcement, in particular, can also be installed through the deep embedment (DE) technique, in which a hole is drilled through the core along the depth of the RC section [Valerio et al. (2009)]. An FRP bar is then inserted and grouted in-place, which acts more like an internal stirrup. The externally bonded FRP, as tension reinforcement for flexural strengthening, can also be prestressed in order to take advantage of the high rupture strain capacities of these materials [Triantafillou and Deskovic (1991)]. There are multiple successful examples of the use of FRP composites in new construction and for existing structures, under all possible techniques. The mechanics, uncertainties, effectiveness, efficiency and feasibility of each of these techniques vary considerably. However, the use of FRP composites for structural rehabilitation (strengthening and retrofitting) is more popular than for new constructions. Within structural rehabilitation, the use of surface mounted non-prestressed externally bonded FRP reinforcement is widely popular compared to the other installation techniques.

1.6 Rationale of research

1.6.1 Motivation

The use of the terms ‘conservative’ or ‘conservativeness’, in a general context of engineering design, and in a particular context of structural design, is commonplace. For example, the latest (third) edition of TR55 (2012) – the design document of the Concrete Society, UK, concerning the use of FRP-based structural strengthening systems for concrete structures – has used the term ‘conservative’ at twenty two
different places well-spread within the document. ACI440.2R (2008) – an analogous strengthening design document of the American Concrete Institute – has used this term at eight places, mainly while narrating the basis of the document to declare that the design criteria in entirety within ACI440.2R (2008) are believed to be conservative. In spite of the frequent use, there is a substantial lack of discussions within the published scientific literature and the practiced design documents on the importance and role of ‘conservativeness’ in engineering design. In general, ‘conservativeness’ is believed to be a technical synonym for ‘safety’, but most commonly it is subjectively used to convey something that is ‘safe enough’ without any tangibility attached to it. Fig. 1.6 shows the number of papers published annually in last 20 years on the themes (safety + engineering design) and (conservative + engineering design). It can be seen that the later is about 10% of the former, and most of them have involved a subjective meaning of the term ‘conservative’. The basic question – Does this subjective connotation of conservativeness portray its true meaning and implications in design? – forms the motive for this thesis.

1.6.2 Scope

The assessment of design processes for FRP-based structural strengthening systems involves a wide spectrum of issues. It is essential, therefore, to have a balanced scope of research so as to ensure that all the necessary research ingredients are included, yet the treatment remains precisely focused on the vital objectives and expectations. The following points summarise the agreed scope of this research:

• Reinforced concrete structures – FRP composites have been demonstrated to be an effective means for strengthening of reinforced concrete, prestressed concrete, steel, masonry and timber structures. However, the scope of this study is limited to RC structures only. Concepts and methodologies proposed in this thesis, however, can
equally be extended to the other types of structures, of course with appropriate contextual modifications.

- **Flexural and shear strengthening design** – This study focuses on flexural and shear strengthening design processes for RC flexural members only. However, it can be extended to include confinement-based axial strengthening of compression members following a similar line of action presented in this study.

- **Surface mounted non-prestressed externally bonded reinforcement** – This study deals with surface mounted (SM) non-prestressed externally bonded FRP reinforcement only. Again, the concept developed within this study can be extended to near surface mounted (NSM) and prestressed FRP reinforcement applications as well.

- **Deterministic approach** – The safety assessment of structures is more popularly presented probabilistically in literature. However, FRP composites are relatively new and offer a range of material types, each with considerably different characteristics. Thus, they suffer from the shortage of statistics needed for reliable and confident probabilistic description of the design variables representing all types of FRP. Furthermore, many of the avenues related to FRP-based structural strengthening are the matters of active research. It is highly likely that existing design criteria will be altered with improved understanding of the mechanics and behaviour of FRP composites. Hence, it is more appropriate to be deterministic while dealing with the present generation of strengthening design guidelines. Therefore, the methodologies in this study are formulated deterministically. However, they can be converted into nondeterministic format (including probabilistic, reliability based, or even artificial intelligence based approaches) at a later date easily using conceptually the same line of action.

- **ACI440 and TR55** – The methodologies developed as a part of this research are calibrated based on the design criteria specified in: (i) document ACI440.2R (2008) of the American Concrete Institute (ACI), hereafter referred to as ACI440, and (ii) technical report 55 – TR55 (2004) second edition of the Concrete Society, UK, hereafter referred to as TR55. Use of these two guidelines is primarily for demonstration purpose, and for the fact that these are two of the important international design guidelines for FRP-based structural strengthening popularly followed worldwide. It is, however, to be noted that this study does not aim for any explicit comparisons between these guidelines. Nevertheless, the methodologies can equally be extended to any design guideline with appropriate calibration.
1.6.3 Objectives

The overall objective of this thesis is to provide an intuitive interpretation of the conservativeness associated with the design of FRP-based structural strengthening systems. In essence, this study aims:

• To provide a framework to track conservativeness within strengthening design processes.

• To demonstrate how the means of producing conservativeness can influence the course of strengthening design process and the quality of the strengthening design solutions.

• To highlight negative consequences of the strategy and means employed to produce additional conservativeness in strengthening design.

1.6.4 Significance

It is argued within this thesis that ‘conservativeness’ in structural engineering design has much wider scope, utility and implications than merely indicating whether something is ‘safe enough’ in subjective terms. It is shown that conservativeness is not only quantifiable, but there exists an exhaustive taxonomy and a systematic hierarchy of ‘conservativeness’ to be exploited within the engineering design process in general, and within the design of structural strengthening systems in particular. It is also shown that an arbitrary inflation of traditional safety factors does not outrightly ensure conservativeness, especially in the context of FRP-based rehabilitation of structures. In fact, there are negative side effects of an unplanned strategy of introducing conservativeness within the design. These are highlighted within this study in terms of the implications of the means and formats used to produce conservativeness on the course of strengthening design process and on the quality of resultant strengthening design solutions. Thus, this study aims at broadening the understanding of the safety format used in design of FRP-based structural strengthening systems. The inferences derived from the findings of this study will provide a basis for rationalising the safety format. This, in turn, will facilitate a more efficient use of FRP composites for structural strengthening.

This study also sets a narrative for investigating how conducive a civil engineering design process is for accommodating relatively newer and lesser time-tested construction materials, e.g. FRP composites, into the mainstream practice, ensuring that the established safety norms are satisfied. From an engineering standpoint, this is an intricate issue that demands a crucial balance between the safety-related risk arising
from being under-conservative in order to be liberal, and the possibility of jeopardising the economics and application potential of the new material arising out of being over-conservative by imposing heavy penalties on them in order to be stringent. Furthermore, such a study can also be used for bringing the broader concepts of robustness and resiliency into the engineering design. This is essential in combating the issues that are faced by the construction industry, e.g. the global infrastructure crisis. Thus, comprehending ‘conservativeness’ as a tangible design characteristic can pay a long-term returns and a great deal of dividends.

1.6.5 Timeliness of this research

The dedicated research efforts, mainly since the 1980’s, towards utilising FRP composites for structural strengthening and retrofitting have led to the appearance of the discrete design criteria in the late 1980’s. This was followed by the first generation strengthening design guidelines in the 1990’s [e.g. BRIJ (1998), ACI440 (1995)]. However, these design guidelines cater only certain aspects of strengthening and/or retrofitting using selected FRP materials and techniques. The early 2000’s saw the emergence of the second generation of strengthening design guidelines [e.g. FIB14 (2001), TR55 (2001), ISIS (2001), ACI440.2R (2002)]. By this time, the interests of the researchers and design engineers in utilising FRP composites towards structural strengthening and retrofitting, and that of the manufacturing industry for tapping its commercial advantages had grown significantly. This led to the emergence of the third generation of strengthening design guidelines very soon from the mid 2000’s until the present time [e.g. ACI440 (2005), TR55 (2004), ACI440 (2008), HKG (2010) and TR55 (2012)].

The design guidelines have consistently preserved the conservative safety formats throughout their evolution. The possible justification for this could partly be the continued lack of time-testimony and deficiency of reliable data for the use of FRP, and partly the reason that still many issues related to the mechanics and behaviour of FRP are under active research consideration. Nevertheless, the safety formats employed by most of these design guidelines remain largely cloned from that used by the local parent design standards for new construction using conventional structural materials. While this appears logical at a first glance, it is to be noted that FRP composites in general, and their use as an externally bonded reinforcement in particular, have distinctions in terms of mechanics and behaviour when compared to the conventional structural materials. Thus, a blind extension of the conventional safety format to FRP-based structural strengthening design could be inappropriate. However, almost all strengthening design guidelines involve this basis, with quantitatively magnified safety
factors compared to those usually prescribed in structural design using conventional materials believed to provide additional conservativeness as an apparent shield against the possible inconsistencies. The implications of such magnifications on the strengthening design process have attracted no attention in the literature. Many societies responsible for producing the strengthening design guidelines are intending to revise their documents from the third to the fourth generation in the near future. Thus, an investigation aimed at systematically studying the implications of conservativeness in strengthening design practice and providing intuitive interpretations based on this will be a very timely exercise.

1.6.6 Structure of thesis and organisation of research

The following specific tasks are devised to meet the objectives of this study:

**Task 1: Identifying issues concerning FRP-based structural strengthening systems**
- To review materials, uncertainties, mechanics and behaviour, design processes and design guidelines (presented in Chapter 2)
- To categorise major commonalities and differences on different aspects of strengthening design, and to highlight important intricacies and design conflicts (presented in Chapter 2)

**Task 2: Developing a representative conservativeness framework for FRP-based structural strengthening systems**
- To contextualise conservativeness in estimation of FRP resistance contribution (presented in Chapter 3)
- To develop detailed taxonomies of uncertainties and safety parameters concerning FRP-based structural strengthening systems (presented in Chapter 3)
- To develop a comprehensive mapping between the uncertainties and safety parameters (presented in Chapter 3)
- To develop a representative model depicting how the safety parameters are integrated within strengthening design (presented in Chapter 3)

**Task 3 and 4: Assessment of flexural and shear strengthening design**
- To develop generic assessment methodologies for flexural and shear strengthening that:
  - can be calibrated to various strengthening design guidelines
  - can produce a range of strengthening design solutions
  - can qualitatively characterise the strengthening design solutions
• To calibrate the assessment methodologies against ACI440 and TR55 specifications for demonstration purpose (presented in Chapters 4 and 5)

• To characterise a range of flexural and shear strengthening design solutions (presented in Chapters 4 and 5)

• To test the clustered flexural and shear strengthening design solutions under different design scenarios to demonstrate the implications of the means of producing conservativeness on (presented in Chapters 4 and 5):
  ▪ the course of flexural and shear strengthening design processes
  ▪ the quality of flexural and shear strengthening design solutions

• To derive implications based on the above analyses from a designer’s and a calibrator’s perspective (presented in Chapters 4 and 5)

• To develop a set of rules that can serve as knowledge-base for decision making during calibration and design (presented in Chapters 4 and 5)

• To propose a set of suggestions and recommendations for structural strengthening design guidelines. These are presented in Chapter 6.

Fig. 1.7 presents an overall picture of the organisation of the research in form of a flowchart.
Figure 1.7
Flowchart showing organisation of research tasks
Review
2.1 Chapter objectives and structure

The purpose of this review is to provide an insight into the commonalities, differences and conflicts within the criteria, processes and safety formats employed in structural strengthening design. This review cuts across major issues that influence design of FRP-based structural strengthening systems, and suggests how this study orients itself in order to interpret conservativeness in strengthening design process and to comprehend its implications. In essence, it sets a narrative for the conservativeness assessment methodologies and their applications proposed within this study.

FRP composites and their application as externally bonded reinforcement for structural strengthening involve peculiar characteristics when compared to conventional structural materials and their applications in civil engineering. These include the type, nature and extent of uncertainties associated with FRP-based structural strengthening systems, and the mechanics and short- and long-term behaviour of these systems. The consequences of these peculiarities are seen in the design criteria and processes for FRP-based structural strengthening systems. In fact, the objectives and working algorithms for strengthening design processes are considerably different than the traditional structural design conventions. Additionally, different strengthening design guidelines employ different analytical models to predict various experimentally observed characteristics of externally bonded FRP. Furthermore, there are areas of conflict amongst individual design guidelines, and between strengthening design guidelines and design standards for new construction. All these disparities, when woven within the fabric of different design philosophies, e.g. limit state design (LSD) or load and resistance factor design (LRFD), add further to the complexities. Thus, a diverse range of issues are involved in a research investigation aiming at tracking conservativeness and its consequences within the strengthening design processes. Five major issues concerning the design of FRP-based structural strengthening systems, along with an appraisal of intricacies and opportunities associated with them, included within this review are:

- Material properties
- Uncertainties and conservativeness
- Mechanics and behaviour
- Design process
- Design guidelines
Focus of Chapter 2: Review
2.2 FRP-based structural strengthening systems: Material properties

A typical FRP-composite comprises of continuous structural fibres that are bound together and held in position by the resin matrix to achieve a unidirectional composite material. Very commonly used structural fibres include glass, carbon and aramid, and very recently basalt fibres are also being used. Based on the fibre material, the resultant FRP is called Glass FRP (GFRP), Carbon FRP (CFRP) or Aramid FRP (AFRP). Each of these fibres carries distinctive mechanical and chemical properties, and hence the long- and short-term properties of the resultant FRP composites made of different types of fibres vary significantly. Many options for the resin exist, of which the most commonly used is epoxy [Teng et al. (2002)]. A summary of the mechanical properties of various FRP composites is presented in Table 2.1.

<table>
<thead>
<tr>
<th>FRP Composite</th>
<th>Modulus of elasticity (GPa)</th>
<th>Tensile strength (MPa)</th>
<th>Rupture strain capacity (%)</th>
</tr>
</thead>
</table>
| CFRP  
General purpose | 220-240 | 2050-3790 | 1.2 |
| High strength | 220-240 | 3790-4820 | 1.4 |
| Ultra high strength | 220-240 | 4820-6200 | 1.5 |
| High modulus | 340-520 | 1720-3100 | 0.5 |
| Ultra high modulus | 520-690 | 1380-2400 | 0.2 |
| GFRP | 69-90 | 1860-4140 | 4.5-5.4 |
| AFRP  
General purpose | 69-83 | 3440-4140 | 2.5 |
| High performance | 110-124 | 3440-4140 | 1.6 |

The following are the advantages of FRP composites over the traditional construction materials [GangaRao et al. (2007)]:

*Versatility:* A wide range of possible fibre-resin combinations presents a spectrum of material properties (e.g. modulus of elasticity, tensile strength and rupture strain) of the FRP composites for the designer to choose from to arrive at a tailor-made strengthening design solution.

*Superiority:* Compared to the conventional structural materials, FRP composites possess high strength-to-weight ratio and negligible corrosion potential. Most FRP composites offer a significantly high rupture strain capacities and high tensile strength compared to the conventional structural materials.
Convenience: The handling and transportation of FRP composites are more convenient compared to that for the traditional construction materials.

Adaptability: The adaptability of FRP composites to suit various sizes, shapes and corners, and the possibility of in-situ manufacturing make them particularly useful for structural rehabilitation.

The limitations of FRP composites over the traditional construction materials include [GangaRao et al. (2007)]:

Lack of plasticity: FRP composites behave linearly elastic right up to the failure, with practically no plastic deformability.

Directional orthotropy: The mechanical properties of FRP in principal direction of fibre and that on the orthogonal directions are substantially different. This raises issues in anchorage-based detailing requirements.

Multiplicity of failure modes: The use of FRP as an externally bonded reinforcement to an existing RC flexural member expands the possibility for a strengthened RC section at ultimate condition to fail. Thus, there are a significantly large number of possible failure modes for an FRP-strengthened RC member.

Temperature susceptibility: The fibre, resin and their composite action are susceptible to temperature change.

Thermal incompatibility: Many types of FRP composite materials do not possess a coefficient of thermal expansion compatible with that of steel and concrete.

Lack of time-testimony: Above all, due to the relatively short history of use in construction industry, the long-term behaviour and response of FRP composites are less confidently known.

Data deficiency: Due to lack of reliable experimental data, the statistical treatment of FRP composite applications is liable to scepticism.

From the above review it can be appreciated that:

- FRP composites produced by combining different fibre and matrix materials offer a wide spectrum of mechanical properties. Thus, there is a possibility of devising a tailor-made, efficient and cost-effective strengthening design solution by selecting an appropriate FRP material that best suits the strengthening requirements.

However, there is a lack of criteria for the designers to arrive at the optimal
material selection. One of the aims of this study is to provide mathematical basis for choosing appropriate FRP materials for ensuring their efficient use.

- While the need of FRP-based strengthening design to be more conservative than usual due to the lack of time-testimony and data deficiency can be appreciated, this study questions the effectiveness of the means employed to produce higher conservativeness within strengthening design.

### 2.3 FRP-based structural strengthening systems: Uncertainties and conservativeness

Uncertainties prevail in every facets of life. Donald Rumsfeld, a US statesman, conveyed this fact rather poetically in 2002, with saying–

\[
\text{As we know,} \\
\text{there are known knowns –} \\
\text{there are things we know we know.} \\
\text{We also know} \\
\text{there are known unknowns –} \\
\text{we know there are some things we do not know.} \\
\text{But there are also unknown unknowns –} \\
\text{the ones we don’t know we don’t know.}
\]

The above quote, in spite of being poetical, very truly reflects the reality that uncertainties are inevitable in any engineering design process. The type and extent of uncertainties, and their design treatment vary from process to process. The more uncertainties in a problem, the less precise we can be in our understanding of that problem [Ross (2010)]. We, the human, due to our inherent fear of the negative consequences of our decisions made under these uncertainties, instinctively tend to be conservative [Kansara and Ramanjaneyulu (2007)]. A brief overview of how uncertainties in engineering design is conceived, and a general literature review of uncertainties associated with FRP-based structural strengthening is presented in this review. This discussion sets a line of action for a more detailed treatment of uncertainties from the subject point of view, which is discussed at length in Chapter 3 of this thesis.

#### 2.3.1 Uncertainties in engineering design

Haldar and Mahadevan (2000) have classified sources of uncertainties in engineering design into cognitive sources and non-cognitive sources. The non-cognitive (or qualitative) uncertainties involve vagueness of the problem arising from intellectual abstractions of reality and can arise from the definitions of certain parameters (e.g.
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structural performance, failure, quality, deterioration, skill and experience of construction workers, environmental impact of projects and condition of existing structure), human factors and definitions of the interrelationships among the parameters of the problem [Ayyub (1994) and Ayyub (1998)]. The non-cognitive uncertainties also arise from the need of meaningful interpretations of the subjective empirical observations, experimental data or analytical outputs [Kansara and Ramanjaneyulu (2007)]. The cognitive (or quantitative) sources of uncertainties, on the other hand, are relatively more crisp and quantifiable, and hence involve much less subjectivity compared to the uncertainties arising from the non-cognitive sources. The non-cognitive uncertainties essentially are implicitly described (or need a specialist treatment to describe them), unlike the cognitive uncertainties, which can be relatively more explicitly described.

In a more general sense, Ayyub and McCuen (2003) and Ayyub and McCuen (2011) have separated uncertainties into aleatory (i.e. related to luck or chance) and epistemic (i.e. related to knowledge). Bulleit (2008) explains that the former arises from the randomness inherent in nature, while the latter is dependent on human knowledge. It is further professed that increasing the knowledge about the area of interest can, in theory, reduce the epistemic uncertainties However, in practice there are uncertainties where it is not obvious how to categorise them. This, probably, means that an aleatory uncertainty for one scientific discipline could be an epistemic uncertainty in context of other scientific disciplines. Thunnissen (2003) has provided a comprehensive summary of the taxonomies of uncertainties from social sciences to economics, through systems, computer, mechanical, civil and structural engineering. The uncertainties pertinent to structural engineering, Thunnissen (2003) debates in light of the discussions provided by Ayyub and Chao (1998) and Melchers (1999), covers phenomenological, decision-related, modelling-related, prediction-related, random, statistical-simplification related or human-factors related uncertainties.

2.3.2 Uncertainties in FRP applications

Zureick et al. (2006) presented statistical characterisation of FRP composite properties, focusing mainly on the random variability of FRP material properties as uncertainties. Atadero and Karbhari (2009) discussed sources of uncertainties associated with the FRP material properties and the process of derivation of their design values. However, their discussion concerns the field-manufactured FRP only. Nevertheless, field manufacturing of FRP is still a popular method used for structural strengthening. Atadero and Karbhari (2009) highlighted that possible variations in the quality control
and variability in field conditions are the most significant sources of uncertainties for the field-manufactured FRP. In addition, uncertainties also result from how the manufacturer determines and specifies the material properties. It was further suggested that the cumulative effect of the variability in field conditions, and that in the specification of the in-situ properties of FRP, could influence the mean and the level of variability (e.g. standard deviation or coefficient of variation). Under this argument, the statistical description of the FRP material properties, and hence their further probabilistic treatment can be contested. This is one of the reasons justifying why the scope of this thesis is limited to deterministic treatment of the problem at hand.

A very large segment of literature discussing uncertainties associated with the FRP-based structural strengthening proceeds from the premise that ignore the uncertainties inherent in pre-strengthening stage. This, in essence, slackens the already poor engineering coordination between the pre- and post-strengthening design phases [Kansara and Ramanjaneyulu (2007)]. Very limited literature is available that considers the inherent uncertainties associated with an RC section being strengthened. Wieghaus and Atadero (2010) presented a discussion on the effect of existing structure and FRP uncertainties on the reliability of the FRP-based repairs, and have shown that the pre-strengthening uncertainties present in the existing structure has a considerable impact on the post-strengthening reliability of the structure. Most design guidelines ignore this aspect, by-and-large.

2.3.3 Uncertainties and conservativeness

The degree of conservativeness in estimation of resistance (or strength) of a structural member is inversely proportional to the confidence in the material used for structural purpose. The confidence in the use of a material itself is inversely proportional to the uncertainties related to its properties and behaviour, and directly proportional to the time-testimony available for the material application. This is notionally presented in Fig. 2.1. The uncertainties are generally accounted for through specifically calibrated safety factors, whereas the lack of time-testimony is addressed indirectly, as there are no primary direct means, in a real sense, to account for it. Nevertheless, an important fact asserted from this narration is that conservativeness forms a primary basis for the safety format in any engineering design.
Conservativeness in resistance estimation

From the above review it can be appreciated that:

- Lack of time-testimony and data deficiency are additional obligations, over-and-above the conventional uncertainties, within the design of FRP-based structural strengthening systems. Amongst all the leading literature on uncertainties, in general, discussion on statistical variability dominates. Lack of knowledge is generally clubbed with the statistical adjustment of FRP material properties, and is believed that conservative models employed within the design will account for this. The discussion on lack of time-testimony is largely absent in the literature. For FRP being relatively new and less time-tested within the construction industry, these are important issues. In fact, most strengthening design guidelines tend to be conservative due to these reasons.

- Detailed classifications of the uncertainties associated with the design of FRP-based structural strengthening systems, and safety parameters accounting for these uncertainties are presented in Chapter 3. An exhaustive mapping between the classified uncertainties and safety parameters is also developed, which provides a basis to present a comprehensive framework depicting not only the interrelation between the uncertainties and safety parameters, but also the role and potential of the safety parameters in producing conservativeness and influencing the strengthening design process.
2.4 FRP-based structural strengthening systems: Mechanics and behaviour

The most important characteristics concerning the mechanics of FRP as an externally bonded reinforcement for structural strengthening include ductility, debonding and anchorage of FRP, whereas durability of FRP composites is an issue that concerns its long-term behaviour. A review of these aspects is presented here.

2.4.1 Ductility

FRP composites behave linearly elastic right up to their rupture exhibiting a lack of plastic deformation. Therefore, an addition of externally bonded FRP reinforcement to an existing RC flexural member towards flexural strengthening poses ductility related implications. Such an addition of reinforcement on the tension side of an RC beam brings section nearer to an over-reinforced strain-state. Indeed, in many design scenarios, the amount of FRP needed for meeting serviceability requirements will lead to an over-reinforced strain-state in post-strengthening phase. An over-reinforced strain state, in fact, relieves strains in the tension steel reinforcement, and therefore, carries substantially lesser sectional ductility. Due to a 5-10 times higher rupture strain capacity of FRP composites compared to the yield strain of steel [GangaRao (2006)], a ductile post-strengthening behaviour of an RC member can still be expected. This possibility depends upon the level of strain in FRP reinforcement at failure. However, an externally bonded FRP derives its reinforcing action by transferring stresses to the concrete substrate through bond mechanisms [Leung (2006)]. The relatively weak tensile stress carrying capacity of the concrete substrate leads to a failure of the bond between FRP and concrete, which limits the strain level in FRP at failure. This bond failure is referred to as debonding of FRP, which is discussed in the following sub-section. Thus, in practice, the strain level in an externally bonded FRP reinforcement remains limited to a considerably low fraction of the rupture strain capacity of FRP material. Consequently, the strain level in tension steel also remains limited leading to a substantial possibility of lack of sectional ductility in the post-strengthening stage.

Various approaches are prescribed in strengthening design to address post-strengthening ductility needs. Matthys and Taerwe (2006) discussed four such approaches. First amongst these approaches involves ensuring minimum deformability by either restricting the depth of compression zone (by limiting the depth of neutral axis to an appropriate upper bound value), or ensuring a minimum strain level in tension steel reinforcement. The second approach involves ensuring sufficient deformability by controlling the governing post-strengthening failure mode. The possibilities of FRP to fail through debonding or rupture, and concrete to crush under compression present a
large number of potential post-strengthening failure modes. Each of these failure modes involves a different potential for deformability, depending upon whether or not these failure modes involve straining of tension steel reinforcement. Therefore, a preference can be arrived at for a certain desirable failure modes involving larger potential for deformability over the rest. The design process needs to be iterated, mainly on a trial-and-error basis, such that a valid strengthening design solution governed by a desired failure mode is reached. The above two are ‘regulatory approaches’ towards addressing post-strengthening ductility needs, and ductility here corresponds to deformability rather than energy dissipation capacity. The third approach discussed by Matthys and Taerwe (2006) is ‘compensatory’, in the sense that it recommends a compensation for the lack of deformability in form of a safety factor that penalises the nominal flexural resistance of a strengthened RC section. However, if a strengthening design solution is governed by serviceability requirements, for example, the amount of FRP required for strengthening may be considerably higher than what is needed for meeting strength requirements. In this case, in spite of obtaining a higher safety margin between the acting design loads and the design resistance, achieving sufficient deformability is difficult. The fourth approach is ‘restrictive’ in the sense that it sets a limit on the degree of strengthening that can be permitted to achieve. This, in turn, restricts the use of high amount of FRP and consequently provides an implicit guarantee of deformability.

- The above review helps visualising the vital importance of ductility within FRP-based strengthening, especially for flexural strengthening. It is rather a tricky design problem that demands a flexural strengthening design solution to include an ‘over-reinforced’ strain-state, and yet to ensure sufficient straining of tension steel reinforcement. It is difficult to accommodate the preference for a desired post-strengthening failure mode to govern a flexural strengthening design solution within the design.

- This study proposes a novel definitions of post-strengthening failure modes, with ductility (in form of deformability through straining of the tension steel) as a central character. These definitions facilitate considering ductility simultaneously with strength requirement within structural strengthening design.

2.4.2 Debonding

The external bonding of FRP reinforcement to the concrete substrate involves two distinct interfaces – the FRP-adhesive interface and the adhesive-concrete interface.
Thus, the strengths of the FRP, adhesive and concrete substrate govern the overall effectiveness of the strengthening system. Typically, FRP composites are much stronger than adhesives and concrete [GangaRao et al. (2006)]. A heavy factor of safety on adhesive materials (of the order of 4.0) [TR55 (2004)] exempts the adhesive strength to govern the effectiveness of the strengthening system. The weakest link in the strengthening system, therefore, is the concrete substrate, and hence the soundness and the tensile strength of the concrete substrate present a serious concern on the structural integrity and effectiveness of the strengthening systems [GangaRao et al. (2006), TR55 (2004)]. There can be a state beyond which the force in the FRP can no longer be sustainably transferred to the concrete substrate. This state is referred to as debonding of the FRP reinforcement under which the FRP reinforcement tend to delaminate leading to a local or significant loss of the composite action and physical integrity of the strengthening system.

Typically, debonding occurs at much lower strain levels in the FRP reinforcement compared to its rupture strain capacity, and hence debonding is considered to be an undesirable (yet inevitable) premature failure mode. Debonding can occur in different forms and can be triggered by a variety of parameters. Fig. 2.2 presents a summary of various possible forms of debonding of an externally bonded FRP reinforcement. Some of the forms of debonding can be effectively mitigated through appropriate detailing prescriptions, imposing stress limitations and/or using mechanical anchorages. However, certain forms of debonding (such as the internal-crack induced debonding, popularly known as the IC-debonding) cannot be regulated through such means and essentially have to be accommodated in the design as a possible failure-mechanism of the externally bonded FRP.

Debonding of an externally bonded FRP reinforcement generally takes place in the regions of high stress-concentrations and is often associated with the material discontinuities and presence of cracks. It presents serious concerns regarding the effectiveness and safety since it is a brittle mode of failure, and may significantly influence the efficiency and effectiveness of the strengthening. Hence, debonding has been the focus of interest of the theoretical and experimental researchers worldwide in last few decades. In the last few years in particular, there has been considerable effort to understand, model and frame design proposals for debonding.
Buyukozturk et al. (2004) have provided a review of the progress made in recent years in understanding and modelling debonding phenomena for FRP-strengthened RC and steel structures. It discusses observations based on experimental investigations and suggests that existing steel reinforcement ratio and type and amount of external FRP reinforcement significantly influence debonding failure behaviour. For a given FRP ratio, the thickness of FRP is also considered to be a significantly influencing parameter. It is also highlighted that improper selection of adhesives may lead to debonding failure. It is further revealed that debonding failure load and ductility decreases with a decrease in the bonded length of FRP reinforcement. It is also suggested that fatigue loading and various environmental exposures including freeze-thaw, wet-dry and temperature cycles and various aqueous solutions also lead to debonding problems in structures. The modelling proposals of debonding were grouped under distinct types based on the approach used such as strength based, fracture based and empirical and semi-empirical approaches.

Leung (2006), towards clarifying some conventional belief, highlighted inapplicability of elastic models for debonding failure predictions and advocated fracture mechanics
based models. The work presents effect of cracking on stresses near the plate end and effect of plate thickness on crack-induced debonding through experimental investigations. Finally, it suggests the use of U-stirrup of FRP covering the longitudinal FRP towards increasing resistance against debonding of longitudinal FRP. An optimal place for such stirrup with respect to the longitudinal FRP plate end is also suggested based on measured bonding stress along the length of FRP.

Aprile and Feo (2007) shown that accuracy of any analytical model aimed at predicting rip-off load of RC beams externally strengthened with FRP is highly dependent upon the evaluation of mean crack spacing in shear zone. They presented a comparative study of different mean crack spacing evaluation models and proposed an analytical model to predict the rip-off failure load under uniform load conditions.

Pesic and Pilakoutas (2003) presented an analysis towards assessing suitability of analytical and finite element approaches for predicting debonding failure. Niu and Wu (2005) presented numerical analysis in form of fracture energy based nonlinear finite element analysis of debonding mechanisms in FRP-strengthened RC beams. They have employed a discrete crack model for concrete crack propagation and a bilinear bond-slip model with softening behaviour for FRP-concrete interface. A parametric study to investigate effects of crack spacing and interface influencing variables such as stiffness, local bond strength and fracture energy on initiation and propagation of the debonding and structural performance has been presented. Based on this, it is concluded that flexural crack spacing and interfacial fracture energy have significant effect on the interfacial debonding mechanism and load carrying capacity and that the stiffness of FRP-concrete interface has insignificant effect on ultimate load carrying capacity. Niu et al. (2006) presented a study on diagonal macro-crack induced debonding failure initiation and propagation based on fracture mechanics based finite element approach.

From the above review, it can be appreciated that:

- Debonding of an externally bonded FRP reinforcement is one of the critical phenomena that affect the strain level that can be achieved in FRP reinforcement at failure. This, in turn, influences the effectiveness and efficiency of FRP-based structural strengthening systems.
- The methodologies proposed within this thesis clearly demonstrate the implications of the design criteria on debonding on the course of strengthening design process.

### 2.4.3 Bond and anchorage

One of the most important peculiar distinctions between the externally bonded FRP reinforcement and the conventional internal steel reinforcement is the bond behaviour. The adhesive bond is essentially inevitable in any external bonding application of FRP not only for the structural reasons but also for the practical reasons. However, the relative importance of adhesive bond for a certain structural configurations of externally bonded FRP reinforcement is higher compared to the others. Usually, most possible structural configurations in flexural strengthening, and the sides-only and U-wrapped configurations in shear strengthening fall into this category, and are classified as bond-critical applications [Foster and Bisby (2008)]. These applications usually require an adhesive bond between the FRP reinforcement and the concrete substrate, and the structural efficiency and efficacy of such applications largely rely on this bond. The strength and condition of the existing concrete substrate are important parameters in such applications [ACI440 (2002)]. Therefore, the quality of workmanship in surface preparation before attaching FRP reinforcement can influence the bond behaviour [ACI440 (2002)]. Additionally, the polymer at the fibre-concrete interface is also an important factor in maintaining this bond. It is to be noted that at a temperature nearer to its glass-transition temperature, a substantial reduction in the mechanical properties of the polymer takes place, which significantly jeopardises the stress-transfer from the concrete substrate to FRP reinforcement through the bond [Foster and Bisby (2005), GangaRao et al. (2006)].

The fully wrapped configuration in shear strengthening (and all configurations for confinement of columns for seismic retrofitting, which are beyond the scope of this thesis) is classified as contact-critical application. Such applications derive structural efficiency and efficacy from the criticality of the contact between the substrate and the FRP reinforcement [Sen et al. (2005)]. The stress transfer between the participating structural components takes place through bearing or horizontal shear transfer at the
interface. The stress in FRP is built up as a consequence of deformation and/or dilation of the concrete section, which is confined by the surrounding FRP reinforcement. The adhesive bond is theoretically not required in such applications, especially for circular column under direct axial compression confined with an FRP jacket. Though an adhesive bond may still be required for a rectangular section fully-wrapped for shear strengthening, a circular column under direct compression and bending confined with an FRP jacket, it is a less important criteria compared to a bond-critical application. Nevertheless, an adhesive bond is generally applied even in contact-critical applications to facilitate installation and to maintain the required configuration [Mirmiran (2004)]. Obviously, the strength of the concrete substrate is not then as important as it was for bond-critical applications. However, surface preparation, that includes removal of low strength low modulus materials (such as plasters), patching of large voids through non-cosmetic repair materials (which is compatible with the existing substrate concrete), and making the surface flat or convex shaped, does influence the effectiveness of such applications [Mirmiran (2004)]. A brief discussion on bond and corresponding anchorage length for flexural and shear strengthening systems is presented here.

(A) **Bond and anchorage in flexural strengthening**

For the conventional internal steel reinforcement, an increase in length of anchorage results into an increased bond failure force. Unlike this, for the externally bonded FRP there exists an upper limit on the increase in the bond failure force with increasing length of anchorage [GangaRao (2006)]. Beyond a certain threshold value, any additional increase in the length of anchorage does not show any corresponding increase in the bond failure force [Neubauer and Rostasy (1997)]. If the actual geometrical constraints do not allow providing the required length of anchorage to activate the maximum bond failure force (or the threshold bond strength), the actual bond force carrying capacity to be considered in design should be reduced in accordance with the provided length of anchorage.

(B) **Bond and anchorage in shear strengthening**

Maeda et al. (1997) and Khalifa et al. (1998) have made experimental observations confirming that the tensile force carried by an FRP reinforcement spanning over a shear crack does not depend upon the bond length of the FRP. It is believed that in the early stage, the tensile load is sustained by the bond available within an area in the vicinity of the tension force, called the active (or effective) bond area. When the delamination of
FRP occurs within this area due to concrete fracture (which is very likely to occur), the bond capacity of FRP within this area is lost and shifted to a new area. This process is repeated until the delamination propagates through the entire FRP bonded length. Thus, the bond stress is transferred only in the active bond area (unlike bond failure mechanism in flexural strengthening). The length of FRP within the active bond area is called the active (or effective) bond length.

(C) Transition of longitudinal shear stress beyond anchorage zone

Since the FRP reinforcement is bonded externally to an existing RC member, it is essential to see that the force transfer between the FRP and concrete substrate is compatible with the stress-transfer mechanics of the concrete and internal steel reinforcement. This would necessitate ensuring two stress-transfer conditions:

- The stress-transfer between the anchorage and non-anchorage zone remains sufficiently gradual.
- The stress-transfer on either-side of a section involving inflection (e.g. points where section property changes, discontinuity in shear force (due to point load, for example) exists and where the internal steel reinforcement undergoes from elastic to post-elastic strain-state).

- The above review highlights the importance of the bond and anchorage length aspects in strengthening design.
- For flexural strengthening these aspects are more important from the detailing point of view. For shear strengthening these aspects form a basis to classify the sides-only, U-wrapped and fully wrapped configurations according to their bond criticality. Each of these configurations is assigned a bond reduction factor based on their potential for reduction in the bond carrying capacity.
- These aspects are the prime focus in the methodologies proposed within this study.

2.4.4 Durability

The durability of FRP composites and that of the FRP-strengthened structures largely influence the quantitative prescriptions of the safety factors in strengthening design guidelines. The mechanical properties of FRP are reported to show sizable reduction under certain environmental exposure including high temperature, humidity and chemicals based on accelerated exposure laboratory tests [ACI440 (2008)]. Karbharai et al. (2003) highlighted some critical gaps in durability of FRP composites. They defined durability of FRP as its ability to resist cracking, oxidation, chemical degradation, debonding, wear, fatigue and the effects of damage from foreign objects for a specified
period of time under appropriate loading conditions and under service exposure conditions. Soudki and Spadea (2006) and Bencardino et al. (2006) suggested need of developing holistic design approach with considering durability and ductility as two sides of a coin.

David and Neuner (2001) discussed environmental durability studies and suggested what conditions of use can be considered as normal conditions for structural strengthening applications. Karbhari (2003) attempted to present some reality on the long-term durability related issues and highlighted that there exists some myths regarding FRP composites’ durability. It has been demonstrated that it is a myth to believe FRP composites as water-proof as moisture can diffuse into composites and can lead to changes in their thermo-physical, mechanical and chemical characteristics. Furthermore, it is shown that the moisture primarily causes both reversible and irreversible changes in the polymer structure of the resins. It also reduces the glass transition temperature and for this reason it is suggested that the FRP should have glass transition temperature above 30°C at the least. Significantly higher reduction (more than 60 %) at the end of a twelve-month exposure period has been reported in the tensile strength of E-glass/Epoxy system based on accelerated test. It has been further demonstrated that most reduction takes place in first 6 months of exposure.

GangaRao and Barger (2001) discussed aging of bond between FRP and concrete towards studying rate of degradation of bond strength under alkaline, acid and freeze-thaw conditions. Almusallam et al. (2013) has presented an experimental investigation on E-glass based strengthening system under different climatic conditions including hot-dry and wet-dry climates, and different environmental exposure including immersion in salt water, alkaline water, normal water, direct and indirect sunlight effect with and without ultra-violet protective coats. Byars et al. (2001) compiled current specifications for FRP applications for strengthening. Karbhari and Abanilla (2007) presented durability prediction approaches for CFRP/Epoxy wet layup systems.

conditions on the flexural capacity of FRP and steel strengthened RC beams. Green et al. (2006) presented behaviour of FRP confined concrete columns under extreme conditions.

Very few models for degradation of mechanical properties of FRP composites are available in the literature. Bank et al. (2001) proposed a model specification for FRP composites for civil engineering applications and have also discussed protocol for accelerated aging and prediction of long-term properties. Karbhari (2002) presented a simple degradation model for FRP composites under freeze-thaw conditions. Most of the experimental researches on degradation of FRP Composites under environmental exposures are conducted at material level. Oliveira and Creus (2004) presented an analytical-numerical framework for aging of FRP composites that is useful for compilation, interpretation and application of experimental data to actual engineering analysis and design. Karbhari and Abanilla (2007) presented methodology to estimate long-term deterioration for a range of material characteristics using two commonly used predictive approaches. Richard et al. (2007) discussed life-cycle performance of composites used in construction. Allen and Atadero (2012) presented a case study on evaluating long-term durability of externally bonded FRP through field assessments.

The mathematical basis behind arriving at a quantitative prescription for the safety factor accounting for the deterioration due to environmental exposure (discussed in Chapter 3 in more details) involves estimation of the long-term response of the FRP composites under accelerated-aging tests. The aim is to determine the reduction in the FRP material properties (and characteristics) from a specimen that is immersed in deionized water under a raised temperature-based forcing-function at the end of a specific exposure period. In accelerated conditions, in a time-temperature superposition approach [Karbhari and Abanilla (2007)], the raised temperature speeds up the moisture uptake rate, which provides us a means to simulate a few hours of accelerated testing duration to an equivalent long-term exposure at normal temperature. Popularly, Arrhenius type diffusion models [Karbhari and Abanilla (2007)], typically of the form expressed by Eq. (2.1) for example, are employed that relates a resultant diffusion coefficient analogous to the rate of deterioration \( r \) with temperature \( T \). Here, \( r_0 \) is a constant, \( R_g \) is the universal gas constant and \( E_a \) is the activation energy [Abanilla and Karbhari (2006)]. Having an established rate of deterioration for FRP, the value of an arbitrary FRP material property \( Y \) at time \( t = t_1 \), after an immersion period on real time-scale at standard temperature (e.g. 23°C), equivalent to a short duration immersion
in accelerated test at a higher than standard temperature, can be expressed through Eq. (2.2). Here, $Y_0$ is the value of $Y$ at time $t = 0$.

$$
\tau = r_0 \exp \left[ \frac{-E_a}{R_g T} \right] \quad (2.1)
$$

$$
Y_{(t_{11})} = F_e \left[ Y_0 \times \left[ r \ln(t_{11}) + Q \right] \right] \quad (2.2)
$$

This approach then can be extended to cover other immersion environment, such as salt and alkali solutions, by introducing appropriate modification factor ($F_e$) that relates rates of deterioration under deionised water to such harsh immersion environments. The ratio of a FRP material property that is retained after a specific equivalent exposure condition and duration ($Y_{(t_{11})}$) to its initial un-deteriorated value ($Y_0$) can be used to derive partial factors of safety accounting for various types of environmental exposures. It is to be noted that while such an approach provides a simple and convenient tool for the comparison of the representative standardised life-times, it does not capture the time dependent changes in the mechanisms particularly as related to the damage progression at the interface and fibre levels.

- It can be seen that due to a short history of use of FRP within the construction industry, the durability aspects of FRP-based strengthening systems are predicted through accelerated aging tests. These tests do not cover all possible environmental conditions and many times extrapolations on the durability aspects of FRP-based strengthening systems are made based on small-sampled accelerated testing of FRP coupons, rather than the full-scale strengthening specimens.
- Also, not all possible types and applications of field- and factory-made FRP are covered within such investigations. Therefore, a specific safety factor accounting for durability aspects of FRP, which is functionally different than the partial factor of safety used in design of concrete structures, is needed.
2.5 FRP-based structural strengthening systems: Design process

A typical structural strengthening design process involves the two major phases. In the order of their execution, these are:

(1) Pre-strengthening assessment of the structure, and
(2) Design of structural strengthening system.

The effectiveness, efficiency and reliability of the strengthening design solution are substantially dependent on the effective execution of these phases. These are discussed in brief in this section.

2.5.1 Pre-strengthening assessment

Pre-strengthening assessment of an existing structure is a crucial part of a structural strengthening programme. It aims at [Kansara and Ramanjaneyulu (2007)]:

- Identifying the prevailing state of the condition of an existing structure (called the condition assessment)
- Evaluating the available capacity of an existing structure (called the strength or capacity assessment)

A meticulous capacity assessment based on the inputs from a detailed condition assessment provides a realistic estimate of the available capacity of the structure. These are discussed in brief here.

**Condition assessment:** Condition assessment is generally performed through reviewing existing drawings and design reports, conducting physical surveys and carrying out visual and/or NDT-based inspections. The outcomes of a condition assessment study facilitates:

- Ascertaining if it is feasible to undertake structural rehabilitation for the existing structure at hand
- If yes, then identifying the extent and type of pre-strengthening treatment needed prior to structural rehabilitation
- If no, then identifying the extent and type of repair needed prior to considering load posting of the structure

ACI311.4R (2005) and AASHTO Manual (2010) provide detailed guidelines for inspection and condition assessment for general concrete structures and bridge structures respectively. ACI364 (1999) provides guidance for evaluation of concrete structures prior to undertaking any rehabilitation exercises. In the same line, TR54
Treatment and preparation of concrete substrate prior to applying FRP is a critical task in pre-strengthening stage of any strengthening exercise. The effectiveness and reliability of strengthening significantly depend on pre-strengthening treatments. In general, pre-strengthening treatments include cleaning surfaces, repairing concrete substrate and preparing surface. TR55 (2004) suggests the concrete surface must be cleaned to remove laitance, loose material, fungal growth, oil or grease, corrosion products, previous coatings and mould release agents and curing membranes (if any). Emmons (1993) provides a concise approach and cover entire range of issues related to concrete repair. ACI Concrete Repair Manual (2013) presents complete and detailed descriptions of evaluation, repair and protection issues of concrete. ICR1320.2R (2009) and ACI503 (1998) provide guidance for selecting repair materials for concrete. ICR1310.1R (2008) presents guidelines for surface preparation for repair of deteriorated concrete structures resulting from corrosion of reinforcing steel, while ICR1320.1R (1996) contains the guidelines for selection of application methods for concrete repair. ACI224 (1998) deals with issues related to repair of cracks in concrete structures. TR22 (1992) provides discussion on non-structural cracks in concrete. ACI546 (2001) provides specific guidance on concrete repair techniques. TR55 suggests some methods for surface preparation in order to receive uniformly thick adhesive layer. It is suggested that the flatness of the surface should be such that the gap between a 1 m straight edge should not be more than 5 mm.

Capacity assessment: Structural evaluation primarily is meant to represent strength evaluation of existing structures, and in certain cases it covers stability analysis as well. The outcomes of a capacity assessment study facilitates:

- Estimating the residual strength of various structural elements
- Identifying the required degree of strengthening needed, if condition assessment has confirmed feasibility of structural rehabilitation
- Load posting the structure, if condition assessment has confirmed unfeasibility of structural rehabilitation

The above review reveals that pre-strengthening assessment carried out using either a design standard for new RC structures or an assessment standard for the existing RC structures forms two distinct schools of thought.

The source, nature and extent of uncertainties to be accounted for within an assessment standard are considerably different compared to a design standard. While the design standards have to deal with a ‘virtual’ structure that is yet to be constructed, a real ‘physical’ structure is available for inspections and verifications while assessing an existing structure.

Consequently, the assessment standards are based on ‘average’ values of the design parameters, unlike the design standards, which are based on the ‘characteristic’ values. Also, relatively less stringent partial safety factors are typically prescribed in an assessment standard compared to a design standard [Kansara and Ramanjaneyulu (2005)], attributed to a relatively less variability-based uncertainties.

Due to this philosophical difference at the fundamental level, the use of a design or an assessment standard for pre-strengthening assessment of the structures would lead to different post-strengthening resistance performance. This, in turn, would lead to inconsistency in terms of safety, risk and reliability.

In order to avoid such an inconsistency, a strengthening design guideline, ideally, needs to be appropriately fused with a concerned pre-strengthening assessment in order to have consistency between pre- and post-strengthening states.

In fact, in spite of most strengthening design guidelines popularly suggesting the use of a design standard for pre-strengthening assessment, in the view of the author, it is more logical to employ the assessment standards instead for this purpose.

However, either ascertaining this or suggesting a remedial measure for this does not fall within the scope of this thesis. This thesis considers the pre-strengthening assessment using the documents as suggested in the considered strengthening design guidelines.

Thus, pre-strengthening assessment using BS8110 or BS5400 instead of BD44 is considered for TR55. Similarly, ACI318 is considered to be the concerned pre-strengthening assessment document for ACI440. The methodologies proposed within this thesis, however, are equally applicable to either of the two schools of thought, with appropriate contextual modifications.
2.5.2 Design of FRP-based structural strengthening systems

After the necessary pre-strengthening treatment, the next phase is to carry out structural design for an appropriate FRP-based strengthening system to achieve the required degree of strengthening estimated during pre-strengthening assessment. Depending upon the case-specific requirements, an existing flexural RC member to be rehabilitated may need either flexural or shear or both strengthening.

Saadatmanesh and Malek (1998) provided design guidelines for flexural strengthening of RC members with externally bonded FRP plates. These guidelines aimed at predicting ultimate strength of flexurally strengthened RC beams using externally bonded FRP plates. These predictions included rupture of the plate, local failure of concrete at the plate end due to stress concentration and debonding of the plate. Rasheed and Pervaiz (2003) presented some closed form design equations for flexural strengthening of RC beams using FRP. However, these equations considered a failure mode involving concrete crushing proceeded by steel yielding, prior to FRP rupture. Pham and Al-Mahaidi (2004a) carried out experimental investigations on flexural retrofitting of RC bridge beams using FRP, with a focus on understanding the end cover separation and shear crack induced debonding of FRP. They indicated that the end cover separation starts from FRP ends and fails in the form of shear failure at steel reinforcement level at the root of the concrete teeth between the shear cracks. Furthermore, they noticed that the shear crack induced debonding is due to the opening of one of those inclined cracks. In another study, Pham and Al-Mahaidi (2004b) provided an assessment of available strength prediction models for FRP-retrofitted RC beams, which included flexural failure, end-debonding and midspan-debonding failure modes. They verified that the beam theory is able to predict the full composite action of beams strengthened with FRP. However, the end and midspan debonding need a conservative design approach, which included limiting FRP stress level and interfacial bond stress between FRP and concrete respectively.

Khalifa et al. (1998) presented discussion on viability of using FRP as the externally bonded reinforcement for shear strengthening of flexural RC members, and proposed a design algorithm to compute contribution of externally bonded FRP shear reinforcement. Khalifa and Nanni (2000) discussed shear performance of RC T-beams under different configurations of externally bonded CFRP, and suggested that the most effective configuration for shear strengthening is U-wrapping with end anchorage. They also proposed design algorithms in ACI and Eurocode formats, which were shown to be conservative and hence acceptable. Chen and Teng (2003) presented work on shear
capacity of FRP-strengthened RC beams with debonding of FRP as the failure mode of interest. Wang and Chen (2003) presented analytical study on the flexural and shear behaviour of RC T-beams retrofitted with CFRP. They proposed an analytical model that was claimed to accurately predict the load-displacement and FRP plate bond stress. Kachlavev and McCurry (2000) discussed behaviour of RC beams retrofitted for shear and flexural strength using CFRP and GFRP based on full-scale experimental behaviour. Aprile and Benedetti (2004) proposed coupled flexural-shear strengthening design using FRP. They also validated the proposed model using an extensive database of more than 100 experimental results. Barros et al. (2007) discussed efficacy of CFRP-based techniques for flexural and shear strengthening of concrete beams. The design processes for FRP-based flexural and shear strengthening systems are discussed here.

(A) **Design for flexural strengthening**

A typical configuration of FRP-based flexural strengthening system is presented in Fig. 2.3. The FRP reinforcement can be in the form of (single or multiple-layered) strips or plates with the principle direction of fibres essentially oriented along the longitudinal axis of the member. The design objective includes determining the cross-sectional area of externally bonded reinforcement, for a given type of FRP composite material, required to safely carry the additional direct-tension induced in it under the revised (increased) loading.

![Figure 2.3](image)

Flexural strengthening of a RC member with externally bonded FRP reinforcement
(Conceptual representation – not to scale)

The design process for the additional flexural strength is philosophically similar to the flexural design of reinforced concrete (RC) elements and generally involves the following major design assumptions [GangaRao et al. (2006)]:

- The plane-sections remain plane in pre- and post-strengthening states.
- The strain along the depth of the section varies linearly, before and after strengthening.
• There is no slip between the FRP and the concrete substrate on which the FRP is bonded.

A flexurally strengthened RC section exhibits an array of failure modes, depending upon the possible combinations of the occurrence of failure in compression concrete and reinforcement systems (i.e. internal tension steel and externally bonded FRP in tension), and the their sequence of occurrence. Some of the possible failure modes for a flexurally strengthened RC section include:

• Compressive crushing failure of concrete after yielding of the steel reinforcement
• Compressive crushing failure of concrete before yielding of the steel reinforcement
• Failure of FRP before yielding of the steel reinforcement
• Failure of FRP after yielding of the steel reinforcement
• Balanced failure-mode involving simultaneous failure of FRP and concrete

Like in conventional RC beam design, in design for flexural strengthening too the failure of compression concrete is assumed to occur when the extreme compression fibre reaches the compression strain limit (typically 0.003 or 0.0035). The possibility of reduction in the sectional ductility in the post-strengthening stage compels the tension steel reinforcement to undergo larger strains at failure compared to its yield strain. Thus, in contrast with the design of RC beams in flexure, two failure strain limits concerning the failure of tension steel reinforcement are considered while designing the FRP-based strengthening systems. One corresponds to the yielding of steel and the other corresponds to its adequate yielding. The latter is generally twice as large compared to the former. The externally bonded FRP reinforcement involves a large number of possible failure mechanisms, mainly due to the possibility of debonding. Some of these failure mechanisms are avoided to occur through prescribing restrictive limits within the design process, or through detailing. However, a few of them are required to be considered within the design as alternative ways for the FRP reinforcement to fail.

Each of the post-strengthening failure modes involves different strain levels in the compression concrete and the reinforcement (i.e. internal tension steel and externally bonded FRP). Since the flexural resistance contributions of concrete and reinforcement are direct functions of their strains, each of the above failure modes involves a different nominal resistance of the strengthened RC section. It should be noted that the strain in the internal tension steel reinforcement corresponds to the sectional ductility (or deformability). Therefore, each of the above failure modes has its own ductility-content. It is clear that setting a preference to a particular failure mode in the design of a flexural
strengthening system should ideally be based on both, strength and ductility, *simultaneously*. However, due to the limitations of the typical strengthening design algorithms that are popularly followed, it is not explicitly possible. These algorithms use initial assumptions – e.g., for FRP-content, depth of neutral axis and/or strain in the compression concrete being equal to its limiting strain value, etc. for the design process to start. These assumptions are revised during the course of strengthening design through trial-and-error based iterations until a satisfactory strengthening design situation is reached. The failure mode corresponding to this satisfactory design situation is then accepted as the governing failure mode for the strengthened RC section, and the sectional ductility is then checked depending upon whether yielding of tension steel reinforcement has occurred or not. This format involves the following limitations:

- With an appropriate penalty on nominal post-strengthening flexural resistance based on the extent of straining in tension steel reinforcement, a large number of strengthening design solutions that meet the strength demand having different sectional ductility qualify to be admissible. Thus, sectional ductility does not form a core requirement in conventional design algorithms, but rather it is a mere consequence. It is up to the designer to repeat the design process with revised initial assumptions to converge towards a better quality strengthening design solution carrying a better combination of strength and ductility.
- If the penalised post-strengthening flexural resistance accounting for the lack of adequate ductility does not meet the strength demand, the design essentially needs to be repeated with revised initial assumptions. This situation is difficult to foresee when making initial assumptions within strengthening design.
- The number of iterations to reach a satisfactory design situation largely depends upon the initial assumptions, and generally the designer needs to rely upon gross simplifications while making these assumptions.
- The possibility for an externally bonded FRP reinforcement to fail under rupture or debonding makes it difficult to arrive at a wise initial guess. A naïve guess can lead to an un-convergent iteration loop, which needs to be aborted, and the whole process then is to be repeated based on the revised initial assumptions. This makes such an approach particularly unsuitable for automating the design process.
- The quality of the strengthening design solution is dictated by the satisfactory design situation, at which the iterations are terminated. Such design situations are often deemed satisfactory merely by them being permissible (based on achieving a
permitted state of force equilibrium that meets the strength and serviceability requirements), and not necessarily for being optimal.

- FRP composites offer the designer a large number of variants in terms of modulus of elasticity, rupture strain capacity and tensile strength. However, with limited control over the failure modes within the design process, the popularly followed design algorithms do not conveniently allow an efficient choice of FRP material to be made.

- An addition of externally bonded FRP reinforcement to an existing RC section brings the post-strengthening strain-state for the section more nearer to an analogous over-reinforced (or compression-controlled) strain-state. This turns the design of FRP-based flexural strengthening systems into a tricky problem, in which the challenge is to derive the required flexural resistance from an analogous over-reinforced strain-state, and yet to ensure sufficient straining of tension steel reinforcement to derive sectional ductility. This is in perfect contrast with the fundamental approach taken up for design of an RC member, and suggests that the logics and strategies conventionally used for minimising the iterations and for accommodating preferences over the failure modes do not directly apply for FRP-based flexural strengthening design.

The design flexural capacity of a strengthened structural element \( M_{\text{design}} \) comprises of the design flexural resistance contributions of the concrete \( M_{\text{conc-design}} \), internal tension steel reinforcement \( M_{\text{st-design}} \), internal compression steel reinforcement \( M_{\text{sc-design}} \), if any, and externally bonded FRP reinforcement \( M_{\text{FRP-design}} \), as symbolically presented by Eq. (2.3). The strengthening design guidelines are primarily concerned with the determination of the design flexural resistance contribution of the FRP component \( M_{\text{FRP-design}} \) only. Although a flexural strength enhancement of up to 160% has been reported in literature [Meier and Kaiser (1991)], the design guidelines prescribe an upper limit on the strengthening, which is discussed later.

\[
M_{\text{design}} = M_{\text{conc-design}} + M_{\text{st-design}} + M_{\text{sc-design}} + M_{\text{FRP-design}} \quad (2.3)
\]

One of the two important side effects of flexurally strengthening an RC section is the increased vulnerability of the strengthened section to fail in shear [GangaRao et al. (2006)]. Therefore, a flexural strengthening design essentially involves a check to see if the shear strength also needs to be supplemented. If so, a shear strengthening scheme needs to be worked out for a flexurally strengthened RC member to avoid the brittle shear failure to govern the post-strengthened RC section. The second side effect is the
reduction in the sectional ductility in the post-strengthening state [GangaRao et al. (2006)]. Therefore, the flexural strengthening design has to ensure an adequate sectional ductility in the post-strengthening state.

- The argument raised within this review suggests a need of a more holistic design philosophy that enables a seamless transition between the failure modes in the pre- and post-strengthening states. This ensures better consistency of safety (in terms of, e.g., probability of failure or reliability) between pre- and post-strengthening stages. However, this does not fall within the scope of this study, and hence is left for the future works. However, the rest of above limitations concern the course of strengthening design, which in turn can influence the quality of flexural strengthening design solutions in terms of strength, ductility and conservativeness. Assessment of these criteria does fall within the direct scope of this study, and hence a method to overcome these limitations needs to be devised.

- One of the important objectives of this study is to check the conservativeness performance of a range of possible strengthening design solutions under various design scenarios. The typical classification of post-strengthening failure modes, as suggested in the beginning of this section, does not allow conveniently capturing the qualitative features of the possible post-strengthening failure modes with a simultaneous focus on strength and ductility. Towards this, a set of novel definitions of the failure modes, called the ductility-based definitions, is developed in this study. It is demonstrated in this chapter that this set of definitions of the failure modes not only provides a better control over failure modes within the strengthening design process, but it also enables us seeing the strength and ductility requirements more cohesively. A detailed discussion on the ductility-based definitions of the post-strengthening failure modes is presented in Chapter 4.

(B) Design for shear strengthening

While more or less a unique structural configuration of the externally bonded FRP reinforcement can be worked out for flexural strengthening, for shear strengthening multiple alternative structural configurations are possible. Three main structural configurations are: fully wrapped, U-wrapped and sides-only configurations [Fig. 2.4]. Based on the possibilities of having either discrete strips or continuous sheet as the externally-bonded shear reinforcement, and aligning the principal direction of the fibres either normal or inclined with respect to the longitudinal axis of the member, widens the
possible variants within these three possible configurations. Each of these configurations carries distinct structural effectiveness, which makes the comprehension of shear strengthening design solutions a difficult problem. However, in any of the configuration, the additional FRP reinforcement is believed to act as externally bonded shear reinforcement.

Figure 2.4
Shear strengthening of a RC member with externally bonded FRP reinforcement (a) Side-only with continuous fabric (b) Side-only with discrete vertical strips (c) Side-only with discrete inclined strips (d) U-wrapping with continuous fabric (e) U-wrapping with discrete vertical strips (f) Full-wrapping with discrete strips (g) Full-wrapping with continuous fabric (Conceptual representation – not to scale)
The possible failure modes for an RC section strengthened in shear are dictated by concrete’s ability to retain integrity, yielding of internal steel stirrups and rupture or debonding of externally bonded FRP reinforcement. From the efficiency and effectiveness points of view, at the ultimate condition, it is desirable to have the internal steel stirrups yielded, at the same time concrete and FRP stirrups have also strained up to their respective governing failure limits simultaneously. However, many design guidelines do not explicitly mention any such check in their prescriptions.

The design process for shear strengthening is philosophically similar to that for shear design of RC elements, and considers the 45° truss-analogy as a valid design assumption for pre- and post-strengthened states [Teng et al. (2002)]. The design objective includes determining the required cross-sectional area of the externally bonded reinforcement (and the longitudinal spacing if relevant for the chosen structural configuration), for a given type of FRP composite material to safely carry the additional direct-tension that will be induced in it under the revised (increased) loading.

The post-strengthened design shear capacity of the strengthened structural element ($V_{\text{design}}$) comprises of the design shear resistance contributions of the concrete ($V_{\text{conc-design}}$), the existing steel stirrups ($V_{\text{st-design}}$) and the externally bonded FRP reinforcement ($V_{\text{FRP-design}}$), as symbolically presented by Eq. (2.4).

$$V_{\text{design}} = V_{\text{conc-design}} + V_{\text{st-design}} + V_{\text{FRP-design}} \quad (2.4)$$

The strengthening design guidelines are primarily concerned with the determination of the design shear resistance contribution of the FRP component ($V_{\text{FRP-design}}$) only. In absence of the synergy of strains, as was the case with flexural strengthening, Eq. (2.4) here represents an algebraic sum of the three independent resistance contributions. Thus, studying the FRP shear resistance contribution independently is possible.

- From the above review it can be appreciated that different possible configurations for shear strengthening an existing structure will present a major hurdle in the assessment for conservativeness.
- Chapter 5 presents a methodology for assessment of shear strengthening design process, which provides a common platform to comprehend all the possible variants in shear strengthening configurations. It captures the qualitative and quantitative implications of the distinctive design specifications for various configurations more tangibly, both for the designer and for the calibrator.
2.6 FRP-based structural strengthening systems: Design guidelines

Significant research during the last few decades has focused on design criteria for FRP-based structural strengthening systems. Seible (2001) presented a general scenario of strengthening design using FRP composites in civil structural environment, and Bank et al. (2001) provided a model specification for use of FRP for civil structures and covered a range of issues including material classification, scope of design specification and protocols for physical and mechanical material properties. Maruyama and Ueda (2001) and Fukuyama et al. (2001) discussed Japanese specifications on use of FRP for strengthening and retrofitting. There has been a considerable work done on FRP-based strengthening issues at the University of Missouri–Rolla, USA, in recent years. Nanni (2001) provided discussion on North American design guidelines for FRP-based strengthening including underlying principles, applications and some unresolved issues. Darby et al. (2007) presented a discussion on gaps in knowledge for strengthening of reinforced concrete structures using FRP composites. Recently, Darby et al. (2009) discussed the influence of changes in the cross-section on the effectiveness of strengthening schemes using externally bonded FRP. Ceroni and Pecce (2009) proposed design provisions for crack spacing and width in RC elements externally bonded with FRP.

The research focusing on developing strengthening design criteria led to the emergence of various design guidelines for structural strengthening using FRP composites. There exist at least ten design-support documents for the FRP-based structural strengthening. Some of the most important international design guidelines include: the document ACI440-2R (2008) of the American Concrete Society (ACI), the technical report TR55 (2004) and TR55 (2012) of the Concrete Society, UK, the technical bulletin FIB14 (2001) of the International Federation of Structural Concrete (FIB) and the design guidelines of the Hong Kong [HKG (2010)], Japan [BRIJ (1998)] and Canada [ISIS (2001)]. These design guidelines are conceptually similar and most of them suggest design criteria for structural strengthening using externally bonded FRP using both, the SM and the NSM techniques. ACI440 and TR55 are the two important international strengthening design guidelines prolifically followed worldwide. Therefore, the concepts and methodologies proposed in this thesis are calibrated for these two guidelines for the demonstration purpose. A review of these guidelines is very much warranted. A comprehensive comparison of the important design criteria for ACI440 and TR55 is presented next in this review. Master Chart 2.1 at the end of this chapter
presents a more detailed summary of important design prescriptions specified in ACI440 (2008) and TR55 (2004).

2.6.1 Development chronology

The ACI440 is a design-support document of the American Concrete Institute’s committee 440.2R. It is in force in its latest form since 2008 superseding the previous 2002 version with significant changes. The TR55 is a design-support document of the Concrete Society, UK. In its present form it is the second revision which is in existence since 2004 superseding earlier 2001 version with sizable changes. In fact, while this thesis is being written, the Concrete Society, UK has already brought in the third edition. The safety formats in the second and the third editions of TR55, by-and-large, are identical, and therefore, this revision does not affect the contention raised within this study. In fact, the findings of this study are equally applicable to the third revision also. However, the second edition of the TR55, technically, is within the scope of this study.

2.6.2 Design philosophy and basis

Some important characteristic features of ACI440 and TR55, pertinent to the design philosophy and basis, are discussed here in:

• **Design philosophy:** ACI440 and TR55 both follow a limit-state design (LSD) philosophy for structural strengthening systems. ACI440, in fact, follows a load and resistance factor design (LRFD) approach, which is ideally a specific variant of the LSD philosophy with a more explicit orientation towards the probability-based reliability method for calibrating safety parameters to account for various uncertainties.

• **Design approach:** The design approach remains grossly cloned from that for the design of new RC structures. The strengthening design is performed at the section and member levels. A strengthening design solution is to be devised such that the suggested structural configuration of the strengthening system satisfies the limit-states of strength and serviceability.

• **Analysis approach:** Both the guidelines deem the elastic methods of analysis with no redistribution as appropriate owing to the linear elastic stress-strain response of the FRP composites. However, the fundamental proportionality between the constitutive material properties of FRP assumed in both these guidelines is different. This aspect is described later in this chapter.

• **Compressive strength of FRP:** Both the design guidelines regard the compressive strength of FRP composites as unreliable and hence advocate that the FRP composites should not be subjected to a direct compressive force.
• **Strengthening limit**: Both the design guidelines specify an upper limit on the extent of strengthening in order to safeguard the strengthened structure against collapse due to the accidental loss of the externally-bonded FRP physically or its efficiency under vandalism, damage and fire.

• **Conservativeness**: The most important fact from the subject point of view is that both of these design guidelines prefer to deem their design framework as conservative.

### 2.6.3 Safety format

The following salient features summerise the safety format employed by ACI440 and TR55.

**(A) Uncertainty management**

The uncertainties within the design for strengthening is managed using the traditional approach of specifying safety factors to account for them. The safety factors accounting for the conventional engineering uncertainties in strengthening design have an *operational resemblance* with those employed in the design of a new RC structure. However, the scope and purposes of the safety factors are significantly different in these two designs, and thus there is a *functional distinction* between them. The uncertainty management between ACI440 and TR55 is conceptually the same, however they carry substantial distinctions both, operationally and functionally. In general, these guidelines attempt to address uncertainties in FRP material properties arising from random, environmental exposure and quality control related variabilities more explicitly. These are discussed below. A more detailed treatment of uncertainties and their design treatment considered within these design guidelines are presented in Chapter 3 in this thesis.

_Random variability:_ The possibility of random variability within FRP material properties is addressed through the concept of setting *characteristic values* for the material properties. Compared to the design of new RC structures, ACI440 and TR55 both are stringent in deriving of the characteristic values of FRP material properties. Against the conventional convention of a reduction of 1.64 times the standard deviation in the mean value to arrive at a corresponding characteristic value, ACI440 and TR55 suggest a reduction of 3 and 2 times the standard deviation respectively while deriving the characteristic values of FRP material properties. Thus, in general, ACI440 is relatively more stringent than TR55 on this ground.
A conformist school of thoughts would profess that due to the data-deficiency for FRP composites in civil engineering, the statistical parameters describing the (assumed) Normal (Gaussian) distribution are not confidently known. Under such a situation, a higher numerical value than 1.64, even for having 5% threshold probability of exceedance, depending upon the number of samples used to determine statistical parameters for the FRP properties is advocated. Guidelines on this are available in standard documents and statistical literature [Elishakoff (1999)]. As a ready reference, Fig. 2.5 presents the belief of BS EN 1990:2002 on this aspect here. Under this assumption, the quantitatively stringent prescription of the multiplier to the standard deviation has to be considered as a compulsory obligation and does not form a means of imparting conservativeness. However, differential quantitative prescriptions of this multiplier for two different design guidelines are surely an indication of conservativeness.

![Figure 2.5](image)

**Figure 2.5**
Correspondence between the multiplier to standard deviation and number of samples [Based on BS EN 1990:2002]

*Environmental exposure*: ACI440 and TR55 both suggests partial factors of safety to account for the time-dependent reduction in the characteristic values of FRP material properties. In this sense, the partial factors of safety in ACI440 and TR55 and those in the design of new RC structures are functionally different, in spite of being operationally similar. ACI440 prescribes a set of reduction factors, called the environmental reduction factor, which are numerically lesser than unity and are used as a multiplier to the characteristic values of the FRP material properties to arrive at the corresponding design values. TR55, on the other hand, prescribes a set of factors of safety, called the partial safety factors, which are numerically greater than unity and are
used as a divisor to the characteristic values of FRP material properties. Following points need attention in this connection:

- Both, ACI440 and TR55 suggest different environmental reduction factors and partial safety factors for different FRP materials. Also, both design guidelines penalise the GFRP and the CFRP composites most stringently and most liberally respectively on this account.

- ACI440 classifies the environmental exposure conditions into three categories – interior, exterior and aggressive – and the environmental reduction factors are prescribed in accordance with these severity of exposure conditions. TR55, on the other hand, does not suggest any such classification of environmental exposure conditions. In the absence of such classification, it is ideal to believe that the partial safety factors suggested by TR55 refers to the possible most aggressive environmental exposure condition. However, there appears no clarification on this either in TR55 or in the supporting literature.

- While ACI440 suggest using a numerically identical environmental reduction factors on tensile strength and rupture strain capacity of FRP, TR55 suggest numerically different partial safety factors on modulus of elasticity and the rupture strain capacity of FRP. This point is further elaborated later in this section.

**Quality control:** Civil engineering applications of FRP employ processes such as wet lay-up, pultrusion and resin infusion, which have lesser quality control compared to processes such as autoclave moulding, etc. that are employed in aerospace and naval engineering applications. TR55 suggests a set of safety factors, called the additional partial safety factors, to account for the possible variation in the in-situ FRP material properties due to type of FRP and the method used for its manufacturing and application. These additional safety factors supplement the stringencies of the partial safety factors accounting for the environmental exposure. ACI440, on the other hand, does not suggest any such factors. The order of stringency (from low to high) considered by TR55 in suggesting the additional partial safety factor according to the FRP manufacturing and application methods include:

- pultrusion, prepregnation and preformation (for FRP plates)
- machine-controlled application, vacuum infusion and wet lay-up (for FRP sheets or tapes)
• filament winding, resin transfer moulding, hand lay-up and hand-held spray application (for prefabricated or factory-made shells)

(B) **Reliability management**

The LRFD philosophy has a possibility of applying safety factors, called the strength reduction factors, on the computed nominal resistance. This possibility is in addition to the safety factors applied on the material properties, and aims at topping up safety-content towards achieving the target reliability index for the design solutions [Kansara and Ramanjaneyulu (2005)]. ACI440, conceived within the LRFD philosophy, does exercise this possibility, and suggests two additional strength reduction factors. One of these two factors is applied on the nominal resistance contribution of the FRP reinforcement, and the other is applied on the nominal resistance of the strengthened RC section. The latter is in accordance with ACI318 (2008) specifications, while the former is suggested to account for the uncertainties: (a). inherent in FRP systems compared with steel reinforced and prestressed concrete applications, (b). pertinent to the difference in statistical evaluation of the variability in material properties, (c). arising from difference in the predicted and the full-scale test results, and (d). related to the variations in field applications. The LSD format of TR55 does not allow the strength reduction factor, and consequently TR55 does not suggest any strength reduction to account for these uncertainties. However, the uncertainties related to variations in field applications are accommodated through a set of additional partial safety factor on FRP material properties in TR55.

(C) **Proportionality in constitutive relationship**

Fig. 2.6 shows the stress-strain relationships for structural concrete, structural steel and FRP composites. From this, the impact of the differences in the approaches for deriving characteristic and design values of the FRP material properties on the constitutive relationships for these materials can clearly be appreciated. With the adopted doctrine of specifying no safety factors on modulus of elasticity of FRP and applying numerically identical environmental reduction factors on tensile strength and rupture strain of FRP, the mean and the design values of the modulus of elasticity of FRP remains identical for ACI440. It can further be inferred from Fig. 2.6 (c) and (d) that the stiffness of FRP, for ACI440, remains completely unaffected from the safety factors. ACI440 does recognise this fact, but does not explain on the reasons. The only apparent explanation behind this could be that under certain circumstances not applying factors of safety on modulus of elasticity is a conservative approach. However, this is not true...
for all the situations. Atadero and Karbhari (2009) explained that the treatment of modulus of elasticity of FRP in ACI440 poses serious gap, particularly for confinement-based seismic retrofitting of columns, in which the design equations are most often directly based on the modulus of elasticity. In such cases, the variation in material property gets neglected. The confinement-based seismic retrofitting is not in the scope of this study.

![Constitutive relationships for structural materials](image)

(a) Structural concrete, (b) Structural steel, (c) FRP composites (ACI440), and (d) FRP composites (TR55)

2.6.4 Design treatment of debonding

(A) Debonding in flexural strengthening design

The most popular approach of accounting for the possibility of debonding of FRP is prescription of a debonding strain limit for FRP reinforcement ($e_{fd-debond}$), signifying the strain in FRP at which debonding triggers. Two most common formats for prescribing the limiting strain $e_{fd-debond}$ exist, and in this study they are referred to as Type I and Type II debonding strain limits. Type I debonding strain limit ($e_{fd-debond(Type I)}$) is in the form of a pre-set constant numerical strain value in FRP, while Type II debonding strain limit ($e_{fd-debond(Type II)}$) is modulus-dependent (in particular, a function of parameter $R = n t_f E_f$). For example, TR55 prescribes a constant strain of 0.008 in FRP as the debonding strain limit [Eq. (2.5)], while ACI440...
specifications suggests a modulus-dependent debonding strain limit with an upper bound of $0.9 \varepsilon_{f_d \text{-rupture}}$ [Eq. (2.6)].

Type I debonding strain limit conforming to TR55 specifications:

$$\varepsilon_{f_d \text{-debond(Type I)}} = 0.008$$  \hspace{1cm} (2.5)

Type II debonding strain limit conforming to ACI440 specifications:

$$\varepsilon_{f_d \text{-debond(Type II)}} = 0.41 \sqrt{\frac{E_b}{R}} \leq 0.9 \varepsilon_{f_d \text{-rupture}}$$  \hspace{1cm} (2.6)

The upper limit specified by ACI440 apparently aims to ensure that the effective debonding strain for FRP always remains numerically lesser than its design rupture strain capacity. However, this upper limit has more significant implications than merely this, which is discussed in Chapter 3 in detail, and demonstrated in Chapter 4.

(B) Debonding in shear strengthening design

Similar to flexural strengthening design, the possibility of debonding within the shear strengthening design is also incorporated through prescribing a debonding strain limit on FRP reinforcement. This debonding strain limit is, typically, seen in conjunction with the bond- or contact-criticality of the shear strengthening configurations. In addition, this debonding strain limit is to be seen simultaneously with the possibilities for concrete to physically disintegrate under diagonal compression, and fracture of FRP shear reinforcement. These possibilities are accounted for through strain limits on FRP at which they occur, represented through $\varepsilon_{f_d \text{-disintegration}}$ and $\varepsilon_{f_d \text{-fracture}}$ respectively.

For the contact-critical configurations (e.g., fully wrapped configuration), in theory, debonding is not a possibility. In this case, the strain value in FRP at debonding can be considered numerically equal to the design rupture strain capacity of FRP ($\varepsilon_{f_d \text{-rupture}}$). In design practice, however, the debonding strain limit for a contact-critical configuration is often prescribed to be numerically lesser than $\varepsilon_{f_d \text{-rupture}}$. Unlike this, debonding is a highly likely practical possibility for the bond-critical configurations (e.g., sides-only and U-wrapped configurations). In the most popular format describing debonding strain limit for a bond-critical configuration, the strain in FRP at which debonding occurs is believed to be modulus-dependent (i.e. a function of parameter $R = n t_f E_{f_d}$). For example, the debonding strain limits conforming to ACI440 and TR55 specifications are presented through Eqs. (2.7) and (2.12) respectively:
Debonding strain limits conforming to ACI440 specifications:

$$\varepsilon_{fd-debond} = \kappa_v \varepsilon_{fd-rupture} \leq \varepsilon_{fd-disintegration} \quad (2.7)$$

where,

$$\kappa_v(bond\ critical) = \frac{\left(\frac{d_f}{L_c}\right)^{7/3}}{11,900 \varepsilon_{fd-rupture}} \times c_{bond-reduction} \times \left[\frac{23,300}{R^{0.58}}\right] \leq 0.75 \quad (2.8)$$

$$\kappa_v(contact\ critical) = 0.75 \quad (2.9)$$

$$c_{bond-reduction} = \begin{cases} \left[\frac{d_f - L_c}{d_f}\right] & \text{[For U-wrapped configuration]} \\ \left[\frac{d_f - 2L_c}{d_f}\right] & \text{[For sides-only configuration]} \end{cases} \quad (2.10)$$

$$\varepsilon_{fd-disintegration} = 0.004 \quad (2.11)$$

Debonding strain limits conforming to TR55 specifications:

$$\varepsilon_{fd-debond} = 0.64 \sqrt{\frac{0.18 (d_f L_c)^{7/3}}{R}} \quad \text{[For bond-and contact-critical configurations]} \quad (2.12)$$

$$\varepsilon_{fd-debond} \leq \varepsilon_{fd-fracture} = 0.5 \varepsilon_{fd-rupture} \quad (2.13)$$

$$\varepsilon_{fd-debond} \leq \varepsilon_{fd-disintegration} = 0.004 \quad (2.14)$$

It can be seen that the TR55 specifications, unlike ACI440 specifications, do not distinguish the debonding strain limit for bond- and contact-criticality of the configurations. Also, unlike TR55 specifications, ACI440 specifications do not explicitly prescribe a possibility for FRP shear reinforcement to fail under fracture.

### 2.6.5 Design treatment of FRP anchorage

#### (A) Anchorage length of FRP in flexural strengthening design

Popularly, the anchorage length ($L_a$) for FRP in flexural strengthening is believed to be modulus-dependent (i.e. a function of parameter $R = n t_f E_{fd}$). For example, the anchorage length models prescribed by TR55 and ACI440 are presented through Eqs. (2.15) and (2.16) respectively.

Anchorage length model conforming to TR55 specifications:
\[ L_{e}^{(\text{flexure})} = 0.7 \frac{R}{\sqrt{0.18 (f_{cu})^{2/3}}} \]  

(2.15)

Anchorage length model conforming to ACI440 specifications:

\[ L_{e}^{(\text{flexure})} = \frac{R}{\sqrt{f'_{e}}} \]  

(2.16)

The similarity between the TR55 and ACI440 formats for prescribing anchorage length of FRP flexural reinforcement can be clearly observed from the above.

(B) **Anchorage length of FRP in shear strengthening design**

Similar to flexural strengthening design, the anchorage length of FRP in shear strengthening is also believed to be modulus-dependent. TR55, in fact, suggests the use of same anchorage length model for FRP flexural and shear reinforcements. Unlike this, ACI440 specifies an anchorage length model for FRP shear reinforcement based on ‘active bond length’ theory, as discussed earlier in this chapter. The anchorage length models for FRP shear reinforcement according to TR55 and ACI440 are presented through Eqs. (2.17) and (2.18) respectively.

Anchorage length model conforming to TR55 specifications:

\[ L_{e}^{(\text{shear})}\text{-Type I} = 0.7 \frac{R}{\sqrt{0.18 (f_{cu})^{2/3}}} \]  

(2.17)

Anchorage length model conforming to ACI440 specifications:

\[ L_{e}^{(\text{shear})}\text{-Type II} = \frac{23,300}{R^{0.58}} \]  

(2.18)

The inverse proportionality of the TR55 and ACI440 anchorage length models to parameter \( R \) can be observed from the above. This is elaborated in more detail in Chapter 5.

From this review the following intricacies concerning the objectives of this study can be summarised:

- Different types of FRP materials, manufacturing and installation routes of FRP, FRP-based strengthening involve a large number of possible design scenarios. The design guidelines have to cater a wide range of possible design scenarios, and it is obvious that the design demands in a certain situations are different than those in
other situations. Within the formats employed by the design guidelines, it is extremely difficult to comprehend the design implications under all the circumstances clearly. Towards this end, the methodologies developed in this study are so developed that enables us to create a range of possible design scenarios. This makes it possible to see the qualitative and quantitative differences within the design solutions under different design scenarios more lucidly.

- The possibility of alternative failure modes and mechanisms significantly influence the course of strengthening design process and the quality of the resultant strengthening design solutions. The format of various design guidelines, in general, makes it very difficult to see the flow of strengthening design process with all the possible options. Towards this end, the methodologies developed within this study present the strengthening design process algorithmically, which lucidly demonstrates the internal architecture and working mechanism of flexural and shear strengthening design processes.

- In spite of being conceptually similar, various strengthening design guidelines have a lack of common basis. This makes their assessment on a common ground very difficult, if not impossible. To overcome this glitch, this study proposes a set of generic methodologies that can be calibrated according to ACI440 and TR55 specifications (or any other conceptually similar design guideline for that purpose) on common terms.

2.7 Summary of differences and conflicts

The important issues concerning the objectives of this study are summarised below:

Differences in safety format:

- Difference in design philosophies adopted by various strengthening design guidelines (e.g. limit state design (LSD) and load and resistance factor design (LRFD) philosophies – by definition LSD will not involve any reduction factors on resistance contribution of FRP reinforcement or post-strengthening resistance)

- Difference in specifying safety factors on the constitutive material properties of FRP (e.g., prescribing factors of safety either on modulus of elasticity and rupture strain capacity of FRP or on tensile strength and rupture strain capacity of FRP – for the latter, modulus of elasticity of FRP will not involve any factor of safety)

- Differences in the quantitative prescription of safety factors on FRP material properties (these are discussed in detail in Chapter 3)
Differences in design treatment of debonding:

- Difference in the format of debonding strain limits (e.g., Type I or Type II debonding strain limit formats) for flexural strengthening
- Strategy of prescribing an upper bound limit on strain in FRP at debonding for Type II debonding strain limit for flexural strengthening
- Strategy of prescribing and not prescribing any safety factors on modulus of elasticity of FRP in conjunction with Type II debonding strain limit

Differences in design treatment of bond length requirement of FRP:

- Strategy of prescribing or not prescribing any safety factors on modulus of elasticity of FRP in context of FRP bond length estimation models for flexural strengthening
- Differences in FRP bond length estimation models for shear strengthening (e.g., Type I and Type II bond length estimation models)

Conflicts in flexural strengthening design:

- Incompatibility between the pre- and post-strengthening failure modes for flexural strengthening (e.g., failure modes in design for flexure for new structures aims to achieve an under-reinforced strain state and does not permit an over-reinforced strain state, while failure modes in design for flexural strengthening of existing structures permits an analogous over-reinforced strain state as well with additional obligations on sectional ductility requirement)

Complexity in shear strengthening design:

- Complexity presented by the multiple possible shear strengthening configurations

The methodologies proposed within this thesis aiming at assessing conservativeness associated with design process of FRP-based structural strengthening systems should investigate implications of the above points on the course of strengthening design process and quality of the resultant strengthening design solutions.
### Master Chart 2.1 Summary of design specifications for ACI440 and TR55

#### A. Concrete

<table>
<thead>
<tr>
<th>Grade Definition</th>
<th>Cylinder crushing strength ($f_{cu}$) in MPa</th>
<th>Cube crushing strength ($f_{cu}$) in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short Term ($E_c$)</td>
<td>$4750 \sqrt{f_{cu}}$</td>
<td>$5.5 \times 10^5 \sqrt{f_{cu}}$</td>
</tr>
<tr>
<td>Long Term ($E_{c-long}$)</td>
<td>---</td>
<td>$0.5 E_c \left( \sqrt{1 + \phi} \right)$</td>
</tr>
<tr>
<td>Partial FoS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ult. Compress. Strain ($\varepsilon_{cc}$)</td>
<td>0.003</td>
<td>0.0035</td>
</tr>
<tr>
<td>Stress-Strain Plot</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### B. Steel

<table>
<thead>
<tr>
<th>Grade Definition</th>
<th>Yield Strength ($f_y$) in MPa</th>
<th>Yield Strength ($f_y$) in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial FoS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>1.15</td>
<td>1.05 (structures after 1997)</td>
</tr>
<tr>
<td>Stress-Strain Plot</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### C. FRP

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Fibre</th>
<th>$C_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>Carbon</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Glass</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Aramid</td>
<td>0.85</td>
</tr>
<tr>
<td>Exterior</td>
<td>Carbon</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Glass</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Aramid</td>
<td>0.75</td>
</tr>
<tr>
<td>Aggressive</td>
<td>Carbon</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Glass</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Aramid</td>
<td>0.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System &amp; Mfg Process</th>
<th>$\gamma_{sys}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plates</td>
<td>Pultruded</td>
</tr>
<tr>
<td></td>
<td>Prepreg</td>
</tr>
<tr>
<td></td>
<td>Preformed</td>
</tr>
<tr>
<td>Sheets/ Tapes</td>
<td>M/C controlled</td>
</tr>
<tr>
<td></td>
<td>Vacuum infusion</td>
</tr>
<tr>
<td></td>
<td>Wet lay-up</td>
</tr>
<tr>
<td>Prefabricat ed Factory-made Shells</td>
<td>Filament winding</td>
</tr>
<tr>
<td></td>
<td>Resin transfer moulding</td>
</tr>
<tr>
<td></td>
<td>Hand lay-up</td>
</tr>
<tr>
<td></td>
<td>Hand-held spray</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP</td>
<td>1.25</td>
</tr>
<tr>
<td>AFRP</td>
<td>1.35</td>
</tr>
<tr>
<td>AR-GFRP</td>
<td>1.85</td>
</tr>
<tr>
<td>E-GFRP</td>
<td>1.95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP</td>
<td>1.10</td>
</tr>
<tr>
<td>AFRP</td>
<td>1.10</td>
</tr>
<tr>
<td>AR-GFRP</td>
<td>1.60</td>
</tr>
<tr>
<td>E-GFRP</td>
<td>1.80</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>$\phi M_{st} \geq 1.1M_{sl1} + 0.75M_{sl2}$</td>
<td>$M_{st} \geq 1.0M_{ek} + 1.0M_{sl2}$</td>
</tr>
</tbody>
</table>

**Sectional Stress Distribution Profile**

$$
\epsilon_i = \frac{1.7 f'_a}{E_i},
$$

$$
k_1 = \frac{3 \epsilon'_s - \epsilon'_s}{3 \beta_i \epsilon'_i},
$$

$$
k_2 = \frac{4 \epsilon'_s - \epsilon'_s}{6 \epsilon'_i - 2 \epsilon'_i}.
$$

**Potential Debonding Limiting Strain**

$$
\epsilon_{f_{debond1}} = 0.41 \frac{f_y}{n \times E_{sy} \times t_f},
$$

$$
\epsilon_{f_{debond2}} = 0.9 \epsilon_{id}.
$$

**Effective Strain in FRP**

$$
\epsilon_{fy} = \min\left(\epsilon_{f_{debond1}}, \epsilon_{f_{debond2}}\right).
$$

**Effective Stress in FRP**

$$
f_{fy} = E_{sy} \times \epsilon_{fy}.
$$

**Penalty Functions for Ductility of Failure Mode**

- By incorporating Strength Reduction Factor $\phi$ based on steel strain $\epsilon_{st}$
- $\phi = 0.90$ (for $\epsilon_{st} \geq 0.005$)
- $\phi = 0.65 + \frac{0.25(\epsilon_{st} - \epsilon_{st0})}{(0.005 - \epsilon_{st0})}$ (For $M_{sl1} < M_{sl2}$)
- $\phi = 0.65 + \frac{0.25(\epsilon_{st} - \epsilon_{st0})}{(0.005 - \epsilon_{st0})}$ (For $\epsilon_{st} < \epsilon_{st0}$)
- $\phi = 0.65$ (for $\epsilon_{st} \geq \epsilon_{st0}$)

**Contribution of FRP to Design Flex. Capacity**

$$
M_{sl1} = \phi M_f \ (\phi = 0.8) \
$$

**Longitudinal Shear Stress between FRP and Concrete**

$$
\tau_{fc} = \frac{V_{f, m_{f, max} A_f (D - x)}}{I_{f, c}} \leq 0.8 \text{ MPa}
$$

May be increased moderately in special cases

**Bond Force**

$$
T_{b, min} = 0.5k_b b_f \sqrt{E_{sb} t_f f_{sb}},
$$

$$
k_b = \begin{cases} 
1.06 & \frac{2}{b_f} > 1.0 \\
1 + \frac{b_f}{100} & \frac{2}{b_f} \leq 1.06 
\end{cases}
$$

$$
f_{sb} = 0.18 f_{sb}^2
$$

**Anchorage Length**

$$
l_{a, min} = \sqrt{\frac{n E_{sb} t_f}{f_{sb}}}
$$

**Max**

$$
0.7 \frac{E_{sb} t_f}{f_{sb}}, \ 500 \text{ mm}
$$

**Reduced Bond Force due to limited Anchorage Length**

$$
T_{b} = T_{b, min} \left(1 - \frac{l_i}{l_{a, min}}\right) 2 - \frac{l_i}{l_{a, min}}
$$

**A. Strengthening Limit Condition**

Upper Limit for Shear Strengthening

\[ V_s + V_f \leq 0.66 \sqrt{f_{ctk}} b d \]

\[ V_{s,max} = \tau_{s,max} b d \]

Max. Permissible Shear Stress in Concrete

\[ \tau_{s,max} \]

For BS-8110

\[ \text{Min}(0.8 \sqrt{f_{ck}}, 5 \text{ MPa}) \]

For BS-5400

\[ \text{Min}(0.75 \sqrt{f_{ck}}, 4.75 \text{ MPa}) \]

Shear Strength Contribution of FRP

\[ V_f = A_f \epsilon_{fs} E_f (\sin \alpha + \cos \alpha) d_f \]

\( \alpha = \text{Angle with longitudinal axis of beam} \)

\( \psi_f = 0.85 \) (For fully wrapped)

\( \psi_f = 0.95 \) (For U & side wraps)

Effective Shear Strain

\[ \epsilon_{fs} = \text{Min}[(\kappa, \epsilon_{fb}), 0.004] \]

\[ \epsilon_{fs} = \frac{0.64 \sqrt{f_{ctk}}}{E_f V_f} \]

**B. Ultimate Limit State Criteria**

Wrapping Coefficient

\[ \kappa_f = \begin{cases} 0.75 & \text{(for fully wrapped)} \\ \frac{L_w}{11900} & (\geq 0.75) \\ \frac{f_{te}}{27} & (\leq 0.75) \end{cases} \]

\[ k_1 = \left( \frac{f_{te}}{27} \right)^{1/2} \]

\[ k_2 = \frac{d_f - L_w}{d_f} \] (for U-wraps)

\[ k_3 = \frac{d_f - 2L_w}{d_f} \] (for side wraps)

Active Bond Length

\[ L_a = \frac{23300}{(n \sqrt{f_{ctk}})^3} \]

\[ L_{a,max} = \text{Max}\left(0.7 \frac{E_f V_f}{f_{ctk}}, 500 \text{ mm} \right) \]

Longitudinal Spacing of FRP Laminates

\[ S_f = b_f + \frac{d_f}{4} \]

\( = 1 \) (for continuous FRP sheet, \( b_f = \cos \beta \))

For strips, \( S_f \leq \text{Min. of –} \)

1. \( 0.8d_f \)
2. \( \frac{n}{3} L_{a,max} \)
3. \( b_f + \frac{d_f}{4} \)
Conservativeness
3.1 Chapter objectives and structure

The objective of this chapter is to contextualise conservativeness in a specific context of the design of FRP-based structural strengthening systems. In particular, it aims at:

- Presenting a tangible definition of conservativeness in the design of FRP-based structural strengthening systems
- Proposing a representative conservativeness framework that –
  - Reveals how the uncertainties and safety parameters comprise the architecture of the safety format used in FRP-based structural strengthening
  - Demonstrates the internal working mechanism of the safety format in accounting for the uncertainties and producing conservativeness
  - Provides a common platform to compare the safety formats employed by strengthening design guidelines

In a nutshell, this chapter can be seen to comprise of three parts. Section 3.2 comprising the first part, presents a conservativeness hypothesis describing the meaning and perception, definition, means and significance of conservativeness in strengthening design. The discussion presented in Chapter 2 has made it clear that uncertainties and safety parameters are of central importance in comprehending conservativeness in strengthening design. Therefore, the second part of this chapter, comprising of sections 3.3 to 3.11, present detailed taxonomies of uncertainties and safety parameters associated with FRP-based structural strengthening systems. A detailed discussion on the safety parameters including their mathematical basis and design prescriptions is also presented. Finally, a representative conservativeness framework for FRP-based structural strengthening is presented in section 3.12 comprising the third part of this chapter. A comprehensive mapping between the uncertainties and safety parameters showing a logical correlation between them is presented here. This mapping also demonstrates the internal architecture of the safety format used in strengthening design, and how various safety parameters are integrated within the strengthening design process. An approach to represent various safety parameters in a condensed form is also discussed here. This approach not only enables us to compare the safety formats used by different strengthening design guidelines, but also facilitates creating representative design scenarios (e.g. most liberal and most stringent prescriptions of safety parameters) to be used in the parametric and sensitivity analyses presented in the next chapters. For illustration, a summary of such condensed safety parameters, based on the safety factors prescribed in ACI440 and TR55, is presented.
Focus of Chapter 3: Conservativeness
3.2 The conservativeness hypothesis

3.2.1 Conservativeness: Perception and meaning

Conservativeness, in general, is a measure of lack of confidence. In context of structural strengthening design, it is a reflection of our apprehension for the consequences of failure of an FRP-based structural strengthening system. This is attributed to the lack of certainty and confidence in the responses, behaviours or outputs that are predicted by the design processes. It is often believed that higher conservativeness means higher safety. While this appears logical, it is only conditionally true as we will see for the externally bonded FRP applications in this thesis. Also, incorporation of higher conservativeness invariably involves a quantitative increase in material requirement, or a need of qualitatively superior material, to meet the same load demand. In fact, an irrationally conservative design format indulges the economics of the rehabilitation scheme. Therefore, higher conservativeness should not automatically be viewed as something that is better.

3.2.2 Conservativeness: Definition

The engineering design processes typically involve estimation of various design parameters through specifically calibrated mathematical models, which can be referred to as the engineered models. A typical engineered model can be seen to consist of two parts. The first part can be called a plain model, which aims to mathematically mimic a physical phenomenon or characteristic. A plain model is formulated based on experimentally measured, empirically observed, logically anticipated or hybrid behaviour. A plain model, due to various reasons, invariably involves uncertainties, which are required to be modelled. Modelling uncertainties includes identifying their sources, and qualifying and quantifying the uncertainties. The second part of an engineered model is a supplement to the plain model, which can be seen as a safety function that is intended to provide a safety cushion in the output predicted by the model. A safety function comprises of safety parameters that are calibrated in accordance with the type and extent of the uncertainties they are accounting for. These safety parameters are a single or a set of multipliers and/or divisors attached to a plain model. Output of a plain model (i.e. without considering any effect of the safety function) is called the nominal output ($P_{\text{nominal}}$), while that of an engineered model (which includes the full effect of the safety function) is called the design output ($P_{\text{design}}$). The absolute difference between these two outputs represents conservativeness ($C$). This is schematically shown in Fig. 3.1.
Figure 3.1
Conservativeness in estimation of a design parameter
(Schematic representation)

Eq. (3.1) represents a generic format of expressing conservativeness associated with an engineered model. A little consideration will show that for an engineered model used in estimation of design load-effects (e.g. the required value of bending moment or shear force produced under design loads), the strategy to achieve conservativeness is by over-estimating the design loads. Thus, for such engineered models the design output ($P_{\text{design}}$) will numerically be greater than the nominal output ($P_{\text{nominal}}$). In contrast, for an engineered model used in estimation of design resistance (e.g. the design moment of resistance or the design shear resistance of a structural member), the strategy to achieve conservativeness is by under-estimating the load carrying capacity. The design outputs ($P_{\text{design}}$) will numerically be lesser than the nominal outputs ($P_{\text{nominal}}$) for such engineered models. The conservativeness corresponding to the engineered models used in estimation of design load-effects and design resistance are expressed through Eqs. (3.2) and (3.3) respectively. An absolute sign in Eq. (3.1) accounts for these possibilities.

Generic format of conservativeness for an engineered model:

$$C = \left| P_{\text{nominal}} - P_{\text{design}} \right|$$  \hspace{1cm} (3.1)

Conservativeness for an engineered model for load-effect estimation:

$$C_L = P_{\text{design}} - P_{\text{nominal}}$$  \hspace{1cm} (3.2)

Conservativeness for an engineered model for resistance estimation:

$$C_R = P_{\text{nominal}} - P_{\text{design}}$$  \hspace{1cm} (3.3)

Here, the subscripts $R$ and $L$ to $C$ distinct the conservativeness in the resistance estimation and load-effect estimation respectively. As a more convenient non-
dimensional format, conservativeness can be expressed as a fraction, called conservativeness index ($I_c$), as shown through Eq (3.4).

$$I_c = \frac{|P_{\text{nominal}} - P_{\text{design}}|}{P_{\text{nominal}}} \quad (3.4)$$

### 3.2.3 Conservativeness: Means

In a broad sense, conservativeness can be brought into the structural design through three primary means. These include over-strength, reserved-strength and redundancy, as summarised in Fig. 3.2.

**Over-strength** refers to the buffer capacity that a structural element is designed to carry for absorbing the likely deviations in the response of an engineered model. It can be obtained by over-estimating the required design capacity (e.g. required design value of post-strengthening flexural resistance), or by under-estimating the available design capacity (e.g. actual design resistance of a structural element).

**Reserved-strength** refers to an additional capacity of a structural element that is specifically reserved for absorbing ill effects of the likely variability in various quantities serving as the inputs for an engineered model. An example of the possible variability is reduction of the efficiency of materials along time under environmental exposure (see Chapter 2). Under a planned strategy, reserved-strength is obtained by under-estimating the material properties that are likely to exhibit variability (i.e. reduction). Such an under-estimation of the material properties results into an underestimated actual design capacity of the structural element, which is believed to set a ‘reserved capacity’ that will be sacrificed towards addressing the concerned variability.

**Redundancy**, on the other hand, refers either to ‘standby’ and/or ‘provision-in-excess’ arrangements, e.g. static indeterminacy, such that it needs a larger amount of energy to be absorbed (i.e. a larger amount of structural damage to occur) within the structural system to set a ‘mechanism’ causing its collapse. It is derived from secondary sources, such as redistribution of forces. The possibility of concrete crushing, or rupture or debonding of FRP to govern an FRP-strengthened structure makes it complicated to accommodate redistribution of forces within strengthening design [Aiello and Ombres (2011)]. In addition to this, typically, the strengthening design is performed at the section and member levels, but not at the system level. This eliminates the possibility of accommodating redistribution of forces, which needs to consider available static indeterminacy at the system level. Under these grounds, ignoring this reserved-strength is believed to be a conservative practice [Tajaddini et al. (2013)]. Therefore, the
presently prevailing conventions on FRP-based structural strengthening do not permit redistribution of forces.

Thus, conservativeness, within this study, primarily refers to over-strength and reserved-strength, while redundancy does not form a subject of interest for this study.

### 3.2.4 Conservativeness: Significance

It can be appreciated from the above discussion that conservativeness forms a central part in engineering design process towards ensuring safety. For FRP-based structural strengthening systems, it is even more important. This is due to the fact that for ensuring that a strengthening design solution is sufficiently safe, the only possibility in the hands of a calibrator of design criteria is to adjust, regulate and/or manipulate the conservativeness associated with resistance contribution of externally bonded FRP reinforcement. It can also be appreciated that uncertainties and their design treatment (in form of safety parameters accounting for the uncertainties) are vital in studying conservativeness. A detailed discussion on uncertainties and safety parameters associated with FRP-based structural strengthening systems is presented in the following sections.

### 3.3 Taxonomy of uncertainties

The basic taxonomy of uncertainties associated with the design of FRP-based structural strengthening systems is presented in Fig. 3.3. Fundamental grouping of uncertainties into *prescriptive* and *non-prescriptive uncertainties* concurs with the general classification of uncertainties suggested by Haldar and Mahadevan (2000) classifying them into those arising from *cognitive* and *non-cognitive* sources, as discussed in Chapter 2. Here, the adjectives prescriptive and non-prescriptive mean that they can or cannot be prescribed in explicit terms using conventional mathematics respectively.
While this fundamental grouping remains valid for any engineering design process in general, the relative dominance of prescriptive and non-prescriptive uncertainties vary from case to case. Each of these two classes of uncertainties, irrespective of their relative dominance, demands the design process to be conservative. The relative degree of conservativeness needed to address each of these two types of uncertainties, of course, depends upon their relative dominance within the design process. It is to be noted, however, that the means employed to provide conservativeness towards addressing the prescriptive uncertainties can be rationalised rather more tangibly compared to those employed for addressing the non-prescriptive uncertainties. This is mainly due to the fact that the prescriptive uncertainties can more explicitly be described and treated mathematically, while no such direct means are available for the non-prescriptive uncertainties. It is discussed later in this chapter that the design of FRP-based structural strengthening systems involves considerably dominant non-prescriptive uncertainties than the conventional structural design using concrete and steel.

It can be seen from Fig. 3.3 that the prescriptive uncertainties are further classified into constitutive and behavioural uncertainties, which are discussed in the subsequent sections. Non-prescriptive uncertainties, on the other hand, comprise of lack of knowledge and time-testimony (together they indicate the lack of confidence in FRP applications in Civil Engineering), and vagueness (i.e. subjectivity or fuzziness). Truly speaking, vagueness can actually be treated analytically, e.g. using fuzzy mathematics [Kansara and Ramanjaneyulu (2007a)]. Therefore, ideally vagueness could be more
appropriately considered as a form of prescriptive uncertainties. However, such approaches still do not form the mainstream convention in structural engineering design and in calibration of standards by-and-large. Dealing with the subjective nature of uncertainties is left for future work, and hence vagueness forms a part of the non-prescriptive uncertainties in Fig. 3.3. The subsequent sections discuss various forms of uncertainties.

### 3.4 Constitutive uncertainties

The mechanical properties of FRP composites, such as rupture strain, modulus of elasticity and tensile strength, inherently carry uncertainties within them. Since constitutive relationships for FRP composites are invariably a function of their mechanical properties, these uncertainties can more appropriately be called constitutive uncertainties. The most common type of constitutive uncertainties is variability, which can either be random or trend-specific, as discussed below. The former is equally applicable to any structural material, while the latter is more specific to FRP composites and their use as externally bonded FRP reinforcement.

#### 3.4.1 Random variability

Random variability is attributed to local variations in production quality due to fluctuations in manufacturing and ambient conditions during production. It exists even under highly controlled production processes. The intra- and inter-production lot variability in FRP material properties are inevitable, under which the mechanical (and even the geometrical) properties of FRP composites can randomly assume any value within an apparent range. A statistical description of the FRP material property is popularly used to designate the random variability, and hence it is also called statistical variability [Montgomery et al. (2009)]. Such statistical descriptions include an assumed statistical distribution expressed by its statistical parameters such as the mean, standard deviation and coefficient of variation [Elishakoff (1999), Elishakoff (1983)]. The statistical mean value of an FRP material property is a quantitative indicator of the qualitative superiority or inferiority of an FRP composite when compared for the intra- and inter-production lots of FRP composite manufactured using the same manufacturing process. However, it does not represent the random variability completely by itself and needs to be supplemented by either an associated standard deviation or a coefficient of variation. A numerically higher value of standard deviation reflects a greater variability in the intra- or inter-production lot quality, resulting in a higher chance of an individual value of the FRP material property to be away from its
representative mean value. Together the statistical mean and standard deviation (or coefficient of variation) express the random variability associated with a FRP material property more completely.

### 3.4.2 Trend-specific variability

Unlike random variability, this type of variability follows a specific trend. Depending upon whether the trend is attributed to a physical phenomenon or is due to a specific manufacturing and/or installation process, trend-specific variability can be thought as either *phenomena-reliant* or *process-reliant* respectively.

**Phenomena-reliant variability** is attributed to the change in material properties under the influence of a specific physical phenomenon, such as environmental deterioration and mechanical degradation of FRP. This type of variability shows a gradual reduction in the mechanical properties of FRP and is invariably a function of time. The *environmental deterioration based variability* of FRP material properties is generally estimated through accelerated aging tests (see Chapter 2), whereas the *mechanical degradation based variability* is estimated through accelerated fatigue tests [Harikrishna *et al.* (2005)]. For an arbitrary FRP material property ($Y$), a ratio of its initial undeteriorated or un-degraded value ($Y_0$), to its value ($Y_t$) that is retained after a certain simulated duration of a specific environmental exposure or mechanical fatigue condition under an accelerated aging test, is used to quantify phenomena reliant variability ($\Delta_Y$) [Eq. (3.5)].

$$\Delta_Y = \frac{y_0}{y_t}$$  \hspace{1cm} (3.5)

**Process-reliant variability** is attributed to the factors that reflect relative superiority or inferiority of the FRP composites produced through different routes of manufacturing and application. Characteristics such as fibre-to-resin ratio, precision of fibre orientation and effectiveness of the resin to hold fibres within it vary significantly for different manufacturing processes of FRP and its installation as an externally bonded reinforcement. Due to this, FRP material properties that are effectively available in in-field conditions are considerably dependent on the application processes of the FRP (covering both, manufacturing and installation). Quality control and uniformity of application vary considerably for factory-made and field-manufactured FRP reinforcement. Similarly, machine-assisted and manual installations also result in differential properties of FRP reinforcement. Different manufacturing processes have possible weaknesses inherent to them. Additionally, certain manufacturing and
installation processes involve higher chances of geometric deviations (that includes deviations in dimensions and fibre-orientations) compared to other processes. The deviations in the relative quantities of resin and fibres can also vary for different manufacturing and installation processes. These characteristics considerably influence the effectiveness of the FRP reinforcement leading to process-reliant variability in FRP material properties. The most logical basis to quantify this class of variability could be the volumetric deviations in fibre content relative to resin associated with different manufacturing and installation processes. As an illustration, an arbitrary FRP material property \((Y)\) can be shown to comprise weighted aggregation of that property of fibre \((Y_{\text{fibre}})\) and matrix \((Y_{\text{matrix}})\). The volumetric fractions of fibres \((v_{\text{fibre}})\) and matrix \((v_{\text{matrix}})\) relative to the total volume of composite \((v_{\text{composite}})\) provide the relative weights in this weighted aggregation [Eq. (3.6)].

\[
Y_{\text{composite}} = \left( \frac{v_{\text{fibre}}}{v_{\text{composite}}} \right) \times Y_{\text{fibre}} + \left( \frac{v_{\text{matrix}}}{v_{\text{composite}}} \right) \times Y_{\text{matrix}} \tag{3.6}
\]

On this basis, the variations in an FRP material property under different manufacturing and installation routes of FRP can be worked out. Such variations directly influence the capability of FRP composites at fibre-, lamina- and laminate-levels [Bank (2006)].

### 3.5 Behavioural uncertainties

In structural strengthening, the behaviour of a strengthened structural element as predicted within the design is measured with reference to either a real (or very likely) behaviour ascertained from experimental observations, or an ideally expected behaviour towards assuring the design to be worthy. Behavioural uncertainties arise due to a deviation between the behaviour as predicted in design from any of these ‘reference’ behaviours, which are discussed below.

#### 3.5.1 Deviation from real behaviour

For practical reasons, the FRP-based strengthening design process has to rest on the following design assumptions [Bank (2006)]:

- The constitutive relationships of the FRP composites are perfectly linear elastic.
- Linear strain distribution exists across the depth of a section, both before and after strengthening.
- No relative slip occurs between the FRP and concrete substrate.
- Shear deformation within the adhesive layer is negligible.
- Externally bonded FRP debonds at a particular strain level in the FRP.
These simplifying assumptions do not completely reflect the realistic mechanics and behaviour of FRP, and consequently lead to behavioural uncertainty [Karbhari (2003), Karbhari and Abanilla (2007)]. Additionally, the mathematical models devised to represent various empirically observed phenomena also carry simplifications and imprecisions. While such assumptions, simplifications and ignorance provide ease in practice, they instigate deviations in the mechanics and behaviour of FRP reinforcement or FRP-strengthened RC member as predicted within the design from its realistic mechanics and behaviour. This deviation amounts to a class of uncertainty, called *behavioural uncertainties*, in this study.

It can be noted that the last three of the above assumptions, in particular, directly or indirectly concern the bond between the FRP and concrete substrate. As discussed in Chapter 2, different structural configurations of flexural and shear strengthening systems exhibit different bond-criticality. Therefore, the impact of these assumptions, and hence the extent of the consequent behavioural deviations, varies for these structural configurations.

### 3.5.2 Deviation from ideal behaviour

A little consideration will show that under the peculiarly distinctive properties of FRP composites, e.g. lack of plastic deformations as discussed in Chapter 2, the behaviour of a strengthened structural element as predicted in design can deviate from the ideally expected behaviour. For example, a worthy structural design involves a relatively high ductility-content to be associated with the governing failure mode. However, FRP composites do not exhibit plastic deformation and, therefore, reduced ductility in the post-strengthening stage for a flexurally strengthened RC section can be expected compared to pre-strengthening [Bencardino et al. (2002)]. This can be seen as a form of deviation from the ideally expected behaviour of an FRP-strengthened RC structure.

### 3.6 Lack of confidence and data

Due to lack of history of use within the construction industry, FRP composites involve lack of time-testimony. Furthermore, there exist some gaps in understanding the mechanics and behaviour [Darby et al. (2007)]. Also, despite a growing interest of researchers and engineers in FRP composites, a shortage of reliable experimental data still prevails. Most experimental investigations focus on the response of a particular type of FRP material tested under limited design scenarios. It is very difficult to ensure that all the possible variants of FRP composites – in terms of their type, form and application in structural rehabilitations – are tested under all possible design scenarios.
Consequently, it is very likely that the analytical models calibrated against small-sample databases are extended to cover untested FRP material types and design scenarios, with rather arbitrary adjustments or on the basis of preliminary tests. Also, the full-scale testing and long-term monitoring of FRP-rehabilitated structures are not yet very common. All these factors contribute to lack of confidence in using FRP composites for structural rehabilitation, which forms the non-prescriptive uncertainty.

3.7 Taxonomy of safety parameters

As stated earlier, the safety format for strengthening design consists of prescribing a set of safety parameters to account for various uncertainties associated with FRP properties, mechanics and behaviour. Within a strengthening design process, these safety parameters can be attached directly onto the inputs (such as FRP material properties) and/or onto the outputs (such as resistance contribution of FRP reinforcement) of the design process. In addition, they can also be embedded within the design process at intermediate design steps. Fig. 3.4 presents classification of the safety parameters on this basis.

![Figure 3.4](image)

Basic taxonomy of the safety parameters associated with FRP-based structural strengthening systems
It can be seen that the safety parameters attached onto FRP material properties are termed as *material safety parameters*, while the safety parameters attached onto resistance contribution of FRP and/or onto resistance of a strengthened RC section are called *resistance safety parameters*. The safety parameters attached at intermediate design steps are termed as *switchers*. These safety parameters are discussed in the following sections.

### 3.8 Material safety parameters

Material properties of FRP composite are the primary inputs in strengthening design process. Safety parameters attached to material properties of FRP are called the *material safety parameters* in this study. These safety parameters are calibrated in accordance with the type and extent of *constitutive uncertainties*. Application of these safety parameters results into under-estimation of FRP material properties towards setting a required *reserved-strength*. Attaching the material safety parameters at the beginning of the design process shows an *active* approach of addressing uncertainties. These safety parameters propagate (in a rather scattered manner) throughout the design process, and hence are capable of influencing the course of strengthening design process mainly by influencing the governing failure modes of strengthened RC members. Therefore, these safety parameters are *failure mode sensitive*, as suggested in Fig. 3.4. Three types of material safety parameters are commonly found in strengthening design process. In this study they are termed as:

- Production quality-reliant safety parameter
- Phenomena-reliant safety parameter
- Application process-reliant safety parameter

#### 3.8.1 Production quality-reliant safety parameter

The production quality-reliant safety parameter accounts for the inherent *random variability* in the material properties of FRP arising out of the practically uncontrollable variations in the quality control measures in the production of FRP. Since such a random variability is inevitable, the prescription of the production quality-reliant material safety parameter is mandatory in strengthening design.

(A) **Mathematical basis**

Following the traditional conventions, a *characteristic value* of a FRP material property accounting for the random variability associated with it is synthesised. In order to
derive a characteristic value of an FRP material property, a product of a certain factor and standard deviation corresponding to that material property is deducted from the statistical mean. This ensures that the characteristic value remains numerically lesser than the statistical mean by a certain amount. The cushion so created between the statistical mean and the characteristic value accounts for the random variability associated with an FRP material property. The multiplier to the standard deviation of the material property is referred to as the production quality-reliant safety parameter ($\gamma_{PQR}$) in this study. For an arbitrary FRP material property ($Y$), with $\bar{Y}$, $\sigma_Y$ and $p_Y$ ($= \sigma_Y / \bar{Y}$) as its statistical mean, standard deviation and coefficient of variation respectively, the synthesis of characteristic value ($Y_{ck}$) is demonstrated through Eq. (3.7). The upper bound for parameter $\gamma_{PQR}$ suggested in Eq. (3.7) is derived to avoid $Y_{ck}$ assuming zero or negative values, which are impractical. The value of parameter $\gamma_{PQR}$ is selected such that the probability ($P_v$) of an individual material property value ($Y_i$) being less than its characteristic value does not exceed a maximum permitted threshold probability ($P_{e-max}$), as demonstrated through Eq. (3.8). From Eqs. (3.7) and (3.8), the role of parameter $\gamma_{PQR}$ in controlling the random variability within the design process can be appreciated. Following the notion of conservativeness presented through Eq. (3.1), the conservativeness produced in process of deriving a characteristic value of an arbitrary FRP material property can be expressed through Eqs. (3.9) and (3.10).

\[
Y_{ck} = \bar{Y} - \gamma_{PQR} \sigma_Y = \bar{Y} \left(1 - \gamma_{PQR} \frac{p_Y}{\bar{Y}}\right) \quad \left[0 \leq \gamma_{PQR} < \left(\frac{\bar{Y}}{\sigma_Y} = \frac{1}{p_Y}\right)\right] \quad (3.7)
\]

\[
P_v \left(Y_i < Y_{ck}\right) \leq P_{e-max} \quad (3.8)
\]

\[
C = \bar{Y} - Y_{ck} = \gamma_{PQR} \bar{Y} \frac{p_Y}{\bar{Y}} = \gamma_{PQR} \sigma_Y \quad (3.9)
\]

\[
I_C = \gamma_{PQR} p_Y \quad (3.10)
\]

Traditionally, the threshold probability ($P_{e-max}$) is considered to be 5%. For a variable statistically described using the Normal (Gaussian) distribution, this value of $P_{e-max}$ corresponds to parameter $\gamma_{PQR}$ being equal to 1.64. Table 3.1 provides the $\gamma_{PQR}$-$P_{e-max}$ correspondence under the assumption of Normal distribution. Inferring a higher value of parameter $\gamma_{PQR}$ as an intention of imparting higher conservativeness is a contestable claim. However, a higher value of parameter $\gamma_{PQR}$ prescribed by one design guideline relative to another certainly indicates an intention to bring higher relative conservativeness into design. In any case, such a prescription not only results in an
increased margin between the characteristic and the mean values of an FRP property, but also reduces the threshold probability $P_{e_{\text{max}}}$. 

<table>
<thead>
<tr>
<th>$\gamma_{PQR}$</th>
<th>$P_{e_{\text{max}}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>---</td>
</tr>
<tr>
<td>1.64</td>
<td>5.00</td>
</tr>
<tr>
<td>2.00</td>
<td>2.28</td>
</tr>
<tr>
<td>3.00</td>
<td>0.14</td>
</tr>
</tbody>
</table>

*For the assumed Normal (Gaussian) statistical distribution of the FRP material properties described using the known statistical parameters obtained based on (theoretically) an infinite number of samples*

It is to be noted that while the parameter $\gamma_{PQR}$ can be shown mathematically to correlate with the threshold probability ($P_{e_{\text{max}}}$), the numerical value of $P_{e_{\text{max}}}$ itself is largely decided subjectively based on engineering judgement and traditional practices [Kansara and Ramanjaneyulu (2005)]. Here, it should not be inferred that mathematics solely is better able to judge safety than engineering judgement. However, directional criteria to arrive at justifiable values of parameter $\gamma_{PQR}$, particularly for externally bonded applications of FRP that involves multiple failure modes, can be of use in rationalising the design guidelines. This is shown later in this thesis.

(B) **Characterisation**

In order to demonstrate the sensitivity of strengthening design process to production quality-reliant safety parameter ($\gamma_{PQR}$) later in this thesis, the quantitative prescription of parameter $\gamma_{PQR}$ is characterised into a set of five qualitative clusters, as shown in Table 3.2.

<table>
<thead>
<tr>
<th>Production Quality Class</th>
<th>$\gamma_{PQR}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>PQ Class 0</td>
<td>0.00</td>
<td>No random variability is addressed (statistical mean and characteristic values are identical) e.g. ACI440 specification for modulus of elasticity of FRP</td>
</tr>
<tr>
<td>PQ Class I</td>
<td>1.00</td>
<td>---</td>
</tr>
<tr>
<td>PQ Class IA</td>
<td>1.64</td>
<td>Traditional value for most structural materials</td>
</tr>
<tr>
<td>PQ Class II</td>
<td>2.00</td>
<td>e.g. TR55 specification for rupture strain, modulus of elasticity and tensile strength of FRP</td>
</tr>
<tr>
<td>PQ Class III</td>
<td>3.00</td>
<td>e.g. ACI440 specification for rupture strain and tensile strength of FRP</td>
</tr>
</tbody>
</table>
Each of these clusters represents a specific production quality class (designated as PQ Class), and reflects one of the possible design scenarios pertaining to the random variability in production quality of FRP. In an increasing order of stringency, these clusters are presented in Table 3.2.

(C) Design prescription

Master Chart 3.1 at the end of this chapter summarises the ACI440 and TR55 design prescriptions for parameter $\gamma_{PQR}$ according to ACI440 and TR55 specifications. These prescriptions should be read in conjunction with the characteristic clusters presented in Table 3.2 above. It can be seen from Master Chart 3.1 that parameter $\gamma_{PQR}$ is applied at Level I within the strengthening design process, which is explained later in this chapter. It can also be seen that according to ACI440, parameter $\gamma_{PQR}$ prescribed for rupture strain and tensile strength of FRP belongs to PQ Class III. According to TR55, on the other hand, parameter $\gamma_{PQR}$ prescribed on rupture strain and modulus of elasticity of FRP belongs to PQ Class II. Thus, amongst ACI440 and TR55, the prescription of parameter $\gamma_{PQR}$ on rupture strain of FRP is relatively stringent for the former compared to the latter. Furthermore, ACI440 does not prescribe parameter $\gamma_{PQR}$ on modulus of elasticity of FRP. Also, neither ACI440 nor TR55 makes any distinction for the type of FRP composite and their manufacturing and application routes when prescribing parameter $\gamma_{PQR}$. Other data in Master Chart 3.1 will be described in subsequent subsections.

3.8.2 Phenomena-reliant safety parameter

The phenomena-reliant material safety parameter deals with the deterioration and degradation of FRP material properties under time-dependent phenomena, and is proposed to absorb the potential risks arising out of phenomena-reliant variability associated with FRP material properties. This variability is mainly attributed to the environmental and/or mechanical influences and leads to slow and gradual reduction in the values of FRP material properties over time. Accordingly, the phenomena-reliant safety parameter can further be of two types:

- Environmental deterioration (EDT) based material safety parameter ($\gamma_{EDT}$)
- Mechanical degradation (MDG) based material safety parameter ($\gamma_{MDG}$)

(A) Mathematical basis

The mechanical wear based degradations, being a consequence of man-made processes, are relatively more controllable than the environmental deterioration based
degradations. Therefore, these are more appropriately accounted for through *restrictive prescriptions* (such as a stress limit in FRP reinforcement under service load conditions) rather than through *absorptive prescriptions* (such as a safety factor accounting for the reduction in values of FRP material properties). Such a restrictive approach for the mechanical degradation based safety parameter does not explicitly influence the ultimate limit state (and consequently the resistance contribution of the FRP component). Therefore, it is not discussed in this study from this point onwards. On the other hand, imposition of such restrictive controls to limit or control deterioration under environmental influences is not possible. This demands prescription of a safety parameter specifically meant to absorb the consequences of material deterioration. The environmental deterioration based safety parameter \((y_{EDT})\) is calibrated in accordance with the phenomena-reliant variability in FRP material properties due to environmental exposure. For an arbitrary FRP material property \((Y)\), it is applied on its characteristic value \((Y_{ck})\) towards obtaining its design value \((Y_d)\), as shown through Eq. (3.11). From the notion of conservativeness presented through Eq. (3.1), the conservativeness produced through this operation can be expressed through Eqs. (3.12) and (3.13).

\[
Y_d = \frac{Y_{ck}}{y_{EDT}} \tag{3.11}
\]

\[
C = Y_{ck} - Y_d = Y_{ck} \left(1 - \frac{1}{y_{EDT}}\right) \tag{3.12}
\]

\[
I_c = \left(1 - \frac{1}{y_{EDT}}\right) \tag{3.13}
\]

Various FRP materials undergo different extents of deterioration under an identical exposure scenario and, therefore, separate environmental deterioration based safety parameters are used for different FRP materials.

**(B) Characterisation**

In order to demonstrate the sensitivity of strengthening design process to environmental deterioration based safety parameter \((y_{EDT})\) later in this thesis, the quantitative prescription of parameter \(y_{EDT}\) is characterised into a set of three qualitative clusters, as shown in Table 3.3. Each of these clusters represents a specific environmental exposure class (designated as EE Class), and reflects one of the possible design scenarios pertaining to the phenomena-reliant variability in FRP material property due to environmental exposure. In an increasing order of stringency, these clusters are presented in Table 3.3. This clustering is based on the ACI440 classification for the environmental exposure conditions.
Table 3.3

<table>
<thead>
<tr>
<th>Environmental Exposure Class</th>
<th>Representative conditions*</th>
</tr>
</thead>
<tbody>
<tr>
<td>EE Class I</td>
<td>Interior exposure</td>
</tr>
<tr>
<td>EE Class II</td>
<td>Exterior exposure</td>
</tr>
<tr>
<td></td>
<td>(e.g. bridges, piers, unenclosed parking, etc.)</td>
</tr>
<tr>
<td>EE Class III</td>
<td>Aggressive exposure</td>
</tr>
<tr>
<td></td>
<td>(e.g. chemical plants, waste-water treatment plants, etc.)</td>
</tr>
</tbody>
</table>

*Devised based on ACI440 classification for environmental exposure

(C) **Design prescription**

As discussed in Chapter 2, ACI440 prescribes an environmental reduction factor \( C_E \), which is numerically less than unity. Factor \( C_E \) is applied as a multiplier to the characteristic value of an FRP material property towards obtaining the corresponding design value. TR55, on the other hand, prescribes a partial safety factor, which is numerically greater than unity. Against the ACI440 strategy of prescribing numerically identical factor \( C_E \) on tensile strength and rupture strain capacity of FRP, TR55 prescribes numerically different partial safety factors on rupture strain and modulus of elasticity of FRP (designated as \( \gamma_E \) and \( \gamma_E \) respectively). The equivalent \( \gamma_{EDT} \) corresponding to factors prescribed in ACI440 and TR55 can be expressed through Eqs. (3.14) and (3.15) respectively.

\[
\begin{align*}
\begin{bmatrix}
\gamma_{EDT-E} \\
\gamma_{EDT-E} \\
\gamma_{EDT-f}^{(ACI440)}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{C_E} \\
1.00 \\
\frac{1}{C_E}
\end{bmatrix}
\end{align*}
\]  

\[
\begin{align*}
\begin{bmatrix}
\gamma_{EDT-E} \\
\gamma_{EDT-E} \\
\gamma_{EDT-f}^{(TR55)}
\end{bmatrix}
= \begin{bmatrix}
\gamma_E \\
\gamma_E \\
\gamma_E \times \gamma_E
\end{bmatrix}
\end{align*}
\]

Master Chart 3.1 at the end of this chapter summarises the ACI440 and TR55 design prescriptions for parameter \( \gamma_{EDT} \). These prescriptions should be read in conjunction with the characteristic clusters presented in Table 3.3 above. It can be seen that, in general, the specifications for parameter \( \gamma_{EDT} \) are more stringent in TR55 than in ACI440. The implications and effectiveness of this observation is demonstrated later in this thesis. Master Chart 3.1 suggests that parameter \( \gamma_{EDT} \) is applied at Level II within the strengthening design process. This fact is discussed later in this chapter.
3.8.3 Application process-reliant safety parameter

The application process-reliant safety parameter ($\gamma_{APR}$) accounts for the application process-reliant variability. As discussed earlier in this chapter, deviations in volumetric proportions of the constituents comprising a FRP composite provide a means to account for the process-reliant variability for different manufacturing and installation processes. These deviations reflect relative superiority or inferiority of the manufacturing and installation processes and can then be used as a basis for deciding on the relative values of the parameter $\gamma_{APR}$.

(A) Mathematical basis

Parameter $\gamma_{APR}$, in a way similar to parameter $\gamma_{EDT}$, is applied on the characteristic value of an FRP material property towards obtaining the corresponding design value. For an arbitrary FRP material property ($Y$), this is shown through Eq. (3.16). In line of notion of conservativeness presented through Eq. (3.1), the conservativeness produced through this operation can be expressed through Eqs. (3.17) and (3.18).

$Y_d = \frac{Y_{ck}}{\gamma_{APR}}$  \hspace{1cm} (3.16)

$C = Y_{ck} - Y_d = Y_{ck} \left(1 - \frac{1}{\gamma_{APR}}\right)$  \hspace{1cm} (3.17)

$I_C = \left(1 - \frac{1}{\gamma_{APR}}\right)$  \hspace{1cm} (3.18)

(B) Characterisation

In order to demonstrate the sensitivity of strengthening design process to application process-reliant safety parameter ($\gamma_{APR}$) later in this thesis, the quantitative prescription of parameter $\gamma_{APR}$ is characterised into a set of four qualitative clusters, as shown in Table 3.4. Each of these clusters represents a specific manufacturing and installation class (designated as M & I Class), and reflects one of the possible design scenarios pertaining to the process-reliant variability in FRP material property. In an increasing order of stringency, these clusters are presented in Table 3.4. This clustering is based on the TR55 classification, which is based on EUROCOMP classification for the manufacturing and installation conditions.
Table 3.4
Characteristic clusters for the parameter $\gamma_{APR}$

<table>
<thead>
<tr>
<th>Manufacturing and Installation (M &amp; I) Class</th>
<th>Represented Application Processes Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>M &amp; I Class I</td>
<td>Form of FRP Composite</td>
</tr>
<tr>
<td></td>
<td>Manufacturing/Installation Process</td>
</tr>
<tr>
<td>Plates</td>
<td>Pultruded, Prepreg</td>
</tr>
<tr>
<td>Sheets/Tapes</td>
<td>Machine-controlled Application</td>
</tr>
<tr>
<td>Prefabricated Shells</td>
<td>Filament Winding</td>
</tr>
<tr>
<td>M &amp; I Class II</td>
<td>Plates</td>
</tr>
<tr>
<td></td>
<td>Preformed</td>
</tr>
<tr>
<td></td>
<td>Sheets/Tapes</td>
</tr>
<tr>
<td></td>
<td>Vacuum Infusion</td>
</tr>
<tr>
<td></td>
<td>Prefabricated Shells</td>
</tr>
<tr>
<td></td>
<td>Resin Transfer Moulding</td>
</tr>
<tr>
<td>M &amp; I Class III</td>
<td>Plates</td>
</tr>
<tr>
<td></td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Sheets/Tapes</td>
</tr>
<tr>
<td></td>
<td>Wet Lay-up</td>
</tr>
<tr>
<td></td>
<td>Prefabricated Shells</td>
</tr>
<tr>
<td></td>
<td>Hand Lay-up</td>
</tr>
<tr>
<td>M &amp; I Class IV</td>
<td>Plates</td>
</tr>
<tr>
<td></td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Sheets/Tapes</td>
</tr>
<tr>
<td></td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Prefabricated Shells</td>
</tr>
<tr>
<td></td>
<td>Hand-help Spray Application</td>
</tr>
</tbody>
</table>

Devised based on TR55 and EUROCOMP classification for application process.

(C) **Design prescription**

As discussed in Chapter 2, ACI440 does not prescribe any factor that accounts for application process-reliant variability in FRP material properties. TR55, on the other hand, prescribes an additional partial safety factor (designated as $\gamma_{nm}$, and which is numerically greater than unity) to account for this class of variability. The equivalent $\gamma_{APR}$ corresponding to factors prescribed in ACI440 and TR55 can be expressed through Eqs. (3.19) and (3.20) respectively.

\[
\begin{align*}
\begin{cases}
\gamma_{APR-E}^{(ACI440)} \\
\gamma_{APR-E}^{(TR55)} \\
\gamma_{APR-f}^{(ACI440)} \\
\gamma_{APR-f}^{(TR55)}
\end{cases}
\end{align*} = 1.00 \\
\gamma_{nm}
\]

(Eq. 3.19)  
(Eq. 3.20)

Master Chart 3.1 at the end of this chapter summarises the ACI440 and TR55 design prescription for parameter $\gamma_{APR}$. These prescriptions should be read in conjunction with the characteristic clusters presented in Table 3.4 above. Master Chart 3.1 suggests that parameter $\gamma_{APR}$ is applied at Level II within the strengthening design process. This fact is discussed later in this chapter.
3.9 Resistance safety parameters

The resistance contribution of FRP reinforcement and the resistance of an FRP-strengthened RC section are the primary outputs of a strengthening design process. The safety parameters attached to these outputs are called the resistance safety parameters in this study. These parameters, ideally, are calibrated in accordance with the type and extent of behavioural uncertainties. Depending upon the format of their prescription, application of these safety parameters results into either under-estimation of design resistance or over-estimation of required capacity of the structural element in post-strengthening state. Either of these sets an over-strength that accounts for the behavioural uncertainties. This set of safety parameters, unlike the material safety parameters, shows a passive approach of addressing uncertainties. Furthermore, since these safety parameters are introduced at the very end of the design process, they are incapable of influencing the course of design process, and hence are failure mode insensitive safety parameters, as mentioned earlier in Fig. 3.4. Three types of resistance safety parameters are commonly found in strengthening design process. In this study they are termed as:

- Compensatory safety parameter
- Punitive safety parameter
- Supplementary safety parameter

3.9.1 Compensatory safety parameter

The compensatory resistance safety parameter ($\gamma_{CMP}$) aims to compensate for the deviation between the behaviour of an externally bonded FRP reinforcement as predicted in design from the real behaviour. This parameter is devised separately for flexural and shear strengthening processes. Different values of this parameter are suggested for different structural configurations in accordance with their bond- or contact-criticality [ACI440 (2002), Bank (2006)].

(A) Mathematical basis

The parameter ($\gamma_{CMP}$) is generally calibrated using reliability analysis so as to achieve a minimum value of the reliability index for a major segment of the range of strengthening design solutions. An increase in the stringency of parameter $\gamma_{CMP}$ (i.e. prescribing a numerically higher value of parameter $\gamma_{CMP}$) enhances reliability of design solutions. More details of the reliability analysis for such purposes can be obtained from Kansara and Ramanjaneyulu (2005), Felder and Spurlin (2005),
Frangopol et al. (1997). Parameter $\gamma_{CMP}$ is applied on the nominal value of resistance contribution of FRP reinforcement to obtain the corresponding modified (i.e. compensated for behavioural deviations pertaining to FRP) value of nominal resistance, as shown in Eq. (3.21). In line of the notion of conservativeness expressed through Eq. (3.1), the conservativeness produced due to this operation can be expressed through Eqs. (3.22) and (3.23).

$$R_{FRP}(\text{nominal-modified}) = \frac{R_{FRP}(\text{nominal})}{\gamma_{CMP}} \quad [\gamma_{CMP} \geq 1]$$  \hspace{1cm} (3.21)

$$C = R_{FRP}(\text{nominal}) - R_{FRP}(\text{nominal-modified}) = R_{FRP}(\text{nominal}) \left(1 - \frac{1}{\gamma_{CMP}}\right)$$ \hspace{1cm} (3.22)

$$I_C = \left(1 - \frac{1}{\gamma_{CMP}}\right)$$ \hspace{1cm} (3.23)

(B) **Characterisation**

A set of three clusters each for flexure and shear strengthening design characterise the quantitative prescription of the parameter $\gamma_{CMP}$, as shown in Table 3.5. Each of these clusters represents a possible class of required compensation (designated as CMP Class). Reading Table 3.5 from top down, these clusters are sorted in an increasing order of the stringency exhibited by them.

<table>
<thead>
<tr>
<th>Strengthening Type</th>
<th>Compensation Class</th>
<th>Compensating condition</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flexural Strengthening</strong></td>
<td>CMP Class F0</td>
<td>No resistance compensation</td>
<td>To be used as a reference</td>
</tr>
<tr>
<td></td>
<td>CMP Class Fl</td>
<td>Compensation for contact-critical applications</td>
<td>FRP plate mechanically anchored without any adhesive bond</td>
</tr>
<tr>
<td></td>
<td>CMP Class FII</td>
<td>Compensation for bond-critical applications</td>
<td>Usually most design situations excluding the above</td>
</tr>
<tr>
<td><strong>Shear Strengthening</strong></td>
<td>CMP Class S0</td>
<td>No resistance compensation</td>
<td>To be used as a reference</td>
</tr>
<tr>
<td></td>
<td>CMP Class SI</td>
<td>Compensation for contact-critical applications</td>
<td>Fully wrapped configuration</td>
</tr>
<tr>
<td></td>
<td>CMP Class SII</td>
<td>Compensation for bond-critical applications</td>
<td>U-wrapped and Sides-only configurations</td>
</tr>
</tbody>
</table>

(C) **Design prescription**

ACI440 prescribes a strength reduction factor ($\Psi_f$), having a value lesser than unity, to be applied as a multiplier to the nominal value of resistance contribution of FRP reinforcement towards accounting for the behavioural uncertainties associated with use of FRP. Factor $\Psi_f$ is prescribed for flexural and shear strengthening in accordance with the bond- or contact-criticality of the structural configurations of the strengthening systems. TR55, on the other hand, does not prescribe any factor that accounts for...
behavioural uncertainties. The equivalent $\gamma_{CMP}$ corresponding to ACI440 and TR55 specifications can be expressed through Eqs. (3.24) and (3.25) respectively.

$$
\gamma_{CMP(ACI440)} = \frac{1}{\Psi_f} 
$$

(3.24)

$$
\gamma_{CMP(TR55)} = 1.00 
$$

(3.25)

Master Chart 3.1 at the end of this chapter summarises the ACI440 and TR55 design prescriptions for parameter $\gamma_{CMP}$. These prescriptions should be read in conjunction with the characteristic clusters presented in Table 3.5 above. It suggests that parameter $\gamma_{CMP}$ is applied at Level III within the strengthening design process. This fact is discussed later in this chapter.

### 3.9.2 Punitive safety parameter

The purpose of a punitive safety parameter ($\gamma_{PNT}$) is to penalise (and not to compensate) for the behavioural uncertainty arising from the deviation between the design prediction for ductility/deformability and the ideal expectation. A worthy design ensures that a strengthened section is governed by flexural failure (either with sufficient ductility or carrying a due penalty for the lack of it), and essentially not by shear failure. Having ensured this condition during the strengthening design for an RC member, practically there is no need to have a ductility-based penalty for shear strengthening. Therefore, such ductility-based penalty is considered for flexural strengthening only and not for shear strengthening.

(A) **Mathematical basis**

Typically, design solutions with low relative ductility are more sensitive to the variations in concrete strength. This makes the statistical distribution of the post-strengthening sectional resistance wider (than that for the pre-strengthening sectional resistance) for the design solutions with low relative ductility [ACI440 (2008)]. Consequently, it leads to an increased probability of failure and reduced reliability of the design solutions carrying lower sectional ductility [ACI440 (2008), Kansara et al. (2009)]. The fact that the flexural resistance and sectional ductility of a strengthened RC section are inversely proportional to each other provides a basis to adjust the resultant conservativeness in the strengthening design solutions. This adjustment is through penalising the flexural resistance in accordance with the ductility-content associated with the flexural strengthening design solution. Parameter $\gamma_{PNT}$ is generally calibrated on this basis to make up for the reduced reliability.
Parameter \( \gamma_{PNT} \) is applied on the nominal value of resistance \( R_{FRP,RC(nominal)} \) of FRP-strengthened section towards obtaining the corresponding design value \( R_{FRP,RC/design} \), as shown in Eq. (3.26). In line of the notion of conservativeness expressed through Eq. (3.1), the conservativeness produced due to this operation can be expressed through Eqs. (3.27) and (3.28).

\[
R_{FRP,RC/design} = \frac{R_{FRP,RC(nominal)}}{\gamma_{PNT}} \quad [\gamma_{PNT} \geq 1]
\]

\[
C = R_{FRP,RC(nominal)} - R_{FRP,RC/design} = R_{FRP,RC(nominal)} \left(1 - \frac{1}{\gamma_{PNT}}\right)
\]

\[
l_c = \left(1 - \frac{1}{\gamma_{PNT}}\right)
\]

**(B) Characterisation**

A set of four clusters characterise the quantitative prescription of the parameter \( \gamma_{CMP} \), as shown in Table 3.6. Each of these clusters represents a possible class of required penalty (designated as PNT Class). Reading Table 3.6 from top down, these clusters are sorted in an increasing order of the stringency exhibited by them.

<table>
<thead>
<tr>
<th>Penalty Class</th>
<th>Representative Conditions</th>
<th>Strain Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNT Class 0</td>
<td>No penalty on flexural or shear resistance</td>
<td>---</td>
</tr>
<tr>
<td>PNT Class I</td>
<td>Penalty on flexural resistance corresponding to adequately-ductile condition</td>
<td>( \varepsilon_{st} \geq \varepsilon_{st-adequate} )</td>
</tr>
<tr>
<td>PNT Class II</td>
<td>Penalty on flexural resistance corresponding to ductile condition</td>
<td>( \varepsilon_{xy} &lt; \varepsilon_{st} &lt; \varepsilon_{st-adequate} )</td>
</tr>
<tr>
<td>PNT Class III</td>
<td>Penalty on flexural resistance corresponding to non-ductile condition</td>
<td>( \varepsilon_{st} \leq \varepsilon_{xy} )</td>
</tr>
</tbody>
</table>

**(C) Design prescription**

Both, ACI440 and TR55 propose a penalty on post-strengthening flexural resistance. However, the form of this parameter and its correspondence with the post-strengthening ductility-content are different in these two design guidelines. Both design guidelines define a particular value of the strain in the tension steel reinforcement, reflecting an *adequate ductility-content* (\( \varepsilon_{st-adequate} \)). The adequate ductility-content for ACI440 and TR55 specifications are presented through Eqs. (3.29) and (3.30) respectively.

\[
\varepsilon_{st-adequate \, (ACI440)} = 0.005
\]
\[ \varepsilon_{st\text{-adequate}}^{(TR55)} = \frac{f_y}{\varepsilon_{st\text{,}m\text{-st}}} + 0.002 \] (3.30)

TR55 suggests an over-strength factor (\(\zeta\)) to augment the required post-strengthening flexural resistance for the flexural strengthening design solutions carrying actual ductility-content less than \(\varepsilon_{st\text{-adequate}}\) [Eq. (3.31)]. However, this over-strength factor is not in direct proportion to the lack of ductility with respect to the adequate ductility-content as seen from Eq. (3.32), which shows an equivalent \(\gamma_{PNT}\) for TR55. ACI440, on the other hand, suggests a strength reduction factor (\(\phi\)) to be applied on the post-strengthened nominal flexural resistance of the member [Eq. (3.33)]. Unlike TR55, this reduction is in proportion to the lack of ductility with respect to the specified \(\varepsilon_{st\text{-adequate}}\) as seen from Eq. (3.34), which shows an equivalent \(\gamma_{PNT}\) for ACI440.

\[ \zeta = \frac{M_{d\text{-required}}}{M_{d\text{-calculated}}} \] (3.31)

\[ \gamma_{PNT}^{(TR55)} = \zeta = \begin{cases} 1.00 & \text{[For } \varepsilon_{st} \geq \varepsilon_{st\text{-adequate}}] \\ 1.15 & \text{[For } \varepsilon_{st} < \varepsilon_{st\text{-adequate}}] \end{cases} \] (3.32)

\[ M_{d\text{-calculated}} = \phi M_{\text{nominal}} \] (3.33)

\[ \gamma_{PNT}^{(ACI440)} = \frac{1}{\phi} = \begin{cases} \frac{1}{0.90} \cong 1.11 & \text{[For } \varepsilon_{st} \geq \varepsilon_{st\text{-adequate}}] \\ \frac{1}{0.65 + 0.25 \frac{\varepsilon_{st\text{-adequate}} - \varepsilon_y}{(\varepsilon_{st\text{-adequate}} - \varepsilon_y)/0.65}} & \text{[For } \varepsilon_{st} < \varepsilon_{st\text{-adequate}} < \varepsilon_y] \\ \frac{1}{0.65} \cong 1.54 & \text{[For } \varepsilon_{st} < \varepsilon_y] \end{cases} \] (3.34)

Master Chart 3.1 at the end of this chapter summarises the ACI440 and TR55 design prescriptions for parameter \(\gamma_{PNT}\). These prescriptions should be read in conjunction with the characteristic clusters presented in Table 3.6 above. Master Chart 3.1 also suggests that the parameter \(\gamma_{PNT}\) is applied at Level IV within strengthening design. This is explained later in this chapter.

### 3.9.3 Supplementary safety parameter

Due to the greater extent of uncertainties and lack of confidence, a drop in overall reliability is expected for FRP-strengthened concrete structures when compared to an equivalent new construction [ACI 440(2008)]. In order to top up the gross safety-content or reliability, the conservativeness associated with the resistance contribution of FRP reinforcement needs to be supplemented. A supplementary safety parameter is employed for this purpose. Such supplementation is essential for both flexural and shear...
strengthening design solutions. Hence, the supplementary safety parameter is specified for both flexural and shear strengthening.

(A) Mathematical basis

The supplementary safety parameter is generally calibrated using reliability analysis. Thus, mathematical basis for supplementary safety parameter is procedurally the same as that for compensatory safety parameter. An increase in the stringency of parameter $\gamma_{SUP}$ (i.e. numerically higher values of parameter $\gamma_{SUP}$) enhances reliability of design solutions. However, unlike compensatory safety parameter, the objective of reliability analysis for supplementary safety parameter is to supplement (and not to compensate) the resultant reliability. Similar to parameter $\gamma_{PNT}$, the parameter $\gamma_{SUP}$ is applied on the nominal value ($R_{nominal}$) of resistance of FRP-strengthened section towards obtaining the corresponding design value ($R_{design}$), as shown in Eq. (3.35). In line of the notion of conservativeness expressed through Eq. (3.1), the conservativeness produced due to this operation can be expressed through Eqs. (3.36) and (3.37).

\[
R_{FRP,RC(\text{design})} = \frac{R_{FRP,RC(\text{nominal})}}{\gamma_{SUP}} \quad [\gamma_{SUP} \geq 1] \quad (3.35)
\]

\[
C = R_{FRP,RC(\text{nominal})} - R_{FRP,RC(\text{design})} = R_{FRP,RC(\text{nominal})} \left(1 - \frac{1}{\gamma_{SUP}}\right) \quad (3.36)
\]

\[
I_C = \left(1 - \frac{1}{\gamma_{SUP}}\right) \quad (3.37)
\]

(B) Characterisation

A set of two clusters for flexure and shear strengthening characterise the quantitative prescription of the parameter $\gamma_{SUP}$, as shown in Table 3.7. Each of these clusters represents a possible class of required supplement (designated as SUP Class). Reading Table 3.7 from top down, these clusters are sorted in an increasing order of the stringency exhibited by them.

<table>
<thead>
<tr>
<th>Strengthening Type</th>
<th>Supplementation Class</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexural Strengthening</td>
<td>SUP Class F0</td>
<td>No supplementation to flexural resistance (to be used as a reference)</td>
</tr>
<tr>
<td></td>
<td>SUP Class F1</td>
<td>Flexural resistance supplemented</td>
</tr>
<tr>
<td>Shear Strengthening</td>
<td>SUP Class S0</td>
<td>No supplementation to shear resistance (to be used as a reference)</td>
</tr>
<tr>
<td></td>
<td>SUP Class S1</td>
<td>Shear resistance supplemented</td>
</tr>
</tbody>
</table>

Table 3.7
Characteristic clusters for the parameter $\gamma_{SUP}$
(C) **Design prescription**

In general, supplementing reliability of strengthening design solutions can be done either through over-estimating required post-strengthening design resistance or under-estimating the actual post-strengthening design resistance. Amongst ACI440 and TR55, the latter does not suggest any factor, either for flexural or shear strengthening, that explicitly serves the purpose of parameter $\gamma_{SUP}$. The former, however, does consider a need of supplementing the reliability of strengthening design solution through a strength reduction factor ($\phi_1$). The equivalent $\gamma_{SUP}$ corresponding to ACI440 and TR55 specifications can be expressed through Eqs. (3.38) and (3.39) respectively.

\[
\gamma_{SUP}^{(ACI440)} = \frac{1}{\phi_1} \quad \text{(3.38)}
\]

\[
\gamma_{SUP}^{(TR55)} = 1.00 \quad \text{(3.39)}
\]

Supplementary safety parameter ($\gamma_{SUP}$) is applied on the nominal resistance of FRP-strengthened structural member, similar to the way the punitive safety parameter ($\gamma_{PNT}$) used in flexural strengthening. Therefore, it is possible to combine the design prescription of these two safety parameters for flexural strengthening. In fact, parameter $\gamma_{PNT}$ for flexural strengthening using ACI440 specifications already contains the effect of parameter $\gamma_{SUP}$. This can be verified by the fact that the value of parameter $\gamma_{PNT}$ is greater than unity, even for PNT Class I that involves the flexural strengthening design solutions carrying actual strain in tension steel greater than or equal to adequate ductility-content. As mentioned earlier, for shear strengthening parameter $\gamma_{PNT}$ is not prescribed, hence supplementing reliability in that case has to be done explicitly by parameter $\gamma_{SUP}$.

Master Chart 3.1 at the end of this chapter summarises the design prescription of parameter $\gamma_{SUP}$ according to ACI440 and TR55 specifications. These prescriptions should be read in conjunction with the characteristic clusters presented in Table 3.7 above. Master Chart 3.1 also suggests that the parameter $\gamma_{PNT}$ is applied at Level IV within strengthening design. This is explained later in this chapter.

### 3.10 Switchers in strengthening design

‘Switchers’ are the design mechanisms intended to accommodate the preferences for failure mechanisms of FRP (e.g., debonding or rupture) within the strengthening design. These switchers, unlike the material and resistance safety parameters, do not account for the uncertainties, and therefore, they are not safety parameters in a true sense. However,
due to their format and strategic locations within the design process, they are able to influence the course of strengthening design process by altering the governing failure mechanism. These switchers, therefore, can be articulated to control the design predictions for failure mechanisms of FRP or post-strengthening failure modes, if so desired. It is to be noted that against an active and a passive approach of addressing uncertainties exhibited by the material and the resistance safety parameters respectively, the switchers exhibit a manipulative approach. A discussion on the switchers in flexural and shear strengthening design process is presented below.

3.10.1 Switchers in flexural strengthening design process

(A) Working mechanism

In a typical flexural strengthening design process, the design rupture strain of FRP numerically competes with the design debonding strain limit in governing the failure mechanism of the externally bonded FRP reinforcement. Based on this fact, a generic format of the switcher in flexural strengthening can be described through Eq. (3.40).

\[ \varepsilon_{fd} = \min[\delta_{fe} \varepsilon_{fd-rupture}, \varepsilon_{fd-debond}] \quad [\delta_{fe} \leq 1] \]  \hspace{1cm} (3.40)

It can be seen that the multiplier \( \delta_{fe} \) to \( \varepsilon_{fd-rupture} \) can be set any value up to and including unity. A little consideration will show that setting the multiplier \( \delta_{fe} \) equal to unity leads to a condition in which the debonding strain limit has to numerically compete with ‘full rupture strain capacity’ of FRP within the above switcher. On the other hand, having a less than unity value for the multiplier \( \delta_{fe} \) leads to a numerical competition between the debonding strain limit and a ‘near rupture strain capacity’ of FRP. Accordingly, the above generic format of the switcher can be presented in two specific formats, as shown below:

The ‘rupture-debonding switch’ (RDS):

\[ \varepsilon_{fd} = \min[\varepsilon_{fd-rupture}, \varepsilon_{fd-debond}] \quad [\:: \delta_{fe} = 1] \]  \hspace{1cm} (3.41)

The ‘near rupture-debonding switch’ (NRDS):

\[ \varepsilon_{fd} = \min[\delta_{fe} \varepsilon_{fd-rupture}, \varepsilon_{fd-debond}] \quad [\:: \delta_{fe} < 1] \]  \hspace{1cm} (3.42)

It is to be noted that the implication of prescribing an NRDS instead of an RDS is not merely limited to the replacement of full rupture strain capacity with near rupture strain capacity of FRP. In fact, the multiplier \( \delta_{fe} \) effectively augments the material safety parameters prescribed on the rupture strain capacity of FRP, and therefore, it is a source

Assessment of conservativeness in design of FRP-based structural strengthening systems
Kunal D. Kansara (2014)
Assessment of conservativeness in design of FRP-based structural strengthening systems
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of conservativeness in addition to the material and resistance safety parameters. This fact largely remains disguised within the design specifications. The following design prescriptions according to ACI440 and TR55 illustrate the switcher in flexural strengthening design.

(B) Design prescription

As discussed in Chapter 2, TR55 suggests a pre-set numerical constant value for debonding strain in FRP (i.e. conforming to Type I debonding strain limit) [Eq. (3.43)], while ACI440 specifies a modulus dependent debonding strain in FRP (i.e. conforming to Type II debonding strain limit) with an upper bound restriction [Eq. (3.44)].

\[ \varepsilon_{f_d, \text{debond (TR55)}} = 0.008 \] (3.43)

\[ \varepsilon_{f_d, \text{debond (ACI440)}} = 0.41 \frac{t_c}{n t_f E_{f_d}} \leq 0.9 \varepsilon_{f_d, \text{rupture (ACI440)}} \] (3.44)

It is the upper bound restriction in Eq. (3.44) that replaces the full design rupture strain capacity of FRP (\( \varepsilon_{f_d, \text{rupture}} \)) with the near design rupture strain capacity (0.9 \( \varepsilon_{f_d, \text{rupture}} \)) within the competition with the debonding strain limit to govern the effective failure strain (\( \varepsilon_{f_d} \)) for ACI440. With the multiplier \( \delta_{f_E} = 0.9 \), the NRDS conforming to ACI440 specifications can be represented through Eq. (3.45).

\[ \varepsilon_{f_d (ACI440)} = \min \left( 0.9 \varepsilon_{f_d, \text{rupture (ACI440)}}, 0.41 \sqrt{\frac{t_c}{n t_f E_{f_d}}} \right) \] (3.45)

In the absence of any analogous upper bound restriction on the debonding strain limit, TR55 specifications conform to an RDS, which can be represented through Eq. (3.46).

\[ \varepsilon_{f_d (TR55)} = \min \left( \varepsilon_{f_d, \text{rupture (TR55)}}, 0.008 \right) \] (3.46)

It can be noted that multiplier \( \delta_{f_E} \) as 0.9 according to ACI440 effectively augments the material safety parameters prescribed on rupture strain capacity of FRP by about 11%.

3.10.2 Switchers in the shear strengthening design process

(A) Working mechanism

For an FRP-based shear strengthening system, there is a substantial chance of concrete loosing its physical integrity beyond a certain strain level [Priestly et al. (1996)]. This generally happens at a considerably low strain level in the FRP compared to its rupture strain capacity. For externally bonded shear reinforcement, this is an extra obligation, in addition to the possibility of its debonding and rupture. Thus, determination of effective...
design failure strain of FRP shear reinforcement involves a numerical competition amongst the three possible failure mechanisms, each of them represented through an empirical strain limit prescribed on FRP. First of these failure strain limit, designated as $\varepsilon_{fd\text{-integrity}}$, corresponds to the ability of concrete to retain physical integrity, which is normally a set constant strain value of the order of 0.4%. Denton et al. (2004) have shown, however, that this empirical strain limit does not necessarily prevent the development of wide cracks. The second strain limit, designated as $\varepsilon_{fd\text{-debond}}$, represents the possibility of debonding, which depends upon the modulus of elasticity of FRP. The possibility of the externally bonded FRP shear reinforcement failing through rupture is captured through the prescription of a strain limit for FRP ($\varepsilon_{fd\text{-rupture}}$) at which FRP fractures across the shear crack. Typically, the strain limit $\varepsilon_{fd\text{-rupture}}$ is a fraction of the design rupture strain capacity ($\varepsilon_{fd\text{-rupture}}$) of FRP as shown in Eq. (3.47) using $\delta_{se}$ as multiplier to $\varepsilon_{fd\text{-rupture}}$ with its value lesser than unity.

$$\varepsilon_{fd\text{-rupture}} = \delta_{se} \varepsilon_{fd\text{-rupture}} \quad [\delta_{se} < 1] \quad (3.47)$$

Based on the above, a generic format of the switcher in shear strengthening can be described through Eq. (3.48).

$$\varepsilon_{fd} = \min \left[ (\delta_{se} \varepsilon_{fd\text{-rupture}}), \varepsilon_{fd\text{-debond}}, \varepsilon_{fd\text{-disintegration}} \right] \quad [\delta_{se} < 1] \quad (3.48)$$

Here, the multiplier $\delta_{se}$ should not be misread to have an analogous purpose to that of the multiplier $\delta_{fe}$ in Eq. (3.40) used for flexural strengthening. While the multiplier $\delta_{fe}$ in flexural strengthening intends to set a preference for one failure mechanism of FRP over the other, multiplier $\delta_{se}$ represents a physical phenomenon. The value of this multiplier $\delta_{se}$ is decided based on whether the behaviour of the regions on the either sides of a shear crack can be characterised as predominantly elastic or rigid body. On this basis, the value of the multiplier $\delta_{se}$ can reportedly be anything from 0.5 to 0.6 [Teng et al. (2002), Chen and Teng (2003), Taljsten (2003)].

In spite of the multiplier $\delta_{se}$ representing a physical phenomenon, the difference in the numerical value of the multiplier $\delta_{se}$ suggested by two different design guidelines can be read as a source to producing differential conservativeness within the strengthening design process. This is illustrated below using TR55 and ACI440 design prescriptions.
(B) Design prescription

Eqs. (3.49)-(3.51), for example, present the disintegration, debonding and fracture strain limits prescribed by TR55. The numerical value of multiplier $\delta_{se}$ as 0.5 in fracture strain limit can be noted from Eq. (3.51).

\[
\varepsilon_{fd\text{-disintegration}}^{(\text{TR55})} = 0.004 \\
\varepsilon_{fd\text{-debond}}^{(\text{TR55})} = 0.64 \sqrt{\frac{0.18(f_{cu})^{2/3}}{n t_f E_f d}} \\
\varepsilon_{fd\text{-fracture}}^{(\text{TR55})} = 0.5 \varepsilon_{fd\text{-rupture}}^{(\text{TR55})}
\]  \hspace{1cm} (3.49) \hspace{1cm} (3.50) \hspace{1cm} (3.51)

TR55, in light of the clarification by Denton et al. (2004), announces the incorporation of strain limit $\varepsilon_{fd\text{-disintegration}}$ [Eq. (3.49)] as a cautious (or conservative) approach. This, however, has a limited rational justification (other than that it has been used across the world for about twenty years). TR55 specifies a straightforward approach for determining effective failure strain in externally bonded FRP shear reinforcement, which is applicable to both the bond- and the contact-critical configurations. It suggests that the minimum of the strain limits $\varepsilon_{fd\text{-fracture}}, \varepsilon_{fd\text{-integrity}}$ and $\varepsilon_{fd\text{-debond}}$ governs the failure mechanism of the externally bonded FRP shear reinforcement, which directly conforms to the format of the switcher presented through Eq. (3.48). In this format, the switcher is called the ‘fracture-debonding-disintegration switch’ (FDDS), which can be expressed through Eq. (3.52).

\[
\varepsilon_{fd}^{(\text{TR55})} = \min \left( 0.5 \varepsilon_{fd\text{-rupture}}^{(\text{TR55})}, 0.004, 0.64 \sqrt{\frac{0.18(f_{cu})^{2/3}}{n t_f E_f d}} \right)
\]  \hspace{1cm} (3.52)

ACI440, on the other hand, prescribes an intricate approach to find the effective failure strain in FRP shear reinforcement. For a contact-critical application, such as a fully wrapped configuration, it appears to suggest that debonding is not a possibility, and that the most likely failure mechanism is concrete loosing its physical integrity. A strain value of 0.004 corresponding to $\varepsilon_{fd\text{-disintegration}}$ is suggested by ACI440 [Eq. (3.53)], which is identical to that for TR55 [Eq. (3.49)] except for the upper limit in Eq. (3.53).

\[
\varepsilon_{fd\text{-disintegration}}^{(\text{ACI440})} = 0.004 \leq 0.75 \varepsilon_{fd\text{-rupture}}^{(\text{ACI440})}
\]  \hspace{1cm} (3.53)
This upper limit can be seen in a format analogous to that used in Eq. (3.47) used to define strain limit $\varepsilon_{fd}$, with the value of multiplier $\delta_{se}$ as 0.75. An explicit consensus to the technical analogy between TR55 and ACI440 prescriptions for determining the strain limit $\varepsilon_{fd}$, shown by Eq. (3.51) and the upper bound in Eq. (3.53) respectively, can always be contested. However, the operational analogy between them cannot be overruled outright. Thus, the value of multiplier $\delta_{se}$ as 0.75 prescribed by ACI440 in the upper limit can be speculated as less conservative when compared to the multiplier $\delta_{se}$ as 0.5 prescribed by TR55. With debonding eliminated from competing to govern design failure strain of externally bonded FRP in a contact-critical configuration, the switch can more appropriately be called the ‘fracture-disintegration switch’ (FDS), which according to ACI440 specifications can be expressed through Eq. (3.54).

$$\varepsilon_{fd,\text{contact-critical}}(\text{ACI440}) = \left[ 0.75 \varepsilon_{fd,\text{rupture}}(\text{ACI440}), 0.004 \right]$$  \hspace{1cm} (3.54)

For a bond-critical application, such as sides-only and U-wrapped configurations, ACI440 considers debonding of FRP as the most likely failure mechanism, which can occur prior to concrete failure due to the loss of physical integrity, as shown through Eq. (3.55).

$$\varepsilon_{fd,\text{debond}}(\text{ACI440}) = \kappa_v \varepsilon_{fd,\text{rupture}}(\text{ACI440}) \leq \varepsilon_{fd,\text{disintegration}}(\text{ACI440})$$  \hspace{1cm} (3.55)

where,

$$\kappa_v = \left( \frac{\varepsilon_{e}}{27} \right)^{2/3} \frac{c_{\text{bond-reduction}} L_e}{11,900 \varepsilon_{fd,\text{rupture}}(\text{ACI440})} \leq 0.75$$  \hspace{1cm} (3.56)

$$c_{\text{bond-reduction}} = \begin{cases} \frac{d_f - L_e}{d_f} & \text{[For U-wrapped configuration]} \\ \frac{d_f - 2L_e}{d_f} & \text{[For sides-only configuration]} \end{cases}$$  \hspace{1cm} (3.57)

$$L_e = \frac{23,300}{(n \varepsilon_f \varepsilon_{fd})^{0.58}}$$  \hspace{1cm} (3.58)

With the multiplier $\kappa_v$ having a value lesser than unity imposed on $\varepsilon_{fd,\text{rupture}}$, Eq. (3.55) may appear to show that the debonding strain limit is a fraction of design rupture strain capacity of FRP. However, the definition of coefficient $\kappa_v$ presented through Eq. (3.56) will show that the debonding strain limit, for a given shear strengthening configuration, is a constant with the strain limit $\varepsilon_{fd,\text{disintegration}}$ as an upper limit, as
shown through Eq. (3.55). In a specific format, the switcher is more appropriately called
the *debonding-disintegration switch* (DDS), which can be expressed through Eq. (3.59).

\[ \varepsilon_{fd-bond-critical(ACI440)} = \min[(\kappa \varepsilon_{fd-rupture(ACI440)}), 0.004] \]  \hspace{1cm} (3.59)

It can be appreciated that the format of FDS and DDS are similar to each other. However, the competing failure mechanisms in FDS and DDS are different. For ACI440 specifications, it can be summarised that a shear strengthening design solution with fully wrapped configuration (i.e. a contact-critical configuration) will involve the FDS, while that with a sides-only or U-wrapped configurations (i.e. bond-critical configurations) will involve the DDS. The value of multiplier \( \kappa \) for the bond-critical configurations cannot exceed the value of multiplier \( \delta_{\varepsilon} \), which is 0.75 for ACI440. For this threshold value, as a particular case, Eq. (3.59) is exactly identical to Eq. (3.54). In light of this fact, it can be observed, based on Eqs. (3.54) and (3.59), that a shear strengthening design solution involving DDS can either be equally or less conservative than that involving FDS.

A direct comparison of conservativeness between ACI440 and TR55 specifications cannot be made for all design scenarios. However, for the shear strengthening design solutions based on TR55 specifications that are not governed by debonding strain limit [i.e. if \( \varepsilon_{f_d-debond} > (\delta_{\varepsilon} \varepsilon_{f_d-rupture}) \), or if \( (\varepsilon_{f_d-debond} > \varepsilon_{f_d-disintegration}) \) ] a comment on relative conservativeness can be made using the following set of switchers:

For contact-critical configurations:

\[ \varepsilon_{fd(\text{TR55})} = \min[(0.5 \varepsilon_{f_d-fracture}), 0.004] \]  \hspace{1cm} (3.60)

\[ \varepsilon_{fd-contact-critical(ACI440)} = \min[(0.75 \varepsilon_{f_d-fracture}), 0.004] \]  \hspace{1cm} (3.61)

For bond-critical configurations:

\[ \varepsilon_{fd(\text{TR55})} = \min[(0.5 \varepsilon_{f_d-fracture}), 0.004] \]  \hspace{1cm} (3.62)

\[ \varepsilon_{fd-bond-critical(ACI440)} = \min[(\kappa \varepsilon_{f_d-fracture}), 0.004] \]  \hspace{1cm} (3.63)

For the same numerical value of limiting strain \( \varepsilon_{f_d-disintegration} \), it can be said that ACI440 provisions, for contact-critical configuration, are relatively less conservative than TR55 provisions. Similarly, ACI440 provisions, for bond-critical configurations, will be relatively less conservative than TR55 provisions if:
Using the definitions of coefficient $\kappa_v$ expressed through Eq. (3.56), the condition necessary for meeting the inequality shown through Eq. (3.64) are:

For sides-only configuration:

$$\left( \frac{L_s(ACI440)}{d_f} \right) < 1 - \left[ \frac{(0.5 \times 11,900) \varepsilon_{f_d-rupture(ACI440)}}{( \ell^2 \chi_2^2) L_s(ACI440)} \right]$$

(3.65)

For U-wrapped configuration:

$$\left( \frac{L_s(ACI440)}{d_f} \right) < \frac{1}{2} - \frac{1}{2} \left[ \frac{(0.5 \times 11,900) \varepsilon_{f_d-rupture(ACI440)}}{( \ell^2 \chi_2^2) L_s(ACI440)} \right]$$

(3.66)

3.11 Quantitative magnification of safety parameters

All of the safety parameters described above are addressing prescriptive uncertainties. The non-prescriptive uncertainties are popularly addressed through quantitatively magnifying the safety parameters accounting for the prescriptive uncertainties. Such a strategy suggests simplicity in handling the non-descriptive uncertainties, and both ACI440 and TR55 appear to employ this strategy.

3.12 Conservativeness framework in strengthening design

The role, interconnectivity and influence of the uncertainties and safety parameters within strengthening design can be more effectively studied by conceiving them in a systematic framework called the conservativeness framework in this study. This is discussed in this section.

3.12.1 Mapping between uncertainties and safety parameters

Master Chart 3.2 at the end of this chapter presents a comprehensive mapping between the identified uncertainties and safety parameters. This chart can be read either from top to bottom or from bottom to top in order to appreciate the functional connectivity between the uncertainties and the safety parameters. This mapping, in a sense, provides an insight into the architecture of safety format used in design of FRP-based structural strengthening systems, and shows how the conservativeness framework is embedded within it. Master Chart 3.2, when read along with Master Chart 3.1, will enable us seeing the ACI440 and TR55 design prescriptions in connection with the conservativeness framework.
The classification of uncertainties and safety parameters in Master Chart 3.2 concur with the taxonomies presented in Figs. 3.3 and 3.4 respectively. It can be seen that parameters $\gamma_{PQR}$, $\gamma_{EDT}$ and $\gamma_{APR}$ comprise the material safety parameter group, while the resistance safety parameter group comprises of parameters $\gamma_{CMP}$, $\gamma_{PNT}$ and $\gamma_{SUP}$. It can further be seen that parameters $\gamma_{PQR}$, $\gamma_{EDT}$ and $\gamma_{APR}$ account for statistical, environmental deterioration based and application process based variability respectively. This possible variability in material properties comprises the class of constitutive uncertainties. As mentioned earlier in this chapter, the mechanical degradation based variability is addressed through prescribing restrictive serviceability criteria and not through any specific safety parameters. Therefore, it is shown distinctively in Master Chart 3.2 through dotted boxes. Parameters $\gamma_{CMP}$, $\gamma_{PNT}$ and $\gamma_{SUP}$ account for deviations, errors, omissions and inconsistencies in prediction of post-strengthening resistance, which constitute the behavioural uncertainties. The constitutive and behavioural uncertainties are prescriptive type of uncertainties arising from cognitive sources. The non-prescriptive uncertainties arising from the qualitative or non-cognitive sources comprises of lack of time-testimony, knowledge and vagueness in descriptions lead to apprehensions in design outputs. These are accounted for through quantitative magnification of the material and resistance safety parameters (in comparison with the traditional factors of safety used for conventional structural materials e.g. concrete and steel).

The material and resistance safety parameters are integrated at four different places along the course of the strengthening design process, each of which indicates a level at which the incorporation of conservativeness within the design process is sought. Parameter $\gamma_{PQR}$ is applied on the mean values of FRP material properties to obtain the corresponding characteristic values. This operation signifies incorporation of conservativeness at Level I within the strengthening design process. Similarly, parameters $\gamma_{EDT}$ and $\gamma_{APR}$ are jointly applied on the characteristic values of FRP material properties towards obtaining the corresponding design values. This signifies incorporation of conservativeness at Level II within the strengthening design process. In the same line, the application of parameter $\gamma_{CMP}$ on the nominal value of contribution of FRP reinforcement signifies incorporation of conservativeness at Level III within the strengthening design process. Lastly, parameters $\gamma_{PNT}$ and $\gamma_{SUP}$ are jointly applied on the nominal resistance of FRP-strengthened structural member to obtain the corresponding design value, which signifies incorporation of conservativeness at Level IV within the strengthening design process. Conservativeness encountered at Level I
and II jointly is referred to as the material conservativeness, which is produced through under-estimating the constitutive material properties of FRP while deriving their corresponding design values. This under-estimation sets a ‘reserved-strength’ in terms of post-strengthening resistance, and refers to an active approach of addressing the constitutive uncertainties. Similarly, the conservativeness encountered at Level III and IV is referred to as resistance conservativeness, which is produced through under-estimating the post-strengthening resistance of FRP reinforcement individually and FRP-strengthened element. This under-estimation sets an ‘over-strength’ in terms of post-strengthening resistance, and refers to a passive approach of addressing the behavioural uncertainties. The material and the resistance conservativeness are discussed in the subsequent subsection.

3.12.2 Material conservativeness

The material safety parameters convert a statistically described material property (through its statistical mean and standard deviation/coefficient of variation) into a deterministic design value. For an arbitrary FRP material property ($Y$), the ratio of its statistical mean ($\bar{Y}$) to its design value ($Y_d$) can be read as the collective effect of the material safety parameters applied at Level I and Level II within the design process. This is expressed through Eq. (3.67), in which the ratio of $\bar{Y}$ to $Y_d$ is called the condensed material safety parameter $[S_Y]$. The square bracket in this notation signifies an aggregative effect of parameters $\gamma_{PQR}$, $\gamma_{EDT}$ and $\gamma_{APR}$ comprising the condensed material safety parameter.

$$ [S_Y] = \frac{\bar{Y}}{Y_d} = \left[ \frac{\gamma_{EDT} \times \gamma_{APR}}{1 - \gamma_{PQR}} \right]_Y $$

(3.67)

Here, the subscript $Y$ indicates the (arbitrary) FRP material property being dealt with, and $p$ is the coefficient of variation. In a sense, the condensed material safety parameter shows the stringency in arriving at the design value of a FRP material property. A numerically higher value of the condensed material safety parameter indicates higher stringency prescribed on an FRP material property in arriving at its design value, and discerns an intention of imparting higher conservativeness to the design solution. A little consideration will show that the design value of a FRP material property can be directly obtained by dividing its statistical mean with the corresponding condensed material safety parameter [e.g. Eqs. (3.68) and (3.70)].

$$ \varepsilon_{f,d,rupture} = \frac{\varepsilon_f}{[S_d]} $$

(3.68)
\[ E_{fd} = \frac{E_f}{[S_E]} \]  
(3.69)

\[ f_{fd} = \frac{f_f}{[S_f]} \]  
(3.70)

Following the notion of conservativeness presented in Eq. (3.1), the material conservativeness for an arbitrary FRP material property \((Y')\) can be expressed through Eqs. (3.71) and (3.72).

\[ C_{\text{material}} = \bar{Y} - Y_d = \bar{Y} \left[ 1 - \frac{1}{[\sigma_Y]} \right] \]  
(3.71)

\[ I_C_{\text{material}} = \left[ 1 - \frac{1}{[\sigma_Y]} \right] \]  
(3.72)

Table 3.8 summarises the condensed material safety parameters derived based on this concept using the ACI440 and TR55 specifications. These values are derived for an assumed coefficient of variation of 0.1, for demonstration purpose. It can be seen that the condensed material safety parameter for modulus of elasticity according to ACI440 specifications is unity. This is due to the fact that ACI440 does not advocate having any material conservativeness at Level I and II on modulus of elasticity of FRP. Consequently, the design value of modulus of elasticity of FRP remains numerically the same as its statistical mean value.

### 3.12.3 Resistance conservativeness

The ratio of the nominal resistance of a strengthened section \((R_{\text{FRP,RC(nom)}})\) to its design resistance \((R_{\text{FRP,RC(des)}})\) can be read as the collective effect of the resistance safety parameters. This ratio is called the condensed resistance safety parameter \([J]\), as shown in Eq. (3.73). It is to be noted here that \(R_{\text{nominal}}\) (and hence \(R_{\text{design}}\)) already include the effect of the material safety parameters. It is also to be noted that the compensatory safety parameter \((\gamma_{\text{CMP}})\) applied at Level III within the strengthening design affects only the FRP resistance contribution. Unlike this, the punitive and supplementary safety parameters \((\gamma_{\text{PNT}} \text{ and } \gamma_{\text{SUP}} \text{ respectively})\), applied at Level IV within the strengthening design, affect the resistance contributions of RC section (i.e. resistance contribution of concrete and steel) and FRP. Therefore, the fractions comprising parameter \([J]\) are different for the reinforced concrete section, designated with a subscript \(RC\) in Eq. (3.74), and that for FRP, designated with a subscript \(FRP\) in Eq. (3.75). The corresponding conservativeness produced due to the resistance safety parameters are expressed through Eqs. (3.76)-(3.79).
\[ U = \frac{R_{FRP,RC(nominal)}}{R_{FRP,RC(design)}} = \frac{R_{RC(nominal)}}{R_{RC(design)}} + \frac{R_{FRP(nominal)}}{R_{FRP(design)}} \]  
\[ (3.73) \]

\[ [J]_{FRP} = \frac{R_{FRP(nominal)}}{R_{FRP(design)}} = [Y_{CMP} \times (Y_{PNT} \times Y_{SUP})] \]  
\[ (3.74) \]

\[ [J]_{RC} = \frac{R_{RC(nominal)}}{R_{RC(design)}} = [(Y_{PNT} \times Y_{SUP})] \]  
\[ (3.75) \]

\[ C_{\text{resistance-FRP}} = R_{FRP(nominal)} - R_{FRP(design)} = R_{FRP(nominal)} \left[ 1 - \frac{1}{[J]_{FRP}} \right] \]  
\[ (3.76) \]

\[ I_{C\text{-resistance-FRP}} = \left[ 1 - \frac{1}{[J]_{FRP}} \right] \]  
\[ (3.77) \]

\[ C_{\text{resistance-RC}} = R_{RC(nominal)} - R_{RC(design)} = R_{RC(nominal)} \left[ 1 - \frac{1}{[J]_{RC}} \right] \]  
\[ (3.78) \]

\[ I_{C\text{-resistance-RC}} = \left[ 1 - \frac{1}{[J]_{RC}} \right] \]  
\[ (3.79) \]

Table 3.9 summarises the condensed resistance safety parameters derived based on this concept using the ACI440 and TR55 specifications. It can be seen that TR55 does not specify any resistance safety parameters except parameter \( Y_{PNT} \) for flexural strengthening.
Table 3.8
Condensed material safety parameters for ACI440 and TR55 specifications

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<tr>
<th>Manufacturing and Installation (M &amp; I) Class</th>
<th>Environmental Exposure Class EE I</th>
<th>Environmental Exposure Class EE II</th>
<th>Environmental Exposure Class EE III</th>
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<td>GFRP</td>
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<td>M &amp; I Class I</td>
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<td>M &amp; I Class II</td>
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<td>M &amp; I Class IV</td>
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Condensed Material Safety Parameter for Rupture strain of FRP [\( \gamma \)]

<table>
<thead>
<tr>
<th>Environmental Exposure Class EE I</th>
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Notes:
- A and T respectively represent ACI440 and TR55 design guidelines.
- Coefficient of variation assumed is 0.1.
- The most liberal prescriptions are highlighted in **F**(ellow).
- The most stringent prescriptions are highlighted in **Y**ellow.

Sample Calculation:
According to TR55: For M & I Class IV and with coefficient of variation as 0.1, for E-GFRP-
Eq. (3.67), with \( \gamma_{FRP(E)} = 2.00 \), \( \gamma_{ED(E)} = 1.80 \), \( \gamma_{APR(E)} = 1.50 \) gives:
\[
\frac{1.80 \times 1.50}{1-(2 \times 0.1)} = 3.75 \approx 3.38
\]
### Condensed resistance safety parameters for FRP ACI440 and TR55 specifications

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<tr>
<th>Strengthening Type</th>
<th>CMP Class</th>
<th>PNT Class</th>
<th>SUP Class</th>
<th>( U_{\text{FRP}} )</th>
<th>( U_{\text{RC}} )</th>
<th>( U_{\text{FRP}} )</th>
<th>( U_{\text{RC}} )</th>
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</table>

**NOTES:**
- The most liberal prescriptions are highlighted in **YELLOW**.
- The most stringent prescriptions are highlighted in **RED**.

:: Sample Calculation ::

According to ACI440: For CMP Class SIB, PNT Class SI & SUP Class SI, Eq. (3.74) gives \( 1.18 \times 1.00 \times 1.33 = 1.57 \).
3.13 Summary of observations

Important observations emerging from this chapter are summarised below:

- Attributed to a relatively less history of use of FRP composites in construction industry, the design of FRP-based structural strengthening systems involves a considerable extent of lack of knowledge (in terms of the quality, mechanics and response of FRP materials), and lack of time-testimony (in terms of their long-term behaviour). These lead to considerable extents of prescriptive (i.e. constitutive and behavioural) and non-prescriptive uncertainties in use of FRP as externally bonded reinforcement for structural strengthening. This, in turn, demands the strengthening design process to be more conservative than usual.

- For FRP-based structural strengthening systems, setting ‘reserved-strength’ and ‘over-strength’ are the main sources of producing conservativeness in terms of post-strengthening resistance. For analytical purposes, conservativeness can be represented as the absolute difference between the nominal and the design output of an engineered model.

- A comprehensive mapping between the identified uncertainties and safety parameters accounting for these uncertainties indicates four major locations (called ‘levels’, in this study) within the strengthening design at which incorporation of conservativeness is sought.

- At Level I, conservativeness while addressing the possible random variability in the FRP material properties is encountered. At Level II, conservativeness while addressing the possible temporal reduction in the FRP material properties due to environmental deterioration, along with the possible variability in the quality of the FRP due to different manufacturing and installation routes, is encountered.

- Prescribing safety factors on the FRP material properties aims to set a ‘reserved-strength’ by under-estimating the constitutive material properties of FRP while deriving their corresponding design values. This refers to an active approach of addressing constitutive uncertainties.

- At Level III and IV, conservativeness while addressing the possible behavioural deviations between the ideally expected or real behaviour of externally bonded FRP reinforcement individually and a strengthened RC member, is encountered.

- Prescribing reduction factors on the resistance contribution of FRP reinforcement and the post-strengthening resistance of an FRP-strengthened component aim to set an ‘over-strength’ by under-estimating the post-strengthening resistance. This refers to a passive approach of addressing behavioural uncertainties.
• In addition to the safety factors on material properties and reduction factors on structural resistance, there exists a set of ‘failure mode switchers’. These switchers are strategically located within the strengthening design process, and play a manipulative role by influencing the design predictions for the failure mechanisms for externally bonded FRP reinforcement. These switchers aim at accommodating the preferences for the desired failure mechanisms of FRP to govern the strengthening design solutions.

• It is shown that an articulated failure mode switcher can magnify the safety factors prescribed on the FRP material properties, which remains a disguised source of inflating conservativeness within the strengthening design without projecting a quantitatively higher set of safety factors on FRP material properties.

• Based on an illustration using the ACI440 and TR55 specifications, it is shown that the failure mode switcher set for flexural strengthening design according to ACI440 inflates the safety factors on FRP material properties by about 11%.

• The material safety parameters according to the TR55 specifications, in general, are more stringent compared to the ACI440 specifications.

• TR55 does not specify any reduction factors on post-strengthening (except the penalty on post-strengthened flexural resistance in proportion with the lack of sectional ductility). This can be speculated as giving less emphasis towards addressing the behavioural uncertainties.

It will be of interest to investigate the implications of the above conclusions on the course of strengthening design processes and on the quality of the resultant strengthening design solutions. This is attempted in Chapters 4 and 5 for flexural and shear strengthening respectively.
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<tr>
<th>Safety Parameter</th>
<th>Type</th>
<th>Applied at</th>
<th>Applied on</th>
<th>To Obtain</th>
<th>Characteristic Cluster</th>
<th>ACI440</th>
<th>TR55</th>
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**Material Safety Parameter**

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**Summary of ACI440 and TR55 design prescriptions for safety parameters**

*Effect accounted in \( Y_{PR} \)*
Mapping of uncertainties and safety parameters showing the architecture of safety format used in design of FRP-based structural strengthening systems.
Flexure
4.1 Chapter objectives and structure

This chapter aims at providing a deeper interpretation of conservativeness associated with the design of FRP-based flexural strengthening systems. It attempts to demonstrate the manipulating capabilities of the means of producing conservativeness within the flexural strengthening design process that are encoded within the strengthening design criteria using different formats, and their implications on the quality of the design outcomes. In particular, this chapter investigates:

- How the means to produce conservativeness propagate within the flexural strengthening design process
- How the means to produce conservativeness perturb the course of flexural strengthening design process by influencing the design predictions for –
  - the failure mechanisms of FRP (e.g., rupture and debonding), and
  - the post-strengthening failure modes
- How the means to produce conservativeness affect the quality of the resultant flexural strengthening design solutions (e.g., in terms of conservativeness in flexural resistance and ductility)

An assessment methodology for the flexural strengthening design process, which is based on a novel set of definitions of the post-strengthening failure modes, is proposed in this chapter. Mathematical equations depicting salient features of these failure modes are derived. In addition, a set of mathematical expressions that facilitates avoiding undesirable failure modes (based on their associated ductility contents) are also presented. This methodology is used to provide an insight into the internal architecture of the flexural strengthening design process and to qualitatively characterise various flexural strengthening design solutions. The implications of different design formats suggested by various strengthening design guidelines on the extents of these clusters are presented. Finally, the sensitivity of the conservativeness of the resultant flexural strengthening design solutions are discussed under different design scenarios. Based on these analyses suitable observations are derived.
Assessment of conservativeness in design of FRP-based structural strengthening systems
Kunal D. Kansara (2014)
4.2 **Assessment methodology for flexural strengthening design**

In flexural strengthening design, the nominal resistance corresponding to the failure modes forms a core design requirement, while the ductility-content associated with them is employed to define a hierarchy of preferences of the failure modes. The methodology developed for assessment of flexural strengthening design involves ductility-based definitions of post-strengthening failure modes. For these definitions, the extent of straining of the tension steel reinforcement forms a prime parameter that defines various ultimate strain-states. A typical post-strengthening ultimate strain-state is presented in Fig. 4.1, which describes the common notations used in deriving the governing equations associated with the proposed ductility-based definitions of the failure modes. Important mathematical relationships based on this strain-state are presented by Eqs. (4.1)-(4.7), which are derived based on the geometry of this notional strain-state. These expressions serve as a handy reference, which can then be particularised for specific strain-states representing the failure modes proposed using the ductility-based definitions as shown later in this section.

![Notional strain-state for ductility-based definitions of failure-modes](image)

**Figure 4.1**
Notional strain-state for ductility-based definitions of failure-modes
(Conceptual representation – not to scale)

\[
K = \frac{d}{d'} \tag{4.1}
\]

\[
S = \frac{d'}{d} \tag{4.2}
\]
\[
\begin{align*}
\varepsilon_c &= \left[ \frac{x_u}{d} \right] \varepsilon_{sc} \\
\varepsilon_{st} &= \left[ \frac{1 - x_u}{1 - \frac{x_u}{d}} \right] \varepsilon_{fe} \\
\varepsilon_{sc} &= \left[ \frac{x_u - S}{1 - \frac{x_u}{d}} \right] \varepsilon_{st} \\
\varepsilon_{fe} &= \left[ \frac{1 - x_u}{1 - \frac{x_u}{d}} \right] \varepsilon_{sc} \\
\varepsilon_{fe} &= \left[ \frac{1 - x_u}{1 - \frac{x_u}{d}} \right] \varepsilon_{sc} = \left[ \varepsilon_{f0} + \varepsilon_{fl} \right] \\
\frac{x_u}{d} &= \frac{1}{K} \left[ \frac{1}{1 + \left( \frac{\varepsilon_{st}}{\varepsilon_c} \right)} \right] \\
\end{align*}
\]
From the force equilibrium for a strengthened section, the FRP-content ($\rho_{\text{FRP}}$) can be calculated [Eq. (4.8)]. Similarly, from moment equilibrium, the nominal moment of resistance of the strengthened section ($M_{\text{nominal}}$) can be calculated [Eq. (4.9)]. Here, $k_1$ and $k_2$ are the stress block parameters [please refer ACI318 or BS8110 for more information]. If tension steel is yielded, then the product ($\epsilon_{st}E_s$) in these equations needs to be replaced with $\left(\frac{f_y}{f_y^-}\right)$. This adjustment also applies if the compression steel is yielded. A little consideration will show that setting $\rho_{\text{FRP}}$ as zero in Eqs. (4.8) and (4.9), with an appropriate depth of neutral axis, refers to a pre-strengthened state. This holds true for the other strain-states discussed later in this section, which shows the adaptability of the proposed definitions of the failure modes to both, the pre- and the post-strengthening states. It is to be noted that the effective strain ($\epsilon_{ef}$) at the soffit, where the externally bonded FRP reinforcement is installed, is the algebraic sum of the initial soffit strain ($\epsilon_{fo}$) at the time of installation and the actual strain in FRP due to loading ($\epsilon_{f1}$) after installation as shown in Eq. (4.6).

$$\rho_{\text{FRP}} = \frac{\left(k_1 \frac{f_{ck} x_u}{Y_{mc} d}\right) + (\epsilon_{sc} E_s \rho_{sc}) - (\epsilon_{st} E_s \rho_{st})}{(\epsilon_{f1} E_{fa})}$$  \hspace{1cm} (4.8)

$$\frac{M_{\text{nominal}}}{bd^2} = \left[ k_1 \frac{f_{ck}}{Y_{mc}} (1 - k_2) \left(\frac{x_u}{d}\right)^2 \right] + \left[ \epsilon_{sc} E_s \rho_{sc} \left(\frac{x_u}{d} - S\right) \right] + \left[ \epsilon_{st} E_s \rho_{st} \left(1 - \frac{x_u}{d}\right) \right] + \left[ (\epsilon_{f1} E_{fa} \rho_{\text{FRP}}) \left(\frac{1}{K} - \frac{x_u}{d}\right) \right]$$  \hspace{1cm} (4.9)

Taking the strain in the tension steel reinforcement as a prime parameter, a reference strain-state, called the critical strain-state has been worked out. Using this reference strain-state, two more strain-states, namely the concrete-controlled and the FRP-controlled strain-states are developed. The failure modes corresponding to critical, concrete-controlled and FRP-controlled strain-states are discussed below. The design equations related to these strain-states are summarised in Tables 4.1 to 4.4 (at the end of this chapter), which should be read along with the following discussion. Table 4.1 presents the basic definitions of these three strain-states in term of sectional ductility.
represented in form of extent of straining of the tension steel reinforcement. Table 4.2 presents the mathematical relations depicting strains in concrete, FRP and compression steel, while Table 4.3 presents the mathematical expressions for depth of neutral axis, FRP-content and moment of resistance corresponding to these strain-states. Table 4.4 summarises sub-clusters of failure modes based on debonding, rupture and ductility references, along with special conditions that facilitate avoiding less preferred failure modes to govern the flexural strengthening design solutions towards achieving optimal design solutions under given conditions.

4.2.1 Critical failure mode

(A) Mathematical description

A hypothetical strain-state in which the concrete and the FRP reinforcement reach their corresponding design failure strain limits simultaneously is called a critical strain-state. The corresponding failure mode is called the critical failure mode. The strain in the tension reinforcement at this state is referred to as the critical tension steel strain \( \varepsilon_{st-crit} \), which technically defines this strain-state [Eq. (4.10)]. The strain in the compression steel strain and the depth of neutral axis for this strain-state are designated as critical compression steel strain \( \varepsilon_{sc-crit} \) and critical depth of neutral axis \( x_{nc-crit} \) respectively (See Table 4.1).

\[
\varepsilon_{st} = \varepsilon_{st-crit} \tag{4.10}
\]

Based on the type of FRP failure mechanism involved, a critical failure mode can either be a ‘debonding-governed’ or a ‘rupture-governed’ critical failure mode, as shown in Eqs. (4.11) and (4.12) respectively. Depending on the strain in the tension steel reinforcement, each of these two types of critical failure modes has three possible variants: a ‘non-ductile’, a ‘ductile’ or an ‘adequately ductile’ critical failure mode, as shown in Eqs. (4.13)-(4.15). The tension steel strain reference value corresponding to the adequately ductile condition is specified by various strengthening design guidelines (see Chapter 2). For the ductile and the adequately ductile critical failure modes, the product \( \varepsilon_{st} E_y \), in corresponding equations in Tables 4.1 to 4.4 has to be replaced with the term \( f_y/y_{ms} \).

\[
\varepsilon_{f-d-debond} < \varepsilon_{f-d-rupture} \quad \Rightarrow \text{Debonding-governed critical failure mode} \tag{4.11}
\]

\[
\varepsilon_{f-d-debond} > \varepsilon_{f-d-rupture} \quad \Rightarrow \text{Rupture-governed critical failure mode} \tag{4.12}
\]
\[
\varepsilon_{\text{st-crit}} < \varepsilon_{\text{sy}} < \varepsilon_{\text{st-adequate}} \Rightarrow \text{Non-ductile critical failure mode} \quad (4.13)
\]
\[
\varepsilon_{\text{sy}} \leq \varepsilon_{\text{st-crit}} < \varepsilon_{\text{st-adequate}} \Rightarrow \text{Ductile critical failure mode} \quad (4.14)
\]
\[
\varepsilon_{\text{sy}} < \varepsilon_{\text{st-adequate}} < \varepsilon_{\text{st-crit}} \Rightarrow \text{Adequately ductile critical failure mode} \quad (4.15)
\]

(B)  
**Condition to avoid non-ductile critical failure mode**

For a critical strain-state, the strain in tension steel is governed by the existing sectional constraints (i.e. geometry of section, tension steel reinforcement content and concrete strength) and the design values of the FRP material properties (in particular, the rupture and debonding strains) intended to be used. The non-ductile critical failure mode can be avoided, if so preferred, by selecting FRP composite with appropriate material properties and/or restricting the FRP-content, such that the criterion specified in Eq. (4.16) is satisfied.

\[
\varepsilon_{\text{st-crit}} \geq \varepsilon_{\text{sy}} \quad (4.16)
\]

Using the mathematical expressions defining \( \varepsilon_{\text{st-crit}} \) and \( x_{\text{u-crit}} \) along with Eq. (4.16), the limiting values of \( \varepsilon_{\text{fd}} \) and \( x_{\text{u-crit}} \) as presented in Tables 4.4 for critical failure mode, can be obtained to avoid non-ductile critical failure mode. Similarly, a limiting value of \( \rho_{\text{FRP-crit}} \), shown in Table 4.4 for critical failure mode, can be obtained by substituting the limiting values of \( x_{\text{u-crit}} \) in expressions for \( \rho_{\text{FRP-crit}} \).

It is to be noted that there is a maximum possible value of \( \varepsilon_{\text{st-crit}} \) that can be achieved under a given design scenario. For the fact that the design failure strain of FRP (\( \varepsilon_{\text{fd}} \)) cannot numerically exceed the debonding strain limit of FRP (\( \varepsilon_{\text{fd-debond}} \)), the maximum possible value of critical tension steel strain, designated as \( \varepsilon_{\text{st-crit}}^{\text{max}} \), can be expressed as:

\[
\varepsilon_{\text{st-crit}}^{\text{max}} = (K - 1) \varepsilon_{\text{cu}} + K (\varepsilon_{f0} + \varepsilon_{\text{fd-debond}}) \quad (4.17)
\]

(C)  
**Most optimal design condition**

It is demonstrated later in this chapter that the flexural strength of a strengthened section varies exponentially and inversely with the strain in tension steel reinforcement \( \varepsilon_{\text{st}} \). Thus, a flexural strengthening design solution with a lower sectional ductility \( \varepsilon_{\text{st}} \), corresponds to a higher flexural strength compared to a design solution with a relatively lesser sectional ductility. However, if the strain \( \varepsilon_{\text{st}} \), for a design solution, is numerically less than the adequate tension steel strain \( \varepsilon_{\text{st-adequate}} \), it will attract
penalty on the nominal moment of resistance of the section. A little consideration, therefore, will show that value of the strain $\varepsilon_{st}$ is most optimal when it is numerically equal to $\varepsilon_{st\text{-adequate}}$, as shown in Eq. (4.18).

$$\varepsilon_{st\text{-crit}} = \varepsilon_{st\text{-adequate}}$$ (4.18)

This fact can serve as the best initial guess in flexural strengthening design. With the concrete and the FRP strained up to their corresponding design failure strains simultaneously, the critical strain-state is the most efficient strain-state amongst the three strain-states in flexural strengthening design. Hence, the optimum value of the tension steel strain equal to the adequate steel strain value is a global optimum condition (here referred to as the most optimal design condition). Eq. (4.18) substituted appropriately in expressions presented in Tables 4.2 and 4.3 leads to expressions for $\varepsilon_{fd}$, $x_u\text{-crit}$ and $\rho_{FRP\text{-crit}}$ corresponding to the most optimal condition as shown in Table 4.4.

The upper bound on the governing design failure strain in FRP ($\varepsilon_{fd}$), which cannot be numerically greater than the debonding strain in FRP ($\varepsilon_{fd\text{-debond}}$), propounds a lower bound on the best initial guess for the critical FRP content, for the given conditions, as shown through Eq. (4.19).

$$\rho_{FRP\text{-crit}}^{\text{min}} = \begin{cases} \left[\frac{f_{ck}}{f_{mc}} \left(1 - \frac{\varepsilon_{st\text{-adequate}}}{\varepsilon_{cu}}\right) + \frac{\varepsilon_{sc\text{-crit}} E_s \rho_{sc}}{\varepsilon_{st\text{-adequate}} E_{frp}} \right] \frac{\varepsilon_{fd\text{-debond}} E_{fd}}{f_y \rho_{st}} \\ \left[\frac{f_{ck}}{f_{mc}} \left(1 - \frac{\varepsilon_{st\text{-adequate}}}{\varepsilon_{cu}}\right) + \frac{\varepsilon_{sc\text{-crit}} E_s \rho_{sc}}{\varepsilon_{st\text{-adequate}} E_{frp}} \right] \frac{\varepsilon_{fd\text{-debond}} E_{fd}}{f_y \rho_{st}} \end{cases}$$ (4.19)

### 4.2.2 Concrete-controlled failure mode

**(A) Mathematical description**

For a given combination of the existing RC section and the externally bonded FRP reinforcement, if the actual strain in tension steel is less than $\varepsilon_{st\text{-crit}}$ [Eq. (4.20)], the
concrete in compression will reach its failure limit prior to FRP in tension reaching its governing failure limit. This is the primary characteristic defining the concrete-controlled strain-state, in which the concrete failure governs the design.

\[ \varepsilon_{st} < \varepsilon_{st-crit} \]  

(4.20)

Based on the ductility reference, the characteristic strain condition presented in Eq. (4.20) can have four possibilities [Eqs. (4.21)-(4.24)], of which the first two possibilities refer to a non-ductile strain-state, the third possibility refers to a ductile strain-state and the last possibility refers to an adequately ductile strain-state.

\[ \varepsilon_{st} < \varepsilon_{st-crit} \leq \varepsilon_{sy} < \varepsilon_{st-adequate} \] (non-ductile strain-state)  

(4.21)

\[ \varepsilon_{st} < \varepsilon_{sy} \leq \varepsilon_{st-crit} < \varepsilon_{st-adequate} \] (non-ductile strain-state)  

(4.22)

\[ \varepsilon_{sy} \leq \varepsilon_{st} < \varepsilon_{st-crit} < \varepsilon_{st-adequate} \] (ductile strain-state)  

(4.23)

\[ \varepsilon_{sy} < \varepsilon_{st-adequate} \leq \varepsilon_{st} < \varepsilon_{st-crit} \] (adequately-ductile strain-state)  

(4.24)

(B) **Condition to avoid non-ductile concrete-controlled failure modes**

An assumed value of \( \varepsilon_{st} \) greater than \( \varepsilon_{sy} \) (but less than \( \varepsilon_{st-crit} \)) will ensure a ductile concrete-controlled strain-state [Eq. (4.25)]. This corresponds to a lower bound on strain in FRP reinforcement under loading, and upper bounds on the depth of neutral axis and FRP-content. These conditions are presented in Table 4.4, which are analogous to those prescribed for the critical strain-state.

\[ \varepsilon_{st-crit} > \varepsilon_{st} \geq \varepsilon_{sy} \]  

(4.25)

For the failure modes involving concrete failure (i.e. the critical and the concrete-controlled failure modes), the compression steel reinforcement will yield, if the depth of neutral axis is satisfying an upper bound presented through the inequality condition Eq. (4.26).

\[ \frac{x_u}{d} \leq \frac{1}{5} \left( 1 - \frac{\varepsilon_{sy}}{\varepsilon_{cr}} \right) \]  

(4.26)

(C) **Optimal design condition**

An assumed value of \( \varepsilon_{st} \) numerically equal to \( \varepsilon_{st-adequate} \) (where admissible under the existing geometrical and material constraints for the given RC section) will serve as an optimal (but not the most optimal) initial guess for the concrete-controlled failure-mode
The strain in FRP reinforcement, depth of neutral axis and FRP-content corresponding to this condition are summarised in Table 4.4.

$$\varepsilon_{st} = \varepsilon_{st\text{-adequate}}$$  \hspace{1cm} (4.27)

### 4.2.3 FRP-controlled failure mode

#### (A) Mathematical description

For a given combination of the existing RC section and the externally bonded FRP reinforcement, if the actual strain in tension steel is greater than $$\varepsilon_{st\text{-crit}}$$ [Eq. (4.28)], the FRP reinforcement in tension will reach its design failure limit prior to the concrete reaching its design compression failure limit. This is the primary characteristic defining the FRP-controlled failure mode, in which the FRP failure governs the design.

$$\varepsilon_{st} > \varepsilon_{st\text{-crit}}$$  \hspace{1cm} (4.28)

Based on the possibilities of rupture and debonding of FRP and using the ductility reference, the characteristic condition presenting the FRP-controlled failure mode can have six possibilities. These are expressed through Eqs. (4.29)-(4.34). Of these, the possibilities expressed through Eqs. (4.31) and (4.32) correspond to an adequately ductile and a ductile FRP-controlled failure modes respectively, whereas the possibilities expressed through Eqs. (4.33) and (4.34) correspond to a non-ductile FRP-controlled failure mode.

\[ \varepsilon_{f_d\text{-debond}} < \varepsilon_{f_d\text{-rupture}} \implies \text{Debonding-governed critical failure mode} \]  \hspace{1cm} (4.29)

\[ \varepsilon_{f_d\text{-debond}} > \varepsilon_{f_d\text{-rupture}} \implies \text{Rupture-governed critical failure mode} \]  \hspace{1cm} (4.30)

$$\varepsilon_{st} > \varepsilon_{st\text{-adequate}} > \varepsilon_{sy} > \varepsilon_{st\text{-crit}}$$ (adequately-ductile strain-state)  \hspace{1cm} (4.31)

$$\varepsilon_{st\text{-adequate}} > \varepsilon_{st} > \varepsilon_{sy} > \varepsilon_{st\text{-crit}}$$ (ductile strain-state)  \hspace{1cm} (4.32)

$$\varepsilon_{st\text{-adequate}} > \varepsilon_{sy} > \varepsilon_{st} > \varepsilon_{st\text{-crit}}$$ (non-ductile strain-state)  \hspace{1cm} (4.33)

$$\varepsilon_{st} > \varepsilon_{st\text{-crit}} > \varepsilon_{sy} > \varepsilon_{st\text{-adequate}}$$ (non-ductile strain-state)  \hspace{1cm} (4.34)

#### (B) Condition to avoid non-ductile FRP-controlled failure mode

An assumed value of $$\varepsilon_{st}$$ greater than $$\varepsilon_{sy}$$ (which is also greater than $$\varepsilon_{st\text{-crit}}$$) will ensure a ductile FRP-controlled strain-state [Eqs. (4.35) or (4.36)]. This corresponds to a lower
bound on the design failure strain of the FRP, and upper bounds on the depth of neutral axis and FRP-content presented in Table 4.4.

\[ \varepsilon_{st} \geq \varepsilon_{sy} > \varepsilon_{st-crit} \]  
\[ \varepsilon_{st} > \varepsilon_{st-crit} \geq \varepsilon_{sy} \]  

(C) **Optimal design condition**

An assumed value of \( \varepsilon_{st} \) numerically equal to \( \varepsilon_{st-adequate} \) (where admissible under the existing geometrical and material constraints for the given RC section) will serve as an optimal (again, not the most optimal) initial guess for the FRP-controlled failure mode [Eq. (4.37)]. The design strain of FRP, depth of neutral axis and FRP-content corresponding to optimal condition are presented in Table 4.4.

\[ \varepsilon_{st} = \varepsilon_{st-adequate} \]  

4.2.4 **Significance of ductility-based definitions of failure modes**

From the discussion it can be appreciated that the value of \( \varepsilon_{st} \) in relation with \( \varepsilon_{st-crit} \) controls the entire mechanism of the ductility-based definitions for the failure modes. They provide not only a means for producing a range of flexural strengthening design solutions, but also a framework for assessing, characterising and rating them. The conservativeness framework, discussed in Chapter 3, provides a means to create possible design scenarios. Therefore, the ductility-based definitions, when used in conjunction with the conservativeness framework, serves an excellent tool for comprehending consequences of the conservativeness within the flexural strengthening design process under different design scenarios.

These definitions portray the entire course of flexural strengthening design process algorithmically, covering all the possible alternate paths the strengthening design process can possibly assume. This, in turn, presents the possible design outcomes comprehensively in a nutshell, and shows a clear picture on how the material safety parameters propagate within the design process. This aspect is discussed in section 4.3.

These definitions can also be used to demonstrate the mechanism behind the design predictions for the failure mechanism of FRP (e.g. rupture or debonding) and post-strengthening failure modes (e.g. concrete- or FRP-controlled). This serves as a methodology to investigate the sensitivity of these design predictions to various formats of design criteria and safety protocols under different design scenarios. The sensitivity
of the design predictions for the failure mechanism of FRP is discussed in detail in sections 4.5 and 4.6, while that for the design predictions for the post-strengthening failure modes is discussed in section 4.7. Furthermore, these definitions can also show the implications of various design parameters and design scenarios on the relative extents of the flexural strengthening design solutions clustered according to their qualitative characteristics. This aspect is presented in section 4.8. Finally, using these definitions, an assessment of conservativeness associated with a possible range of flexural strengthening design solutions under different design scenarios can be carried out. This aspect is discussed in section 4.9.

### 4.3 Architecture of the flexural strengthening design process

Fig. 4.2, in light of the ductility-based definitions of the failure modes, presents the design possibilities for flexural strengthening design process algorithmically. It can be seen that a flexural strengthening design solution can be reached through one of the fifteen alternate paths – each leading to a qualitatively different class of flexural strengthening design solution. The possibility for the flexural strengthening design solutions to be governed by either of the concrete-controlled, critical or FRP-controlled failure modes sets three major bifurcations within the course of the flexural strengthening design process. These bifurcations depend upon the actual value of strain in the tension steel reinforcement ($\varepsilon_{st}$) relative to its critical value ($\varepsilon_{st-$crit$}$). The possibility of FRP to fail through rupture (or near rupture) or debonding sets further two bifurcations within the flexural strengthening design process. These bifurcations are applicable to FRP-controlled and critical failure modes only, and depend upon the value of the mean rupture strain capacity of FRP ($\varepsilon_f$) relative to its limiting value ($\varepsilon_{f-limit}$), which is explained later in this chapter. For the above five bifurcations, the value of $\varepsilon_{st}$ relative to ductility strain references $\varepsilon_{sy}$ and $\varepsilon_{st-adequate}$ sets further three bifurcations – each leading to non-ductile, ductile or adequately ductile design solutions. These three bifurcations apply to all three possible failure modes. Considering the possible bifurcations by accounting for the full- and near-rupture (arising from an RDS and an NRDS respectively, see Chapter 3) and the pre-set constant and modulus-dependent strain in FRP at debonding (arising from Type I and Type II debonding strain limits respectively, see Chapter 2) along with the ductility-based definitions of the failure modes, the total possible types of design solutions are 27, as shown in Fig. 4.2. Here, the applicable material, resistance and switcher safety parameters for each of these types of design solutions are highlighted.
Figure 4.2 Internal architecture of flexural strengthening design process
Following points emerge from this figure:

- It can easily be observed that each of the 27 types of flexural strengthening design solutions is differently sensitive to the material safety parameters prescribed on the rupture strain capacity of FRP. It can also be seen that these material safety parameters are not applicable for the flexural strengthening design solution involving debonding of FRP. Thus, the design solutions involving debonding of FRP are more vulnerable in terms of conservativeness in flexural resistance compared to other types of design solutions.

- It can also be observed that the material safety parameters on the modulus of elasticity of FRP are applicable to all the types of design solutions except the ones involving debonding of FRP with Type I debonding strain limit (i.e. the preset constant value of strain in FRP at debonding). Thus, the strengthening design guidelines opting for not prescribing the material safety parameters on modulus of elasticity of FRP (e.g., ACI440) are loosing a possible scope of producing conservativeness in post-strengthening flexural resistance.

- Most resistance safety parameters are applicable to all types of flexural strengthening design solutions – even for those involving debonding of FRP. However, for the strengthening guidelines opting for not specifying the resistance safety parameters [e.g., TR55 (2004) and TR55 (2012)], no means for producing conservativeness are available for the flexural strengthening design solutions involving debonding of FRP.

- It can be appreciated that the ductility-based definitions, in the algorithmic format presented in Fig. 4.2, are highly conducive for automating the strengthening design process more conveniently.

- The applicability of the switcher parameter ($\delta_{fe}$) for the failure modes involving near-rupture of FRP can also be observed. The material safety parameters prescribed on rupture strain of FRP get magnified for this type of strengthening design solutions, as stated in Chapter 3.

- It can also be appreciated that the popularly followed design algorithms can lead to any one of these possible types of flexural strengthening design solutions. However, a designer, in this case, has a very limited control on the design process. Against this, the ductility-based definitions of the failure modes, along with the mathematical basis provided to avoid undesirable design conditions and to promote the optimal design conditions, provide a designer much better control over the
strengthening design process. The possibilities of achieving a quality design solution and ensuring the efficient use of the FRP composites can, thus, be assured.

4.4 Assessment approach

The important factors that influence the course of flexural strengthening design process and the quality of the resultant flexural strengthening design solutions include:

- The rupture strain capacity of the FRP ($\varepsilon_f$) and the stringency of the material safety parameters prescribed on it (in terms of condensed safety parameter $[S_e]$ – see Chapter 3),

- The value of strain in FRP at debonding and the format of prescribing debonding strain limit (e.g., Type I and Type II debonding strain limit), and

- The absence or presence of the switcher parameter ($\delta_{fe}$) in the process of determining the design failure strain of FRP.

These factors affect the course of flexural strengthening design process by influencing the design predictions for the failure mechanisms of FRP (e.g., full- or near-rupture, and debonding) and for the post-strengthening failure modes (e.g., concrete- or FRP-controlled failure modes) governing the flexural strengthening design solutions. Therefore, an assessment of the impact of these factors on the course of flexural strengthening design process should investigate the sensitivity of:

- The process of determining the design failure strain of FRP
- The design predictions for the failure mechanisms of FRP
- The design predictions for the post-strengthening failure modes

Various failure mechanisms of FRP and the post-strengthening failure modes are differently sensitive to the prescribed safety parameters, and different types of flexural strengthening design solutions involve different flexural resistance and ductility. Thus, the above factors also affect the quality of the resultant flexural strengthening design solutions. An assessment of the impact of these factors on the quality of flexural strengthening design solutions should focus on the sensitivity of:

- The extents of the strengthening design solutions governed by different failure modes
- The residual conservativeness of the flexural strengthening design solutions

The sensitivity and parametric analyses devised for these purposes are discussed in the following sections.
4.5 Sensitivity of design failure strain of FRP

4.5.1 Basis of assessment

The flexural strengthening switch, presented in Chapter 3, can be used to assess the sensitivity of design failure strain of FRP to prescribed material safety parameters, and type of debonding strain limit. For convenience, the flexural strengthening switcher is expressed here through Eq. (4.38), along with Type I and Type II debonding strain limits conforming to TR55 and ACI440 specifications respectively. The conservativeness associated with the design rupture strain capacity of FRP is expressed through Eq. (4.41) in form of a conservativeness index ($I_C(\varepsilon_{fd})$). Here, subscripts ‘Plain’ and ‘Engineered’ refer to omission of all material safety parameters (and producing a plain response) and inclusion of all applicable material safety parameters (and producing an engineered response) respectively while determining the design failure strain of FRP. A discussion on the results of this assessment is presented in next subsection.

\[
\varepsilon_{fd} = \min \left[ \delta_{fe} \frac{e_f}{[S_d]}, \varepsilon_{fd-debond} \right] \quad [\delta_{fe} \leq 1] \quad (4.38)
\]

\[
\varepsilon_{fd-debond} \text{(Type I)} = 0.008 \quad \text{[e.g., conforming to TR55]} \quad (4.39)
\]

\[
\varepsilon_{fd-debond} \text{(Type II)} = 0.41 \left( \frac{t_f}{n t f E_{fd}} \right) \quad \text{[e.g., conforming to ACI440]} \quad (4.40)
\]

\[
I_C(\varepsilon_{fd}) = \left[ \frac{\varepsilon_{fd} \text{(Plain)} - \varepsilon_{fd} \text{(Engineered)}}{\varepsilon_{fd} \text{(Plain)}} \right] \quad (4.41)
\]

4.5.2 Results and discussions

Figs. 4.3 and 4.4 illustrate the sensitivity of ($\varepsilon_f$-$\varepsilon_{fd}$) relation to the material safety parameters prescribed on rupture strain capacity of FRP under the two formats of debonding strain limits. Fig. 4.3 represents Type I debonding strain limit, with an assumed numerical strain value of 0.008 as the debonding strain limit, for demonstration. The solid curves in Fig. 4.3 are based on $\delta_{fe}$ as unity, i.e. corresponding to a rupture-debonding switch (RDS), while the dotted curves are based on $\delta_{fe}$ less than unity (here, 0.9 for demonstration purpose), i.e. corresponding to a near rupture-debonding switch (NRDS). Fig. 4.4 represents Type II debonding strain limit. The abscissa in Fig. 4.4 consists of parameter $R (= n t_f E_{fd})$, presented in log-scale to cover a large range of possible combinations of modulus of elasticity of FRP, effective thickness of FRP ply and number of FRP plies used.
In Fig. 4.3, the red curves correspond to a condensed material safety parameter \([S_e]\) equal to unity, and represent the *plain response* (i.e. ignoring all the material safety parameters on rupture strain of FRP, prescribed at Level I and II within strengthening design) while determining \(\varepsilon_{fd}\). The curves corresponding to a condensed material safety parameter greater than unity represent the *engineered response* (i.e. considering the applicable material safety parameters on rupture strain capacity of FRP at Level I and II). Thus, the difference between a plain and corresponding engineered response (see Fig. 4.3, for an example), under identical conditions, reflects conservativeness in the spirit of discussion presented in Chapter 3.

The rupture strain capacity of FRP is clustered into: Low (i.e. \(\varepsilon_f < 10,000\) micro-strain), Medium (i.e. \(10,000 \leq \varepsilon_f \leq 20,000\) micro-strain) and High (i.e. \(\varepsilon_f > 20,000\) micro-strain). This arrangement is based on the author’s engineering judgement for the ease in deriving qualitative inferences, and applies to the entire thesis, unless otherwise specified.

Notation \(\varepsilon_{fd(\text{Condition1})}\) in these plots refers to the design failure strain of FRP corresponding to the most optimal condition for a critical failure mode, while notation \(\varepsilon_{fd(\text{Condition2})}\) refers to the condition required to avoid non-ductile strain-state for critical failure mode. In Table 4.4, the former is presented as an inequality condition, while the latter is presented as an equality condition. Both of these conditions are based on an assumed initial soffit strain \(\varepsilon_{f0}\) equal to 600 micro-strain. These two conditions provide ductility references in comprehending the sensitivity of \((\varepsilon_f - \varepsilon_{fd})\) relation. The design failure strain in FRP above the \(\varepsilon_{fd(\text{Condition2})}\) line suggests ductile design solutions (i.e. involving yielding of tension steel reinforcement), while that coinciding with \(\varepsilon_{fd(\text{Condition1})}\) line suggests the most optimal design solutions (i.e. the strain in tension steel reinforcement being equal to adequate strain, \(\varepsilon_{st\text{-adequate}}\)). The difference between the \(\varepsilon_{fd(\text{Condition1})}\) for ACI440 and TR55 is mainly due to the different values of specified \(\varepsilon_{st\text{-adequate}}\) specified in these two guidelines.
Figure 4.3
Sensitivity of design failure strain of FRP for Type I debonding strain limit

Fig. 4.4 (Continued on next page)
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Figure 4.4
(a). For \([S_{e}]=1.00\), (b). For \([S_{e}]=2.00\), (c). For \([S_{e}]=3.00\), and (d). For \([S_{e}]=4.00\)
Sensitivity of design failure criteria of FRP for Type II debonding strain limit
(Developed for \(f'_{c}\) of 30 MPa)
The conservativeness associated with the design failure strain of FRP is presented in Fig. 4.5. The abscissa in this figure represents mean rupture strain capacity normalised with design debonding strain limit \( \left( \frac{\varepsilon_f}{\varepsilon_{fd-debond}} \right) \). This arrangement is for the convenience of representing the possible ranges of the mean rupture strain of FRP and the design debonding strain limits through a single parameter. The solid curves in this figure correspond to \( \delta_{fe} \) as unity, while the dotted curves correspond to \( \delta_{fe} \) less than unity (for illustration, \( \delta_{fe} \) is 0.9 in Fig. 4.5 conforming to ACI440 specifications). While this conservativeness is not directly representative of the quantitative conservativeness associated with either the flexural resistance contribution of FRP reinforcement or the post-strengthening flexural resistance of the beam, it does give a clear idea qualitatively.

![Conservativeness corresponding to design failure strain of FRP](image)

Figure 4.5
Conservativeness corresponding to design failure strain of FRP

Following observations derived from Figs. 4.3-4.5 help us appreciating the impact of material safety parameters and formats of debonding strain limit on design failure strain of FRP:

- Type I and Type II debonding strain limits produce similar effects in restricting the numerical value of the design failure strain of FRP. However, the latter exhibits an inverse proportionality between modulus of elasticity of FRP and numerical value of the strain in FRP at debonding. Due to this, a prescription of the material safety parameters on modulus of elasticity of FRP leads to an increase in the numerical value of strain in FRP at debonding. Thus, especially for the low modulus FRP
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materials, Type II debonding strain limit produces considerably higher numerical values for strain in FRP at debonding compared to Type I debonding strain limit.

- It can be seen from Fig. 4.3 that the design failure strain of FRP, before attaining its highest possible value (which is equal to the debonding strain limit), differs marginally for the specifications involving an RDS (e.g., TR55) and an NRDS (e.g., ACI440), even for a same condensed material safety parameter. This margin of difference is signified by the set value of switcher parameter $\delta_{fe}$.

- From Fig. 4.4(a), in particular, it can be seen that even for $[S_c]$ as unity, the values of $\tilde{\varepsilon}_f$ and $\varepsilon_{fd}$ do not coincide with each other. This is due to parameter $\delta_{fe}$ set to be less than unity in an NRDS. A numerically lower value of parameter $\delta_{fe}$ in an NRDS will increase the difference between $\tilde{\varepsilon}_f$ and $\varepsilon_{fd}$, signifying a higher magnification of the material safety parameters on rupture strain of FRP.

- Conservativeness associated with the design failure strain of FRP reduces with an increase in the normalised mean rupture strain capacity of FRP (i.e. with an increase in the actual mean rupture strain capacity of FRP or with a decrease in the design debonding strain limit). This can be confirmed by comparing the plain and the engineered responses in Figs. 4.3 and 4.4.

- It can also be seen, from Fig. 4.5 in particular, that there exists an upper bound on the conservativeness that can be achieved under a given conditions. This corresponds to a design situation involving $\left(\tilde{\varepsilon}_f / \varepsilon_{fd-debond}\right)$ equal to unity (i.e. at which the numerical value of mean rupture strain capacity equal to the design debonding strain limit).

- Conservativeness corresponding to a design situation at which the design rupture strain capacity is numerically equal to or greater than the design debonding strain limit (i.e. for the strengthening design solutions involving FRP failure through debonding and not through rupture) is zero.

- The design failure strain in FRP determined through an NRDS results into higher conservativeness compared to that determined through an RDS.

- It can also be seen from Fig. 4.3 that the entire plain response curve for RDS and most part of the NRDS plain response curve are above their respective most optimal value of design failure strain of FRP represented by $\tilde{\varepsilon}_{fd(Condition1)}$. An increased stringency of the condensed material safety parameter or that of the debonding strain limits tends to flatten these curves. This, in effect, restricts the strain in tension steel reinforcement signified by an increase in the extents of the
design failure strain of FRP below the $\varepsilon_{fd}(\text{Condition1})$ and $\varepsilon_{fd}(\text{Condition2})$ reference lines.

- As a specific observation for ACI440 and TR55, it can be seen that for the high and most part of the medium rupture strain capacity FRP materials, the ACI440 format allows relatively higher design failure strains of FRP compared to that allowed by TR55 format for parameter $[S_e]$ of up to 2. These values fall within a comparable range only for the value of the condensed material safety parameter beyond 3.

4.6.3 Inferences and implications

Following inferences and implications of the above observations can be derived based on the sensitivity analyses presented in this section:

- The format of prescribing Type II debonding strain limit needs reconsideration. The most popular choice for Type I debonding strain limit with a preset constant value of 8,000 micro-strain [e.g., TR55 (2004)] itself is speculated as too liberal. Compared to this value, the possible numerical outcomes of Type II debonding strain limit, especially for low modulus FRP materials, can be considerably higher. An upper bound value for the numerical outcomes of the Type II debonding strain limit is, therefore, suggested.

  - ACI440, in fact, suggests Type II debonding strain limit with an upper bound on its outcome. However, this upper bound is in terms of a fraction of design rupture strain capacity of FRP (see section 2.6.4). It is a part of a tactful strategy to ensure that the design failure strain of FRP will invariably be governed by the debonding strain limit, and not by its full design rupture strain capacity. It restricts the numerical outcomes of Type II debonding strain limit relative to the design rupture strain capacity, and not in absolute terms. For example, for an FRP material with design rupture strain capacity of 15,000 micro-strain, the corresponding maximum possible value of strain in FRP at debonding resulting from the upper bound on Type II debonding strain limit according to ACI440 is 13,500 micro-strain. This is considerably higher than 8,000 micro-strain resulting from Type I debonding strain limit. Thus, the ACI440 format of upper bound prescribed on Type II debonding strain limit does not serve the required purpose.

  - In fact, such a format of upper bound has a side effect in terms of conservativeness associated with strengthening design solutions, which largely goes unnoticed. As shown in Chapter 3, an upper bound in terms of a fraction of the design rupture strain capacity of FRP leads to an NRDS. The parameter $\delta_{fe}$ in an NRDS, in effect, acts like a disguised factor that unintentionally ‘inflates’ the stringency of the
material safety parameters on rupture strain capacity of FRP, without projecting any rise in the safety factors on the rupture strain capacity of FRP.

- While an NRDS enhances the stringency of the material safety parameters, the corresponding increase in the conservativeness concerns only the strengthening design solutions that involve FRP failure through rupture (and not through debonding) as predicted within the strengthening design. Thus, an NRDS does not contribute towards producing conservativeness associated with the strengthening design solutions involving FRP failure through debonding.

- Since Type II debonding strain limit exhibits an inverse proportionality with the modulus of elasticity of FRP, prescribing material safety parameters on the modulus of elasticity of FRP is a deleterious strategy. In this sense, the ACI440 approach of specifying Type II debonding strain limit, with an absence of material safety parameters prescribed on the modulus of elasticity of FRP, is a wise approach.

4.6 Sensitivity of design predictions for failure mechanisms of FRP

The previous section has demonstrated that conservativeness associated with the strengthening design solutions differs significantly based on whether they involve failure of FRP through rupture or debonding. Therefore, it will be of interest to investigate how the design predictions for the failure mechanisms of FRP are influenced by influential factors summarised in section 4.4. A detailed study aiming at assessing the sensitivity of the design predictions for the failure mechanisms of FRP is presented in this section.

4.6.1 Basis of assessment

In order to capture the sensitivity of the design prediction of the failure mechanism of FRP, a hypothetical condition at which the design rupture strain capacity of the FRP becomes numerically equal to the design debonding strain limit, is devised [Eq. (4.42)]. A little consideration will show that a design solution will be governed by debonding of FRP, if the design rupture strain or near-rupture strain is numerically greater than the debonding strain limit, and vice-versa [Eqs. (4.43) and (4.44)].

\[
\delta_{Te} \leq 1 
\]

Eq. (4.42)

\[
\delta_{Te} \geq \epsilon_{f_{d-ruputre}} > \epsilon_{f_{d-debond}} \Rightarrow \text{Debonding of FRP governs}
\]

Eq. (4.43)

\[
\delta_{Te} \leq \epsilon_{f_{d-ruputre}} < \epsilon_{f_{d-debond}} \Rightarrow \text{Rupture/Near-rupture of FRP governs}
\]

Eq. (4.44)
These possibilities can be used to assess sensitivity of the design prediction for the FRP failure mechanism, from a calibrator’s and a designer’s perspective as discussed below.

(A) **Calibrator’s perspective**

From a calibrator’s perspective, it would be more interesting to see how the quantitative prescription of material safety parameters influences design predictions for the FRP failure mechanism. Towards this, Eq. (4.42) is presented in elaborated form, as shown in Eq. (4.45). Setting $\delta_{fe}$ as unity in this and the following equations will correspond to an RDS, otherwise it will correspond to an NRDS. The particular values of the safety parameters $\gamma_{PQR-e}$, $\gamma_{EDT-e}$, $\gamma_{APR-e}$ and $[S_e]$ can be determined such that the condition presented by Eq. (4.42) is satisfied. Denoted through subscript ‘limit’ in their notations, these limiting values can be used to demonstrate influence of quantitative prescription of the material safety parameters on design predictions for the debonding and rupture of FRP, as shown through Eqs. (4.46)-(4.51).

\[
\delta_{fe} \frac{\varepsilon_f}{[S_e]_{\text{limit}}} = \delta_{fe} \frac{\varepsilon_f}{\left[\frac{\gamma_{EDT-e} \times \gamma_{APR-e}}{\gamma_{PQR-e}}\right]_{\text{limit}}} = \varepsilon_{fd-debond} \tag{4.45}
\]

For given $\varepsilon_{fd-debond}$ and material safety parameters at Level II (i.e. $\gamma_{EDT-e}$ and $\gamma_{APR-e}$):

\[
\left(\gamma_{PQR-e}\right)_{\text{limit}} = \frac{1}{\varepsilon_{fe}} \left[1 - \left(\frac{\gamma_{EDT-e} \times \gamma_{APR-e}}{\gamma_{PQR-e} \times \varepsilon_{fd-debond}}\right)\right] \tag{4.46}
\]

Such that:

\[
\begin{cases}
\gamma_{PQR-e} > \left(\gamma_{PQR-e}\right)_{\text{limit}} \Rightarrow \varepsilon_{fd-rupture} < \varepsilon_{fd-debond} \Rightarrow \text{Rupture/Near-rupture governs} \\
\gamma_{PQR-e} < \left(\gamma_{PQR-e}\right)_{\text{limit}} \Rightarrow \varepsilon_{fd-rupture} > \varepsilon_{fd-debond} \Rightarrow \text{Debonding governs}
\end{cases} \tag{4.47}
\]

For given $\varepsilon_{fd-debond}$ and material safety parameter at Level I (i.e. $\gamma_{PQR-e}$):

\[
\left[\gamma_{EDT-e} \times \gamma_{APR-e}\right]_{\text{limit}} = \left[1 - \gamma_{PQR-e} \varepsilon_{fe}\right] \times \delta_{fe} \left[\frac{\varepsilon_f}{\varepsilon_{fd-debond}}\right] \tag{4.48}
\]

Such that:
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If

\[
\begin{align*}
[Y_{\text{EDT-}\epsilon} \times Y_{\text{APR-}\epsilon}] & > [Y_{\text{EDT-}\epsilon} \times Y_{\text{APR-}\epsilon}]_{\text{limit}} \Rightarrow \varepsilon_{\text{f-d-rupture}} < \varepsilon_{\text{f-d-debond}} \Rightarrow \text{Rupture/Near-rupture governs} \\
[Y_{\text{EDT-}\epsilon} \times Y_{\text{APR-}\epsilon}] & < [Y_{\text{EDT-}\epsilon} \times Y_{\text{APR-}\epsilon}]_{\text{limit}} \Rightarrow \varepsilon_{\text{f-d-rupture}} > \varepsilon_{\text{f-d-debond}} \Rightarrow \text{Debonding governs}
\end{align*}
\]

(4.49)

For given \(\varepsilon_{\text{f-d-debond}}\) :

\[
[S_e]_{\text{limit}} = \delta_f \frac{\varepsilon_f}{\varepsilon_{\text{f-d-debond}}}
\]

(4.50)

Such that:

\[
\begin{align*}
[S_e] & > [S_e]_{\text{limit}} \Rightarrow \varepsilon_{\text{f-d-rupture}} < \varepsilon_{\text{f-d-debond}} \Rightarrow \text{Rupture/Near-rupture governs} \\
[S_e] & < [S_e]_{\text{limit}} \Rightarrow \varepsilon_{\text{f-d-rupture}} > \varepsilon_{\text{f-d-debond}} \Rightarrow \text{Debonding governs}
\end{align*}
\]

(4.51)

(B) Designer’s perspective

A designer would probably be interested in producing a more competitive design solution within the specified design framework. This would involve finding how best the prescribed specifications can be arrayed to achieve the design objectives. The choice of the FRP material and application process forms a critical design decision to be made, and involves subjective judgements to arrive at a suitable choice. Any justifiable basis to help the designer in making this decision is of considerable importance.

Based on the condition narrated by Eq. (4.42), a limiting value for the mean rupture strain capacity of FRP can be synthesised, as shown through Eq. (4.52). If a designer opts to use a FRP composite having a mean rupture strain capacity greater than the limiting value, the resultant design solution will invariably involve FRP debonding predicted within the design, and vice-versa [Eqs. (4.53) and (4.54)]. These equations apply to the design solutions governed by a failure mode involving FRP failure.

\[
\varepsilon_{f-\text{limit}} = \frac{1}{\delta_f} [S_e] \varepsilon_{\text{f-d-debond}} = \frac{1}{\delta_f} \left(\frac{Y_{\text{EDT-}\epsilon} \times Y_{\text{APR-}\epsilon}}{1 - \left[Y_{\text{FQR-}\epsilon}\right]_{\text{limit}} \times p_f}\right) \times \varepsilon_{\text{f-d-debond}}
\]

(4.52)

Such that:

\[
\varepsilon_f > \varepsilon_{f-\text{limit}} \Rightarrow \text{Debonding governs}
\]

(4.53)

\[
\varepsilon_f < \varepsilon_{f-\text{limit}} \Rightarrow \text{Rupture/Near-rupture governs}
\]

(4.54)
4.6.2 Results and discussion

(A) Calibrator’s perspective

Fig. 4.6 is developed based on Eqs. (4.48) and (4.49) to demonstrate the above concept more clearly. The mean rupture strain capacity of FRP, normalised with debonding strain limit, is presented on abscissa. This plot can be used to determine the limiting value of safety parameters prescribed at Level II within the strengthening design process corresponding to the given other design parameters. The solid lines in this plot use $\delta_{fr}$ as unity and correspond to an RDS, while the dotted lines use $\delta_{fr}$ less than unity (0.9 in this plot for demonstration) and correspond to an NRDS. The ‘most liberal’ and ‘most stringent’ prescriptions for the material safety parameters at Level II (i.e. $\gamma_{EDT-\varepsilon}$ and $\gamma_{APR-\varepsilon}$), as specified in ACI440 and TR55 for illustration, are superimposed on this plot for reference.

![Figure 4.6](image)

An illustrative key to read this plot is presented in Fig. 4.6, which shows that the limiting value of $[\gamma_{EDT-\varepsilon} \times \gamma_{APR-\varepsilon}]_{\text{limit}}$ is about 1.58 for the normalised mean rupture strain capacity of FRP of 2.5, using the dotted line representing $\gamma_{PQR-\varepsilon}$ as 3. This illustrative key holds good for reading other plots presented later in this section. The significance of this limiting value can be appreciated based on Eq. (4.49). Accordingly, design prediction for the failure mechanism of FRP will invariably be debonding of FRP, for a hypothetical design guideline specifying the safety factors on rupture strain capacity of FRP at Level II such that the product $(\gamma_{EDT-\varepsilon} \times \gamma_{APR-\varepsilon})$ has a value lesser
than 1.58. An assessment of the possible design predictions for rupture or debonding of FRP based on the material safety parameters prescribed on the rupture strain capacity of FRP can be made using this concept.

Figs. 4.7 to 4.12 present the sensitivity plots, which, for illustration, are developed based on ACI440 and TR55 specifications assuming an initial soffit strain of 600 micro-strain. Figs. 4.7 to 4.9 are developed for $e_{fd-debond}$ equal to 8,000 micro-strain, using a Type I debonding strain limit that conforms to TR55. Figs. 4.10 to 4.12 are developed using Type II debonding strain limit conforming to ACI440, using $R$ equal to 42 kN/mm (which corresponds to the debonding strain in FRP as 11,000 micro-strain for $f'_c$ of 30 MPa). Suitable specific variations of $\gamma_{PQR-e}$ (covering PQ Class 0 to III), $\gamma_{EDT-e}$ (covering EE Class I to III) and $\gamma_{APR-e}$ (covering M & I Class I to IV) are appropriately considered within these figures. Please see Chapter 3 for the characteristic classes for the material safety parameters. The shaded regions in these figures suggest the range of material safety parameters bound by their ‘most liberal’ and ‘most stringent’ values prescribed by TR55 and ACI440.

Fig. 4.7 shows an illustrative key to read these plots, which should be read in conjunction with Eqs. (4.46) and (4.47). This key holds good for Fig. 4.8 to 4.12 also, in conjunction with appropriate equations.
Figure 4.7
$\gamma_{PQR-\text{limit}}$ plots for TR55 specifications (Developed for $\varepsilon_f = 600$ micro-strain)
(a). For CFRP, (b). For AFRP, (c). For AR-GFRP, (d). For E-GFRP
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Figure 4.8
$(\eta_{EDT,\varepsilon} \times \eta_{APR,\varepsilon})_{limit}$ plots for TR55 specifications (Developed for $\varepsilon_{f0} = 600$ micro-strain)
(a). For CFRP, (b). For AFRP, (c). For AR-GFRP, (d). For E-GFRP
Figure 4.9
\([S_e]_{\text{limit}}\) plots for TR55 specifications (Developed for \(\varepsilon_f = 600\) micro-strain)
(a). For CFRP, (b). For AFRP, (c). For AR-GFRP, (d). For E-GFRP
Figure 4.10

$\gamma_{PQR}$ plots for ACI440 specifications

(Developed for $\epsilon_f = 600$ micro-strain and $R = 42$ kN/mm)

(a). For CFRP, (b). For AFRP, and (c). For GFRP
Figure 4.11

\((\gamma_{E DT - \varepsilon} \times \gamma_{APR - \varepsilon})_{\text{limit}}\) plots for ACI440 specifications

(Developed for \(\varepsilon_f = 600\) micro-strain and \(R = 42\) kN/mm)

(a). For CFRP, (b). For AFRP, and (c). For GFRP
Figure 4.12

$[S_e]_{\text{limit}}$ plots for ACI440 specifications
(Developed for $\varepsilon_{f0} = 600$ micro-strain and $R = 42$ kN/mm)

(a). For CFRP, (b). For AFRP, and (c). For GFRP
Following observations and inferences can be derived based on Figs. 4.7 to 4.12. Although the quantitative inferences from these observations are specific for the ACI440 and TR55 specifications, the qualitative inferences are generic.

- Consider, as an example, a flexural strengthening design solution governed by a failure mode that involves failure of FRP, and either CFRP or AFRP with the rupture strain capacity of 20,000 micro-strain is used for this design solution:
  - As evident from Fig. 4.7, TR55 will invariably predict debonding of FRP, if this design solution belongs to M & I Class III. For the same M & I Class, the design prediction would have invariably been rupture of FRP, if the design solution had involved GFRP instead of CFRP or AFRP.
  - Similarly, it can be seen from Fig. 4.8(a) that the design prediction for this design solution would have invariably been rupture of FRP, if it had belonged to M & I Class IV. Interestingly, the design prediction would change to debonding of FRP for the same design solution, should TR55 have prescribed the parameter $\gamma_{PQR}$ as 3.00 instead of 2.00.
- The range of material safety parameters prescribed on rupture strain capacity of FRP (i.e. the difference between the most liberal and most stringent specifications for these parameters) is considerably larger for TR55 when compared to ACI440.
- It can be seen from Figs. 4.7 to 4.9 that the curves for M & I Class I and II, as per TR55 specifications, are sitting very close to each other. This reflects the fact that the material safety parameters prescribed at Level II are very similar for these classes. While such a distinction between the M & I classes brings more qualitative clarity, it does not appear to be a wise approach. This is attributed to the sensitivity of design predictions for rupture and debonding of FRP, and that of the consequent conservativeness associated with the design solutions involving rupture or debonding of FRP, as demonstrated above.
- It can be seen that a value of parameter $\gamma_{PQR}$ greater than 3, is barely of any practical significance in terms of a positive impact on probability $P_{e_{-max}}$. While the basis of prescribing a stringent value of parameter $\gamma_{PQR}$ is always open for speculations, the TR55 specified value of 2 for the parameter $\gamma_{PQR}$ is a reasonable choice, if at all a stringent value for this parameter, compared to the traditionally accepted value of 1.64, is to be prescribed.
These illustrations clearly describe the potential of the quantitative prescriptions for material safety parameters in influencing the design predictions for failure mechanisms of FRP reinforcement, irrespective of the reality. Since conservativeness associated with the failure modes involving rupture and debonding of FRP is different, the sensitivity of the design predictions for FRP failure mechanism would consequently influence the conservativeness of flexural strengthening design solutions. Apparently, these aspects are grossly ignored while calibrating the design criteria and safety parameters.

(B) Designer’s perspective

Fig. 4.13 presents a $\varepsilon_{\text{f-limit}}$ plot, which is developed based on Eqs. (4.52)-(4.54). It serves as a designer’s aid in selecting a suitable FRP material towards ensuring an efficient use of the FRP rupture strain capacity within the strengthening design. The solid curves in this plot are based on an RDS (i.e. with $\delta_{\text{f}}$ as unity), while the dotted curves are based on an NRDS (with $\delta_{\text{f}}$ as 0.9, for illustration). For a given debonding strain limit for FRP, and for a particular value of the condensed material safety parameter [$S_{\varepsilon}$] signifying a specific design scenario, a corresponding limiting value of the mean rupture strain capacity ($\varepsilon_{\text{f-limit}}$) can be readily obtained from this plot. This limiting value provides a reference in selecting an FRP material suitably. An FRP material with the mean rupture strain capacity higher than this limiting value will invariably involve debonding of FRP (and not rupture) should the design solution involve failure of FRP. The rupture strain capacity beyond this limiting value is not only superfluous, but also promotes design prediction for debonding of FRP over its rupture (or near-rupture). Thus, it is not a wise choice.
Figure 4.13

$\varepsilon_{f\text{-limit}}$ plot as a designer's aid

For demonstration purposes, Figs. 4.14 and 4.15 are developed, which present the design predictions for debonding or rupture of FRP to govern FRP failure based on the TR55 and ACI440 specifications respectively. Following observations emerge from these figures:

- Fig. 4.14 reveals that, according to TR55, a choice of CFRP with a mean rupture strain capacity greater than 12,500 micro-strain would invariably predict FRP debonding in the design (should the design solution be governed by a failure mode involving FRP failure) for all the four M & I classes. Thus, a choice of CFRP material with a mean rupture strain capacity greater than 12,500 micro-strain would involve no benefits of the higher rupture strain capacity.

- Similarly, it can also be seen that a choice of the low and the moderate rupture strain capacity GFRP materials would be a better choice (for all M & I classes) since it will predict FRP rupture in the design compared to a high rupture strain capacity GFRP.

- It is to be noted that the design predictions portrayed in Fig. 4.15 remain fixed, since TR55 specifies a constant strain of 8,000 micro-strain in FRP at debonding based on the Type I debonding strain limit. Unlike this, the design predictions portrayed in Fig. 4.15, for ACI440, are for a specific $R$ value. The use of FRP with either different modulus of elasticity or effective thickness of FRP reinforcement, or both, would alter the relative extents of the design predictions for debonding or near-rupture failure mechanisms to govern failure of FRP for ACI440.
<table>
<thead>
<tr>
<th>FRP Material</th>
<th>M &amp; I Class</th>
<th>Safety Parameters</th>
<th>$\gamma_{PQR}$</th>
<th>$\gamma_{APR}$</th>
<th>$[S_e]$</th>
<th>$\varepsilon_{f,\text{limit}}$</th>
<th>Mean Rupture Strain Capacity of FRP ($\varepsilon_f$) (in micro-strain unit)</th>
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</tbody>
</table>

**Key to read**

- $\varepsilon_f < \varepsilon_{f,\text{limit}} \Rightarrow$ Rupture governs FRP failure mechanism
- $\varepsilon_f > \varepsilon_{f,\text{limit}} \Rightarrow$ Debonding governs FRP failure mechanism

Figure 4.14
Design implications of the quantitative prescription of the material safety parameters for TR55 specifications
(Developed for $\varepsilon_{f,d,\text{debond}} = 8,000$ micro-strain)
### Table: Mean Rupture Strain Capacity of FRP ($\varepsilon_f$) (in micro-strain unit)

<table>
<thead>
<tr>
<th>FRP Material</th>
<th>EE Class</th>
<th>$Y_{PRB}$</th>
<th>$Y_{f,c}$</th>
<th>$[S]$</th>
<th>$\bar{\varepsilon}_f$-limit</th>
<th>Low (5,000)</th>
<th>Medium (15,000)</th>
<th>High (25,000)</th>
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</thead>
<tbody>
<tr>
<td><strong>CFRP</strong></td>
<td>I</td>
<td>3.00</td>
<td>1.05</td>
<td>1.5038</td>
<td>18380</td>
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<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td></td>
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<td>1.6807</td>
<td>20542</td>
<td>Green</td>
<td>Green</td>
<td>Yellow</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>3.00</td>
<td>1.18</td>
<td>1.6807</td>
<td>20542</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td><strong>AFRP</strong></td>
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<td>1.05</td>
<td>1.5038</td>
<td>18380</td>
<td>Green</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td></td>
<td>II</td>
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<td>1.33</td>
<td>1.9048</td>
<td>23281</td>
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<td>Green</td>
<td>Yellow</td>
</tr>
<tr>
<td></td>
<td>III</td>
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<td>24943</td>
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<tr>
<td><strong>GFRP</strong></td>
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<td>3.00</td>
<td>1.33</td>
<td>1.9048</td>
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<td>2.8571</td>
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</tbody>
</table>

**Key to read**

- If $\varepsilon_f < \bar{\varepsilon}_f$-limit ⇒ Near-rupture governs FRP failure mechanism
- If $\varepsilon_f > \bar{\varepsilon}_f$-limit ⇒ Debonding governs FRP failure mechanism

**Figure 4.15**

Design implications of the quantitative prescription of the material safety parameters for ACI440 specifications
(Developed for $R = 42$ kN/mm and $f'_c = 30$ MPa, i.e. $\varepsilon_{f,debond} \approx 11,000$ micro-strain, for demonstration purpose only)
4.9 Sensitivity of design predictions for post-strengthening failure modes

Previous sections have demonstrated that the failure of FRP through its rupture and debonding are differently sensitive to the prescribed material safety parameters on rupture strain capacity of FRP, and that the design predictions for rupture and debonding of FRP are sensitive to the quantitative prescriptions of these material safety parameters. It is of interest now to investigate how sensitive are the design predictions for the post-strengthening failure modes to the influential factors summarised in section 4.4. An assessment aiming at addressing this question is presented in this section.

4.9.1 Basis of assessment

The critical strain-state, by definition, provides a direct reference to classify the concrete- and the FRP-control failure modes. Thus, design predictions for the post-strengthening failure modes are fundamentally dependent upon the critical strain-state. Sensitivity of the critical strain-state, therefore, is of central importance in this assessment.

4.9.2 Results and discussion

Fig. 4.16 demonstrates sensitivity of the ductility representative parameters (i.e. the depth of neutral axis (\(x_{u-crit}/d\)) and the sectional ductility-content (\(\varepsilon_{st-crit}\))) to the mean rupture strain capacity of FRP (\(\varepsilon_f\)) and the material safety parameters prescribed on \(\varepsilon_f\). For illustration purpose, Fig. 4.16(a) is developed for \(\varepsilon_f\) as unity, while Fig. 4.16(b) is developed for \(\varepsilon_f\) equal to 10,000 micro-strain and \(\delta_{fe}\) equal to 0.9. Thus, Fig. 4.16(a) corresponds to an RDS with Type I debonding strain limit (e.g., to conform to TR55 specifications), while Fig. 4.16(b) corresponds to an NRDS with Type II debonding strain limit (e.g., to conform to ACI440 specifications).

In spite of Figs. 4.16(a) and (b) being qualitatively similar, the RDS and NRDS, and the Type I and Type II debonding strain limits have quantitative implications on the critical strain-state. These are summarised in the following observations:

- It can be observed that the maximum possible value of the critical tension steel strain (\(\varepsilon_{st-crit}^{max}\)) is directly dependent upon the numerical value of the strain in FRP at debonding.
Figure 4.16

Sensitivity of ductility representative parameters for critical strain-state
(a). For $\delta_{fe} = 1$ and $\varepsilon_{fd-debond} = 8,000$ micro-strain (e.g., RDS and Type I debonding strain limit conforming to TR55 specifications), (b). For $\delta_{fe} = 0.9$ and $\varepsilon_{fd-debond} = 10,000$ micro-strain (e.g., NRDS and Type II debonding strain limit conforming to ACI440 specifications)

- An increase in the permissible numerical value of strain in FRP at debonding results into an increase in the numerical value of $\varepsilon_{st-crit}^{max}$ and thus shifts the $\varepsilon_{st-crit}$ curves upwards.
- Since Type I debonding strain limit is a set constant numerical value of the strain in FRP at debonding, the value of $\varepsilon_{st-crit}^{max}$ remains fixed for the strengthening design guidelines prescribing Type I debonding strain limit (e.g., TR55).
• Unlike the above, the numerical value of strain in FRP at debonding, and hence the value of $\varepsilon_{st-crit}^{max}$ is not a fixed constant but depends upon the modulus of elasticity of FRP, for the strengthening design guidelines prescribing Type II debonding strain limit (e.g., ACI440).
• An increase in the numerical value of parameter $[S_e]$ tends to shift the $\varepsilon_{st-crit}$ curves rightwards, which in turn tends to decrease the area under the $\varepsilon_{st-crit}$ curves. This reduction signifies reduction in the extent of the flexural strengthening design solutions governed by FRP-controlled failure mode. This is discussed in more detail later in this chapter, through the solution grids.

Conservativeness associated with the $\varepsilon_{st-crit}$, designated as $C(\varepsilon_{st-crit})$, can be obtained from comparing the $\varepsilon_{st-crit}$ curves for plain and engineered responses, as indicatively shown in Fig. 4.16(b). This conservativeness qualitatively follows the same pattern as shown in Fig. 4.5.

4.8 Sensitivity of the extents of post-strengthening failure modes

Quality of a flexural strengthening design solution is jointly depicted by the post-strengthening failure mode governing it, involvement of rupture or debonding of FRP, ductility- and conservativeness-content associated with it. Using the ductility-based definitions, it is possible to produce a range of possible flexural strengthening design solutions under a given design scenario, and classify them according to their qualitative characteristics into different clusters. Each cluster is, thus, a representative of a specific characteristic quality of the strengthening design solutions. Implications of influential factors on the relative extents of these clusters, therefore, are the indicators of change in the quality of strengthening design solutions. An assessment aimed at this is presented in this section.

4.8.1 Basis of assessment

A little consideration will show that a set of strain values $\varepsilon_f$ and $\varepsilon_{st}$, under a given design scenario, presents a unique flexural strengthening design solution in context of the ductility-based definitions of failure modes. Ranges of $\varepsilon_f$ and $\varepsilon_{st}$, therefore, present a universe of possible flexural strengthening design solutions under a given design scenario. These design solutions can be presented in the form of a solution grid, such as the one shown in Fig. 4.17 for illustration. It comprises of a $\varepsilon_{st}$ axis covering a possible range of sectional ductility, and a $\varepsilon_f$ axis covering a possible range of mean rupture strain capacity of FRP composites. It can be seen that the possible range of sectional
ductility is classified into: non-ductile (i.e. involving $\varepsilon_{st} < \varepsilon_{sy} < \varepsilon_{st-adequate}$), ductile (i.e. involving $\varepsilon_{sy} < \varepsilon_{st} < \varepsilon_{st-adequate}$) or adequately-ductile (i.e. involving $\varepsilon_{sy} < \varepsilon_{st-adequate} \leq \varepsilon_{st}$). Similarly, the range of mean rupture strain of FRP (in micro-strain unit) is classified into: low ($\bar{\varepsilon}_f < 10,000$), medium ($10,000 < \bar{\varepsilon}_f < 20,000$) and high ($\bar{\varepsilon}_f > 20,000$). These classifications characterise the universe of flexural strengthening design solutions into nine qualitatively distinct regions. The $\bar{\varepsilon}_f$ axis can be transformed into a corresponding $\varepsilon_{fd-rupture}$ axis using the condensed material safety parameter $[S_e]$ applicable for that design scenario. Fig. 4.17 involves, for illustration, $[S_e]$ value of 2 for a hypothetical design scenario.

![Figure 4.17](image.png)

**Figure 4.17**

Typical solution-grid with the indicative characteristic clusters of the flexural strengthening design solutions

A demarcation on the $\varepsilon_{fd-rupture}$ axis can be placed based on the numerical value of $\varepsilon_{fd-debond}$ applicable for that design scenario, as shown through a thick blue (horizontal) line in Fig. 4.17. This demarcating line, when extended onto the $\bar{\varepsilon}_f$ axis, refers to the limiting value of the mean rupture strain capacity ($\bar{\varepsilon}_f-limit$). This line, thus, classifies the flexural strengthening design solutions into those involving debonding of FRP (above the blue line) and rupture or near-rupture of FRP (below the blue line).

It is shown earlier that the numerical value of $\varepsilon_{st}$ for a design solution relative to the critical sectional ductility $\varepsilon_{st-crit}$ under a given design scenario can be used to characterise a design solution on the basis of if it is governed by FRP- or concrete-
controlled failure mode. Thus, imposing a \( \varepsilon_{st-crit} \) curve, applicable for that design scenario, on the solution grid classifies the flexural strengthening design solutions into those involving concrete- and FRP-controlled failure modes. A thick red curve in Fig. 4.17 indicates the \( \varepsilon_{st-crit} \) curve. Design solutions onto the left of this curve are governed by concrete-controlled failure mode, while those onto the right of this curve are governed by FRP-controlled failure mode.

This arrangement classifies the universe of flexural strengthening design solutions into three characteristically different clusters, called Clusters A, B and C, as shown in Fig. 4.17. Cluster A consists of the flexural strengthening design solutions governed by concrete-controlled failure modes. Clusters B and C consist of flexural strengthening design solutions governed by FRP-controlled failure mode involving rupture (or near-rupture) and debonding of FRP respectively.

For a given design scenario and for a given FRP composite material, the demarcations based on \( \varepsilon_f-limit \) and \( \varepsilon_{st-crit} \), and hence the extents of Clusters A, B and C remain fixed. A specific flexural strengthening design solution, represented by a pair of coordinates \( (\varepsilon_{st}, \varepsilon_f) \), can belong to either of these three clusters. Its position on the solution grid, thus, determines its qualitative features in terms of post-strengthening failure modes, failure mechanism of FRP and ductility-content.

The point marked by a symbol, which is at the intersection of these three clusters, has specific significance and utility. The regions on the left- and right-hand sides of this point represent concrete- and FRP-controlled design solutions respectively, while the regions above and below of this point represent debonding and rupture (or near-rupture) of FRP respectively. Thus, ‘contrasting characteristics’ exist on either sides of this point, and therefore, it is more appropriately called the point of dual contra-characteristics (PDCC). The coordinates of a PDCC are \( (\varepsilon_{st-crit}^{max}, \varepsilon_f-limit) \). The abscissa of PDCC presents a limiting value of critical tension steel strain, while the ordinate presents a limiting value of the mean rupture strain capacity of FRP for the given conditions. These coordinates determine the extents of Clusters A, B and C of the flexural strengthening design solutions, and remain fixed for a given design scenario based on a given strengthening design guideline. Therefore, sensitivity of the coordinates of PDCC to various influences provides vital information related to the design predictions for the post-strengthening failure modes and the failure mechanism of FRP. These predictions, in turn, control the extents of design solutions involving...
these failure modes and mechanisms, and the ductility-content associated with these design solutions.

4.8.2 Results and discussion

In order to understand the implications of various design formats used in flexural strengthening design on the extents of type of flexural strengthening design solutions, solution grids under different design scenarios are devised (Figs. 4.18 to 4.20). The point of dual contra-characteristics (PDCC) is highlighted through a ♦ mark in these plots. The extents of Cluster A (consisting of flexural strengthening design solutions governed by concrete-controlled failure mode), Cluster B (consisting of design solutions governed by FRP-controlled failure mode involving rupture/near-rupture of FRP) and Cluster C (consisting of design solutions governed by FRP-controlled failure mode involving debonding of FRP) are highlighted in these grids. Following observations emerge from these figures:

• From Fig. 4.18 it can be seen that a numerically lower value of the permissible strain in FRP at debonding substantially increases the extent of Cluster C. This is due to the fact that a lower value of strain in FRP at debonding reduces the values of $\varepsilon_{\text{max}}^{\text{st-crit}}$ and $\varepsilon_{f, \text{limit}}$, which in turn shift the PDCC leftwards and downwards, resulting in an increase in the extent of Cluster C. This increase in the extent of Cluster C is complemented by a decrease in the extent of Cluster B, and signifies an increased chance of the design predictions for the design solutions governed by FRP-controlled failure modes involving debonding of FRP.

• In light of the above point, the possible range of numerical values for strain in FRP at debonding resulting from a Type II debonding strain model are more influential on the extents of Cluster B and C.

• From Fig. 4.18 is can also be seen that an NRDS, when prescribed instead of an RDS, tends to reduce the extent of Cluster C. This is due to the fact that the value of parameter $\delta_{f \varepsilon}$ less than unity in an NRDS tends to increase the value of $\varepsilon_{f, \text{limit}}$, which shifts the PDCC upwards, resulting in a decrease in the extent of Cluster C. This reduction in extent of Cluster C signifies a decreased chance of design predictions for the design solutions governed by FRP-controlled failure modes involving debonding of FRP.

• The change in the extents of the clusters due to an NRDS (prescribed instead of an RDS) is marginal compared to that due to a Type II debonding strain limit prescribed (instead of Type I debonding strain limit).
Figure 4.18
Sensitivity of the extents of clusters of flexural strengthening design solutions
(a) $\varepsilon_{f_d-debond} = 8,000$ micro-strain and $\delta f_e = 1$, (b) $\varepsilon_{f_d-debond} = 6,000$ micro-strain and $\delta f_e = 1$, and (c) $\varepsilon_{f_d-debond} = 8,000$ micro-strain and $\delta f_e = 0.9$
Figure 4.19
Sensitivity of the extents of clusters of flexural strengthening design solutions for Most Liberal Combination (MLC) of material safety parameters for $\varepsilon_{td,debond} = 8,000$ micro-strain

(a) For TR55 (i.e. $\delta_{fe} = 1$), and (b) For ACI440 (i.e. $\delta_{fe} = 0.9$),
Sensitivity of the extents of clusters of flexural strengthening design solutions for Most Stringent Combination (MSC) of material safety parameters for $\varepsilon_{\text{f-debond}} = 8,000$ micro-strain (a) For TR55 (i.e. $\delta_{FE} = 1$), and (b) For ACI440 (i.e. $\delta_{FE} = 0.9$)

- For the design guidelines prescribing Type I debonding strain limit, values of $\varepsilon_{\text{f-limit}}$ and $\varepsilon_{\text{f-limit, max}}$ and hence position of the PDCC on a solution grid, remain fixed for a given design scenario. Thus, the extents of clusters of strengthening design solutions, in this case, are solely dependent upon the material safety parameters prescribed on the rupture strain capacity of FRP.

- Unlike the above, for the design guidelines prescribing Type II debonding strain limit, values of $\varepsilon_{\text{f-limit, max}}$ and $\varepsilon_{\text{f-limit}}$, and hence position of the PDCC on a solution grid, are sensitive to the modulus of elasticity of FRP. Thus, the extents of clusters of strengthening design solutions, in this case, are dependent upon the modulus of...
elasticity of FRP (and the material safety parameters prescribed on it) in addition to the material safety parameters prescribed on the rupture strain capacity of FRP.

- From Fig. 4.19 it can be seen that the MLC condition results into similar extents of the clusters for TR55 and ACI440 specifications. However, the plots for ACI440 in this figure are developed for numerically identical value of strain in FRP at debonding compared to TR55. A different choice of FRP material and/or its effective thickness can alter the numerical value of strain in FRP at debonding according to ACI44. Such choices can lead to a considerable difference between the extents of characteristic clusters for ACI440 and TR55, for both MLC and MSC conditions.

- From Table 3.8 it can be seen that the values for parameter $[S_{e}]$ under the MLC condition are 1.50 and 1.64 as prescribed by ACI440 and TR55 respectively. This suggests that TR55 specification is relatively less stringent than ACI440 specification on this ground. However, this is rather an illusion. The parameter $\delta_{Fe}$ equal to 0.9 effectively raises the value of $[S_{e}]$ for ACI440 from 1.50 to 1.67, which, in fact, is marginally more stringent than the TR55 specified value of $[S_{e}]$ as 1.64.

- From Fig. 4.20 it can be seen that the resulting extents of clusters are considerably different for these two guidelines under MSC condition, even when these plots are developed for an identical value of strain in FRP at debonding.

4.9 Sensitivity of conservativeness in flexural resistance

This section exhibits how the total mechanism of safety format embedded within the strengthening design process influence conservativeness associated with flexural resistance of the strengthening design solutions.

4.9.1 Basis of assessment

Fig. 4.21 shows a typical sectional ductility-moment of resistance relation based on the ductility-based definitions of the failure modes. For a given FRP material and for a design scenario, a particular value of strain $\varepsilon_{st}$ represents a unique flexural strengthening design solution. Thus, $\varepsilon_{st}$ axis in Fig. 4.21 represents a range of flexural strengthening design solutions under a given design scenario. Dotted part of the curve in this figure notionally indicates the penalised moment of resistance for the design solutions exhibiting a lack of adequate ductility. Such a plot can serve as a tool to comprehend conservativeness in the flexural resistance by comparing the change in moments of resistance produced under different design scenarios.
The conservativeness associated with a flexural strengthening design solution under a given design scenario can be presented in terms of a residual conservativeness index ($RCI$), through Eq. (4.55). It can be seen that Eq. (4.55) is a specific variant of representing conservativeness in the notion expressed earlier through Eqs. (3.1) and (3.4).

$$RCI = \frac{(M_{bdz})_{\text{Plain}} - (M_{bdz})_{\text{Engineered}}}{(M_{bdz})_{\text{Plain}}}$$

Equation (4.55)

Here, subscript ‘Engineered’ refers to the engineered version of moment of resistance, i.e. using all the applicable safety parameters on FRP while computing $(M_{bdz})$, as shown in Table 4.5. Subscript ‘Plain’ indicates omission of all the safety parameters prescribed on FRP.

### Table 4.5
Conservativeness combinations

<table>
<thead>
<tr>
<th>Type of conservativeness</th>
<th>Safety parameter combination ‘Engineered’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>None</td>
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<tr>
<td>Level I+II</td>
<td>Material Conservativeness (MC)</td>
</tr>
<tr>
<td>Level I+II+III+IV</td>
<td>Total Conservativeness (TC)</td>
</tr>
</tbody>
</table>

Remarks:
- Material safety parameters refer to rupture strain capacity of FRP.
- Applicable factors of safety on steel reinforcement and concrete are included in each of the above combinations.
Two representative design scenarios – one involving the most liberal combination (MLC) and the other involving the most stringent combination (MSC) of the material safety parameters on rupture strain capacity of FRP – have been created, for illustration using the ACI440 and TR55 specifications using Table 3.8. For each of these two design scenarios, two representative cases, one involving rupture of FRP and the other involving debonding of FRP, are created. The results of this study are discussed below.

4.9.2 Results and discussion

Figs. 4.22 and 4.23 present sensitivity of post-strengthening resistance of a range of flexural strengthening design solutions to different combinations of safety parameters under MLC and the MSC scenarios respectively. In the same line, Fig. 4.24 presents sensitivity of residual conservativeness in terms of RC1. These plots are developed for ACI440 and TR55 specifications to capture the influence of an NRDS and Type II debonding strain limit (as specified by ACI440) over an RDS and Type I debonding strain limit (as specified by TR55). It is also to be noted here that TR55 does not suggest any resistance safety parameters, except $\gamma_{PNT}$. Following observations emerge from these plots:

- An obvious consequence of prescribing material safety parameters on rupture strain capacity of FRP is to increase the conservativeness-content of the strengthening design solutions involving rupture (or near-rupture) of FRP. However, the strengthening design solutions involving debonding of FRP or those governed by concrete-controlled failure modes are not affected by these material safety parameters.

- Resistance safety parameters, on the other hand, are more useful for producing conservativeness in post-strengthening flexural resistance.

- A principal implication of the material safety parameters prescribed on rupture strain capacity of FRP is reduction in numerical value of the critical tension steel strain ($\varepsilon_{st\text{-crit}}$). This reduction is signified in Fig. 4.22 (a) and (b) through a leftward ‘shift’ of $\varepsilon_{st\text{-crit}}$ for the engineered response curves compared to the plain response curve. Axiomatically, this shift in $\varepsilon_{st\text{-crit}}$ is observed only if the FRP-controlled failure mode involved rupture/near-rupture, and not debonding, of FRP.

- Since the strain $\varepsilon_{st\text{-crit}}$ demarcates the flexure strengthening design solutions governed by concrete- and FRP-controlled failure modes, the leftward shift of $\varepsilon_{st\text{-crit}}$ signifies an increase in the extent of the flexural strengthening design solutions governed by FRP-controlled failure mode. This increase is complemented by a
decrease in the extent of the flexural strengthening design solutions governed by concrete-controlled failure mode.

- Consequently, a bunch of flexural strengthening design solutions, towards the tail end of the plain response curve in these plots (which originally were involving concrete-controlled failure mode) transform to involve FRP-controlled failure mode for the engineered response curve. This signifies the potential of material safety parameters prescribed on rupture strain capacity of FRP in altering the design prediction for the post-strengthening failure mode.

- The extent of the bunch of flexural strengthening design solutions involving such a change in design predictions for the post-strengthening failure modes increases with an increase in the numerical value of the material safety parameters prescribed on rupture strain of FRP. This is evident from Fig. 4.23 (a) and (b).

- Parameter $\delta_{fe}$, as shown earlier, numerically inflates the material safety parameters prescribed on rupture strain capacity of FRP. Therefore, it also contributes towards the leftward shift of $e_{st-crit}$.

- The quantitative implications of the conservativeness plots shown in Fig. 4.24 are case-specific for the assumed data. The quantitative variances within these plots are attributed to the differences in conservativeness associated with the post-strengthening flexural resistance contributions of the RC section and FRP, which arise from the differences in design criteria and formats. However, the qualitative implications of these are generic and apply to all design scenarios represented within Fig. 4.24.
Figure 4.22
Flexural resistance plots for the most liberal combination (MLC) for TR55 and ACI440 specifications
(a) and (b) For FRP-rupture or near-rupture governed design solution, (c) and (d) For FRP-debonding governed design solution
Figure 4.23
Flexural resistance plots for the most stringent combination (MSC) for TR55 and ACI440 specifications
(a) and (b) For FRP-rupture or near-rupture governed design solution, (c) and (d) For FRP-debonding governed design solution
Figure 4.24
Residual conservativeness plots for TR55 and ACI440 specifications
Design solution governed by (a) FRP-rupture (MLC), (b) FRP-debonding (MLC), (c) FRP-rupture (MSC), (d) FRP-debonding (MSC),
4.10 Summary of observations

Important observations based on the discussion presented in this chapter are summarised below:

- It is shown that the ductility-based definitions offer considerable advantages over the conventional flexural strengthening design process. An assessment methodology based on these definitions of failure modes enables us:
  - classifying the post-strengthening failure modes more objectively into concrete- or FRP-controlled failure modes, and to systematically cover all the possible variants for these failure modes based on their ductility-content and involvement of debonding or rupture/near-rupture of FRP.
  - setting a logical hierarchy of preferences over the design failure modes. Supporting mathematical expressions for various post-strengthening failure modes enable us avoiding undesirable failure modes and deriving optimum strengthening design solutions tailored to specific needs of the prevalent design scenario, thus promoting an efficient use of FRP materials.
  - seeing flexural resistance and sectional ductility as a simultaneous design objectives.
  - having a clear and complete picture of the flexural strengthening design process algorithmically, which provides an insight into the internal architecture of this process.
  - producing a range of possible flexural strengthening design solutions under a given design scenario, which can be characterised according to their salient qualitative characteristics.

- The failure mode switcher sets important bifurcations within the flexural strengthening design process. The format of this switcher influences the design predictions for debonding and rupture/near-rupture of FRP.
  - It is shown that the strengthening design solutions involving debonding or rupture/near-rupture of FRP have different fractions of the prescribed material safety parameters applicable to them. This indicates that the strengthening design solutions involving debonding and rupture/near-rupture of FRP involve different extents of conservativeness in terms of post-strengthening flexural resistance.
  - The failure mode switcher is sensitive to the material safety parameters, which means that the design predictions for debonding or rupture/near-rupture of
FRP are sensitive to the quantitative prescriptions for the material safety parameters.

- The format of prescribing debonding strain limit for FRP influences the working of failure mode switcher and the flexural strengthening design process.
  - Type I debonding strain limit (i.e. involving a pre-set constant value of strain in FRP at debonding, e.g. TR55 debonding strain limit) is completely insensitive to the prescribed material safety parameters. Thus, the prescribed material safety parameters on FRP material properties in this case do not contribute towards producing conservativeness in terms of flexural resistance for the strengthening design solutions involving debonding.
  - Type II debonding strain limit (i.e. involving an empirical model, e.g. ACI440 debonding strain limit) is inversely proportional to the modulus of elasticity of FRP. Thus, material safety parameters prescribed on modulus of elasticity FRP leads to negative conservativeness in terms of post-strengthening resistance for the design solution involving debonding of FRP. In this context, the ACI440 strategy of not prescribing any material safety parameters on modulus of elasticity of FRP avoids generating negative conservativeness.
  - A substantially low value of strain in FRP at debonding according to either Type I debonding strain limit or Type II debonding strain limit for high modulus FRP materials does not allow taking advantage of FRP materials with high rupture strain for flexural strengthening.
  - ACI440 approach of imposing an upper bound limit of 0.9 $\varepsilon_{fd,\text{rupture}}$ on the numerical value of strain in FRP at debonding ceases the possibility of predicting full-rupture of FRP within the design. The design predictions in such a case involve either debonding or near-rupture of FRP. Also, the multiplier 0.9 comprising the upper bound on debonding strain limit inflates the prescribed material safety parameters by about 11%.
  - The upper bound limit on strain in FRP at debonding, e.g. as prescribed by ACI440, might be useful in controlling the strain in FRP at debonding for low rupture strain capacity FRP materials. However, the numerical value of strain in FRP at debonding arriving out of Type II debonding strain limit compared to that for Type I debonding strain limit can be substantially high for low modulus high rupture strain capacity FRP materials.
  - The bond length estimation model for flexural strengthening involves a direct proportionality with the modulus of elasticity of FRP. On this account, the ACI440 strategy of not prescribing any material safety parameters on modulus of elasticity
of FRP leads to unconservativeness in the required bond length of FRP reinforcement predicted by ACI440 bond length model relative to TR55 bond length model.

- The design predictions for the concrete- or FRP-controlled failure modes are sensitive to the quantitative prescriptions of the material safety parameters. The material safety parameters prescribed on the FRP material properties do not contribute towards producing conservativeness in terms of post-strengthening flexural resistance for the strengthening design solutions involving concrete-controlled failure mode. For such design solutions, only the prescribed resistance safety parameters contribute towards producing conservativeness. On this account, the TR55 strategy of not prescribing most resistance safety parameters loses an opportunity to produce conservativeness in terms of post-strengthening flexural resistance.
Table 4.1 Summary of design equations for ductility-based definitions of failure modes: Basic definitions of strain-states

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Critical Failure Mode</th>
<th>Concrete-controlled Failure Mode</th>
<th>FRP-controlled Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain-state</td>
<td>$\varepsilon_{ct} = \varepsilon_{st-crit}$</td>
<td>$\varepsilon_{ct} &lt; \varepsilon_{st-crit}$</td>
<td>$\varepsilon_{ct} &gt; \varepsilon_{st-crit}$</td>
</tr>
</tbody>
</table>
| Defining condition | $\varepsilon_{st-crit} = \left\{\begin{array}{c}
\frac{1 - \frac{\varepsilon_{st-crit}}{\varepsilon_{cu}}}{1 + \frac{\varepsilon_{st-crit}}{\varepsilon_{fu}}} \varepsilon_{cu} \\
\frac{1}{K} \left[ \frac{1}{\varepsilon_{st-crit}} \left( \varepsilon_{st-crit} + \varepsilon_{fd} \right) \right] \end{array}\right.$ | $\varepsilon_{st} < \varepsilon_{st-crit}$ | $\varepsilon_{st} > \varepsilon_{st-crit}$ |
| $\frac{x_{u-crit}}{d} = \left\{\begin{array}{c}
\frac{1}{K} \left[ \frac{1}{\varepsilon_{st-crit}} \left( \varepsilon_{st-crit} + \varepsilon_{fd} \right) \right] \end{array}\right.$ | $\varepsilon_{ct} < \varepsilon_{ct-crit}$ | $\varepsilon_{ct} > \varepsilon_{ct-crit}$ | $\varepsilon_{ct} > \varepsilon_{ct-crit}$ |

Limiting values for critical strain-state

- Maximum possible value of design failure strain in FRP: $\varepsilon_{fd}^{max} = \varepsilon_{fd-debond}$
- Maximum possible value of critical strain in tension steel: $\varepsilon_{st-crit}^{max} = (K - 1) \varepsilon_{cu} + K (\varepsilon_{fu} + \varepsilon_{fd-debond})$
- Maximum possible value of critical depth of neutral axis: $\left(\frac{x_{u-crit}}{d}\right)_{max} = \frac{1}{K} \left[ \frac{1}{\varepsilon_{st-crit}} \left( \varepsilon_{st-crit} + \varepsilon_{fd-debond} \right) \right]$
Table 4.2 Summary of design equations for ductility-based definitions of failure modes: Strains in components

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Critical Failure Mode</th>
<th>Concrete-controlled Failure Mode</th>
<th>FRP-controlled Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_c$</td>
<td>$\varepsilon_{cu}$</td>
<td>$\varepsilon_{cu}$</td>
<td>$\varepsilon_{cu}$</td>
</tr>
<tr>
<td>$\varepsilon_c = \varepsilon_{cu}$</td>
<td>$\varepsilon_c = \varepsilon_{cu}$</td>
<td>$\varepsilon_c &lt; \varepsilon_{cu}$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{fe}$</td>
<td>$(\varepsilon_{fo} + \varepsilon_{fd})$</td>
<td>$(\varepsilon_{fo} + \varepsilon_{ft}) = \frac{1}{\frac{1 - x_u}{d} - 1} \varepsilon_{st}$</td>
<td>$(\varepsilon_{fo} + \varepsilon_{fd})$</td>
</tr>
<tr>
<td>$\varepsilon_{ft} = \varepsilon_{fd}$</td>
<td>$\varepsilon_{ft} &lt; \varepsilon_{fd}$</td>
<td>$\varepsilon_{ft} = \varepsilon_{fd}$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{sc}$</td>
<td>$\varepsilon_{sc-crit} = \left{ \begin{array}{ll} \left[ 1 - \frac{S}{X_{sc-crit}} \right] \varepsilon_{cu} \ \text{OR} \ \left[ \frac{X_{sc-crit} - S}{1 - X_{sc-crit}} \right] \varepsilon_{st-crit} \end{array} \right}$</td>
<td>$\varepsilon_{sc-crit} = \left{ \begin{array}{ll} \left[ 1 - \frac{S}{X_{sc-crit}} \right] \varepsilon_{cu} \ \text{OR} \ \left[ \frac{X_{sc-crit} - S}{1 - X_{sc-crit}} \right] \varepsilon_{st-crit} \end{array} \right}$</td>
<td>$\varepsilon_{sc-crit} = \left{ \begin{array}{ll} \left[ 1 - \frac{S}{X_{sc-crit}} \right] \varepsilon_{cu} \ \text{OR} \ \left[ \frac{X_{sc-crit} - S}{1 - X_{sc-crit}} \right] \varepsilon_{st-crit} \end{array} \right}$</td>
</tr>
<tr>
<td>$\varepsilon_{sc} = \varepsilon_{sc-crit}$</td>
<td>$\varepsilon_{sc} &gt; \varepsilon_{sc-crit}$</td>
<td>$\varepsilon_{sc} &lt; \varepsilon_{sc-crit}$</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.3 Summary of design equations for ductility-based definitions of failure modes: Depth of neutral axis, FRP-content and moment of resistance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Critical Failure Mode</th>
<th>Concrete-controlled Failure Mode</th>
<th>FRP-controlled Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_u / d )</td>
<td>( x_u - x_u\text{-crit} / d )</td>
<td>( x_u &gt; x_u\text{-crit} / d )</td>
<td>( x_u &lt; x_u\text{-crit} / d )</td>
</tr>
<tr>
<td>( \rho_{FRP} )</td>
<td>( \rho_{FRP\text{-crit}} )</td>
<td>( \rho_{FRP &gt; \rho_{FRP\text{-crit}}} )</td>
<td>( \rho_{FRP &lt; \rho_{FRP\text{-crit}}} )</td>
</tr>
<tr>
<td>( M_{nominal} / bd^2 )</td>
<td>( \frac{M_{nominal\text{-crit}}}{bd^2} = \frac{k_1 f_{ck}}{f_{mc} (1 - k_2) \left( \frac{x_u - x_u\text{-crit}}{d} \right)^2} )</td>
<td>( \frac{M_{nominal}}{bd^2} &gt; \frac{M_{nominal\text{-crit}}}{bd^2} )</td>
<td>( \frac{M_{nominal}}{bd^2} &lt; \frac{M_{nominal\text{-crit}}}{bd^2} )</td>
</tr>
</tbody>
</table>

Concrete Contribution

\[ \left( e_{sc} E_s \rho_{sc} \right) \left( \frac{x_u - x_u\text{-crit}}{d} \right) \]

Compression Steel Contribution

\[ \left( e_{st} E_s \rho_{st} \right) \left( 1 - \frac{x_u - x_u\text{-crit}}{d} \right) \]

Tension Steel Contribution

\[ \left( e_{st} E_s \rho_{st} \right) \left( 1 - \frac{x_u - x_u\text{-crit}}{d} \right) \]

FRP Contribution

\[ \left( e_{fR} E_{fR} \rho_{fR} \right) \left( \frac{1}{K} - \frac{x_u - x_u\text{-crit}}{d} \right) \]
Table 4.4 Summary of design equations for ductility-based definitions of failure modes: Sub-clusters, avoiding non-ductile strain-states and optimal condition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Critical Failure Mode</th>
<th>Concrete-controlled Failure Mode</th>
<th>FRP-controlled Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debonding-governing</td>
<td>( \varepsilon_{d,debond} &lt; \varepsilon_{d,rupture} )</td>
<td>---</td>
<td>( \varepsilon_{d,debond} &lt; \varepsilon_{d,rupture} )</td>
</tr>
<tr>
<td>Rupture-governing</td>
<td>( \varepsilon_{d,debond} &gt; \varepsilon_{d,rupture} )</td>
<td>---</td>
<td>( \varepsilon_{d,debond} &gt; \varepsilon_{d,rupture} )</td>
</tr>
<tr>
<td>Non-ductile</td>
<td>( \varepsilon_{st,crit} &lt; \varepsilon_{sy} &lt; \varepsilon_{st,adequate} )  AND  ( \varepsilon_{st,crit} &lt; \varepsilon_{st} &lt; \varepsilon_{st,crit} &lt; \varepsilon_{st,adequate} )  AND  ( \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{st,adequate} )</td>
<td>( \varepsilon_{st} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,adequate} )  AND  ( \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{st,adequate} )</td>
<td>( \varepsilon_{st} &gt; \varepsilon_{st,adequate} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} )</td>
</tr>
<tr>
<td>Ductile</td>
<td>( \varepsilon_{sy} &lt; \varepsilon_{st,crit} &lt; \varepsilon_{st,adequate} )  ( \varepsilon_{sy} &lt; \varepsilon_{st} &lt; \varepsilon_{st,crit} &lt; \varepsilon_{st,adequate} )  ( \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{st,adequate} )</td>
<td>( \varepsilon_{st} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,adequate} )  AND  ( \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{st,adequate} )</td>
<td>( \varepsilon_{st} &gt; \varepsilon_{st,adequate} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} )</td>
</tr>
<tr>
<td>Adequately ductile</td>
<td>( \varepsilon_{sy} &lt; \varepsilon_{st,adequate} &lt; \varepsilon_{st,crit} )  ( \varepsilon_{sy} &lt; \varepsilon_{st} &lt; \varepsilon_{st,crit} &lt; \varepsilon_{st,adequate} )  ( \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{st,adequate} )</td>
<td>( \varepsilon_{st} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,adequate} )  AND  ( \varepsilon_{st,crit} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} &gt; \varepsilon_{st,adequate} )</td>
<td>( \varepsilon_{st} &gt; \varepsilon_{st,adequate} &gt; \varepsilon_{sy} &gt; \varepsilon_{st,crit} )</td>
</tr>
</tbody>
</table>

Conditions to avoid non-ductile strain-states

\[
\varepsilon_{st,crit} \geq \varepsilon_{sy}, \quad \varepsilon_{st,crit} > \varepsilon_{st} \geq \varepsilon_{sy}, \quad \varepsilon_{st} > \varepsilon_{st,crit} \geq \varepsilon_{sy}, \quad \varepsilon_{st} \geq \varepsilon_{sy} > \varepsilon_{st,crit}
\]

\[
\varepsilon_{d} \geq \frac{\varepsilon_{sy} - (K - 1)\varepsilon_{cu}}{K} - \varepsilon_{f0}
\]

\[
\varepsilon_{f1} \geq \frac{\varepsilon_{cu} + \varepsilon_{sy}}{K} - (\varepsilon_{cu} - \varepsilon_{f0})
\]

\[
\varepsilon_{d} \geq \frac{\varepsilon_{cu} - 1}{\varepsilon_{d} - 1} \varepsilon_{sy} - \varepsilon_{f0}
\]

Conditions to avoid non-ductile strain-states

\[
\frac{X_{u,crit}}{d} \leq \left[\frac{1}{1 + \left(\frac{\varepsilon_{sy}}{\varepsilon_{cu}}\right)}\right] \left[1 - \left(\frac{\varepsilon_{sy}}{(\varepsilon_{f0} + \varepsilon_{f1})}\right)\right] \left[1 - \left(\frac{\varepsilon_{sy}}{(\varepsilon_{f0} + \varepsilon_{d})}\right)\right]
\]

\[
\frac{X_{u}}{d} \leq \left[\frac{1}{1 + \left(\frac{\varepsilon_{sy}}{\varepsilon_{cu}}\right)}\right]
\]

\[
\frac{X_{u}}{d} \leq \left[\frac{1}{1 + \left(\frac{\varepsilon_{sy}}{\varepsilon_{cu}}\right)}\right]
\]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Critical Failure Mode</th>
<th>Concrete-controlled Failure Mode</th>
<th>FRP-controlled Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{f\text{-crit}} \leq \left[ \left( k_1 \frac{f_{ck}}{f_{mc}} \right) \left( 1 - \frac{1}{1 + \frac{\varepsilon_{st\text{-crit}} E_s \rho_{st}}{f_y / f_{mc} \rho_{st}}} \right) + \left( \varepsilon_{st\text{-crit}} E_s \rho_{st} - \frac{f_y}{f_{mc} \rho_{st}} \right) \right] (\varepsilon_{fd} E_{fd}) \right]$</td>
<td>$\rho_f \leq \left[ \left( k_1 \frac{f_{ck}}{f_{mc}} \right) \left( 1 - \frac{1}{1 + \frac{\varepsilon_y}{\varepsilon_{cu}}} \right) + \left( \varepsilon_{sc} E_s \rho_{sc} - \frac{f_y}{f_{mc} \rho_{st}} \right) \right] \left( \left( \frac{f_{cu} + \varepsilon_y}{K} \right) - (\varepsilon_{cu} - \varepsilon_{f0}) \right) E_{fd}$</td>
<td>$\rho_f \leq \left[ \left( k_1 \frac{f_{ck}}{f_{mc}} \right) \left( 1 - \frac{1}{1 + \frac{\varepsilon_y}{\varepsilon_{cu}}} \right) + \left( \varepsilon_{sc} E_s \rho_{sc} - \frac{f_y}{f_{mc} \rho_{st}} \right) \right] \left( \frac{K \varepsilon_{u} - \varepsilon_{f0} \left( \frac{K - 1}{K} \right) \varepsilon_{sy} - \varepsilon_{f0} \right) E_{fd}$</td>
</tr>
<tr>
<td>Conditions to avoid non-ductile strain-states (continued)</td>
<td>$\left[ k_1 \frac{f_{ck}}{f_{mc}} \left( 1 - \frac{1}{1 + \frac{\varepsilon_y}{\varepsilon_{cu}}} \right) \right] + (\varepsilon_{sc} E_s \rho_{sc}) - \left( \frac{f_y}{f_{mc} \rho_{st}} \right)$</td>
<td>OR</td>
<td>$\left[ k_1 \frac{f_{ck}}{f_{mc}} \left( 1 - \frac{1}{1 + \frac{\varepsilon_y}{\varepsilon_{cu}}} \right) \right] + (\varepsilon_{sc} E_s \rho_{sc}) - \left( \frac{f_y}{f_{mc} \rho_{st}} \right)$</td>
</tr>
<tr>
<td>Optimal condition</td>
<td>$\varepsilon_{st\text{-crit}} = \varepsilon_{st\text{-adequate}}$</td>
<td>$\varepsilon_{st} = \varepsilon_{st\text{-adequate}}$</td>
<td>$\varepsilon_{st} = \varepsilon_{st\text{-adequate}}$</td>
</tr>
<tr>
<td>Defining conditions</td>
<td>$\varepsilon_{fd} = \left( \frac{K}{K} \right) \varepsilon_{cu} - \varepsilon_{f0}$</td>
<td>$\varepsilon_{f1} = \left( \frac{\varepsilon_{cu} + \varepsilon_{st\text{-adequate}}}{K} \right) - (\varepsilon_{cu} - \varepsilon_{f0})$</td>
<td>$\varepsilon_{fd} = \left( \frac{1 - \frac{X_d}{d}}{1 - \frac{X_d}{d}} \right) \varepsilon_{st\text{-adequate}}$</td>
</tr>
</tbody>
</table>
Table 4.4 (Continued) Summary of design equations for ductility-based definitions of failure modes: Sub-clusters, avoiding non-ductile strain-states and optimal condition (continued)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Critical Failure Mode</th>
<th>Concrete-controlled Failure Mode</th>
<th>FRP-controlled Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{\text{FRP-crit}} =$</td>
<td>$\rho_{\text{FRP}} =$</td>
<td>$\rho_{\text{FRP}} =$</td>
</tr>
<tr>
<td></td>
<td>$\left( k_{1} \frac{f_{\text{ec}}}{f_{\text{mc}}} \right) \left( 1 + \frac{1}{\varepsilon_{\text{st-adequate}}/\varepsilon_{\text{cu}}} \right) + (\varepsilon_{\text{sc}} \cdot E_{s} \cdot \rho_{\text{sc}}) = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right)$</td>
<td>$\left( k_{1} \frac{f_{\text{ec}}}{f_{\text{mc}}} \left( \frac{1}{1 + \varepsilon_{\text{st-adequate}}/\varepsilon_{\text{cu}}} \right) \right) + (\varepsilon_{\text{sc}} \cdot E_{s} \cdot \rho_{\text{sc}}) = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right)$</td>
<td>$\left( k_{1} \frac{f_{\text{ec}}}{f_{\text{mc}}} \left( \frac{1}{1 + \varepsilon_{\text{st-adequate}}/\varepsilon_{\text{cu}}} \right) \right) + (\varepsilon_{\text{sc}} \cdot E_{s} \cdot \rho_{\text{sc}}) = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
<td>$\left( \varepsilon_{\text{cu}} \cdot E_{s} \cdot \rho_{\text{sc}} = \left( \frac{f_{y}}{f_{\text{mc}}} \cdot \rho_{\text{st}} \right) \right)$</td>
</tr>
</tbody>
</table>
Shear
5.1 Chapter objectives and structure

In line similar to that of Chapter 4, this chapter aims at providing a deeper interpretation of conservativeness associated with the design of FRP-based shear strengthening systems. In particular, it presents an assessment methodology that demonstrates:

- The internal architecture of the shear strengthening design process
- The sensitivity of important design variables to the safety parameters
- The effectiveness of safety parameters on producing conservativeness of the shear strengthening design solutions

An assessment methodology for the shear strengthening design process is proposed in this chapter. It presents the shear resistance contribution of FRP reinforcement in a normalised non-dimensional format, which is conducive for carrying out parametric and sensitivity analyses. For illustration purpose, this methodology is calibrated according to ACI440 and TR55 specifications. A detailed insight into the internal architecture of the shear strengthening design process, based on the proposed methodology, is then presented. The influence of orientation of different fibre orientation and wrapping configurations are discussed. A qualitative characterisation of shear strengthening design solutions based on the assessment methodology is presented. This characterisation suggests that a range of the possible shear strengthening design solutions under a given design scenario can belong to one of the four qualitatively distinct clusters, each cluster exhibiting unique qualitative characteristics. The implications of different design formats suggested by various strengthening design guidelines on the extents of these clusters are presented. Finally, the sensitivity of the conservativeness of the resultant shear strengthening design solutions are discussed under different design scenarios. Based on these analyses suitable observations are derived.
Focus of Chapter 5: Shear
5.2 Assessment methodology for shear strengthening design

5.2.1 Basic mechanics of an FRP shear reinforcement

Fig. 5.1 represents definitions of important design parameters for an arbitrary shear strengthening scheme. The conceptual mechanics of an arbitrary FRP shear reinforcement working towards arresting the shear crack in a beam is presented through Fig. 5.2.

![Figure 5.1 Schematic representation of an arbitrary shear strengthening scheme (Conceptual representation only – not to scale)](image1)

![Figure 5.2 Conceptual resistance mechanism of an FRP shear reinforcement](image2)

Attributed to its orthotropy, as discussed in Chapter 2, an FRP reinforcement can produce resisting tensile force along the principal direction of fibres only. This tensile force represents the nominal shear resistance capacity of an FRP reinforcement ($V_{FRP, nominal}$), which can be represented as a product of design effective failure strain ($\varepsilon_{fse}$), modulus of elasticity ($E_{fa}$) and effective cross-sectional area of FRP ($A_{FRP}$) [Eq. (5.1)]. Strain $\varepsilon_{fse}$ is explained later in this section. With $H$ as the axial rigidity of FRP
in tension, \( V_{\text{FRP, nominal}} \) can be represented in a convenient non-dimensional format as shown in Eq. (5.2).

\[
V_{\text{FRP, nominal}} = \varepsilon_{fse} E_{fd} A_{\text{FRP}}
\]  
(5.1)

\[
\left( \frac{V_{\text{FRP, nominal}}}{h} \right) = \varepsilon_{fse}
\]  
(5.2)

For an RC flexural member strengthened in shear using FRP, an ideal failure expectation would involve a rupture-based mature failure of the FRP, in which the FRP material is strained up to its full rupture strain capacity. However, a part of the fibres in an FRP reinforcement engaged in arresting the shear crack can possibly be under-strained compared to the rest of the fibres that have already reached to their rupture strain capacity. Consequently, the strain in FRP reinforcement is likely to be non-uniform [Khalifa et al. (1998)], and thus, even a failure mechanism involving rupture of FRP does not utilise the full rupture strain capacity of the entire fibres that are engaged in arresting a shear crack. This characteristic is incorporated within design through a strain limit defined for FRP in form of an average rupture strain in FRP shear reinforcement (\( \varepsilon_{fd-\text{fracture}} \)), which is considerably lesser than the design rupture strain capacity of FRP. This average rupture strain depends upon the behaviour of the member, which can be characterised by either predominantly elastic deformations [Taljsten (2002)] or predominantly rigid body movements of the regions [Chen and Teng (2003)] of the member on either sides of a shear crack.

In order to incorporate the possibilities of debonding of FRP and physical disintegration of concrete (and thus to diminish its shear carrying capacity), two additional strain limits are imposed on FRP reinforcement. The limiting strain \( \varepsilon_{fd-\text{debond}} \) represents the strain in FRP at which debonding occurs, while the limiting strain \( \varepsilon_{fd-\text{disintegration}} \) represents the strain in FRP at which concrete starts to physically disintegrate. Thus, the three strain limits for FRP, \( \varepsilon_{fd-\text{fracture}} \), \( \varepsilon_{fd-\text{debond}} \) and \( \varepsilon_{fd-\text{disintegration}} \), numerically compete to govern the design effective failure strain of FRP (\( \varepsilon_{fse} \)), within a fracture-debonding-disintegration switch (FDDS), as shown in Chapter 3.

A little consideration will show that the mechanics of an FRP shear reinforcement is influenced by the orientation of the FRP reinforcement relative to the direction of shear crack and type of shear strengthening configuration. Accordingly, most strengthening design guidelines suggest the nominal shear resistance contribution of FRP to be corrected for:
• Orientation of FRP reinforcement
• Wrapping configuration

(A) **Modification for FRP orientation**

The resisting tensile force produced in an FRP reinforcement can be resolved, using the principles of statics, into two components with reference to an arbitrary direction of a shear crack. One of these components is normal to the shear crack plane, and the other is tangential to the shear crack plane. Angle $\alpha$ or angle $\beta$ with reference to the longitudinal axis of a beam can be used for this force resolution. Of these two components, the one normal to the shear crack plane that is effective in resisting the tension induced in concrete due to the opening of a shear crack. The value of this component depends upon the orientation of the FRP reinforcement relative to the anticipated shear crack direction, which determines the effectiveness of FRP reinforcement in arresting a shear crack. This characteristic is represented through a *fibre orientation factor* ($\eta_1$), which is a trigonometric function of angle $\alpha$ (or angle $\beta$), as shown in Eq. (5.3) and in Fig. 5.1.

\[
\eta_1 = (\sin \alpha + \cos \alpha) = (\cos \beta + \sin \beta) \tag{5.3}
\]

It can be seen that the geometric relation between angles $\alpha$ and $\beta$ ($\beta = 90^\circ - \alpha$) leads to a trigonometric similarity, and hence the use of either of these angles leads to the same value of factor $\eta_1$.

(B) **Modification for wrapping configuration**

The effectiveness of wrapping configuration is accounted for by considering the effective bonded length of FRP available after allowing for a reduction based on the bond- or contact-criticality of the wrapping configuration [Khalifa *et al.* (1998)]. Per unit longitudinal spacing of FRP shear reinforcements ($S_f$), this characteristic is represented through a *wrapping effectiveness factor* ($\eta_2$), as shown in Eq. (5.4).

\[
\eta_2 = \frac{(d_f - \kappa_{eq} L_c)}{S_f} \tag{5.4}
\]

Here, $\kappa_{eq}$ is the bond-reduction coefficient, which depends upon the bond- or contact-criticality of the wrapping configuration. A little adjustment enables presenting $\eta_2$ in a form that comprises of two non-dimensional parameters – the depth of FRP shear reinforcement ($d_f$) and the bond length ($L_c$), both normalised with the centre-to-centre longitudinal spacing of FRP reinforcement ($S_f$) – as shown through Eq. (5.5).
\[ \eta_2 = \lambda \left[ \frac{d_f}{s_f} - \kappa_{PS} \frac{l_2}{s_f} \right] \]  

(5.5)

Here, parameter \( \lambda \) is an equaliser, which is introduced to equalise the difference in the formats for defining the bond characteristics (i.e. bond length and bond reduction coefficient) of FRP prescribed by one strengthening design guidelines compared to the other. For a strengthening scheme involving continuous FRP sheet instead of the discrete strips, the equivalent width of FRP stirrup \( (b_f) \) will be either \( \sin \alpha \) or \( \cos \beta \), both of which are trigonometrically identical for angle \( \beta \) being equal to \( (90^\circ - \alpha) \).

**C) Generic model for shear resistance contribution of FRP reinforcement**

Allowing for the above two corrections, the effective nominal shear resistance contribution of FRP reinforcement can be presented through Eq. (5.6).

\[
\left( \frac{V_{FRP, nominal}}{H} \right) = \eta_1 \eta_2 \epsilon_{fse} 
\]  

(5.6)

In order to keep the left-hand side of the above equation free from the material safety parameters, Eq. (5.6) can be represented in the following form:

\[
\left( \frac{V_{FRP, nominal}}{H_0} \right) = \frac{1}{[S_E]} \eta_1 \eta_2 \epsilon_{fse} 
\]  

(5.7)

where,

\[
H = \frac{H_0}{[S_E]} 
\]  

(5.8)

\[
H_0 = A_{FRP} E_f 
\]  

(5.9)

\[
A_{FRP} = 2 t_f b_f 
\]  

(5.10)

It is to be noted that the nominal shear resistance contribution of FRP reinforcement presented through Eq. (5.7) is for a pair of FRP reinforcements available on both of the vertical sides of an RC beam being strengthened. Eq. (5.10) assumes both vertical sides of an RC beam to receive identical FRP reinforcement.

The design value of shear resistance contribution of FRP reinforcement \( (V_{FRP, design}) \) can be obtained by including the applicable resistance safety parameters, as shown through Eq. (5.11).

\[
\left( \frac{V_{FRP, design}}{H_0} \right) = \frac{1}{[S_E][J]_{FRP}} \eta_1 \eta_2 \epsilon_{fse} 
\]  

(5.11)
Various parameters comprising $V_{\text{FRP,nominal}}$ in the above generic format can be calibrated based on the design criteria prescribed by various strengthening design guidelines. As an illustration, this generic model is calibrated against the TR55 and ACI440 design specifications. It is also to be noted that while the generic model is mathematically homogeneous (i.e. the left- and right-hand sides are dimensionally correct), upon calibration this mathematical homogeneity may not be retained due to the use of empirical definitions for parameters comprising the right-hand side of Eq. (5.11).

### 5.2.2 Calibration

The fibre orientation factor ($\eta_1$), as presented through Eq. (5.3), is solely geometry dependent, and requires no calibration. Design variables comprising the wrapping effectiveness factor ($\eta_2$) are presented through Eqs. (5.12)-(5.25). For a strengthening scheme involving a continuous FRP sheet instead of discrete strips, the value of $S_f$ is unity.

$$L_e(\text{Type I}) = 0.7 \sqrt{\dfrac{R}{0.18 f_{cu}^{2/5}}} \quad \text{[For TR55]}$$

$$L_e(\text{Type II}) = \dfrac{23.300}{R^{0.56}} \quad \text{[For ACI440]}$$

$$R = n t_f E_{fd} = \dfrac{\sigma}{[S_e]} \quad \text{(5.14)}$$

$$\bar{R} = n t_f \bar{E}_f$$

$$\lambda = \begin{cases} \dfrac{L_e}{11900 E_{fd-rupture}} & \text{[Sides-only and U-wrapped]} \\ 1.00 & \text{[Fully wrapped]} \end{cases}$$

$$\delta_{se(ACI440)} = 0.75$$

$$\kappa_{\nu s} = \begin{cases} 2/3 & \text{[sides-only]} \\ 1.00 & \text{[U-wrapped]} \\ 0.00 & \text{[Fully wrapped]} \end{cases}$$

The effective strain in an FRP stirrup ($\varepsilon_{fse}$) for ACI440 is expressed through Eq. (5.19), while that for TR55 is expressed through Eq. (5.20).

$$\varepsilon_{fse(ACI440)} = \min \left[ \varepsilon_{f,\text{rupture(ACI440)}}, \varepsilon_{f,\text{disintegration(ACI440)}} \right]$$

(5.19)
\[ \varepsilon_{fs}(TR55) = \min \left[ \varepsilon_{fd\text{-fracture}(TR55)}, \varepsilon_{fd\text{-debonding}(TR55)}, \varepsilon_{fd\text{-disintegration}(TR55)} \right] \] (5.20)

where,

\[ \varepsilon_{fd\text{-disintegration}(TR55)} = 0.004 \] (5.21)

\[ \varepsilon_{fd\text{-disintegration}(ACI440)} = 0.004 \] (5.22)

\[ \varepsilon_{fd\text{-fracture}(TR55)} = \delta_{se}(TR55) \varepsilon_{fd\text{-rupture}} \] (5.23)

\[ \delta_{se}(TR55) = 0.5 \] (5.24)

\[ \varepsilon_{fd\text{-debonding}(TR55)} = 0.64 \sqrt{\frac{0.18 (f_{cu})^{2/3}}{R}} \] (5.25)

As discussed in Chapter 3, ACI440 does not explicitly consider the failure possibility through debonding of FRP reinforcement. However, it does specify that the maximum effective failure strain an FRP reinforcement can attain for any wrapping configuration is limited to \((\delta_{se} \varepsilon_{fd\text{-rupture}})\). An upper bound on factor \(\eta_2\), as shown through Eq. (5.26), represents this restriction.

\[ \eta_2\text{-upper-bound} \leq \frac{\delta_{se(ACI440)}}{S_f} \] (5.26)

\[ \delta_{se(ACI440)} = 0.75 \] (5.27)

The restrictions imposed in terms of maximum permissible longitudinal spacing of FRP reinforcement are expressed through a lower bound presented in Eq. (5.28). Here, \(b\) and \(d\) are the width and effective depth of the RC beam section respectively.

\[ \left( \frac{d_f}{S_f} \right) \geq \max \left\{ x_1, x_2 \right\} \] (5.28)

where,

\[ x_1 = \begin{cases} 1.00 & \text{[For ACI440]} \\ 1.25 & \text{[For TR55]} \end{cases} \] (5.29)

\[ x_2 = \left[ \frac{4 t_1}{1 + 4 t_1 t_2^2} \right] \] (5.30)

\[ t_1 = \frac{d_f}{d} \] (5.31)
Using Eq. (5.28) it is possible to work out a lower bound limit for factor $\eta_2$, expressed through Eq. (5.33), corresponding to the spacing limitations prescribed by ACI440 and TR55. Such a lower bound implies that the design solutions carrying the numerical value of the factor $\eta_2$ less than $\eta_2$-spacing are violating the maximum permitted longitudinal spacing of FRP stirrups criteria and hence are not valid design solutions.

$$\eta_2(i) \leq \eta_2\text{-spacing}$$

(5.33)

where,

$$\eta_2\text{-spacing} = \begin{cases} 
\lambda \left[ \left( \max \left\{ \frac{x_1}{x_2} \right\} - \frac{k_{\text{vs}}}{x_2} \right) \right] & \text{[For ACI440 and TR55]} \\
1.00 & \text{[For TR55]} 
\end{cases}$$

(5.34)

### 5.2.3 Significance of shear strengthening assessment methodology

The following points summarise the significance of the shear strengthening assessment methodology:

- **Non-dimensional format:** It can be appreciated that the generic model representing the shear resistance contribution of FRP reinforcement is non-dimensional. Thus, the conflicts arising from different unit systems followed by different strengthening design guidelines are avoided. Of course, many parameters comprising the generic model are defined empirically, which demands consistency in terms of the unit system to be followed while calibrating the generic model against a particular strengthening design guideline.

- **Normalised format:** It can be seen that the left-hand side of generic model in Eqs. (5.7) and (5.11) depends on geometric dimensions of FRP reinforcement and mean value of the modulus of elasticity of FRP - both of these are independent of any design guideline specifications. The right-hand side terms in these equations include the influential design parameters, such as effectiveness of the FRP orientation and wrapping configuration and effective failure strain in FRP. This allows seeing the influential parameters more discretely, and studying their sensitivity independently.

- **Algorithmic format:** Methodology allows seeing the course of shear strengthening design process algorithmically. This facilitates demonstrating the internal
architecture of the shear strengthening design process indicating how the conservativeness propagates within the strengthening design process.

- **Conducive for parameterisation**: Non-dimensional geometric parameters \( \left( \frac{d}{s_f} \right) \) and \( \left( \frac{L}{s_f} \right) \) comprising the wrapping effectiveness parameter \( \eta_2 \) can be used as independent variables. For a given wrapping configuration and other factors, a pair of values of these two independent variables represents a unique shear strengthening design solution. Thus, a range of these two independent variables presents a universe of possible shear strengthening design solutions. These strengthening design solutions, for a given design scenario, can be classified into different qualitative clusters. These clusters of strengthening design solutions can be studied under different design scenarios.

### 5.3 Architecture of the shear strengthening design process

The generic model presented within this chapter enables seeing the shear strengthening design process as a systematic algorithm, which depicts the architecture of the shear strengthening design process clearly. This makes possible to demonstrate how the conservativeness, which is imparted through various safety parameters that are integrated at various levels, propagates within the design process. For illustration, Fig. 5.3 and 5.4 present the course of shear strengthening design process based on the generic model calibrated against TR55 and ACI440 specifications respectively. These figures represent the shear strengthening design process as a conceptual network, which comprises of *conservativeness propagation paths* (CPPs). Each of these CPPs leads to a unique shear strengthening design solution involving one of the possible failure modes of FRP reinforcement. The design shear resistance contribution of FRP reinforcement can be represented as shown through Eq. (5.35).

\[
\frac{V_{\text{FRP,design}}}{h_0} = \frac{ARF}{[S_e]^m [S_f]^n [J]_{\text{FRP}}} \tag{5.35}
\]

Here, the numerator \( (ARF) \) is the absolute resistance function, which represents the shear resistance contribution of FRP reinforcement for a particular failure mode without considering any material or resistance safety parameters. The denominator represents the total conservativeness associate with the shear resistance contribution of FRP reinforcement. The total conservativeness comprises of applicable fractions of the material and resistance conservativeness represented through the powers \( m, n \) and \( q \) of \([S_e], [S_f]\) and \([J]_{\text{FRP}}\) respectively. Table 5.1 presents various \( ARF \) along with the
potential and the actually prescribed total conservativeness by TR55 and ACI440. These powers should be read in conjunction with Eq. (5.11) and the applicable definition of parameter $\lambda$ presented through Eq. (5.16). Following observations emerge from Figs. 5.3 and 5.4 and Table 5.1:

- It can be seen that the fracture-debonding-disintegration switch (FDDS) sets the primary diversions within the strengthening design process. The design specifications of TR55, for example, shows three possible design paths designated in Fig. 5.3 as CPP1, CPP2 and CPP3. Compared to this, the ACI440 specifications lead to two major design paths (designated in Fig. 5.4 as CPP1 and CPP2), and each of these two paths have two sub-routes (designated as CPP1-1, CPP1-2 and CPP2-1 and CPP2-2).

- For a given design scenario, a shear strengthening design solution can be reached through one of the possible paths. Each of the so reached design solutions will have a unique failure mode involved, which has a unique combination of the material safety parameters, and hence a unique effective material conservativeness, associated with it. The absolute resistance function and associated total conservativeness for each of the CPP’s are presented in Table 5.1.

- It can be noted that for TR55 the absence of resistance safety parameters is reflected in actually prescribed power $q$ of $[J]_{FRP}$ as zero against its potential value of unity. This observation signifies a loss of opportunity in producing conservativeness in shear resistance contribution of FRP reinforcement. This is of particular importance for the CPP’s involving minimal or no contributions (e.g., CPP2 and CPP3 for TR55) from the material safety parameters in producing conservativeness.

- Similarly, for ACI440 the absence of material safety parameters prescribed on modulus of elasticity of FRP (e.g., for CPP2-1 and CPP2-2) is reflected in actually prescribed power $n$ of $[S_E]$ as zero against its potential non-zero value.

- Interestingly, it can be seen that CPP1-2 for ACI440 is carrying a negative value for power $m$ of $[S_g]$. This signifies that for CPP1-2, the prescription of material safety parameters on rupture strain capacity of FRP, in fact, is negatively affecting the conservativeness associated with the shear resistance contribution of FRP reinforcement.

- The CPP’s involving the least degree of total conservativeness are highlighted with red colour.
It can be clearly appreciated that the qualitative and quantitative differences in prescribing the safety parameters and switchers can influence the course of shear strengthening design process, and ultimately the quality of shear strengthening design solutions. Investigations on these aspects are discussed in the subsequent sections in this chapter.
Figure 5.3
Architecture of shear strengthening design process (based on TR55 specifications)
Figure 5.4
Architecture of shear strengthening design process (based on ACI440 specifications)
<table>
<thead>
<tr>
<th>Conservativeness Propagation Path (CPP)</th>
<th>Absolute Resistance Function (ARF)</th>
<th>Total Conservativeness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Potential</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prescribed</td>
</tr>
<tr>
<td>CPP1</td>
<td>([\sin \alpha + \cos \alpha] \left( \frac{d_f}{S_f} - \kappa_{vs} \frac{L_e}{S_f} \right) [\delta_{se(\text{TR55})} \varepsilon_f] )</td>
<td>([S_\varepsilon]^1 [S_E]^1 [U]_{\text{FRP}}^1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^0 [S_E]^0 [U]_{\text{FRP}}^0 )</td>
</tr>
<tr>
<td>CPP2</td>
<td>([\sin \alpha + \cos \alpha] \left( \frac{d_f}{S_f} - \kappa_{vs} \frac{L_e}{S_f} \right) [0.64 \frac{f_{ctm}}{R}] )</td>
<td>([S_\varepsilon]^0 [S_E]^{0.5} [U]_{\text{FRP}}^{0.5} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^0 [S_E]^0 [U]_{\text{FRP}}^0 )</td>
</tr>
<tr>
<td>CPP3</td>
<td>([\sin \alpha + \cos \alpha] \left( \frac{d_f}{S_f} - \kappa_{vs} \frac{L_e}{S_f} \right) ) [0.004]</td>
<td>([S_\varepsilon]^0 [S_E]^1 [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^0 [S_E]^1 [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td>CPP1-1</td>
<td>(17.62 \frac{1}{R_0 (f_c)^{1/2}} \sin \alpha + \cos \alpha \left( \frac{d_f}{S_f} - \kappa_{vs} \frac{L_e}{S_f} \right) ) [\delta_f ] [0.004]</td>
<td>([S_\varepsilon]^{-1} [S_E]^0 [U]_{\text{FRP}}^{-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^{-1} [S_E]^0 [U]_{\text{FRP}}^{-1} )</td>
</tr>
<tr>
<td>CPP2-2</td>
<td>(0.07 \frac{1}{\varepsilon_f R_0 \frac{f_c}{(f_c)^{2/3}}} \sin \alpha + \cos \alpha \left( \frac{d_f}{S_f} - \kappa_{vs} \frac{L_e}{S_f} \right) ) [\delta_f ] [0.004]</td>
<td>([S_\varepsilon]^{1} [S_E]^{1} [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^{1} [S_E]^{1} [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td>CPP2-1</td>
<td>([\sin \alpha + \cos \alpha] \left( \frac{d_f}{S_f} \right) ) [\delta_{se(\text{ACI440})} \varepsilon_f] [0.004]</td>
<td>([S_\varepsilon]^{1} [S_E]^{0} [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^{0} [S_E]^{0} [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td>CPP2-2</td>
<td>([\sin \alpha + \cos \alpha] \left( \frac{d_f}{S_f} \right) ) [\delta_{se(\text{ACI440})} \varepsilon_f] [0.004]</td>
<td>([S_\varepsilon]^{1} [S_E]^{0} [U]_{\text{FRP}}^{1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([S_\varepsilon]^{0} [S_E]^{0} [U]_{\text{FRP}}^{1} )</td>
</tr>
</tbody>
</table>

The CPP’s involving the least degree of total conservativeness are highlighted in RED.
5.4 **Assessment approach**

A little consideration will show that the parameters $\eta_1$, $\eta_2$ and $\epsilon_{fse}$ comprising the generic model influence the course of the shear strengthening design process and the quality of the resultant shear strengthening design solutions. Hence, the parametric and sensitivity analyses presented within this chapter aims to study the influence of these parameters on strengthening design process.

5.5 **Influence of fibre orientation**

The effectiveness of orientation of the fibres in arresting a shear crack can be captured through the fibre orientation factor ($\eta_1$). As seen from Eq. (5.3), the factor $\eta_1$ is solely a function of principal direction of the fibres (either $\alpha$ or $\beta$). Fig. 5.5 presents the variation in factor $\eta_1$ with change in angle $\beta$ (i.e. the orientation of the fibres with respect to an axis normal to the longitudinal axis of the member).

![Figure 5.5](image_url)

Following are the salient observations based on this plot:

- It can be seen that, for a shear crack assumed to orient at 45° with respect to the longitudinal axis of the beam, angle $\beta$ of 45° is the most effective orientation for an FRP reinforcement in arresting a shear crack. At this angle, the factor $\eta_1$ attains its maximum possible numerical value, which is about 1.41.
- It is to be noted, however, that the direction of a shear crack can vary from 22° to 45° under gravity loading conditions. Thus, it might not always be feasible to have the FRP reinforcement oriented at the efficient angle.
• Also, under seismic conditions, there exists a possibility for a shear crack to reverse its direction. In this case the direction of the shear crack will coincide with the principal direction of fibres, which in turn will reduce the effectiveness of the FRP reinforcement in arresting the shear crack to near zero.

• For practical ease, FRP reinforcements are often oriented normal to the longitudinal axis of the beam, i.e. with angle $\beta$ as 0°. The corresponding numerical value of the factor $\eta_1$ at this orientation is unity. Any orientation of FRP reinforcement with angle $\beta$ less than 0° will lead to a numerically less than unity value of factor $\eta_1$.

• It can also be seen that an orientation of FRP reinforcement at an angle $\beta$ less than $-45^\circ$ results in negative value of factor $\eta_1$. Since the factor $\eta_1$ is a multiplier to the rest of the terms in deriving the shear resistance contribution of FRP, its value less than unity or zero will reduce the shear resistance contribution of FRP.

Thus, the detailing requirement concerning the orientation of FRP reinforcement with respect to the anticipated direction of the shear crack directly affects the shear resistance contribution of FRP reinforcement quantitatively.

5.6 Influence of wrapping configuration

Unlike factor $\eta_1$, factor $\eta_2$ is a function of many parameters. The influence of the wrapping configuration is measured in terms of the sensitivity of parameter $\eta_2$ through the following sensitivity and parametric analyses:

• Sensitivity of bond length ($L_e$) estimation models
• Influence of bond reduction coefficient ($\kappa_{re}$)
• Variation of wrapping effectiveness parameter ($\eta_2$)

5.6.1 Sensitivity of bond length estimation models

Two popular formats for describing anchorage bond length ($L_e$) of FRP shear reinforcement are discussed in Chapter 2. One of these two formats, referred to as Type I bond length model in this study (e.g., the one specified by TR55 for shear), involves a direct proportionality of the bond length $L_e$ with the modulus of elasticity of FRP (through parameter $\bar{R} = n t f \bar{E}_f$). Against this, the Type II bond length model (e.g., the one specified by ACI440 for shear) involves an inverse proportionality of $L_e$ with modulus of elasticity of FRP. Type II bond length model specified by ACI440 is based on the active bond length concept. Active bond length is the length of FRP
reinforcement in contact with concrete substrate over which the majority of the bond stress is maintained [Trantafillou (1998a) and Khalifa et al (1998)]. Fig. 5.6 presents sensitivity of Type I and Type II bond length models to parameter $R$ and material safety parameter $S_E$. In order to cover the extremely low and high values of modulus of elasticity and effective thickness of FRP (based on the practical values suggested by TR55) logarithmic scale is used to represent $R$. Following are the observations derived based on this plot:

- The inverse proportionality to modulus of elasticity of FRP exhibited by Type I and Type II bond length models can clearly be observed from Fig. 5.6. An increase in the values of estimated bond length with an increase $R$ is observed for Type I bond length model. Against this, Type II bond length model shows a decrease in the estimated bond length values with an increase in $R$.

- Also, an increase in the stringency of safety parameter $S_E$ results in a reduction in the estimated bond length values for Type I bond length model. Unlike this, an increase in the estimated bond length is observed with an increase in the stringency on $S_E$ for Type II bond length model.

- Thus, prescription of material safety parameters on modulus of elasticity is an unconservative strategy for Type II bond length model. The ACI440 strategy of not prescribing material safety parameters on modulus of elasticity, for this reason, is advantageous.

Figure 5.6
Sensitivity of Type I and Type II bond length models in shear strengthening
Eq. (5.16) suggests that the equalising parameter \( \lambda \) for sides-only and U-wrapped configurations for ACI440 is a function of \( L_e \). Thus, format of \( L_e \) does influence the value of parameter \( \lambda \) within the shear strengthening assessment methodology. For TR55 parameter \( \lambda \) has a unit value for all shear strengthening configurations. For sides-only and U-wrapped configurations for ACI440, \( \lambda \) is a function of \( \varepsilon_{fd,\text{rupture}} \), and active bond length \( (L_e) \), which in turn is a function of parameter \( R \). For the fully-wrapped configuration according to ACI440, parameter \( \lambda \) is equal to \( \delta_{\varepsilon f(ACI440)} = 0.75 \). Figs. 5.7 and 5.8 present the sensitivity of parameter \( \lambda \) to parameter \( R \) and \( \varepsilon_{fd,\text{rupture}} \) respectively, for the given other conditions.
Following implications emerge based on these plots:

- It can be seen that parameter $\lambda$ for sides-only and U-wrapped configurations according to ACI440 decreases exponentially with an increase in $\bar{R}$ and $\varepsilon_f$.
- Like Type II model for bond length estimation, parameter $\lambda$ for sides-only and U-wrapped configuration according to ACI440 also exhibits an increase in the value of parameter $\lambda$ with an increase in the values of material safety parameters $[S_E]$ and $[S_\varepsilon]$.
- It also can be seen that the value of parameter $\lambda$ remains numerically lesser than unity for the most part of the range of $\bar{R}$.

Since parameter $\lambda$ is a multiplier to the factor $\eta_z$, its numerical value being greater or smaller than unity is of concern for many of the CPP’s shown in Fig. 5.4. However, it is essential to reiterate that seeing any single parameter in isolation within this assessment methodology would not give a complete picture.

5.6.2 Sensitivity of bond reduction coefficient

The bond reduction coefficient ($\kappa_{\nu\phi}$) within the assessment methodology represents a penalty imposed on bond length of FRP reinforcement, which accounts for the possible reduction in bond capacity of the FRP reinforcement. As stated in Chapter 2, the bond- and contact-criticality of the shear strengthening configuration provides the basis for prescribing this penalty. Accordingly, it is obvious for the bond-critical configurations (such as sides-only and U-wrapped configurations) to attract a higher penalty compared to the contact-critical configuration (such as fully wrapped configuration) on this account. In theory, the contact-critical configurations do not depend upon the bond behaviour, and hence does not require any penalty to be imposed on the bond carrying capacity of the FRP reinforcement. Eq. (5.18) representing the coefficients $\kappa_{\nu\phi}$ according to TR55 and ACI440 specifications for fully wrapped configuration testifies this fact.

In contrast with this, the bond-critical configurations exhibit different susceptibility of reduction in bond carrying capacity of FRP shear reinforcement. Since the sides-only and U-wrapped configurations involve one and two end regions per side of FRP reinforcement respectively (i.e. a sides-only configuration involves twice as many end regions compared to an U-wrapped configuration), the former involves double the chances of reduction in bond carrying capacity of FRP than the latter. In view of this, the sides-only configuration is penalised twice as much compared to the U-wrapped
configuration. This is evident from Eq. (5.18) representing $\kappa_{ps}$ derived based on TR55 and ACI440 specifications, for example.

It can be appreciated that the format of the assessment methodology enables comparing the effectively applicable reductions in bond carrying capacity as specified by different strengthening design guidelines more conveniently. This, otherwise, is not possible due to the different formats employed by these guidelines. It can be seen that the coefficients $\kappa_{ps}$ for sides-only and U-wrapped configurations are more stringently prescribed in ACI440 compared to TR55. The higher relative stringency, here, denotes higher reduction in bond carrying capacity of FRP reinforcement. A more detailed discussion on the implication of this fact is presented later in this chapter.

Again, seeing the above facts in isolation for the coefficients $\kappa_{ps}$ solely by no means provides a full justification and a complete picture. However, such a study provides a clearer insight into the internal mechanics of the design frameworks suggested by ACI440 and TR55 for estimating shear resistance contribution of FRP. This can also be useful in calibration or fine-tuning of these design guidelines.

### 5.6.3 Sensitivity of wrapping effectiveness factor

Figs. 5.9 and 5.10 presents typical variation in factor $\eta_2$ under different conditions. Figs. 5.9 is developed for sides-only configuration for TR55 and ACI440 respectively, under identical conditions. It is to be noted that, unlike for TR55, factor $\eta_2$ for ACI440 is sensitive to safety parameter $[S_e]$ due to the dependency of parameter $\lambda$ on rupture strain capacity of FRP. In order to demonstrate this fact, Figs. 5.9 (b) and (c) are developed for $[S_e]$ as unity and 3 respectively. Following salient observations emerge from these plots:

- It can be seen that effectiveness of wrapping configuration, represented through factor $\eta_2$, decreases with an increase in the bond length for sides-only.
- The plots for U-wrapped configuration are not included here. However, the U-wrapped configuration shows a trend similar to the sides-only configuration, but with a less steep $\eta_2$ curves compared to the sides-only configuration.
- Unlike the above, factor $\eta_2$ remains constant for the fully wrapped configurations. This observation will hold good for all strengthening design guidelines involving coefficients $\kappa_{ps}$ as zero.
- In the light of above, the differences arising from different formats for estimating the bond length of FRP shear reinforcement (e.g. Type I and Type II model) does
not affect the shear strengthening design solutions involving fully wrapped configuration.

- It can be seen that the certain combinations of the parameters \( \left( \frac{d_f}{S_f} \right) \) and \( \left( \frac{\varepsilon_x}{S_f} \right) \) for the sides-only (and also for the U-wrapped) configuration involve a possibility for the factor \( \eta_2 \) to be less than unity. This substantially reduces the shear resistance contribution of FRP reinforcement.

- In fact, for the sides-only and U-wrapped configurations, certain combinations of the parameters \( \left( \frac{d_f}{S_f} \right) \) and \( \left( \frac{\varepsilon_x}{S_f} \right) \) lead to zero or negative values of factor \( \eta_2 \), and that of shear resistance contribution of FRP, in turn. The criteria leading to this condition are explained later in this section.

- For the fully wrapped configuration the possibility for the factor \( \eta_2 \) to be less than zero does not exist.

- For ACI440, it can be seen that an increased value of \([S_x]\) leads to an increase in the values of factor \( \eta_2 \). This is mainly due to the parameter \( \lambda \) being inversely sensitive to \( \varepsilon_{f_d-rupture} \), as shown in Eq. (5.16).

- It can be seen from Fig. 5.10 that the factor \( \eta_2 \) remains constant for the entire range of independent variables for either of the two design guidelines. Figs. 5.10 (b) and (c) are developed for ACI440 with the upper bound limit suggested in Eq. (5.27) omitted and included respectively. It can be said that under identical conditions, even on omitting the upper bound limit on factor \( \eta_2 \), the values of factor \( \eta_2 \) are smaller for ACI440 than TR55. Thus, the nominal value of the shear resistance contribution produced according to ACI440 specifications will quantitatively be lesser than that produced according to TR55 specifications. This indicates that, in general, the basic format of shear resistance contribution of FRP according to ACI440 is more stringent compared to that of TR55.
Assessment of conservativeness in design of FRP-based structural strengthening systems
Kunal D. Kansara (2014)

Figure 5.9
Sensitivity of wrapping effectiveness factor $\eta_2$ for sides-only configuration
(a) For TR55 (b) For ACI440 with $[S_e] = 1.00$ (c) For ACI440 with $[S_e] = 3.00$
Figure 5.10
Sensitivity of wrapping effectiveness factor $\eta_2$ for fully wrapped configuration
(a) For TR55 (b) For ACI440 with upper bound limit omitted (c) For ACI440 with upper bound limit included
Based on Eq. (5.5) two limiting values for non-dimensional parameter \((\frac{L_e}{S_f})\) and coefficient \(K_{ps}\) can be derived, as shown in Eqs. (5.36) and (5.38) respectively. These specific values present a set of possibilities for factor \(\eta_2\), and in turn for \(V_{FRP}\), as shown in Eqs. (5.37) and (5.39). These possibilities apply to all types of shear strengthening design solutions for ACI440 and TR55, except those reached through CPP2-1 and CPP2-2 for ACI440.

\[
(\frac{L_e}{S_f})_{crit} = \left[ \frac{1}{K_{ps}} \frac{d_f}{S_f} \right] \quad (5.36)
\]

Such that:

\[
\begin{align*}
\text{If} & \quad \left(\frac{L_e}{S_f}\right)_{actual} < \left(\frac{L_e}{S_f}\right)_{crit} \Rightarrow \eta_2 > 0 \Rightarrow \frac{V_{FRP}}{K_0} > 0 \\
\text{If} & \quad \left(\frac{L_e}{S_f}\right)_{actual} = \left(\frac{L_e}{S_f}\right)_{crit} \Rightarrow \eta_2 = 0 \Rightarrow \frac{V_{FRP}}{K_0} = 0 \\
\text{If} & \quad \left(\frac{L_e}{S_f}\right)_{actual} > \left(\frac{L_e}{S_f}\right)_{crit} \Rightarrow \eta_2 < 0 \Rightarrow \frac{V_{FRP}}{K_0} < 0
\end{align*}
\]

\[
K_{ps-crit} = \left(\frac{d_f}{S_f}\right)_{ACI440} \quad (5.38)
\]

Such that:

\[
\begin{align*}
\text{If} & \quad K_{ps} < K_{ps-crit} \Rightarrow \eta_2 > 0 \Rightarrow \frac{V_{FRP}}{K_0} > 0 \\
\text{If} & \quad K_{ps} = K_{ps-crit} \Rightarrow \eta_2 = 0 \Rightarrow \frac{V_{FRP}}{K_0} = 0 \\
\text{If} & \quad K_{ps} > K_{ps-crit} \Rightarrow \eta_2 < 0 \Rightarrow \frac{V_{FRP}}{K_0} < 0
\end{align*}
\]

For the bond length models prescribed by ACI440 and TR55, Eqs. (5.36) and (5.38) can further be elaborated as follows:

For ACI440:

\[
\begin{align*}
\text{If} & \quad \bar{R} > \bar{R}_{crit} \Rightarrow \left(\frac{L_e}{S_f}\right)_{actual} < \left(\frac{L_e}{S_f}\right)_{crit} \Rightarrow \eta_2 > 0 \Rightarrow \frac{V_{FRP}}{K_0} > 0 \\
\text{If} & \quad \bar{R} = \bar{R}_{crit} \Rightarrow \left(\frac{L_e}{S_f}\right)_{actual} = \left(\frac{L_e}{S_f}\right)_{crit} \Rightarrow \eta_2 = 0 \Rightarrow \frac{V_{FRP}}{K_0} = 0 \\
\text{If} & \quad \bar{R} < \bar{R}_{crit} \Rightarrow \left(\frac{L_e}{S_f}\right)_{actual} > \left(\frac{L_e}{S_f}\right)_{crit} \Rightarrow \eta_2 < 0 \Rightarrow \frac{V_{FRP}}{K_0} < 0
\end{align*}
\]

where,

\[
\bar{R}_{crit(ACI440)} = \left[ 23,300 \frac{K_{ps(ACI440)}}{d_f} \right]^{1/0.58} [S_E]_{(ACI440)} \quad (5.41)
\]
For TR55:

\[
\begin{align*}
R > R_{\text{crit}} & \Rightarrow \left( \frac{\varepsilon}{S_f} \right)_{\text{actual}} > \left( \frac{\varepsilon}{S_f} \right)_{\text{crit}} \Rightarrow \eta_2 < 0 \Rightarrow \frac{V_{\text{FRP}}}{K_0} < 0 \\
R = R_{\text{crit}} & \Rightarrow \left( \frac{\varepsilon}{S_f} \right)_{\text{actual}} = \left( \frac{\varepsilon}{S_f} \right)_{\text{crit}} \Rightarrow \eta_2 = 0 \Rightarrow \frac{V_{\text{FRP}}}{K_0} = 0 \\
R < R_{\text{crit}} & \Rightarrow \left( \frac{\varepsilon}{S_f} \right)_{\text{actual}} < \left( \frac{\varepsilon}{S_f} \right)_{\text{crit}} \Rightarrow \eta_2 > 0 \Rightarrow \frac{V_{\text{FRP}}}{K_0} > 0
\end{align*}
\]

where,

\[
R_{\text{crit(TR55)}} = \left[ \frac{d_f}{0.7 \kappa_{\text{eq(TR55)}}} \right]^2 \left[ S_E \right]_{\text{(TR55)}} \times 0.18 \left( f_{\text{cu}} \right)^2 / 3
\]

The difference in the implications of the conditional relations between $R$ and $R_{\text{crit}}$ for ACI440 and TR55 is to be noted in Eqs. (5.40) and (5.42). The possibilities expressed by above set of equations facilitate classifying shear strengthening design solutions into different characteristic clusters, which is discussed later in this chapter. To a designer, this set of equations helps making design decision for selecting a FRP composite involving a more efficient material use, while to a calibrator this helps in creating design scenarios and better tuning the safety format. Fig. 5.11 shows $R_{\text{crit}}$ plots for sides-only (S) and U-wrapped (U) configurations under identical conditions, for ACI440 and TR55. The fully wrapped (W) configuration for either of the two design guidelines would always result into a positive value of factor $\eta_2$. 

![Figure 5.11](image_url)
For the given other conditions, a typical design scenario can be depicted by a pair of co-ordinates \((d_f, \bar{R})\). An appropriate \(\bar{R}_{\text{crit}}\) plot developed for a design guideline and a shear strengthening configuration in conjunction with this co-ordinate can serve as a designer’s as well as a calibrator’s tool in determining whether or not a design scenario will involve positive or negative wrapping effectiveness factor. For example, as evident from Fig. 5.11, for a sides-only configuration for TR55, two design scenarios represented by co-ordinates (400 mm, 1100 kN/mm) and (700 mm, 100 kN/mm) will lead to a negative and a positive wrapping effectiveness factor \(\eta_2\) respectively. For a sides-only configuration for ACI440, on the other hand, two design scenarios represented by co-ordinates (500 mm, 1 kN/mm) and (700 mm, 10 kN/mm) will lead to a negative and a positive value of factor \(\eta_2\). Thus, a \(\bar{R}_{\text{crit}}\) plot provides an upper bound limit for TR55 (beyond which a design solution will involve a negative \(\eta_2\)), while for ACI440 it provides a lower bound limit (below which a design solution will involve a negative \(\eta_2\)).

5.6.4 Sensitivity of effective failure strain of FRP stirrup

Eqs. (5.19) and (5.20) defines the effective failure strain of FRP reinforcement \(\varepsilon_{\text{fse}}\) within the assessment methodology. Using these equations, a typical sensitivity plot (Fig. 5.12) for \(\varepsilon_{\text{fse}}\) is developed, which illustrates the sensitivity of \(\varepsilon_{\text{fse}}\) to \([S_e]\) and \(\bar{R}\) for ACI440 and TR55. For demonstration purpose, this plot is developed for an FRP having mean rupture strain capacity of 10,000 micro-strain.

![Figure 5.12](image)

\(\varepsilon_{\text{fse}}\) plot for ACI440 and TR55
It can be clearly appreciated that the strain limit for FRP shear reinforcement, attributed to the possibility of physical disintegration of concrete, forms a stringent restriction on $\varepsilon_{f_{se}}$. Consequently, the numerical value for $\varepsilon_{f_{se}}$, even for a high rupture strain FRP material, remains confined to a very low value.

5.7 Characterisation of design solutions

5.7.1 Basis for assessment

It can be seen that the non-dimensional parameters $\left( \frac{d_f}{S_f} \right)$ and $\left( \frac{l_e}{S_f} \right)$ comprise the core of the assessment methodology. For given conditions, a pair of values of these non-dimensional parameters defines a specific value of the factor $\eta_2$, which represents a unique shear strengthening design solution. Therefore, appropriate ranges of these two non-dimensional parameters represent a universe of possible shear strengthening design solutions. These design solutions can be qualitatively characterised based on the concept presented within sub-section 5.6.3. For a particular shear strengthening configuration, Eqs. (5.36) and (5.37) can produce a solution surface within the universe of shear strengthening design solution confined by the considered ranges of non-dimensional parameters. Fig. 5.13 shows a typical solution surface, with parameter $\left( \frac{d_f}{S_f} \right)$ varying within a range (~0 to 20), while parameter $\left( \frac{l_e}{S_f} \right)$ varying within a range (0 to 10).

Out of the possible combination, a certain combinations of coordinates $\left( \frac{d_f}{S_f}, \frac{l_e}{S_f} \right)$ correspond to equal or less than zero value of factor $\eta_2$, and hence that of $V_{FRP}$. Design solutions involving such combinations of coordinates $\left( \frac{d_f}{S_f}, \frac{l_e}{S_f} \right)$ are referred to as unproductive design solutions. In contrast, the design solutions involving combinations of coordinates $\left( \frac{d_f}{S_f}, \frac{l_e}{S_f} \right)$ leading to a positive value of factor $\eta_2$, and hence that of $V_{FRP}$, are referred to as productive design solutions. Thus, the factor $\eta_2$ provides a basis to split a solution surface into two qualitatively distinct primary characteristic clusters – one comprising of the unproductive and the other comprising of the productive design solutions. The former and the latter are referred to as Cluster A Cluster B respectively, which are the two primary clusters. A little consideration will show that the extent of each of these two clusters, within a universe of design solutions, complements each other.
Figure 5.13
Characteristic clustering of shear strengthening design solutions

The upper and lower limits imposed on factor $\eta_2$, expressed through Eqs. (5.26) and (5.34) respectively, further classifies the design solutions belonging to Clusters A and B. Thus, two more clusters, namely Cluster C and Cluster D, are introduced, making a total of four qualitatively distinct clusters. Clusters C and D are, in fact, the sub-clusters of the primary Clusters A and B. The indicative Clusters A, B, C and D are shown in Fig. 5.13, and are discussed here. Criteria defining these clusters are summarised in Table 5.2.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Criteria</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\eta_2 \leq 0$</td>
<td>Unproductive design solutions</td>
</tr>
<tr>
<td>B</td>
<td>$\eta_2 &gt; 0$</td>
<td>Productive design solutions</td>
</tr>
<tr>
<td>C</td>
<td>$\eta_2 \geq \eta_2$-upper-bound</td>
<td>Productive design solutions, with restricted productivity</td>
</tr>
<tr>
<td>D</td>
<td>$\eta_2 &lt; \eta_2$-spacing</td>
<td>Productive or Unproductive design solutions, violating maximum longitudinal stirrup spacing criteria</td>
</tr>
</tbody>
</table>

$\eta_2$-upper-bound and $\eta_2$-spacing defined by Eqs. (5.26) and (5.34) respectively.

Cluster A: It involves inefficient pairing of the parameters $\left( \frac{d_f}{s_f} \right)$ and $\left( \frac{L_d}{s_f} \right)$ that leads to the unproductive design solutions having less than or equal to zero values of factor $\eta_2$. 
and hence that of $V_{FRP}$. In Fig. 5.13 a horizontal plateau centred at zero output signifies Cluster B. For the sake of convenience, the negative values of $V_{FRP}$ are plotted as zero in Fig. 5.13.

**Cluster B:** It comprises the productive design solutions that involve efficient pairing of parameters $\left( \frac{d_t}{s_t} \right)$ and $\left( \frac{l_w}{s_t} \right)$ leading to positive $\eta_2$ values. This cluster is signified by an inclined surface within the universe in Fig. 5.13.

**Cluster C:** There exists a subclass of design solutions, mainly within Cluster B, carrying the productive design solutions, but with a restricted productivity in terms of $V_{FRP}$. It can be observed that for the given fibre orientation, FRP material and other conditions, factor $\eta_1$ and design failure strain ($\epsilon_{fse}$) remain constant for both, ACI440 and TR55. Thus, an additional upper bound limit on factor $\eta_2$ restricts the productivity of the design solutions. Cluster C comprises of this type of the design solutions.

An upper bound of 0.75, as prescribed by ACI440 on the numerical value of the bond reduction coefficient for FRP, gets transcribed into an upper bound limit on the factor $\eta_2$ within the generic model. Thus, ACI440 includes a possibility of existence of Cluster C design solutions, and any adjustment on the multiplier 0.75 on bond reduction coefficient will alter the extent of Cluster C. For example, an increased stringency in prescribing upper bound limit on the bond reduction coefficient (i.e. a numerical value lesser than, let’s say 0.75) will result into an increase in the extent of Cluster C. On the other hand, a complete dilution of the upper bound limit on the bond reduction coefficient (i.e. a numerical value of unity) will seize the possibility of existence of Cluster C. This is the case with TR55, and hence it does not include Cluster C under any design scenarios.

**Cluster D:** Restrictions on the maximum permitted longitudinal spacing of FRP stirrups gets transcribed into the lower bound on factor $\eta_2$ in the generic model, as shown in Eq. (5.34). Cluster D typically involves relatively lower values (up to say, 2) of the parameter $\left( \frac{d_t}{s_t} \right)$, and represents a class of design solutions that are not valid on the ground of violating the maximum permitted longitudinal spacing criteria specified by the design guidelines. This class of design solutions may or may not involve positive values of factor $\eta_2$. However, any such value of the factor $\eta_2$ does not lead to a valid design solution in accordance with Eq. (5.33). It is to be noted that the design solutions belonging to this cluster are independent of material properties and wrapping configurations.
5.7.2 Results and discussion

Figs. 5.14 and 5.15 show typical shear strengthening design solution surfaces for two design scenarios, one representing the most liberal and the other representing the most stringent combination of material safety parameters respectively and the applicable resistance safety parameters for ACI440 and TR55. A detailed study of the solution surfaces, developed under a range of possible design scenarios for all wrapping configurations, has been carried out as a part of this study. Following observations emerge from this study:

- This study revealed that for a particular design guideline the extents of primary clusters (i.e. Clusters A and B), within a universe of shear strengthening design solutions, remain constant for a given wrapping configuration using a particular strengthening design guideline. Thus, the relative extents of these clusters for a given wrapping configuration using different design guidelines can be used as a measure of the relative inefficiency (or efficiency) of the shear strengthening design formats employed by them.

- It is seen that under identical design scenarios, the extent of Cluster A comprising the unproductive design solutions is the highest for sides-only configuration, followed by U-wrapped configuration for both, ACI440 and TR55. Cluster A is absent for the fully wrapped configuration for both the guidelines.

- For TR55, for MLC and MSC both, the maximum possible numerical values of the shear resistance contribution of FRP remains the same for all the three configurations for the given other identical conditions. The differences in the solution surfaces for these configurations include the extent of productive (or unproductive) design solutions and the steepness in variation of the shear resistance contribution of FRP along the \( \frac{\mu L}{S_f} \) axis. The variation of shear resistance contribution of FRP along the \( \frac{\mu L}{S_f} \) remains unchanged for all three configurations for TR55.

- The sides-only and U-wrapped configurations for ACI440 show the same characteristics to TR55. However, the fully wrapped configuration for ACI440 involves a higher maximum value of the shear resistance contribution of FRP compared to that for the sides-only and U-wrapping configurations for ACI440.
Figure 5.14
Shear strengthening design solution-surfaces for the most liberal combination (MLC) of the material safety parameters
Figure 5.15
Shear strengthening design solution-surfaces for the most stringent combination (MSC) of the material safety parameters
5.7.3 Implications

The qualitative characterisation of the shear strengthening design solutions lead to following two implications:

(A) **Extents of primary clusters**

As an important observation, it is seen that the extent of Cluster A having unproductive design solutions is significantly larger for ACI440 compared to that for TR55 under identical conditions. This is in spite of the fact that both, ACI440 and TR55, follow conceptually similar shear strengthening design formats. A set of parametric studies devised to investigate this finding in more detail have revealed that there exists a unique correspondence between the bond reduction coefficient \((\kappa_{ps})\) and the extents of primary clusters \((\xi)\) of shear strengthening design solutions. This correspondence is presented in Fig. 5.16, which can be expressed through predominantly linear mathematical correlations presented through Eqs. (5.44) and (5.45).

![Figure 5.16](image)

**Figure 5.16**

The \(\xi - \kappa_p\) correlation

\[
\begin{align*}
\xi_{(\text{Cluster A})}\% &= 25.25 \kappa_{ps} - 2.02 \\
\xi_{(\text{Cluster B})}\% &= -25.25 \kappa_{ps} + 102.02 \\
\xi_{(\text{Cluster A})}\% + \xi_{(\text{Cluster B})}\% &= 100 \%
\end{align*}
\]

Various wrapping configurations have different vulnerability of having a reduced bond length. Further, the possibility of employing additional mechanical anchorages to reinforce the ends of the FRP stirrups can alter this vulnerability, and thereby alter the
effectiveness of the bond characteristics of FRP shear reinforcement. This, in turn, suggests that the bond reduction coefficients can take any value, greater than or equal to zero. Thus, a range of variation in the bond reduction coefficient, from 0 to 2, is considered in Fig. 5.16. The implications of the ACI440 and TR55 prescriptions on \( \kappa_{ps} \) for sides-only and U-wrapped configurations are clear from Fig. 5.16. Fig. 5.17 represents the relative extents of the primary clusters, i.e. Cluster A and Cluster B, for the three configurations for ACI440 and TR55. The coefficients of bond reduction for sides-only and U-wrapped configurations prescribed by ACI440 (without any consideration of additional mechanical anchorages) are three-times larger than those prescribed by TR55. It is this fact that makes a significant difference in the extents of primary clusters for these two guidelines.

![Figure 5.17](image)

Relative extents of primary clusters of shear strengthening design solutions for ACI440 and TR55

(B) **Conservativeness of shear strengthening design solutions**

It can also be observed that for MLC, estimated design shear resistance contribution of FRP is relatively higher for TR55 compared to ACI440. For MSC, on the other hand, ACI440 estimates numerically higher values of the design shear resistance contribution of FRP compared to TR55. These, of course, are due to the fact that the material safety parameters (especially those attached at Level II with the strengthening design process) involved in the most liberal and most stringent combinations are numerically different for ACI440 and TR55. The most liberal condition for TR55 is less stringent than that for ACI440, while the most stringent condition for TR55 is more stringent than that for
ACI440. The implication of this observation, in terms of conservativeness, is discussed in the next section.

5.8 Sensitivity of conservativeness of the design solutions

5.8.1 Basis of assessment

The net impact of the material and resistance conservativeness for a shear strengthening design solution with a particular wrapping configuration can be obtained, individually and cumulatively, in form of a residual conservativeness index \( RCI \) through Eq. (5.48).

\[
RCI = \frac{(\frac{V_{FRP}}{H_0})_{Plain} - (\frac{V_{FRP}}{H_0})_{Engineered}}{(\frac{V_{FRP}}{H_0})_{Plain}}
\] (5.48)

Here, subscript ‘Engineered’ refers to the engineered version of the shear resistance contribution of FRP reinforcement \( (\frac{V_{FRP}}{H_0}) \), i.e. considering all the applicable material and resistance safety parameters while computing \( (\frac{V_{FRP}}{H_0}) \). Subscript ‘Plain’ indicates omission of a class of or all safety parameters on FRP while computing \( (\frac{V_{FRP}}{H_0}) \). Various combinations of safety parameters are presented in Table 5.3.

<table>
<thead>
<tr>
<th>Type of conservativeness</th>
<th>Safety parameter combination for ( \left(\frac{V_{FRP}}{H_0}\right)_{Engineered} )</th>
<th>Safety parameter combination for ( \left(\frac{V_{FRP}}{H_0}\right)_{Plain} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level I+II Material Conservativeness (MC)</td>
<td>(\gamma_{PQR}, \gamma_{EDT}, \gamma_{APR})</td>
<td>---</td>
</tr>
<tr>
<td>Level III+IV Resistance Conservativeness (RC)</td>
<td>(\gamma_{CMP}, \gamma_{PNT}, \gamma_{SUP})</td>
<td>(\gamma_{PQR}, \gamma_{EDT}, \gamma_{APR})</td>
</tr>
<tr>
<td>Level I+II+III+IV Total Conservativeness (TC)</td>
<td>(\gamma_{PQR}, \gamma_{EDT}, \gamma_{APR}) (\gamma_{CMP}, \gamma_{PNT}, \gamma_{SUP})</td>
<td>---</td>
</tr>
</tbody>
</table>

Remarks:
- Material safety parameters refer to rupture strain capacity and modulus of elasticity of FRP.
- Applicable factors of safety on steel reinforcement and concrete are included in each of the above combinations.

A particular shear strengthening design solution when assessed using different strengthening design guidelines leads to different design shear resistance contributions.
of FRP. A comparison of shear resistance contributions of FRP so obtained under identical conditions can provide an estimate of relative conservativeness in terms of relative residual conservativeness index ($RRCI$) as shown in Eq. (5.49). Here, subscripts DG1 and DG2 represent two different arbitrary design guidelines.

$$RRCI = \frac{\left(\frac{\sigma_{FRP}}{\eta_0}\right)_{DG1}}{\left(\frac{\sigma_{FRP}}{\eta_0}\right)_{DG2}} = \frac{\eta_2(DG1)}{\eta_2(DG2)} \times \frac{\varepsilon_{fse(DG1)}}{\varepsilon_{fse(DG2)}}$$

(5.49)

The sensitivity analysis of conservativeness indicators $RCI$ and $RRCI$, for a range of shear strengthening design solutions under different design scenarios for ACI440 and TR55, is presented in the next subsection.

### 5.9.2 Results and discussion

Figs. 5.18 and 5.19 present $RCI$ plots for the most liberal combination (MLC) and the most stringent combination (MSC) of the material safety parameters respectively and the applicable resistance safety parameters for ACI440 and TR55. The sensitivity of $RCI$ to the material safety parameters attached to Level II in the strengthening design process is portrayed in Figs. 5.20 and 5.21. The former shows variation in $RCI$ for different M & I Classes for TR55 and the latter shows that for different EE Classes for ACI440. Following observations emerge from these plots:

- **Uniformity of residual conservativeness:** Productive design solutions within the $RCI$ plots for ACI440 and TR55 exhibit a uniform distribution of residual conservativeness for sides-only, U-wrapped and fully wrapped configurations under MLC and MSC scenarios. This uniformity can be confirmed from Table 5.1 indicating the constant extent of total conservativeness associated with a given failure mode. The unproductive design solutions involve less than or equal to zero values of $RCI$. For better visual clarity, the negative $RCI$ values are plotted as zero in Figs. 5.18 and 5.19.

- **Equality of residual conservativeness:** Productive design solutions within the $RCI$ plots for MLC and MSC design scenarios shows equal residual conservativeness for all three configurations for TR55. Unlike the above, the fully wrapped configuration for ACI440, for both the design scenarios, involve lesser residual conservativeness compared to that for the sides-only and U-wrapped configurations. This is the implication of the fact that the ACI440 design solutions with fully wrapped configuration have a higher maximum value of the shear resistance contribution of FRP compared to that for sides-only and U-wrapped configurations under identical conditions, as pointed in section 5.7.2.
Figure 5.18
The residual conservativeness index (RCI) plots for the most liberal combination (MLC) of the material safety parameters
Figure 5.19
The residual conservativeness index ($RCI$) plots for the most stringent combination (MSC) of the material safety parameters.
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Fig. 5.20
RCI values for the different M & I Classes for TR55

Fig. 5.21
RCI values for the different EE Classes for ACI440

Fig. 5.22 presents the RRCI plots without considering any material and resistance safety parameters prescribed by ACI440 and TR55, while Fig. 5.23 presents the RRCI plots for MLC and MSC design scenarios. It is to be noted that the RRCI is the ratio of the shear resistance contribution of FRP for ACI440 to that for TR55. The RRCI, being the ratio of the shear strengthening resistance contribution of FRP for ACI440 to that for TR55, its value lesser than unity reflects higher residual conservativeness associated with an ACI440 design solution compared to that associated with a TR55 design solution under identical conditions. Following observations emerge from these plots:

...
• Fig. 5.22 indicates that the $RRCI$ values for sides-only and U-wrapped configurations remain lesser than unity. This signifies that these design solutions are more conservative for ACI440 compared to TR55, when the material and the resistance safety parameters are ignored while estimating the shear resistance contribution of FRP. The fully wrapped configuration shows a marginally higher than unity values of $RRCI$, signifying that these design solutions are approximately equally conservative for ACI440 and TR55, under the identical conditions.

• From Fig. 5.23 it can be observed that the relative residual conservativeness remains uniform for the fully wrapped configurations under MLC and MSC scenarios. For the sides-only and U-wrapped configurations, however, this uniformity is not apparent in the $RRCI$ plots. This apparent non-uniformity is merely due to different extents of the cluster of productive design solutions. The common regions of the productive design solutions for ACI440 and TR55 within the solution surfaces do show a uniform distribution of relative residual conservativeness.

• It can also be observed that the $RRCI$ values for MLC design scenario remains lesser than unity for the entire range of possible design solutions for the sides-only and U-wrapped configurations. This indicates that the TR55 design solutions under the MLC scenario are less conservative than the ACI440 design solutions for the sides-only and U-wrapped configurations.

• For the fully wrapped configuration under MLC design scenario $RRCI$ is marginally above unity. The fully wrapped design solutions for ACI440 and TR55 under the MLC scenario are approximately equally conservative.

• On the other hand, for a major fraction of the possible design solutions under MSC design scenario, the $RRCI$ values are observed to be considerably greater than unity, for sides-only and U-wrapped configurations. This signifies that a major fraction of possible TR55 design solutions for the sides-only and the U-wrapped configurations under MSC design scenario are considerably conservative compared to ACI440.

• The $RRCI$ values for fully wrapped configuration for the entire range of the possible design solutions under the MSC scenario remain significantly greater than unity. Also, the entire range of possible TR55 design solutions for the fully wrapped configuration under MSC scenario are significantly conservative compared to ACI440.
Figure 5.22

$RRCI$ plots without considering the material and the resistance safety parameters.
Figure 5.23
The relative residual conservativeness index (RRCI) plots for the most liberal and stringent combinations of the material safety parameters
5.9 Summary of observations

Important observations based on the discussion presented in this chapter are summarised below:

- It is shown that the shear resistance contribution of FRP reinforcement can be represented as a function of effective failure strain of FRP reinforcement, with modifications for fibre orientation and wrapping configuration. This format provides a means to assess different formats of shear strengthening design criteria on a common platform. The significance and the applicability of this approach have been demonstrated by calibrating it against TR55 and ACI440 design specifications.

- The internal architecture of the shear strengthening design process reveals that it comprises of different conservativeness propagation paths, each of these paths represents a unique failure mode. The design value of the shear resistance contribution of FRP for each of these failure modes is shown to comprise of an absolute resistance function and total conservativeness (consisting of applicable fractions of the material and the resistance safety parameters). Conservativeness propagation paths involving the least extent of the total conservativeness are highlighted.

- It is quantitatively demonstrated that an FRP reinforcement is most efficient when oriented at 45° with respect to the anticipated direction of the shear crack. However, for many reasons the FRP reinforcement oriented vertically (i.e. with principle direction oriented normal to the longitudinal axis of the beam) is shown to be the most practical orientation.

- The inverse proportionality of the Type I (e.g., the one prescribed by TR55) and Type II (e.g., the one prescribed by ACI440) empirical bond length estimation models is highlighted. It is shown that the ACI440 strategy of prescribing Type II bond length model and not prescribing material safety parameters on modulus of elasticity of FRP avoids a conflicting situation in which a numerically higher value of material safety parameters on modulus of elasticity of FRP reduces the conservativeness.

- TR55 prescribes the same empirical bond length estimation model along with a minimum bond length requirement of 500 mm to be used for flexural and shear strengthening both. This minimum bond length requirement can be practical for flexural strengthening. However, it is impractical for most practical shear strengthening design situations. For shear strengthening, this minimum bond length
requirement should be removed. The methodology presented in this chapter does not consider this minimum bond length requirement.

- Relation between the quantitative prescription of the bond reduction coefficient and the susceptibility of reduction in bond carrying capacity of FRP reinforcement is established. The least susceptibility of bond reduction exhibited by the fully wrapped configuration, owing to it being a contact-critical application, is revealed from the zero value for the bond reduction coefficient for this configuration.

- U-wrapped and sides-only configurations, being the bond-critical applications, exhibit susceptibility of reduction in bond carrying capacity. The numerical values of bond reduction coefficients reveal that the susceptibility of reduction in bond carrying capacity for sides-only configuration is twice as high compared to that for U-wrapped configuration.

- In particular, it is highlighted that the bond reduction coefficients for the bond-critical configurations for ACI440 are thrice as high compared to those for TR55 under identical conditions.

- It is shown that a choice of FRP material or wrapping configuration for shear strengthening can lead to either a productive (i.e. carrying a positive value of the shear resistance contribution of FRP) or an unproductive (i.e. carrying a zero or negative value of the shear resistance contribution of FRP) shear strengthening design solution. A set of mathematical rules is proposed that predict productivity of a shear strengthening design solution based on the choice of FRP material or wrapping configuration.

- In particular, it is demonstrated that the inverse proportionality of the Type I (e.g., for TR55) and Type II (e.g., for ACI440) bond length models to the modulus of elasticity of FRP exhibits contrasting predictions for the productivity of a shear strengthening design solution.

- It is shown that a range of possible shear strengthening design solutions for a given design scenario can be characterised into four distinct clusters with a solution surface. The clusters of productive and unproductive design solutions form the primary clusters, and their extents within a solution surface are demonstrated to be directly dependent upon the numerical value of the coefficient of bond reduction. Mathematical correlations between the extents of productive (and unproductive) design solutions to the coefficient of bond reduction are synthesised.

- With a value of coefficient of bond reduction as zero, the fully wrapped configuration does not involve a cluster of unproductive shear strengthening design solutions. U-wrapped and sides-only configurations do involve the cluster of
unproductive shear strengthening design solutions. It is shown that the extent of the
cluster of unproductive shear strengthening design solutions for sides-only
configuration is considerably large compared to that for U-wrapped configuration.

• The thrice as high values of the coefficients of bond reduction for ACI440
compared to those for TR55 result into substantially large extents of the clusters of
unproductive design solutions for ACI440 compared to TR55 under identical
conditions. This observation strongly suggests a need of reconsidering the ACI440
design prescriptions for the coefficient of bond reduction.

• For TR55, the cluster of productive design solutions exhibits uniformity and
equality of residual conservativeness for all the three shear strengthening
configurations.

• For ACI440, uniformity of residual conservativeness is observed for all three
configurations for the cluster of productive design solutions. However, only the
sides-only and U-wrapped configurations exhibit equality of residual
conservativeness. The fully wrapped configuration for ACI440 shows lesser
residual conservativeness compared to the sides-only and U-wrapped
configurations under identical conditions.

• With ignoring all the applicable material and resistance safety parameters, sides-
only and U-wrapped configurations for ACI440 are demonstrated to have a higher
conservativeness relative to TR55 under identical conditions. Fully wrapped
configuration for ACI440 and TR55 are approximately equally conservative under
identical conditions.

• The most liberal combinations of the material safety parameters lead to relatively
higher residual conservativeness for the productive design solutions of ACI440
compared to TR55. In contrast, the most stringent combinations of the material
safety parameters lead to relatively higher residual conservativeness for the
productive design solutions of TR55 compared to ACI440.
Conclusions
6.0 Conclusions and recommendations

The content of this study has provided a comprehensive framework to track conservativeness within the flexural and shear strengthening design processes. The assessment methodologies proposed within this study have provided a detailed insight into the internal architecture of flexural and shear strengthening design processes, and have served as a means to characterise range of possible flexural and shear strengthening design solutions into distinctive qualitative clusters. The implications of identified differences and conflicts in design specifications on the course of strengthening design process and quality of resultant strengthening design solutions have been demonstrated. The research objectives set in Chapter 1, thus, have been successfully achieved. A set of important conclusions derived based on this study along with appropriate suggestions or recommendations are presented here:

Conclusion 1: A vast array of material properties offered by various types and forms of FRP materials can be utilised to provide a custom-made strengthening design solution by making the most cost-effective choice on type and form of FRP material for a given design condition. However, there is a lack of rational guidance on such decision making involved in structural strengthening design.

Recommendation: There is a substantial scope and need of developing a comprehensive expert system enabling decision making required within design of FRP-based strengthening systems. The mathematical rules derived within this study can serve as a knowledge-base not only for devising optimal strengthening design solutions ensuring an efficient use of FRP materials, but also for fine-tuning and/or calibrating design criteria for structural strengthening.

Conclusion 2: The choice of employing an assessment standard for existing structures or a design standard for new construction for quantifying the required degree of strengthening sets a design conflict. Under this conflict, the resultant safety targets (e.g., in terms of probability of failure or reliability index) can potentially be different based on the use of a design or an assessment standard.

Recommendation: As an interim measure, it is suggested that the loading specifications for the new constructions should be used for calculating the design load-effects for structural strengthening. However, an applicable assessment standard shall be used for estimating the residual strength of an existing structure. Any fall in safety-content due to the use of these combinations of estimated design load-effects and design
available structural resistance shall be absorbed by appropriate adjustment of various factors of safety. A detailed investigation of the reliability profiles of strengthening design solutions under different design scenarios is recommended. This study can serves as a basis for such investigations.

**Conclusion 3:** Accounting for the non-prescriptive uncertainties arising from the prevailing lack of knowledge and time-testimony in using FRP materials for structural strengthening is an additional obligation for design of FRP-based structural strengthening systems. This fact serves as a justification for having a higher degree of conservativeness in terms of post-strengthening resistance.

**Conclusion 4:** The extent of prescriptive uncertainties arising from the variability in constitutive FRP material properties and deviations between the design predictions and the ideal and real behaviour of a strengthened structure is considerably large compared to that for design using conventional structural materials. While this fact serves as a justification for having a higher degree of conservativeness in terms of post-strengthening resistance, there is a lack of rationale behind the strategy of prescribing higher safety factors (relative to the conventional design norms) towards achieving higher conservativeness in post-strengthening resistance.

**Recommendation:** The qualitative and quantitative findings presented within this study can serve as a basis for rationalising the safety factors employed within various strengthening design guidelines.

**Conclusion 5:** The mapping between the identified prescriptive uncertainties and safety parameters has demonstrated that the behavioural uncertainties are is primarily accounted for by the safety factors prescribed on post-strengthening resistance. The use of limit state design (LSD) or load and resistance factor design (LRFD) is not a justifiable basis for having a strategy of not prescribing safety factors on resistance. Such a strategy, therefore, implies ignorance towards accounting for the behavioural deviations.

**Recommendation:** It is suggested that the strategy of not prescribing the safety factors on resistance contribution of FRP and post-strengthening structural resistance employed by the strengthening design guidelines (e.g., TR55) be reconsidered under the light of approaches suggested within this study.
**Conclusion 6:** The possible bifurcations within the course of strengthening design processes have been demonstrated by transcribing the design criteria in form of various ‘failure mode switchers’. These switchers are of strategic importance and can play a manipulative role within strengthening design process.

**Conclusion 7:** It is shown that amongst the failure modes competing within a switcher to govern a strengthening design solution, the prescribed safety factors on FRP material properties are applicable only to the rupture-based failure modes. The debonding-based failure modes, by-and-large, do not include these safety factors. Thus, the strengthening design solutions involving rupture- or debonding-based failure mode will involve differential conservativeness in terms of post-strengthening resistance. It is also demonstrated that the format of upper bound limit suggested on the value of strain in FRP at debonding (e.g., as prescribed by ACI440) replaces the possibility of involving full-rupture of FRP with near-rupture of FRP. It is further demonstrated that the prescribed safety factors on FRP material properties get quantitatively inflated for the near-rupture condition compared to its full-rupture. This fact remains disguised within the strengthening design process.

**Recommendation:** Mathematical rules depicting design predictions for debonding and rupture/near-rupture of FRP provided within this study can be used to articulate the failure mode switchers towards ensuring a uniform residual conservativeness associated with the strengthening design solutions involving different failure modes competing within a failure mode switcher.

**Conclusion 8:** The low order of strain in FRP at debonding arising from debonding strain limits makes the use of high rupture strain capacity FRP material an inefficient choice.

**Recommendation:** Mathematical rules depicting design predictions for debonding and rupture/near-rupture of FRP enables to workout a threshold value of mean rupture strain capacity of FRP under a given condition, beyond which any higher rupture strain capacity of FRP will be superfluous. This approach provides a basis for an efficient use of FRP materials for given conditions. Additionally, inclusion of design prescriptions for debonding strain limits for mechanically anchored FRP reinforcements can bring considerable improvement on this account.
Conclusion 9: Type II debonding strain limit (e.g., as specified by ACI440) involves an inverse proportionality between the strain in FRP at debonding and modulus of elasticity of FRP. The strategy of not prescribing any material safety parameters on modulus of elasticity is avoids unconservative implications in estimation of strain in FRP at debonding. However, the bond length estimation models used in flexural strengthening involve a direct proportionality between the required bond length and modulus of elasticity. A strategy of not prescribing any material safety factors on the modulus of elasticity of FRP, in such a situation, leads to unconservative estimation of the required bond length of FRP reinforcement.

Recommendation: A reduction factor, used as a multiplier to the required bond length for FRP estimated through the bond length model for flexural strengthening, is suggested for the design guidelines opting not to prescribe any material safety factors on the modulus of elasticity of FRP.

Conclusion 10: The strain in FRP at debonding estimated through Type II debonding strain limit (e.g., as specified by ACI440), especially for low modulus high rupture strain capacity FRP materials, can be substantially high compared to the order of the strain in FRP at debonding estimated through Type I debonding strain limit (e.g., as specified by TR55).

Recommendation: A pre-set constant value as an upper bound limit on the numerical value of strain in FRP at debonding estimated through Type II debonding strain limit is suggested. The order of this pre-set constant upper bound could be similar to the strain in FRP at debonding estimated through Type I debonding strain limit.

Conclusion 11: The prescribed safety factors on FRP material properties do not contribute towards producing conservativeness for the flexural strengthening design solutions involving concrete-controlled failure modes or those involving FRP-controlled failure modes with debonding of FRP. For such design solutions, the strength reduction factors prescribed on the post-strengthening resistance are the only means of producing conservativeness. A strategy of not prescribing any strength reduction factors on post-strengthening resistance (e.g., according to TR55 specifications) presents a substantial gap in producing conservativeness towards ensuring required safety targets.

Recommendation: Strength reduction factors are suggested for the flexural strengthening design solutions involving concrete-controlled failure modes or FRP-controlled failure modes with debonding of FRP.
controlled failure mode with debonding of FRP. Mathematical rules depicting design predictions for concrete- and FRP-controlled failure modes can serve as a basis for this.

**Conclusion 12:** Attributed to the inability of FRP composites to undergo plastic deformations, the fundamental design objective of ensuring a ductile post-strengthening failure mode to govern the flexural strengthening design solutions has to rely upon an analogous over-reinforced strain state. This is in perfect contrast with the conventional norms used in flexural design of new structures. This leads to an obvious possibility of having differential safety targets in pre- and post-strengthening stages.

**Recommendation:** This issue demands a more holistic treatment for design for strength for new construction and design for additional strength for strengthening existing structures that ensures a seamless transition between the pre- and post-strengthening failure modes. The premise of this study has set a narrative for this.

**Conclusion 13:** In shear strengthening design, Type II bond length model (e.g., according to ACI440 specifications) involves an inverse proportionality between the required bond length of FRP and modulus of elasticity of FRP. A strategy of not prescribing any safety factors on modulus of elasticity of FRP, in such a case, avoids unconservative estimation of the required bond length of FRP.

**Conclusion 14:** TR55 specification suggests a unique bond length model for FRP reinforcement, along with a minimum required bond length of 500 mm, for flexural and shear strengthening design. This minimum required bond length might be practically feasible in flexural strengthening design. However, in most common shear strengthening design configurations, this is not feasible to achieve.

**Recommendation:** The minimum required bond length of 500 mm is to be removed for shear strengthening design.

**Conclusion 15:** The numerical values of the bond length reduction coefficients suggested by ACI440 are thrice as high compared to those suggested by TR55 under identical conditions. This leads to a substantially large proportion of the unproductive shear strengthening design solutions for ACI440 compared to TR55 under identical conditions.
**Recommendation:** A revision of the numerical values for the bond reduction coefficients prescribed by ACI440 is suggested. As a first remedy, these values should be corrected to one-third of their existing values.

**Conclusion 16:** The low order of failure strain in FRP corresponding to the physical disintegration of concrete makes the use of high rupture strain capacity FRP for shear strengthening an inefficient choice of FRP material. This being the lowest amongst all the other strain limits for FRP governs the most shear strengthening design solutions, the prescribed safety factors on FRP material properties, for the shear strengthening design solutions involving physical disintegration of concrete, do not contribute towards conservativeness in terms of post-strengthening shear resistance. The strength reduction factors are the only means for such strengthening design solutions to produce conservativeness in terms of post-strengthening resistance.

**Recommendation:** Strength reduction factor for the shear strengthening design solutions involving physical disintegration of concrete is suggested to compensate for the fall in required safety targets due to not applicability of safety factors prescribed on FRP material properties.
Future scope
7.1 General

The philosophy, concepts and methodologies presented within this thesis can be extended and used in more than one ways. This chapter provides a summary of the future scope this work. In principle, there two major approaches that can be taken from this point forward.

7.2 Non-deterministic treatment and development of an expert system

It is possible to transform the proposed methodologies from its present deterministic format to appropriate non-deterministic formats using the same conceptual line of actions. Two such possibilities arise in this direction. The first possibility includes the probability-based reliability analysis. Considering the conservativeness associated with the load-effect estimation also, the limit state of conservativeness can be treated probabilistically to write the safety-contents in terms of probability of failure or reliability index.

The second possibility includes a fuzzy-mathematical treatment of the subjective uncertainties involved in the strengthening design process. It is clear from these study that FRP-based structural strengthening design involves a considerable extent of subjectivity, such as qualitative classifications of environmental exposure conditions, manufacturing and installation methods, empirical debonding strain limits, etc. Fuzzy mathematics is a powerful tool to comprehend the subjective uncertainties, and the mathematical rules developed within this thesis can be converted in a fuzzy-mathematical format in form of a Fuzzy Inference Mechanism (FIM). Such a FIM, then, can be used to facilitate justifiable design (and even calibration) decisions making. The author has developed such a FIM serving as a brain (i.e. a rule-based decision maker) for a Bridge Management System (BMS) that is being used by the Indian Railways. The concepts developed for this BMS, can be extended to cover FRP-based structural rehabilitation as well. In itself, the FIM developed for structural rehabilitation can serve as an Expert System, not only for design and calibration, but also for teaching of engineering students. Expanding this to integrate field-based experimental testing can also serve as a tool for performance evaluation of rehabilitated existing structures. A proposed protocol for this is presented here for reference, which is self-explanatory.
Figure 7.1
Phase I of the FIM for expert system
Figure 7.2
Phase II of the FIM for expert system
Figure 7.3
Phase III of the FIM for expert system
Figure 7.4
Phase IV of the FIM for expert system
Figure 7.5
Phase V of the FIM for expert system
7.3 **Conservativeness for robustness and resiliency in engineering design**

Another direction in which this study can be extended is rather more holistic, and deals with the civil engineering design process at large. The following brief discussion will give an overview of this approach. In fact, this approach, though intended to deal with the structural design process, can be extended to any engineering design for a complex systems with a contextual modification and alterations.

**Background:** The socio-economic-strategic importance of the civil engineering infrastructure systems compels them not only to remain functional during the disasters but their services are continuously needed over long period of time, often significantly far beyond their initially envisaged life. Consequently, they are expected to absorb extreme operational variations and adapt to large functional, operational, condition-based and compliance-based changes. Engineering design processes to ensure structural safety of infrastructures include design for strength while constructing new infrastructure, and design for additional strength while rehabilitating an existing underperforming infrastructure. Both of these have some fundamental limitations. First, these processes are mono-directional and focus on immediate needs that are assumed to prevail forever. For example, design for a new infrastructure completely neglects the possible rehabilitation at a later date. The rehabilitation is typically summoned on an ad hoc ‘when needed’ basis. Thus, there is a serious lack of engineering coordination between these two design processes. Further, these processes are unable to incorporate new materials and techniques easily to adapt to the changes. All these limitations lead to a cyclic situation, in which the new infrastructure being constructed now will face the same (or similar) problems in future that the existing infrastructures are facing at present. This inadequacy keeps the engineers and designers unproductively busy by allowing the problems to occur at first place, and then resolving them for the immediate needs, cyclically.

**Objective:** There are ample examples, in the UK and worldwide, of the infrastructures that were constructed centuries ago, but are still in service. One of the common characteristics in these infrastructures is the inherent conservativeness in terms of strength, which enabled them to absorb extreme variations in demands (e.g. earthquakes, aging, etc.) and yet keeping sufficient margin available towards adapting to the changes in their operations and functions. This, however, was not a planned strategy, but was rather a limitation of the materials and methods that were available during that time. Nevertheless, it unintentionally facilitated keeping the options open to withstand uncertain variations and adapt to unpredictable changes. I intend to
demonstrate that a design philosophy, with calculated and rationalised conservativeness, can serve as a fabric to weave robustness (i.e. the ability to withstand variability) and resiliency (i.e. the ability to adapt to the changes) within the traditional risk-reliability based design formats.

**Motivation:** The natural systems apparently have a strong inclination towards conservativeness. For example, the human body, as an evolutional outcome, with ‘one heart and two kidneys’ suggests that the nature intends to tailor a certain vital organs of the system for an enhanced capacity, while some vital organs are preferred to have an enhanced redundancy. Perhaps, such a distribution of over-strength and redundancy acting in tandem towards a common goal provides enough robustness to a complex system, such as a human body, to perform reliably over its individual lifetime. On the other hand, the ecosystems retain their resiliency over considerably longer periods compared to the individual lifetimes of its member organisms, perhaps from their surroundings that have ample buffer capacities to absorb disturbances in general. From these, it can be inferred that: (a) enhanced capacity, redundancy and buffer capacity (which can be seen as forms of conservativeness) helps deriving robustness and resiliency, (b) robustness is a relatively short-term objective compared to resiliency, (c) an ability to renew part of a complex system to refurbish robustness repeatedly over a series of short-periods serves as a building-block to exercise resiliency over a long period, and (d) robustness rests on the premise that the hazards are recognisable, while resiliency warrants keeping options open to adapt to the unexpected. Can these observations be mathematised to comprise an algorithm that can then be applied to the engineering design of infrastructures and other complex systems? I strongly believe, yes.

**Strategy:** This thesis has demonstrated that conservativeness in capacity design of infrastructure systems can be represented at material, member and system levels through over-strength (i.e. sacrificial buffer margin in capacity), reserved-strength (i.e. mobilisable buffer margin of capacity from the systems capability of redistributing the overload within the members or subsystems) and redundancy (i.e. exploitable alternate paths such that the physical absence of a few elements or members does not lead to a gross loss of structural form). This concept can be further extended to derive a network that maps the conservativeness of individual members within the system to achieve robustness and resiliency requirements. An inbuilt capability for adapting to newer materials and methods of in-service rehabilitations, and having sufficiently modular infrastructure service-life can bring in radical changes to the ways the civil engineering
infrastructure are designed and managed. Modularity helps in segregating the performance objectives of the infrastructure systems for short-term (pertinent to robustness) and long-term (pertinent to resiliency), and deriving a hierarchy of robustness and resiliency at different levels within and for the infrastructure system.
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Journal Papers (Published and under preparation)

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Conference Papers

Research Reports
