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¹ **Using multiple donor sites for enhanced flood**
² **estimation in ungauged catchments**

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3 **Abstract.** A new generalised method is presented enabling the use of mul-
4 tiple donor sites when predicting an index flood variable in an ungauged catch-
5 ment using a hydrological regression model. The method is developed from
6 the premise of having an index flood prediction with minimum variance, which
7 results in a set of optimal weights assigned to each donor site. In the model
8 framework presented here, the weights are determined by the geographical
9 distance between the centroids of the catchments draining to the subject site
10 and the donor sites. The new method was applied to a case study in the United
11 Kingdom using annual maximum series of peak flow from 602 catchments.
12 Results show that the prediction error of the index flood is reduced by us-
13 ing donor sites until a minimum of six donors have been included, after which
14 no or marginal improvements in prediction accuracy are observed. A com-
15 parison of these results is made with a variant of the method where donor
16 sites are selected based on connectivity with the subject site through the river
17 network. The results show that only a marginal improvement is obtained by
18 explicitly considering the network structure over spatial proximity. The eval-
19 uation is carried out based on a new performance measure that accounts for
20 the sampling variability of the index flood estimates at each site. Other re-
21 sults compare the benefits obtained by adding relevant catchment descrip-
22 tors to a simple regression model with those obtained by transferring infor-
23 mation from local donor sites.

1. Introduction

24 Estimating the magnitude of design floods (here defined as the discharge associated
25 with a pre-defined return period) in ungauged basins is an important practical problem in
26 applied hydrology, and one which has also received considerable attention in the scientific
27 literature; in particular as part of the IAHS Prediction in Ungauged Basin (PUB) decade
28 [*Blöschl et al.*, 2013]. A class of methods which has found favour among practitioners and
29 academics is regional flood frequency analysis, where, using statistical analysis of samples
30 of extreme flood or rainfall data from a geographical region, estimates at ungauged sites are
31 obtained through transfer of data from gauged sites. Numerous techniques for undertaking
32 regional frequency analysis have been reported in the literature [*Cunnane*, 1988; *Blöschl*
33 *et al.*, 2013]. One method in particular, the index flood method, described by *Dalrymple*
34 [1960] and more recently made popular in the L-moment version described by *Hosking*
35 *and Wallis* [1997], has been the subject of numerous studies [e.g. *Pearson*, 1991; *Vogel*
36 *et al.*, 1993; *Parida et al.*, 1998; *Kachroo et al.*, 2000; *Lim and Voeller*, 2009; *Yang et al.*,
37 2010; *Salinas et al.*, 2013].

38 Underpinning the index flood method is a set of assumptions of which the most promi-
39 nent, but simplistic, is that within a homogeneous region the flood series from different
40 sites are independent and identically distributed apart from a scale parameter, the index
41 flood, often defined as the mean or median annual maximum flood [*Hosking and Wallis*,
42 1997; *Institute of Hydrology*, 1999]. The estimation of the index flood at ungauged sites
43 is typically implemented using a regression-type model relating the index flood at gauged
44 sites to a set of relevant physiographic, geomorphologic and climatic catchment descrip-

tors such as catchment area, mean annual rainfall, soil type, urban extent etc. [Glover
et al., 2002]. Estimation of the regression model parameters can be undertaken using,
for example, techniques such as ordinary, weighted or general least squares (OLS, WLS
or GLS) depending on the level of complexity adopted in the model building phase [Ste-
dinger and Tasker, 1985]. Alternatively, the model parameters can be estimated using
maximum-likelihood [Kjeldsen and Jones, 2009; Mediero and Kjeldsen, 2014] or Bayesian
[e.g. Reis et al., 2005; Haddad et al., 2012] techniques.

The standard errors of estimates obtained at ungauged sites using this type of regres-
sion model are generally relatively large. For example, the regression model developed by
Kjeldsen and Jones [2009] for estimating the median annual flood at ungauged sites in the
UK was based on annual maximum peak flow data from 602 gauged catchments and has a
factorial standard error (fse) of 1.431. Another example is the study by Meigh et al. [1997]
developing regression models linking the mean annual flood (MAF) to catchment descrip-
tors in regions from around the world. They reported values of fse in the range between
1.36 (24 basins in South Korea larger than 1000km²) and 2.88 (162 arid and semi-arid
basins worldwide). In recognition of this high level of uncertainty, the guidance provided
in the UK on flood frequency estimation in ungauged catchments [Institute of Hydrology,
1999] suggests that pure regression-based estimates should be adjusted, where possible,
through data transfer from hydrologically similar gauged donor catchments. This strategy
is in keeping with similar conclusions drawn by other researchers. For example, Merz and
Blöschl [2008] highlighted the benefit of incorporating local knowledge and data into flood
frequency analysis. Conceptually, the use of local data to adjust the regression-based esti-
mate at a particular site can be viewed as an attempt to compensate for exclusion of local

68 flood controlling factors in the explanatory variables used in the regression model [*Kjeld-*
69 *sen and Jones*, 2010]. The rules provided by *Institute of Hydrology* [1999] for selection of
70 suitable donor catchments were heuristic and included sites that were considered hydro-
71 logically similar in terms of catchment area, mean annual rainfall and soil type. Assessing
72 the benefit of data transfer in the UK, *Kjeldsen and Jones* [2010] found that superior per-
73 formance was achieved when donor sites were selected based on geographical proximity
74 rather than hydrological similarity as measured by catchment descriptors. A similar con-
75 clusion was reached by *Merz and Blöschl* [2005] and *Viglione et al.* [2013]. In a separate
76 study involving spatial generalization of flood statistics, *Morris* [2003] found that where
77 donor sites were upstream or downstream of the subject site, utilising their location on the
78 river network relative to the subject site (primarily determined by similarity of catchment
79 area) could significantly enhance the performance of the data transfer methodology. Other
80 researchers [*Skøien et al.*, 2005; *Guse et al.*, 2009; *Ganora et al.*, 2013] have also reported
81 benefits when including river network geometry in flood frequency regionalization studies

82 By studying the error structure of a hydrological regression model, *Kjeldsen and Jones*
83 [2009, 2010] developed an optimal procedure for transferring data from a single gauged
84 site to an ungauged subject site utilising a functional relationship between the spatial
85 correlation of regression model errors and the geographical distance between catchment
86 centroids. Here, optimality is defined as minimising the prediction variance of the adjusted
87 estimates of the index flood at the ungauged sites. *Kjeldsen and Jones* [2010] found that
88 the prediction error obtained when using estimates derived from the non-optimal data
89 transfer procedure presented by the *Institute of Hydrology* [1999] in the Flood Estimation
90 Handbook (FEH) is about twice as large as the optimal data transfer procedure. The

91 FEH method is to choose "similar" catchments as donors, and these are not necessarily
 92 geographically close.

93 This paper presents a novel and generalised method for adjusting regression-based es-
 94 timates of the index flood at ungauged sites in the UK using data transfer from multiple
 95 gauged donor sites. The analytical results are also extended to develop a new performance
 96 metric for the evaluation of regional models that filters out the effect of at-site sampling
 97 noise. Based on a case study from the UK considering an existing model for predicting
 98 an index flood in ungauged catchments, the benefit of using data transfer from gauged
 99 donor catchments is illustrated and quantified.

2. A framework for data transfer from multiple sites

100 The optimal data transfer procedure is intimately related to the structure of the regional
 101 regression model used for predicting the index flood based on catchment descriptors only.
 102 Thus, before discussing the data transfer procedure, the details of the regression model
 103 are described.

2.1. A regression model for predicting the index flood at ungauged sites

104 Consider a region where annual maximum series (AMS) of peak flow events are available
 105 from $i = 1, \dots, n$ different gauged catchments. The median of each individual AMS is
 106 denoted m_i and the corresponding log-transformed value is y_i . Following *Stedinger and*
 107 *Tasker* [1985] the sample estimate of y_i can be written in terms of a regression model as

$$y_i = \mathbf{x}_i^T \boldsymbol{\theta} + \eta_i + \varepsilon_i = \xi_i + \varepsilon_i \quad (1)$$

where subscript i refers to catchment number, ξ_i is the true (but unknown) value of the log-transformed median, ε_i is the sampling error of the log-transformed index flood (y_i) and is assumed to be normally distributed with mean zero and a covariance matrix Σ_ε . The value of individual elements of Σ_ε depends on: i) record-length, ii) the assumed distribution of the AMS series and, iii) for non-diagonal elements, the distance between catchment centroids and length of overlapping record. An example of the sampling covariance matrix Σ_ε of the log-transformed median annual maximum flood can be found in *Kjeldsen and Jones* [2009]. The regression model parameters are denoted by θ , and \mathbf{x}_i is a $(q + 1)$ vector of q catchment descriptors for the i th site with a value of one in the first location. The term $\mathbf{x}_i^T \theta + \eta_i$ is then the true value expressed as a linear regression model based on catchment descriptors plus a model error terms due to the inability of a simple regression model to represent complex basin hydrology. The model error, η , is assumed to have zero mean and the elements of the covariance matrix Σ_η are defined as

$$\Sigma_{\eta,ij} = \text{cov}(\eta_i, \eta_j) = \begin{cases} \sigma_\eta^2 & i = j \\ \sigma_\eta^2 r_{\eta,ij} & i \neq j \end{cases} \quad (2)$$

where $r_{\eta,ij}$ is the correlation between model errors which *Kjeldsen and Jones* [2009] related to the geographical distance between catchment centroids, d_{ij} , as

$$r_{\eta,ij} = \varphi_1 \exp[-\varphi_2 d_{ij}] + (1 - \varphi_1) \exp[-\varphi_3 d_{ij}] \quad (3)$$

108 where φ_1 , φ_2 , and φ_3 are model parameters that must be estimated along with the
 109 regression model parameters θ and the model error variance σ_η^2 . The model and sampling
 110 errors are assumed to be mutually independent but cross correlated within each set, and
 111 the spatial correlation of the model errors is a key part of the data transfer scheme. The
 112 introduction of correlated model errors is an extension to the GLS model presented by

113 *Stedinger and Tasker* [1985], who assumed negligible or no correlation between model
 114 errors. *Kjeldsen and Jones* [2009] provide a more detailed discussion of the difference
 115 between the two error types.

116 In the next section a new procedure will be developed which allows for the estimates of
 117 the index flood obtained using a regional model, such as Eq. (1), to be moderated using
 118 local data from neighbouring sites.

2.2. Optimal data transfer from multiple sites

119 Suppose that a set of estimated regression model parameters, $\hat{\theta}$, such as those in Table
 120 1, is available which enables the prediction of the log-transformed index flood, y , at any
 121 site in the region of interest. The estimate \hat{y}_s is given as

$$\hat{y}_s = \mathbf{x}_s^T \hat{\theta} \quad (4)$$

122 where the subscript s indicates a specific site, and the estimate in Eq.(4) therefore
 123 constitutes the regression-only estimate at a specific site. Subsequently, this first estimate
 124 should be adjusted by using data transfer from multiple donors. The regression-only
 125 estimate at the site of interest, \hat{y}_s from Eq. (4), is adjusted using residuals from p nearby
 126 and gauged donor sites as a weighted average: a weight α (to be determined) is applied
 127 to the regression residual $(y_i - \hat{y}_i)$ at each of the p sites. The resulting adjusted estimate
 128 is denoted \tilde{y}_s and is derived as

$$\tilde{y}_s = \hat{y}_s + \sum_{i=1}^p \alpha_i (y_i - \hat{y}_i)$$

$$\begin{aligned}
&= \underbrace{\mathbf{x}_s^T \hat{\theta}}_{\hat{y}_s} + \sum_{i=1}^p \alpha_i \left(\underbrace{\mathbf{x}_i^T \theta + \eta_i + \varepsilon_i}_{y_i} - \underbrace{\mathbf{x}_i^T \hat{\theta}}_{\hat{y}_i} \right) \\
&= \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^p \alpha_i \left(\mathbf{x}_i^T (\theta - \hat{\theta}) + \eta_i + \varepsilon_i \right)
\end{aligned} \tag{5}$$

129 The set of weights α_i is specified so as to ensure the prediction error variance of \tilde{y}_s
130 is as small as possible. Note that there is no requirement for the weights α_i to sum to
131 unity. For example, if only one donor site is used ($p = 1$) then the weight should decrease
132 towards zero as the distance between the donor and the subject site increases [*Kjeldsen*
133 *and Jones*, 2010]. Given that a reasonably large number of gauged sites is used in the
134 estimation of the regression model parameters, a reasonable assumption is $\theta \approx \hat{\theta}$, which
135 reduces the complexity of the equation above to

$$\tilde{y}_s \approx \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^p \alpha_i (\eta_i + \varepsilon_i). \tag{6}$$

136 The prediction error of \tilde{y}_s is denoted e_s and will be derived by utilising the fact that
137 the true value at the subject site, ξ_s , is defined as $\xi_s = \mathbf{x}_s^T \theta + \eta_s$ as discussed previously
138 (Eq. 1). Thus, e_s is defined as

$$\begin{aligned}
e_s &= \tilde{y}_s - \xi_s \\
&= \underbrace{\mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^p \alpha_i (\eta_i + \varepsilon_i)}_{\tilde{y}_s} - \xi_s \\
&= \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^p \alpha_i (\eta_i + \varepsilon_i) - \mathbf{x}_s^T \theta - \eta_s \\
&\approx -\eta_s + \sum_{i=1}^p \alpha_i (\eta_i + \varepsilon_i) = -\eta_s + \alpha^T (\eta + \varepsilon)
\end{aligned} \tag{7}$$

139 which, as above, assumes that $\theta \approx \hat{\theta}$. The vector α contains the weights assigned to
 140 each of the p donor sites, and similarly η and ε denote vectors of the model and sampling
 141 errors. Next, the variance of this prediction error e_s is defined as

$$\begin{aligned} \text{var}(e_s) &= \text{var} \left\{ -\eta_s + \alpha^T (\eta + \varepsilon) \right\} \\ &= \sigma_\eta^2 + \alpha^T \text{var}(\eta + \varepsilon) \alpha - 2\alpha^T \mathbf{b} \\ &= \sigma_\eta^2 + \alpha^T \mathbf{\Omega} \alpha - 2\mathbf{b}^T \alpha \end{aligned} \quad (8)$$

142 The $p \times p$ covariance matrix of the combined error terms $(\eta_i + \varepsilon_i)$ is denoted by $\mathbf{\Omega}$.
 143 The vector containing the covariance between the model error at the subject and each
 144 of the donor sites $\text{cov}(\eta_s, \eta_i)$ is denoted by \mathbf{b} where the p elements are obtained directly
 145 from Eq. (3) considering the geometric distance, d_{ij} , between the subject site s and
 146 each of the $i = 1, \dots, p$ donor sites. In deriving Eq (8), independence between sampling
 147 and model errors is assumed. Next, the minimum prediction variance is found through
 148 straight-forward differentiation of Eq.(8) with respect to the weights α as

$$\frac{\partial \text{var}(e_s)}{\partial \alpha} = 0 \Rightarrow \alpha^T (\mathbf{\Omega}^T + \mathbf{\Omega}) - 2\mathbf{b}^T = 0 \quad (9)$$

149 As $\mathbf{\Omega}$ is symmetric, the optimal set of weights α can finally be derived by isolating α in
 150 Eq. (9) as

$$\alpha = \mathbf{\Omega}^{-1} \mathbf{b} \quad (10)$$

151 If only one donor site is selected, the solution to Eq.(10) reduces to the corresponding
 152 analytical solution for a single donor presented by *Kjeldsen and Jones* [2010]. Thus, the

153 result obtained in Eq.(10) constitutes a new and more general framework for including
 154 local information into the estimation of the index flood variable at ungauged sites.

155 Given that the variance of the model errors are typically much larger than the variance
 156 of the sampling errors, it is likely that the weights will primarily be determined by the
 157 model errors (i.e. how well the regression model describes the data) rather than the
 158 sampling errors.

3. A revised performance measure

159 A common approach for assessing the performance of a regional method is the mean
 160 squared error, or root mean squared error (RMSE), based on the sum of squared errors,
 161 S . The version, S_C that compares the regional estimate with the sample estimate for each
 162 of the n gauged site in the region in turn is

$$S_C = \sum_{s=1}^n (\tilde{y}_s - y_s)^2 = (\tilde{\mathbf{y}} - \mathbf{y})^T (\tilde{\mathbf{y}} - \mathbf{y}) \quad (11)$$

163 where $(\tilde{\mathbf{y}} - \mathbf{y})$ is a vector containing the n residuals. However, the at-site sample esti-
 164 mates are themselves only best estimates of the true values, ξ , and thus what would be
 165 more interesting to know is the value of

$$S_T = \sum_{s=1}^n (\tilde{y}_s - \xi_s)^2 = (\tilde{\mathbf{y}} - \xi)^T (\tilde{\mathbf{y}} - \xi) \quad (12)$$

166 In the following, the relationship between S_C and S_T will be derived, allowing an esti-
 167 mate of S_T rather than S_C to be used for assessing the performance of the data transfer
 168 method. As before, the difference between θ and $\hat{\theta}$ is ignored (i.e. $\theta \approx \hat{\theta}$) which is the
 169 same as ignoring the sampling error of θ , a reasonable assumption when a large number

of sites are included in the regression analysis. This analysis starts from a slightly revised version of the earlier expression in Eq.(6), where the adjustment to the index flood estimate is represented here as a weighted sum of the residuals from all n sites in the national dataset. This leads to Eq.(6) being replaced by

$$\tilde{y}_s = \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^n \alpha_{si} (y_i - \hat{y}_i) \quad (13)$$

but where the weights α_{si} are mostly zero except for the p sites chosen for the data transfer for the site s . The difference between the donor adjusted estimate and the at-site estimate and true value, respectively, can be expressed by combining Eqs.(1) and (4) with the assumptions listed above. First, consider the difference between the adjusted estimate (\tilde{y}_s , Eq.13) and the at-site sampling value, (y_s , Eq.1):

$$\begin{aligned} \tilde{y}_s - y_s &= \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^n \alpha_i (y_i - \hat{y}_i) - y_s \\ &= \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^n \alpha_i \left(\underbrace{\mathbf{x}_i^T \theta + \eta_i + \varepsilon_i}_{y_i} - \underbrace{\mathbf{x}_i^T \hat{\theta}}_{\hat{y}_i} \right) \\ &\quad - \underbrace{(\mathbf{x}_s^T \hat{\theta} + \eta_s + \varepsilon_s)}_{y_s} \\ &= \sum_{i=1}^n \alpha_i (\eta_i + \varepsilon_i) - (\eta_s + \varepsilon_s) \end{aligned} \quad (14)$$

which in a vector format considering all n sites is given as

$$\hat{\mathbf{y}} - \mathbf{y} = (\mathbf{A} - \mathbf{I}) \boldsymbol{\eta} + (\mathbf{A} - \mathbf{I}) \boldsymbol{\varepsilon} \quad (15)$$

180 where \mathbf{A} is a matrix of the weights α_{si} , \mathbf{I} is an identity matrix, and η and ε are $(n \times 1)$
 181 vectors of model and sampling errors, respectively. Analogous to Eq.(7) the difference
 182 between the adjusted estimate and the true value is derived as

$$\begin{aligned}
 \tilde{y}_s - \xi_s &= \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^n \alpha_{si} (y_i - \hat{y}_i) - \xi_s \\
 &= \mathbf{x}_s^T \hat{\theta} + \sum_{i=1}^n \alpha_{si} \left(\underbrace{\mathbf{x}_i^T \theta + \eta_i + \varepsilon_i}_{y_i} - \underbrace{\mathbf{x}_i^T \hat{\theta}}_{\hat{y}_i} \right) \\
 &\quad - \underbrace{(\mathbf{x}_s^T \hat{\theta} + \eta_s)}_{\xi_s} \\
 &= \sum_{i=1}^n \alpha_{si} (\eta_i + \varepsilon_i) - (\eta_s)
 \end{aligned} \tag{16}$$

183 Again, the corresponding vector notation considering all n sites simultaneously is

$$\hat{\mathbf{y}} - \xi = (\mathbf{A} - \mathbf{I}) \eta + \mathbf{A} \varepsilon \tag{17}$$

184 Considering the two different definitions of residuals in Eqs. (14) and (16) it can be
 185 seen that comparing the predictions of the index flood with the observations directly
 186 (Eq. 14) will contaminate the residuals with a sampling error, ε_s of the observed index
 187 flood. The result is that performance measures such as RMSE will be inflated by this
 188 sampling error. As the sampling error depends on the record-length of the at-site record,
 189 the RMSE will also become a function of record-length and it is not possible to ascertain
 190 how much of the difference between predictions and observations is caused by model
 191 deficiency and how much is down to sampling noise in the at-site observations. However,
 192 the framework developed here allows for a revised performance measure to be developed
 193 where the influence of the at-site sampling noise is removed.

194 From the matrix representation of the differences in Eqs.(15) and (17), the sum of
 195 squares defined in Eqs.(11) and (12) can now equally be written in matrix form as

$$S_C = \eta^T (\mathbf{A} - \mathbf{I})^T (\mathbf{A} - \mathbf{I}) \eta + \varepsilon^T (\mathbf{A} - \mathbf{I})^T (\mathbf{A} - \mathbf{I}) \varepsilon - 2\eta^T (\mathbf{A} - \mathbf{I})^T (\mathbf{A} - \mathbf{I}) \varepsilon \quad (18)$$

$$S_T = \eta^T (\mathbf{A} - \mathbf{I})^T (\mathbf{A} - \mathbf{I}) \eta + \varepsilon^T \mathbf{A}^T \mathbf{A} \varepsilon - 2\eta^T (\mathbf{A} - \mathbf{I})^T \mathbf{A} \varepsilon \quad (19)$$

197 By subtracting the two expression in Eq.(18) and Eq.(19) the following relationship
 198 between S_C and S_T is obtained

$$S_C - S_T = 2\eta^T (\mathbf{A} - \mathbf{I})^T \varepsilon + \varepsilon^T (\mathbf{I} - \mathbf{A}^T - \mathbf{A}) \varepsilon \quad (20)$$

199 Next, the mean value of the difference between the two sums of squares is

$$\begin{aligned} E\{S_C - S_T\} &= E\{2\eta^T (\mathbf{A} - \mathbf{I})^T \varepsilon + \varepsilon^T (\mathbf{I} - \mathbf{A}^T - \mathbf{A}) \varepsilon\} \\ &= 0 + E\{\varepsilon^T (\mathbf{I} - \mathbf{A}^T - \mathbf{A}) \varepsilon\} \\ &= \text{tr}\{(\mathbf{I} - \mathbf{A}^T - \mathbf{A}) \Sigma_\varepsilon\} \\ &= \text{tr}\{\Sigma_\varepsilon\} - 2\text{tr}\{\mathbf{A} \Sigma_\varepsilon\} \end{aligned} \quad (21)$$

where tr is a trace function and, as previously, the model and sampling errors are assumed to be independent. For computational convenience, Eq.(21) is written as sums, i.e.

$$E\{S_C - S_T\} = \sum_{s=1}^n \Sigma_{\varepsilon,ss} - 2 \sum_{s=1}^n \sum_{i=1}^n a_{si} \Sigma_{\varepsilon,si} \quad (22)$$

200 The term in Eq.(22) can be considered a bias-correction term to be subtracted from the

201 calculated sum of squares, S_C to obtain an improved estimate of S_T denoted \hat{S}_T

4. A case study using UK data and methods

202 The method developed above for using multiple donor sites to adjust a regression es-
203 timate for an ungauged site was tested using annual maximum series of instantaneous
204 peak flow available at 602 non-urban catchments located throughout the UK as shown in
205 Figure 1. The dataset is part of the Hiflows-UK dataset, consisting of peak flow series
206 where the gauging authorities have sufficient confidence in the rating curves that these
207 stations can be recommended for use in flood studies.

208 FIGURE 1

209 Using the same dataset, *Kjeldsen and Jones* [2009] adopted a maximum-likelihood
210 method to estimate the regression model parameters as well as the parameters in Eq.(3)
211 controlling the relationship between model error correlation and distance between catch-
212 ment centroids, and the results are replicated in Table 1.

213 TABLE 1:

214 The four catchment descriptors used in the regression model are catchment area (AREA)
215 in km², standard annual average rainfall as measured from 1961-1990 (SAAR) in mm, an
216 index of flood attenuation due to online reservoirs and lakes (FARL) which takes values
217 between zero and one, (where a value of one indicates no attenuation), and BFIHOST
218 which is related to the hydrological properties of the catchment soils and can take values
219 between zero (impermeable) and one (completely permeable). Each of these descriptors
220 is transformed as shown in Table 1. A detailed description of the model development and
221 performance is reported in *Kjeldsen and Jones* [2009] and not repeated here.

222 The relative performance of the proposed donor transfer method was initially inves-
 223 tigated by comparing the performance for predicting in ungauged catchments based on
 224 three different cases:

- 225 1. Using the regression model only
- 226 2. Identify the geographically nearest gauged catchments
- 227 3. Identify donors connected to the subject site by being located on the same river
 228 network

229 Finally, an investigation was conducted into the relative value of the donor transfer
 230 method using local data by contrasting the performance to that of a number of existing
 231 regional models of varying complexity.

For all investigations the performance of the models will be reported in terms of the factorial standard error (fse), which is defined as the exponential of the standard error as derived from the sum of squared errors as

$$\text{fse} = \exp \left(\sqrt{\frac{S}{m - (q + 1)}} \right) \quad (23)$$

232 where S is the sum of square errors defined either as S_C from Eq.(18) or the bias
 233 corrected version S_T , m is the total number of catchments, and q is the number of catch-
 234 ment descriptors in the regional regression model. The RMSE is defined from Eq.(23)
 235 as $\text{RMSE} = \log(\text{fse})$, but the advantage of the fse measure is that it can more easily be
 236 translated into confidence intervals than the RMSE measure, e.g. *Kjeldsen* [2014].

4.1. Nearest geographical neighbours

237 For the dataset at hand, the potential number of donors for each subject site is 601
 238 when selecting based on geographical distance only. Evaluating the performance of the

239 adjustment procedure, the performance criteria $fse_C = \exp\left(\sqrt{S_C/(602 - 5)}\right)$ and $fse_T =$
 240 $\exp\left(\sqrt{\hat{S}_T/(602 - 5)}\right)$ were both evaluated as a function of the number of donor sites
 241 used to adjust the regression-only estimates of the index flood at the subject sites. The
 242 results are plotted as a function of the number of donor-sites in Figure 2 for a range of
 243 different cases: i) the regression-only estimates (no donor adjustment) , ii) donor-adjusted
 244 estimates with and without neglecting sampling errors when calculating weights using the
 245 matrix $\mathbf{\Omega}$ in Eq. (10), and iii) donor-adjusted estimates using the revised performance
 246 measure in combination with the full solution.

247 FIGURE 2

248 For the dataset used in this study, a minimum level of fse (RMSE) values was reached
 249 when using 6 or more donors, and the corresponding fse (RMSE) values are reported in
 250 Table 2 below. It is also evident from the Figure that in the case presented here, the
 251 omission of the sampling errors when calculating the weight assigned to each donor site
 252 has a relatively minor effect on the performance.

253 TABLE 2

254 It is clear that the inclusion of multiple donors is beneficial in terms of reducing the pre-
 255 diction variance. The results suggest that the prediction accuracy is relatively insensitive
 256 to the actual number of donors, as long as this number exceeds about five.

4.2. Drainage network structure

257 Results presented by *Morris* [2003], *Skøien et al.* [2005], *Guse et al.* [2009] and *Ganora*
 258 *et al.* [2013] suggest that the connectivity of the site of interest and different gauging
 259 stations as determined by the river network structure can be an important source of
 260 information when estimating the index flood at ungauged sites. Thus, an assessment of

261 the benefit from selecting donor sites connected to the target site via the river network
262 was undertaken. Based on the available dataset of 602 AMS, river network based donors
263 were selected by first searching downstream from the subject site until a gauging station
264 was encountered or the sea was reached, and then searching in an upstream direction up
265 every tributary in turn, in each case stopping when a gauging station was encountered
266 or the head of the tributary was reached. A maximum of seven donors was allowed as
267 this was considered to be fully adequate for the needs of the original FEH statistical
268 procedures. There are only ten UK river reaches where the number of eligible gauges
269 exceeds seven, and here the preferred upstream donors are those whose catchment area is
270 closest to that of the subject site. At sites with fewer than seven network-based donors,
271 additional donors are selected on the basis of proximity, provided that their catchments
272 are sufficiently similar to that of the subject site. Because of the need for catchment
273 similarity, not all of the subject sites have seven donors. Of the 602 locations in this
274 study, 29 had no donors and 53 had between one and five donors.

275 TABLE 3:

276 Comparing the results in Table 3 with the corresponding results in Table 2 obtained
277 without consideration of the river network structure, it can be observed that the resulting
278 RMSE and fse values are generally reduced slightly when incorporating information on
279 river network structure, especially when not considering the influence of the sampling
280 errors of the at-site estimates. Removing the error contribution from the at-site samples,
281 the resulting RMSE values are almost similar, suggesting that the actual network structure
282 does not add additional information over and above that already contained in the distance
283 between catchment centroids. This conclusion is derived from analysis of AMAX data, and

284 it is of course possible that analysis of other types of hydrological data and indices would
285 provide more insight into the role of the drainage network geometry and its influence on
286 runoff processes.

4.3. A regional model versus local data

287 A comprehensive set of catchment descriptors are available for each of the catchments
288 included in the UK case study. However, other regions might have only a smaller subset of
289 descriptors available, and in this section the value of local data will be evaluated for two
290 cases of catchment descriptor availability. The performance of the data transfer scheme
291 developed in this study will be compared when combined with a full regional model, i.e.
292 the model in Table 1 based on four different catchment descriptors (AREA, SAAR, FARL
293 and BFIHOST) with a fse value equal to 1.43, and a second more simple model using
294 catchment area (AREA) only as a covariate and a fse value of 2.76. The catchment area
295 (AREA) only model was developed previously by *Kjeldsen and Jones* [2009]. The two
296 models were contrasted in *Kjeldsen and Jones* [2010] who showed that the omission of
297 catchment descriptors in the simple model resulted in a much higher degree of model
298 error correlation. Based on these results it was argued that the existence of high model
299 error correlation increases the value of data transfer as a compensation of the lack of
300 explanatory power of the simple regression model.

301 Figure 3 shows a comparison between the fse values obtained using the full and the
302 simple regression model, respectively, in combination with the data transfer scheme de-
303 veloped in this study. A third model based on AREA and SAAR is also shown, and this
304 model has an fse value of 1.92 [*Kjeldsen and Jones*, 2009].

305 FIGURE 3

306 The horizontal lines represent the fse value for the three regression models, i.e. using
307 only catchment descriptors but without the use of data transfer. Clearly the fse of the
308 simple AREA only model is much higher than for the two more complex models, showing
309 the importance of additional catchment descriptors beyond catchment area for describing
310 the between-catchment variation in the index flood.

311 When incorporating local data through use of the data transfer scheme, the drop in
312 fse observed in connection with the simple AREA only model is much larger than the
313 corresponding reduction in fse when using the full model. In fact, the fse values obtained
314 using the AREA only model in combination with data transfer are lower than the fse values
315 obtained when using a more complex regional model based on both AREA and SAAR
316 without data transfer. However, even with data transfer, the fse values obtained for the
317 simple AREA-only model do not reach the low level associated with the four-descriptor
318 model without data transfer. Nonetheless, these results suggest that there is potentially a
319 large gain to be had in predictive power when using data transfer in combination with a
320 simple regional model developed using catchment area only.

321 A more detailed assessment of the link between the complexity of the regional model
322 and the benefit of data transfer can be made by studying more closely the behaviour of the
323 differences between the log-transformed at-site values, and corresponding log-transformed
324 values predicted using the regionalised models with and without data transfer (residuals).
325 The top row of Figure 4 shows the residuals (grey points) of the full regression model
326 plotted against the four catchment descriptor values used in the model (AREA, SAAR,
327 FARL and BFIHOST), and the bottom row shows the corresponding residuals from the
328 area-only model. Note that the scale of the y -axis is the same in both rows to better

329 enable the comparison. In each of the eight plots the spread of the residuals is indicated
330 by a convex hull spanning 95% of all the points (solid line). A second convex hull (dashed
331 line) spans 95% of the second set of residuals obtained when comparing at-site values
332 with estimates obtained by using data transfer with five donor sites. For this second set
333 of residuals only the convex hull is shown but not the actual points. For both sets of
334 residuals the 95% convex hull was chosen to limit the influence of outliers on a visual
335 assessment of the general behaviour of the residuals.

336 FIGURE 4 here

337 Considering first the residuals of the full model in the top row, the second convex hull
338 (dashed line) generally has the same shape and occupies an area that is marginally smaller
339 than the convex hull spanning the regression model only residuals (solid line). This is in
340 line with the results shown in Figure 3, and suggests that the utility of data transfer is
341 more limited when the regression model accounts for the differences in the index flood
342 values observed in different types of catchments. In contrast, the difference between the
343 two convex hulls is much more pronounced when considering the simpler area-only model
344 (bottom row). Firstly, the area spanned by the residuals obtained using data transfer is
345 much smaller than the residuals from the regression model only and more similar to the
346 shape observed for the full model in the top row. Secondly, there are clear structures
347 in the regression-only residuals (grey points), especially when plotted against SAAR and
348 BFIHOST, with underestimation in wet and impermeable catchments and, conversely,
349 overestimation in dry and more permeable catchments. However, using local flood records
350 largely removes this stratification from the residuals. For FARL (reservoirs and lakes) the
351 results are less clear, but this is caused by the relatively limited number of catchments

352 with low FARL values (stronger influence of upstream reservoirs and lakes) in the dataset.
353 These results show that using data transfer can (i) improve the performance of a simple
354 regression model, and (ii) effectively remove most of the structure in the performance of
355 regional models caused by not incorporating key catchment descriptors into the model.

5. Discussion and conclusion

356 The procedure outlined in this study is a formal framework that will enable hydrologists
357 to incorporate local data when estimating an index flood variable at an ungauged site
358 using a generalised regional procedure. The adjusted estimates are shown to have a
359 lower fse than those obtained from regression models only. This is particularly important
360 in regions where the available lumped catchment descriptors cannot be considered to
361 adequately capture local flood controlling processes and mechanisms. The methodology
362 was developed for use with index flood estimates in the UK, where the optimal number
363 of donors was found to be five or more.

364 The study also presented a generalized procedure for removing the effects of sampling
365 error from RMSE (fse) and hence the effects of sample length available at each site. This
366 is potentially an important result as it will allow comparison of performance of methods
367 between datasets from different regions.

368 Interestingly, the derivations presented in this study and in *Kjeldsen and Jones* [2010]
369 show that if the correlation between model errors is neglected (i.e. $r_{\eta,ij} = 0$ in Eq.3) and
370 not considered as part of the initial construction of the hydrological regression model, then
371 there is seemingly no benefit associated with transfer of data to an ungauged catchment
372 from nearby gauged catchments (as the regression model is then assumed to explain all
373 between-catchment variation). In most practical settings this would be an untenable

374 position, and thus model error correlation should be considered an integral part of models
375 attempting to predict hydrological variables in ungauged catchments.

376 Comparing the performance of regional regression models (in terms of predictive ability)
377 when combined with data transfer from donor catchments showed that the benefit of data
378 transfer in ungauged catchments, when used in combination with a simple regional model
379 using catchment area only outperforms the more advanced regional model using both
380 catchment area and mean annual rainfall as covariates if data transfer is not used. Thus,
381 careful consideration of model error correlation in the model building phase can help
382 to address poor model performance originating from access to only a limited subset of
383 catchment descriptors. As such, the proposed method is considered a valuable addition
384 to the toolbox available for regional hydrological models.

385 The results obtained in this study are based on flood data from a relatively dense
386 gauging network in the UK. A case study by *Mediero and Kjeldsen* [2014] using the
387 GLS framework to develop a model linking at-site estimates of a 100-year design flood to
388 catchment descriptors in a north-east Spain identified model error correlation, suggesting
389 that the method might also be useful in other geographical regions. However, further
390 research is needed to verify the extent to which these conclusions are valid. In particular,
391 the influence of reservoirs and lakes on flood characteristics should be further examined.
392 Finally, this study considered only the index flood variable, but similar analysis could
393 have been undertaken in connection with any hydrological variable where a regionalisation
394 model is the basis for prediction in ungauged catchments, including statistical moments
395 of high and low flow series as well as parameters in rainfall-runoff models.

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Table 1. Summary statistics for regression model linking the log median annual maximum peak flow to catchment descriptors (from Kjeldsen and Jones, 2009).

Coefficient	Parameter	St. error	t-value	p-value
Intercept	2.1170	0.1172	18.06	0.000
Ln[AREA]	0.8510	0.0114	74.35	0.000
(SAAR/1000) ⁻¹	-1.8734	0.0968	-19.35	0.000
Ln[FARL]	3.4451	0.2654	12.98	0.000
BFIHOST ²	-3.080	0.1158	-26.60	0.000

Table 2. RMSE (fse) when using 6 donor sites for estimating the index flood at each target site.

Incl. sampling errors in Ω	Apply bias correction?	RMSE	fse
Yes	No	0.311	1.365
Yes	Yes	0.303	1.354
No	No	0.312	1.366
No	Yes	0.304	1.355

Table 3. RMSE and fse when selecting donors site based on shared network structure.

Incl. sampling errors in Ω	Apply bias correction?	RMSE	fse
Yes	No	0.308	1.360
Yes	Yes	0.304	1.356
No	No	0.308	1.360
No	Yes	0.304	1.355

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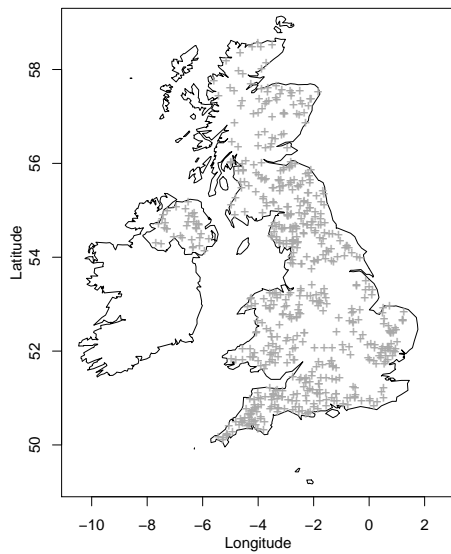


Figure 1. Location of the 602 gauging stations used in this study.

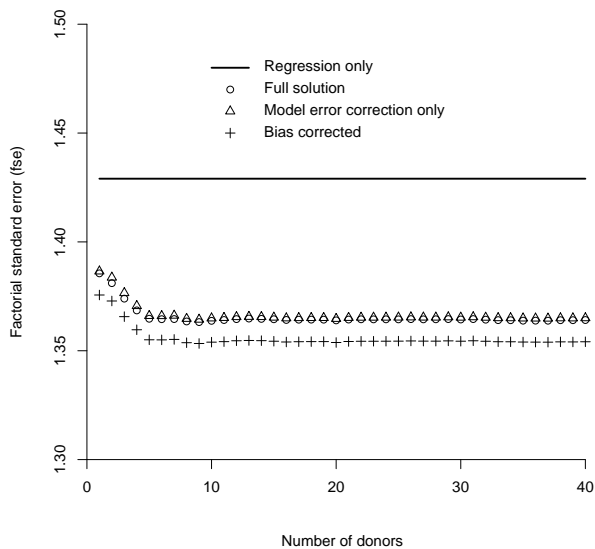


Figure 2. Factorial standard error (fse) plotted as a function of the number of donor sites for i) the regression-only estimates (no donor adjustment), ii) donor-adjusted estimates with and without neglecting sampling errors when calculating weights, and iii) donor-adjusted estimates with and without using the revised performance measure.

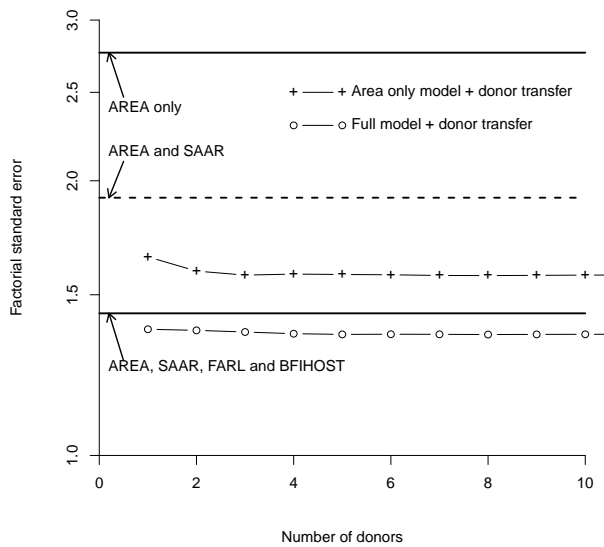


Figure 3. Factorial standard error (fse) plotted as a function of the number of donor sites for the regression-only estimates (no donor adjustment) and donor-adjusted estimates for i) the full model and ii) the simple catchment area only model. The fse for an intermediate regression-only model based on catchment area and mean annual rainfall is shown as a dashed line.

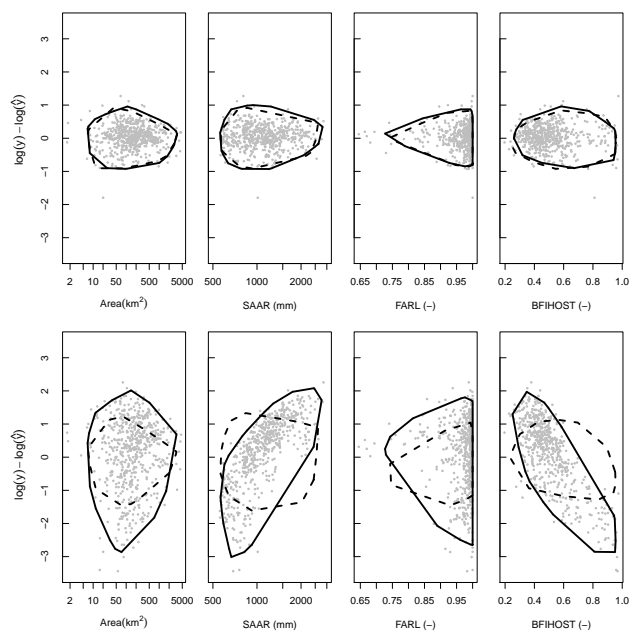


Figure 4. Residuals plotted against key catchment descriptors. 95% convex hull for regression model residuals (solid line - points shown in grey), and residuals obtained using regression model with data transfer from six donor sites (hatched line - points not shown). Top row shows results for full model and bottom row shows results for area-only model.