Numerical modelling of masonry arch bridges: investigation of spandrel wall failure

submitted by

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of the

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Junzhe Wang
Abstract

Masonry arch bridges still play an important role in the transportation infrastructure today in the United Kingdom. Previous research has mainly focused on the load carrying capacity in the span direction. The three dimensional behaviour is often investigated by simplifying into two dimensions with modified arch parameters but these simplified analyses cannot represent all aspects of behaviour. Spandrel wall failure in some railway masonry arch bridges has raised concerns recently, and this is one aspect which cannot be modelled in two dimensions. This thesis presents a research which attempts to model the interaction behaviour between arch, backfill and spandrel wall with the aim of representing the three dimensional nature of real bridges. It mainly focuses on the spandrel wall defects under increasing load, including crack development across the wall and longitudinal cracks in the arch barrel underneath spandrel wall.

Experimental results from the laboratory tests on engineering blue brick and a hydraulic premixed mortar as well as brickwork masonry specimens are presented. Numerical analysis was initially performed on these brickwork masonry specimens for validation. Three dimensional FE models were proposed for both small and large scale bridges. The general behaviour of the small scale bridge under rolling load and large scale bridge under increasing load were studied. Reasonable agreement between the FE analyses and experimental results from previous literature was obtained, indicating the model validated for small masonry specimens could be scaled up to full-scale bridges.

A series of computer models were constructed to investigate the relationship between a range of geometric and material parameters and the lateral behaviour of arch bridges. The backfill depth and spandrel wall thickness have greatest impact on both bridge strength and lateral behaviour. The fill properties also have an importance influence on the load carrying capacity. This provides an indication of which bridge should be more closely monitored for spandrel wall defects. Separate FE models was constructed to simulate existing longitudinal cracks found in the arch barrel for old bridges and the influence of strengthening of spandrel wall with tie bars. The general behaviour under a concentrated load is studied and discussed. It has been demonstrated that it is possible to effectively model the three dimension behaviour of masonry arch bridges and in particular, spandrel wall failures.
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Chapter 1

Introduction

1.1 Background

Masonry arches are one of the earliest structural forms: stone and brick arches have been used for thousands of years as parts of bridges, cathedrals, castles, etc. It is widely accepted that the use of the arch as a structural form developed independently in China and Middle East more than 5000 years ago (Van beek, 1987), and masonry arch bridges still play a significant role in the transportation infrastructure today in the United Kingdom (Figure 1-1). The majority of arch bridges were constructed between the 17th and 19th centuries, while in the 18th century there were important developments in British bridge design such as the use of relieving arches, and stiffened arch with longitudinal walls, which became common usage thereafter (McKibbins et al., 2006). It is estimated that there are over 40000 masonry arch bridges still in service in the UK at present and the continuing use of these bridges shows the durability of masonry structures. However, the gradual deterioration of materials over time and the increase in loading from modern road and rail vehicles make the assessment and maintenance of these structures important in order to ensure safety and serviceability. Masonry arch bridges are now carrying much heavier loads than their designers would have expected. The maximum allowable axial load was increased from 9.0 tons to 10.0 tons in 1984 under new European Commission Directives and has further increased to 11.5 tons since January 1999. A total replacement of the old bridges with modern bridges is almost impossible, and even strengthening is difficult because of local authority regulations and access.
As it is essential to have effective techniques for the assessment of masonry arch bridges to ensure safety and avoid wasted funds on replacement, several methods have been developed in recent decades for the assessment of load carrying capacity of masonry arch bridges.

The two traditional methods of analysis are either the Military Experimental Establishment (MEXE) method or the mechanism method (Harvey, 1988). The detailed procedures of MEXE method can be found in Department of Transport standard (2001) which is part of the Design Manual for Road and Bridges. In recent years, with the wide applications of computers in engineering, several numerical methods have been developed which could be used for the analysis of masonry arch bridges, such as rigid block analysis, the finite element method.
(FEM) and the discrete element method (DEM). The details of these assessment methods and their application and deficiencies are discussed in Chapter 2. Many small scale and large scale tests have been carried out on masonry arches to obtain a better understanding of the behaviour of such structures. However, the majority of previous research has focused on the carrying capacity of the arch barrel in the span direction with little research effort on spandrel wall failure problems. Although the results of a spandrel wall collapse may be very serious, the transverse behaviour of masonry arch bridges has not well understand.

In the UK context, rail bridges were largely constructed during Victorian times (between 1840s and 1900s) and comprise either stone or brick masonry with a weak lime mortar. Only a few new masonry arch bridges are built nowadays because the construction of masonry arch bridges requires a high standard of design and craftsmanship if they are to achieve a satisfactory appearance. Though the traffic loads are much heavier now than during historic use, it is rare for a masonry arch bridge to experience global collapse of the arch barrel. However many masonry arch bridges have experienced incidents over the last few years where spandrel wall failures have affected the track support (Sowden, 1990). Sometimes the spandrel wall may fail suddenly without indicating any precursors. There are generally two transverse effects which need to be considered in masonry arch bridges. One is the outward movement of spandrel wall under lateral earth pressure. The other is the transverse bending of the arch barrel under traffic load, which is constrained at the edge by relatively stiff spandrel wall. A survey carried out by Page et al. (1991) on 98 arch bridges showed 90% of the investigated bridges experienced a spandrel wall defect of some kind. Site visits have been conducted by the author on several masonry arch rail bridges, and there are generally two types of spandrel wall defects have been observed: one is the longitudinal cracks in the arch barrel underneath the spandrel wall (Figure 1-2(a)), which could lead to the separation between the spandrel wall and arch barrel. The other form of defects is cracks developed along mortar joints in the spandrel wall as shown in Figure 1-2(b). These types of spandrel wall defects have also confirmed by Page’s survey. Given the conditions that current assessment standards do not contain any guidance on quantitative evaluation and recommends a qualitative approach for bridges with spandrel defects, an investigation of the potential reasons for this type of failure is necessary in order to prolong the use of UK’s transport infrastructure.
1.2 Aims and Objectives

The aim of this research project is to improve fundamental understanding of the underlying mechanism(s) that lead to failure of the spandrel walls in masonry arch bridge structures, especially longitudinal cracks in the arch barrel and diagonal cracks in spandrel wall. The particular objectives of this thesis were:

- To review the current assessment methods for masonry arch bridges and discuss their deficiencies with respect to spandrel wall failure;
• To collect experimental data on material performance characteristics related to spandrel wall failures;

• To develop 3D numerical models to analyse the behaviour of masonry arch bridges including spandrel wall failure under service and ultimate load conditions.

• To obtain further understanding on the relationship between a range of geometric and material properties and lateral behaviour through parametric studies.

• To provide advice on further research and analysis into the performance of spandrel walls

1.3 Research approach

When considering the failure of spandrel wall in masonry arch bridge, it is generally accepted that the key issues lie in the complex three dimensional interactions under loading between the backfill, arch barrel structure and the spandrel walls. Because of the environmental limitations and the cost, it was not feasible to perform experimental tests on real masonry arch bridges. Numerical modelling techniques have been selected as the tool to analyse the behaviour of masonry arches under different load conditions. The work initially focuses on developing fundamental understanding of the contact behaviour between masonry units, mortar and backfill. Experimental testing was undertaken on brick and mortar units to provide essential material properties for the numerical analysis. 3D finite/discrete element models based on the commercial finite element package, ANSYS, have been utilised and the numerical models are to be supported by small scale lab tests, field observations (case studies) and monitoring data.

1.4 Layout of thesis

Chapter 1 provides a brief introduction to masonry arch bridges and the background information to the research project. It also presents the aims and objectives and provides a discussion about the proposed approach for the study.
Chapter 2 reviews published literature and is divided into four sections: overview of the history and construction of masonry arch bridges; analysis and assessment methods; experimental tests on masonry arch bridges; research into spandrel walls.

The experimental work on brick and mortar units as well as masonry specimens is presented in Chapter 3. It aims to provide essential information about material properties that can be used for the numerical modelling work described in later chapters.

In Chapter 4, a brief review of the modelling techniques for masonry structures is included. Numerical models were constructed for modelling of small masonry structures with validation purpose. The verification work consisted of comparing the analytical results for the masonry to results found during the material testing programme. Several modelling techniques were proposed and discussed in order to obtain better simulation of load displacement relationships of these structures.

In Chapter 5, the results of 3D finite element analysis of small/large scale masonry arch bridges are detailed. The finite element models were constructed using an available commercial finite element package and the results obtained from numerical analysis are presented and compared to experimental measurements. This includes the arch deflections and pressures on the extrados under rolling load for the small scale model bridge. The load displacement relationships for large scale bridge under increasing load and crack patterns in spandrel wall were investigated.

In Chapter 6, a series of computer models was constructed to investigate the relationship between a range of geometric and material parameters and the lateral behaviour of arch bridges. A separate finite element model was constructed to simulate the existing longitudinal cracks found in the arch barrel for old bridges and strengthening of spandrel wall with steel tie bars. The general behaviour under a concentrated load was studies and discussed.

Finally, Chapter 7 presents the main conclusions and achievements of the work, and it also gives a discussion about the recommendations for further research in this field.
Chapter 2

Literature review

This review of literature gives a brief introduction of the history construction process of masonry arch bridges. Different assessment methods for masonry arch bridges are reviewed together with their advantages, disadvantages and applications in practice. Former experimental tests on large/small scale models are discussed. Research findings related to the influence of spandrel walls on the behaviour of masonry arch bridges and different failure modes of spandrel walls are included in the last section.

2.1 Overview of masonry arch: history and construction

2.1.1 A brief history

It is generally accepted that masonry structures are one of the most durable structures in the world. It is difficult to tell when and where the first arch bridges were built but the use of arch as a structural form can date back to China and Middle East more than 5000 years ago (Van beek, 1987). Arches have been a popular structural form in China, and the most famous arch bridges still in use in China is the Zhao Zhou Bridge located in Hebei province (Figure 2-1). The bridge was built in Sui Dynasty about 1400 years ago with a span of 37.02 m and a span/rise ratio of 5.25. The design of this bridge shows good agreement with
modern design thinking and reflects the wisdom of ancient people. It has great importance in the development of bridge design theory in China (Qian, 1987).

Figure 2-1: Zhao Zhou Bridge in China (designed by Li Chun between 605-618 A.D.)

In Europe, the first flourish of arch bridge construction was during the government of the Roman Empire. Most of the bridges were built for military purposes, and one of the most famous and influential Roman bridges was the one over the Thames at Southwark in London (Cook, 1998). New materials and construction techniques were developed and many are still impressive even by today’s standards. During the Renaissance, bridge design experienced great advances as new geometry of arch bridges was introduced and much higher span/rise ratio could be achieved. In the UK, the 18th century was considered as a significant period for arch bridges since it saw a number of developments in bridge design. The Westminster Bridge (Figure 2-2), which was originally designed and built by Charles Labelys and rebuilt by Thomas Page in 1853 (Mare, 1975), introduced several innovations which influenced British bridge design and construction. After the construction of Westminster Bridge, a number of concepts were adopted in subsequent bridge design (Colla et al., 2002), such as a high span/pier thickness ratios, the use of relieving arches, and stiffened arch with longitudinal walls.

The concept of mass production of arch bridges developed in the second half of 18th century and made great contributions of the standardisation of construction. A large number of bridges with spans under 10 m were built in this period (McKibbins et al., 2006). At the same time, some impressive masonry bridges with multi spans or large spans were constructed, for example, the Wharncliffe viaduct and bridge over the Thames at Maidenhead which were
designed by Brunel in 19th century (Owen, 1976). In the 20th and 21st century, steel and concrete have taken the place of masonry in the construction of bridge. With the development of design theory for concrete and steel structures, masonry arch bridges are now rarely built as it requires a high standard of design and craftsmanship in order to achieve a satisfactory appearance. However, masonry arch bridges still play a vital part in the transport infrastructure of the UK and other countries, and their maintenance and repair are important for the continued economic development.

### 2.1.2 Construction of arch bridges

There are a variety of options in terms of arch geometry for construction, which include semicircular, pointed, segmental, parabolic, elliptical and combinations of circular segments. The main components of a typical masonry arch bridge are presented schematically in Figure 2-3. The arch barrel is the main structural element and is placed on the abutments. It usually consists of several rings for brick arches or one or two rings for stone arches. The spandrel wall lies on each side of arch barrel. Its main function is to retain the fill material, and it also believed to have increased the stiffness and strength of arch barrel. Frequently, the arch was backfilled with materials whatever materials at hand locally. It has great influence on the overall behaviour of arch bridges as it helps the distribution of traffic load and prevents movement of arch barrel.

The construction process for masonry arch bridges has not changed much over decades, and a detailed procedure is described by Heyman (1982). The process involved a basic surveying of the construction site and preparation of all materials. A platform which consists of some type of grillage was built above either raft or pile foundations. The construction started with the piers and abutments, and a timber truss in the shape of an arch which is known as the ‘centring’ was set up to define the intrados of the arch barrel. The spandrel walls were then built off the barrel and they were then backfilled. However, sometimes the spandrel wall was built after the removal of the centring in order to minimise the potential cracking in the spandrel walls due to the deformation of arch barrel (Ruddock, 2000). The timing of centring removal was critical during construction, and it could be left for several weeks or months after the completion of the arch barrel. There are also a few masonry arch bridges with multi spans and skewed
CHAPTER 2. LITERATURE REVIEW

Figure 2-2: Original design of Westminster Bridge by Labelys in 1853
CHAPTER 2. LITERATURE REVIEW

Figure 2-3: Structural components of a typical masonry arch bridges (Sowden, 1990)
arches with faces of the arch not perpendicular to its abutments and a plan view being a parallelogram, rather than the rectangular plan view of a regular arch. The construction of these structures follows the same manner but other details need to be considered, such as the interaction of adjacent spans for multi span bridges and the laying patterns for a skewed arch bridge. A detailed description about the construction of these bridges is given by Ryall et al. (2000). Though masonry arch bridges have rarely been built in the last century, several methods have been developed for the construction of arches with unreinforced concrete. A flexible concrete arch, developed by Taylor et al. (2006), and the TechSpan System (Smith, 1995), developed by the Reinforced Earth Company, both uses a series of precast concrete block for the construction of the arch barrel. The arch can be erected by crane without the need for centring, and it is time efficient in term of construction period compared with conventional masonry arch bridges. As it does not require curing for the mortar, so the construction of subsequent parts (spandrel walls, parapet) can start immediately after the arch is set up.

2.2 Analysis and assessment methods

Although the application of masonry arches can be traced back to ancient times, the accurate assessment of their strength remains a difficult job for engineers. The behaviour of masonry arch bridges is complicated since it is a combination of the interactions between individual bricks, mortars and fills. Several methods for assessing arch bridges have been established, and all the methods come with advantages but also limitations. There are still some gaps between the theory and reality, and it remains a challenge to apply these theories in real projects.

2.2.1 Elastic and MEXE method

Navier was a pioneer who worked on the development of an elastic method for the analysis of masonry arches in the 19th century. His most important contribution to masonry arch theory was the introduction of stress analysis (Kurrer, 2012). A law for the distribution of pressure across a surface was developed by Navier, which could be adopted for the analysis of masonry arches.
It was concluded that no tensile stress develops in the arch ring if a thrust line could be found that lies in the middle third of the arch thickness. This would ensure the arch is in safe condition. An elastic method was developed by Castigliano (1879) based on minimum strain energy, and he concluded that the arch ring would behave as a continuous elastic rib if the line of thrust fell within the middle third of the arch ring (Page, 1993). A series of experimental studies were carried out by Pippard et al. (1936; 1938; 1948; 1952). The analytical model developed by Pippard is based on a parabolic arch, which has a span to rise ratio of four with both abutments pinned. The arch was loaded at the crown with a transverse line load as shown in Figure 2-4.

![Figure 2-4: Pippard’s real and analytical arch model (Heyman, 1982)](image)

A expression has been derived by Pippard between a safe axle load and the geometry profile of the arch bridge, including arch ring thickness, arch span and fill depth at crown. The Military Engineering Experimental Establishment (MEXE) developed a nomogram based on Pippard’s work using the following formula (Equation 2.1) for a quick assessment of strength of masonry arch bridges

$$ Provisional \text{ axle load} = \frac{(d + h)^2}{L^{1.3}} $$

(2.1)
Where $L$ is the span, $d$ is the arch ring thickness, and $h$ is the depth of fill at the crown. The provisional axle load (PAL) in then modified with a series of factors (Department of Transport, 2001) by taking the influence of material, general conditions of structure and other factors into account. Specifically, they are:

- Span/rise factor: bridge with deep arches are known as stronger than shallow arches, so it considers $F_{sr} = 1$ for an arch with span/rise of 4.0 or less. A higher span rise ratio indicates a small value for this factor.

- Profile factor $F_p$: the shape of the arch has great impact on the load capacity, the ideal profile is considered to be parabolic and for this shape the rise at the quarter points, $r_q = 0.75r_c$, where $r_c$ is the rise at the crown. Any arch profile different of this is modified by this factor.

- Material factor $F_m$: this factor takes the type of backfill and arch ring materials into account.

- Joint factor $F_j$: this is affected by the conditions and size of mortar joints.

- Condition factor $F_{cm}$: this factor reflects the general judgement in assessing the arch condition based on quantitative information from a close inspection of the structure. It has a value ranging from 0 to 1.0. Zero is applicable to a bridge in poor condition with a lot of defects while 1.0 may be taken for an arch barrel in good condition with no defects.

$$Modified\ axis\ load = PAL \times F_{sr} \times F_p \times F_m \times F_{cm}$$ \hspace{1cm} (2.2)\

A modified axle load can then be calculated by Equation 2.2. Details of the values of these modification factors may be found in standard BA 16/97 (2001) for the assessment of highway bridges and structures. The MEXE method is a quick and easy way for the assessment of arch bridges and is still recommended by the current standard. However, it is largely an empirical tool and has several limitations. The analysis is based on a parabolic arch with a span/rise ratio of 4 and is only applicable to bridges with a span less than 18 m and should not be used for flat arches. It should be noted only the arch is assumed to be structural in this method. The load carrying capacity is mainly determined by the span
of the arch, ring depth and fill depth at the crown, while the influence of arch geometry and spandrel wall as well as any strengthening are not considered. The results of MEXE assessment is heavily dependent on the experience of assessor and is generally considered as conservative (McKibbins et al., 2006).

2.2.2 Mechanism method

The mechanism method uses the geometrical properties of the arch and equilibrium to analyse the collapse of such structures. It is believed that the mechanism methods were developed at the same time as the MEXE method and the use of the basic mechanism method was first demonstrated by Pippard and Baker (1957). The fundamental standpoint of the theory is that the collapse of arch bridges will happen when the line of thrust reaches the intrados or extrados of the arch and converts the structure into a mechanism by the formation of hinges (Figure 2-5). These hinges are necessary, in this method, to turn the arch into a statically determinate structure. Heyman (1969) developed an approximate approach based on plastic analysis, which enabled a quick assessment to be made for the strength of a given bridge. The following assumptions were introduced in his method: sliding between voussoirs cannot occur; masonry cannot resist tensile stress and the masonry has infinite compressive strength.

Figure 2-5: Four hinges failure mechanism

Figure 2-6 shows the dimensions of the arch model and the corresponding line of thrust as well as the positions of hinges that has been used by Heyman for his analysis. The worst loading is assumed at quarter span, and the road surface is assumed horizontal. The fill is assumed to have no strength and the live load is transmitted to the arch barrel without dispersion. The following expression
(Equation 2.3) is then derived using statical equations of equilibrium to calculate the load that transforms an arch into a hinged mechanism (Heyman, 1982). This method takes the geometry of the arch into account in terms of $\alpha$, $\beta$ and $\tau$, and the dead weight of the fill and arch ring are considered during analysis by assuming the same unit weight for them. When using this type of analysis, for a given live load, a minimum arch thickness is required for including a line of thrust in equilibrium within it, and the level of safety for the structure can be evaluated by comparing the real arch ring thickness with the minimum arch thickness.

$$
P = \frac{W_2x_2[\alpha + (1 - 1/4k)\tau] - (W_1x_1 + 1/4W_2)[(1 - \alpha) - (1 + 1/4k)\tau]}{(3 - 2\alpha) - (2 + k)\tau} \quad (2.3)
$$

Figure 2-6: Heyman’s arch model for analysis
The application of the mechanism method to masonry arch bridges started in the 1970s according to Heyman (1996). Heyman’s method does not consider the effects of fill, but later research showed the fill have great influence on the arch strength. The basic mechanism method was therefore modified by subsequent researchers and several computer programmes based on this approach were developed (Crisfield and Packham, 1987; Gilbert, 2001; Taylor and Mallinder, 1993; Hughes and Hee, 2002).

Crisfield and Packham (1987) developed a computer program based on a mechanism method. Figure 2-7 shows the analytical model used by Crisfield, it was assumed that the abutment are completely rigid and calculations are based on a unit thickness of arch with no tensile stress. Compared with Heyman’s model, there are two options for the load distribution through the fill. The first procedure applies a uniform pressure over a horizontal line at the level of intersection with the arch, directly under the load. The second has a linearly varying distribution between two points. The influence of lateral earth can be considered by including an allowance of movement for block 3 of the fill. Also, a concept of ‘yield blocks’ at each hinge point was introduced for the allowance of material damage instead of the assumption of infinite compressive strength in previous models. Significant improvements were made using this approach, however, the success of its application is restricted by some unquantifiable parameters involved during the analysis because of limited experimental validation.

![Figure 2-7: Mechanism with equilibrating forces](image)

The basic formulations of limit analyses were modified by Livesley (1978) and a rigid block approach was developed for the analysis of masonry arches using the lower bound theorem of plasticity. In this method, the arch was divided
into a series of discrete rigid blocks which were connected via zero thickness and zero tensile strength joints. As shown in Figure 2-8, the block interfaces are assumed to be normal to the centre plane of the arch. The applied live and dead loads were distributed and represented by three forces in terms of $q_i$, $s_i$ and $t_i$, which are two normal forces at the edge, and a frictional force acting along the interface. A collapse load could be determined by considering the virtual displacements of the system of rigid blocks using the linear programming technique. The main improvement of this method is that instead of the four hinge failure in a basic mechanism analysis, much more complex structures and mechanisms could be taken into account, such as multi-span arches and sliding between blocks. The method was extended by Livesley (1992) for the analysis of three dimensional structure to allow consideration of the effects of spandrel wall. It was concluded that a three dimensional collapse mechanism is difficult to visualize. Due to the complexity of the theory for three dimensional behaviour, it is confined to the academic community and not ready for a wide application (McKibbins et al., 2006).

A further rigid block method was developed by Gilbert and Melbourne (1994) for the determination of the plastic collapse load of masonry structures. The arch was treated as consisting of a large number of discrete rigid blocks as in Livesley’s model, and sliding and rotation between adjacent blocks were introduced, which made it possible for the consideration of ring separation in the arch. The arch was assumed to be subjected to a unit vertical displacement at the position of the applied load. A work equation was derived based on the geometric constraints imposed against the movement of deformed arch, and an upper bound solution of the collapse load was calculated using linear programming techniques.

Figure 2-8: Notation for the analysis of a plane single span arch (Livesley, 1992)
Experimental tests were carried out by Melbourne et al. to investigate the collapse behaviour of multi span arch bridges (Melbourne et al., 1997) and multi ring brickwork arch bridges (Melbourne and Gilbert, 1995). The experimental results were used for the verification of the rigid block analysis method and showed good agreement. This method was further developed by Gilbert (2001) into a computer programme which includes the influence of horizontal backfill properties by introducing uniaxial fill elements.

It is noted that so far almost all the existing research works into mechanism methods have included a simplification of reality and can generally only be applied to two dimensional analysis of arch bridges. The predication of collapse is significantly affected by the deformation of arch barrel and serviceability is not included. Further studies are required on transverse behaviour to extend analysis to three dimensions.

2.2.3 Finite element method

The finite element method (FEM) has been widely used for the analysis of structural problems in the last few decades due to the development of more powerful computers. The FEM is a numerical technique for finding approximate solutions of partial differential equations. In this method, a structural continuum is divided into a number of small elements which consists of several nodes, the complete structure is represented by the interaction of these finite elements. The stiffness matrix and load vectors are then determined for each element and assembled to produce the overall stiffness matrix, once a certain boundary condition is given, the complete system can be solved by the solution of a set of energy equations to determine the force and deflections properties of unknown nodal values. The details of finite element theory and the applications in engineering are described by Bathe (1982), Hinton and Owen (1989) and Zienkiewicz (1971).

Research has been conducted by various researchers in the application of finite element analysis for arch bridges. The first application of the FEM to masonry arches is claimed to be performed by Tower and Sawko (Towler and Sawko, 1982; Towler, 1985). A one dimensional model specifically for brickwork arches was developed and the numerical solutions were compared with
experimental results on brickwork model arches. Their work demonstrated the potential of a non-linear finite element program which could be used not only to model ultimate limit state but also provide data on the extent of cracking under abnormal loading. However the interaction between the masonry and fill was not considered in their model which was a limitation.

A one dimensional curved beam element was developed by Crisfield (1984; 1985) for analysing masonry arch barrels based on the mechanism method proposed by Pippard and Baker (1957). This method incorporated the conventional mechanism assumptions where sliding between voussoirs and ring separation could not occur with a brickwork arch and the arch was considered as a smeared continuum with a unit width. The main improvement of this model was the introduction of horizontally acting fill elements (Figure 2-9) for lateral fill effects. This fill restrains the displacements beyond the abutments at some distance $\Delta x_f$, and it was assumed to act only when it is in compression. Later work showed the significant effect of the backfill passive pressure for the overall strength of the arch. Non-linear springs were used in finite element models to simulate the lateral resistance of the backfill (Crisfield and Wills, 1986). The model is relatively simple and shows a lack of consideration of many important factors (geometry, materials). The assumption of unit width of arch barrel has ignored the influence of arch thickness which has been considered as important for arch strength.

![Figure 2-9: Arch with fill elements (Crisfield, 1985)](image)

A one dimensional finite element method was developed by Choo et al. (1991b) to simulate the behaviour of a masonry arch using a tapered beam element. The arch material exhibited an elastic-plastic behaviour under compression and had no tensile strength. The model enabled the consideration of material cracking and crushing under loading. As the nodal cracks developed, the tension zone of each element was assumed to be ineffective and was neglected, while the crushing zone was considered to contribute strength but have no structural
stiffness. The remaining uncracked and uncrushed portion of the arch, which is the so called ‘effective arch ring’, was represented by a tapered beam as shown in Figure 2-10. The total load acting on the arch consisted of its self-weight and applied live load. The imposed loading was on the fill and distributed to the arch through the fill. The spread of the load was governed by a vertical angle which varied from $0^\circ$ to $45^\circ$. A similar fill element as employed by Crisfield (1984) was used to simulate the lateral resistance of the fill. The collapse of arch can be derived by this method based on an incremental-iterative technique. The proposed model was initially applied to two circular segmental brick arches tested by Towler (1982) and compared with modelling results obtained by Crisfield (1985). The program was then used to assess the behaviour of three real stone arch bridges which had been loaded to collapse. Good agreement was found between the experimental tests and finite element analysis in terms of the load displacement relationship, however it was noticed that the Young’s modulus and compressive strength during the analysis were not the laboratory test values. Further studies were performed by Choo et al. (1991a) for the development of a two dimensional model using eight-node quadrilateral elements to model masonry arch bridges. The proposed model was able to analyse the behaviour of masonry arches with common defects, such as initial cracks and ring separations. The behaviour of skewed arches was also examined by Choo and Gong (1995) in a later study. However, the numerical analysis has a lack of validation with experimental results using laboratory tested material properties.

Figure 2-10: Analytical model for masonry arch (Choo et al., 1991b)

A two dimensional model developed by Loo and Yang (1991) incorporated several additional concepts. The brick and mortar were not treated separately in their analysis and average masonry properties were used in the model. Masonry was assumed as an isotropic material with prior cracking. A tensile failure criteria for plain concrete proposed by Chen and Saleeb (1982) was adopted
and a simplified Von Mises criteria for crushing failure was used. As the cracking of masonry is significant for masonry structures, the ‘smeared crack’ model was used in their study. The cracked masonry was assumed to remain as a continuum rather than including a single discrete crack, and cracks were represented as a change in the material property of the elements. The backfill and arch ring are simulated using two different types of elements as shown in Figure 2-11. The interface between the fill and arch barrel was represented by a series of hinges with the aim of including horizontal and vertical forces produced by the fill. Parametric studies were carried out using the proposed method for a semi-circular arch. The influence of the support movement was analysed and the results indicated that small support movements could cause serious distress in fixed end structures. However, there was a lack of verification of the model against experimental tests. The use of hinges for the simulations of interface between fill and arch ring and the concrete material model for masonry structures were not well recognised at that time.

Cavicchi and Gambarotta (2006) published their work on a two dimensional procedure to evaluate the load carrying capacity of multi span masonry bridges based on the Kinematic Theorem of Limit Analysis (Jirasek and Bazant, 2002; Cohn et al., 1979; Chen and Liu, 1990). The arch and piers of the bridges were modelled using two node beam element, and the masonry was assumed to have no tensile resistant and be ductile in compression. The fill was represented as a cohesive frictional material and discretized into triangular elements. Linear programming technique was used to obtain the collapse load which is an upper bound solution. The advantage of this approach is a comprehensive description of
the collapse mechanism, which includes the interaction between adjacent arches for multi span bridges. It also takes the arch fill interaction into account during the analysis. Two examples were analysed by the proposed approach, the authors claimed good agreement with experimental results with reduced computational cost. However, the plain strain assumption for the fill material in the proposed needs further review. The model showed a lack of ability for the prediction of corresponding deformations.

Compared with traditional mechanism analysis, there are several advantages for the finite element method:

- Complex bridge geometry, such as skewed multi span, open spandrel and internal spandrel can be investigated by using 3D models. The effects of the spandrel walls and backfill could be considered;
- The effects of specific defects such as ring separation and longitudinal cracks could be considered;
- The dynamic effects of loading can be simulated;
- The displacements, strains and stress conditions of structures at certain load level can be evaluated.

There are also some disadvantages, including:

- It is time-consuming if the problem contains large amount degrees of freedom (DOF);
- It does not perform well for problems with large displacements;
- Results are highly sensitive to the input parameters and the presence of cracks can influence results.

Almost all of these masonry arch specific finite element models have been concerned with two-dimensional ‘slices’ through the arch, and some research work that considered three dimensional effects is discussed in section 2.4.

2.2.4 Discrete element method

The discrete element method (DEM), also called the distinct element method is a numerical method for computing the motion of a large number of particles. DEM is becoming widely used as an effective method of addressing engineering
problems in granular and discontinuous materials, especially in granular flows, powder mechanics and rock mechanics. For masonry structures which are formed by discrete units (stones or bricks) separated by joints, their primary feature is the composite nature. The mechanical behaviour of masonry structure is determined both by the properties of the blocks and the discontinuities between them (mortar or frictional contact). The DEM model treats the structures as a collection of separate elements; each of which has the ability to move and deform independently. Contact detection and the ability of analysing the interactions between blocks are the heart of DEM. Compared with traditional finite elements methods which are mainly continuum-based, DEM methods are more accurate in reflecting the nature of masonry structures. Since DEM incorporates all the features described for the finite element methods, it is considered as the most suitable approach for the analysis of masonry structures. The efficiency of DEM contact detection and the avoidance of equilibrium calculations allow DEM simulations to predict failure, collapse and post-failure behaviour of masonry structures (Brook, 2010).

Cundall is considered a pioneer in the development of discrete element modelling. He originally proposed a method for modelling fractured rocks and later developed into a well know commercial discrete element software (Cundall, 1971; Cundall and Strack, 1979; Cundall, 1990; Cundall and Hart, 1993). Various research work has been undertaken by subsequent researchers. Lemos (2007) discussed the main assumptions and principles of discrete element modelling techniques and reviewed the computational developments of these techniques as well as their application in engineering practice. A numerical analysis of a historic stone arch bridges built about 150 years ago in Japan was reported by Jiang and Esaki (2002) using the discrete element method. They produced a two dimensional model to represent the real structure as accurately as possible (Figure 2-12). The mechanical behaviour of the stability of the whole structure was mainly controlled by load distributions and material properties of the contact planes. Analysis was performed to investigate the displacement under static truck loading. The modelling results were compared with experimental observation. Their work was verified by Toth et al. (2009) and extended for the analysis of multi span masonry arches. The impacts of the material properties of the backfill on the load carrying capacity of the arch bridge were examined in their study. The main advantage of the model used by Jiang and Toth is that it provides
a better representation of the real structure with discontinuities at the mortar joint. However unlike previous two dimensional models, the backfill was ignored rather than the spandrel wall during the analysis, and the live load was applied directly on the spandrels, which is not consistent with reality.

![Figure 2-12: Distinct element model of the arch (Jiang and Esaki, 2002)](image)

The bearing capacity of the Bridgemill masonry arch bridge was studied by Rouxinol et al. (2007) using a mixed discrete element method. A two dimensional model of rigid discrete elements, including masonry block and fill particles has been produced to accurately simulate its limit behaviour. The bridge was analysed in four different conditions, two with the isolated arch, and two combining the arch with fill (Figure 2-13a) and the spandrel walls (Figure 2-13b). These solutions have been used for the estimation of a three dimensional collapse load. Modelling results show that the existence of fill materials and spandrel walls give a much higher collapse load than an isolated arch, however the two combined models with backfill and spandrel wall do not show much difference in terms load-displacement relationships. A comparison with a previous study by Rouxinol et al. (2006) showed that a fine mesh of the fill particles will increase the predicted collapse load.

Giordano et al. (2002) published their research work on modelling of a historical masonry structure using different approaches. The first approach is the standard finite element modelling strategy using the commercial code ABAQUS. It is based on the concepts of homogenized material and a smeared cracking constitutive law. The second approach used is the finite element method using
discontinuous elements with the programme Visual CASTEM. Finally, a discrete element method using UDEC software was used. The ABAQUS model was found to be unable to predict cyclic behaviour, and further research work was identified for the development of masonry oriented constitutive model. The main difficulty encountered with the CASTEM model was the establishment of effective contacts when large displacements were involved during the analysis. The limits of these models are overcome by the DEM method handling large displacement problem. The disadvantage is the stress conditions across the structures can not be well demonstrated, and there is generally lack of a sophisticated constitutive models for the internal elements when deformable blocks are used in an analysis. Encouraging results have been found between experimental tests and numerical predictions. Their work provides essential information for the selection different of methods when analysing masonry structures with specific need. However, the material properties used during the analysis are mainly estimated value rather than experimental measured value. The work was mainly carried out at two dimensional level and focused on the load carrying capacity. The load distribution and stress development across the structure were not included in their numerical analysis.

Figures 2-13: Discrete element mesh (a) arch backfill model (b) arch spandrel model (Rouxinol et al., 2007)
2.3 Experimental tests on arch bridges

Though the masonry arch has been used as a structural form since ancient times, some aspects including the transverse strength of arch bridges and the assessment of strengthened bridges are not well understood and documented. In the past few decades, a number of tests have been carried out on full-scale bridges, small-scale and large-scale laboratory models in order to establish realistic methods of assessing the strength of masonry arch bridges. There are many parameters that contribute to the overall behaviour of a masonry arch bridge, including geometric factors (span, rise, arch thickness, depth of fill, spandrel wall thickness), material properties (different type of brick/stone, mortar and fill materials) and load conditions (ground conditions, drainage). It is impossible to include all these parameters in an experimental programme and only the key factors could be considered in one programme. An overview of the experimental work that have been undertaken previously by various researchers is presented in the following section.

A series of tests was carried out by Pippard et al. (1936; 1939) on model arches. The arch model in the first set of test consists of fifteen steel voussoirs to ensure an elastic behaviour, with a span and rise of 1219 mm and 305 mm respectively. The arches were tested under two arrangements, with the ends pinned (Figure 2-14a) or mounted on skew-backs, where one support was fixed in position and the other was attached to a saddle which could move on ball bearings in a horizontal direction (Figure 2-14b). Pippard concluded that both voussoir arches behaved elastically within a limiting load. For the two-pinned arch, it reached an ultimate load with the formation of a third pin either at the arch intrados or extrados between the load point and the arch center, depending on the abutment movement. While for the fixed end arches, additional pins was found at either the extrados or intrados near the skew-back to form a four hinge mechanism. In a later test programme (Pippard and Ashby, 1939), seven series of tests were undertaken using mass concrete voussoirs, while non-hydraulic lime mortar and rapid hardening Portland cement mortar were used as joint material. The arch models had a span of 3048 mm and rise of 762 mm. It was concluded from these tests that the arch behaved as an elastic continuum until the formation of the first hinge. The arch still maintained strength after the first cracking of a mortar joint before any instability occurred. Pippard concluded that it was also
safe to adopt a middle half rule rather than the more conventional middle third rule.

![Figure 2-14: Test arrange for model arch (a) two pined arch (b) fixed end arch (Pippard et al., 1936)](image)

Chettoe and Henderson (1957) conducted a test programme on 13 arch bridges under service states. The tested bridges were in a good condition, and the load was applied by means of vehicles with a maximum loading of 90 tonnes. Their work concerned the following aspects in a masonry bridge: load dispersion through fill materials; the effects of abutment movement and strength contribution of the fill, parapet and spandrel wall. They concluded that a dispersal angle of 45° could be assumed for fill materials, and it should be reduced for poor fill and might reasonably be increased for good fill. Abutment movement was found in most of the bridges, with considerable movements and reduction in strength expected for bridges with weak backing. However, there was no direct evidence showing that the backfill and spandrel wall increased the strength during their tests.

In the 1980s, the Transport and Road Research Laboratory (TRRL) had conducted a test programme to improve the understanding of the behaviour of masonry arch bridges, and examine the validity of the MEXE method for the assessment of arch bridge capacity. The programme involved eight tests on redundant bridges and was reported by Page (1987; 1988; 1989) and Hendry et al. (1985; 1986). Two full scale laboratory tests were also carried out by Harvey et al. (1989) and Melbourne et al. (1990). A summary of the geometries of the eight bridges are given in Table 2.1. Most of the bridges have a segmental arch, except that of the Preston bridge which has an elliptical arch while the Bridgemill bridge is a parabolic arch. These bridges were subjected to a line load across the whole width of the bridge at the quarter span, with the exception of Preston bridges where the load was applied at the third span position. These tests were mainly focused on load displacement relationships, collapse mechanism.
and collapse load. A problem with these tests is that the bridges were not in a
good condition and the material properties were not well known. In addition, the
existing defects in the bridges were not well considered and all the bridges were
considered as two dimensional arches, with the transverse behaviour of the arch rings ignored. It was concluded by Page that the spandrel walls provided little
contribution to the load capacity of the bridge as spandrel wall separation was
found in most of these bridges.

Table 2.1: Bridges tested by the TRRL (Page, 1987;1988;1989 and Hendry et al.,
1985;1986)

<table>
<thead>
<tr>
<th>Name</th>
<th>Span</th>
<th>Rise</th>
<th>Width</th>
<th>Depth of fill above crown</th>
<th>Ring thickness</th>
<th>Collapse load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(kN)</td>
</tr>
<tr>
<td>Preston</td>
<td>5180</td>
<td>1640</td>
<td>5700</td>
<td>380</td>
<td>600</td>
<td>2110</td>
</tr>
<tr>
<td>Prestwood</td>
<td>6550</td>
<td>1430</td>
<td>3800</td>
<td>165</td>
<td>340</td>
<td>228</td>
</tr>
<tr>
<td>Torksey</td>
<td>4900</td>
<td>1150</td>
<td>7800</td>
<td>246</td>
<td>380</td>
<td>1080</td>
</tr>
<tr>
<td>Shinafoot</td>
<td>6160</td>
<td>1180</td>
<td>7020</td>
<td>215</td>
<td>365</td>
<td>2524</td>
</tr>
<tr>
<td>Strathmashie</td>
<td>9420</td>
<td>2990</td>
<td>5810</td>
<td>410</td>
<td>Varies</td>
<td>1325</td>
</tr>
<tr>
<td>Barlae</td>
<td>9860</td>
<td>1690</td>
<td>9800</td>
<td>295</td>
<td>400</td>
<td>2900</td>
</tr>
<tr>
<td>Bridgemill</td>
<td>18300</td>
<td>2850</td>
<td>8300</td>
<td>125</td>
<td>711</td>
<td>3100</td>
</tr>
<tr>
<td>Bargower</td>
<td>10000</td>
<td>5180</td>
<td>8680</td>
<td>160</td>
<td>558</td>
<td>5600</td>
</tr>
</tbody>
</table>

Two large scale arches were constructed and tested under laboratory
conditions. The first was a semi-circular arch with a span of 4 m, barrel thickness
of 0.25 m and depth of fill over crown of 0.2 m (Harvey et al., 1989). The
pressures at soil structure interfaces were monitored using 48 soil pressure cells
mounted on the extrados. The test results showed that the maximum pressure
was found beneath the load line during the test to failure. It was concluded
that the spread of load through the fill material is small, while the arch ring
exhibited a remarkable ability to disperse the applied load over the entire ring.
The stabilising force required was mainly provided by the spandrel wall rather
than the backfill. The second large model was a much flatter arch with a 6 m span,
1 m rise, and consisting two rings of brickwork (Melbourne and Walker, 1990).
Load was applied across the full width of the bridge, and the fill pressure was
recorded by 34 pressure cells. Test results showed that the bridge failed due to
a four hinge mechanism. Both the backfill and spandrel wall provided significant
restraint to the arch barrel. The separation of the spandrel wall from the arch
barrel was observed at 30% of the failure load. The load deflection response for the arch barrel was initially linear until the formation of the hinges. A comparison of the load displacement curves between the tests on redundant bridges and full scale model bridges showed that the redundant bridges experienced much larger deformation as load applied until failure. This could be due to the deterioration since construction and the support conditions in the field being different to those in the laboratory.

A series laboratory and field tests were supervised by Melbourne to study the behaviour of multi-span masonry arch bridges. Ring separation and spandrel wall separation were considered for square arches, and the behaviour of skewed arches was also discussed. In the multi-ring arch bridge test programme (Melbourne and Gilbert, 1995), seven segmental single span brickwork arch bridges were built with a span of either 3 m or 5 m, and a span to rise ratio of 4. These bridges were constructed with certain defects such as ring separation, spandrel wall detachments or both of these. Ring separation was achieved by using damp sand rather than mortar between the rings. Spandrel wall detachment was achieved by building the wall so that a narrow gap was present between the inner face of the wall and the edge of the barrel. Loading was applied at the quarter point via two hydraulic jacks bearing on a loading beam and providing a line load across the width. All the model bridges failed by the formation of hinged mechanisms. It was also found that the ring separation was highly unpredictable and significantly reduced the strength and stiffness of the structures. The load capacity of the bridge was significantly improved by the horizontal backfill pressure as the arch barrel swayed into the backfill.

A later test programme comprised a series of three large-scale model multi-span bridges (Melbourne et al., 1997). Each bridge contained three spans, each span was of the same geometry as the 3 m span single-span bridges also tested by the authors. Some of the spans were constructed with the defects initially allowing spandrel wall detachment. A similar hydraulic load system as for the single span bridge was employed during the testing. All the model bridges failed in mechanisms which involved both the loaded span and adjacent spans. Experimental results provided evidence to support the theory that the spandrel wall has a strengthening effect. It was found that the critical loading position was near to the arch crown for multi-span arch bridges rather than the quarter point for single span segmental arches. Horizontal backfill pressures were found to increase.

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carrying capacity, but had less influence than in the case of single span bridges.

The behaviour skewed arch bridges was examined by Melbourne and Hodgson (1995; 1996). Four segmental single span skew arches were built with a nominal square span of 3 m and rise at the crown of 750 mm. The nominal skew of the arches was 45°. Load was applied parallel to the abutments as a line load across the whole width of the bridge. Experimental results showed that the failure of a skew arch involved a three dimensional mechanism, and more than four hinges were required to form a kinematically admissible mechanism. They also reported that although the stiffness of a skew arch might be greater than a straight bridge, the strength was not increased compared with the equivalent square-span barrel. It is also found that the passive backfill pressure is related to the deformation of the arch and that the skew arch cannot mobilise the same level of backfill pressure as the equivalent right arches.

Fairfield and Ponniah (1994) carried out a series of tests on model arches to examine the effects of fill on buried arches. The test programme involved a semicircular arch with a span of 700 mm constructed in timber with 45 voussiers and another arch with span to rise ratio of four with 25 voussiers. To minimize the effects of friction, a gap was introduced between the arch and the 4 mm thick glass side wall through which the two dimensional behaviour could be viewed. These walls were supposed to have no structural function and do not affect the collapse load. Strips of polythene sheet were used to retain the fill and minimise the friction between the fill and sidewall. The fill was medium, uniformly graded dry silica sand with rounded particles. A total of 88 tests were performed in four stages. The first three tests were used for the determination of appropriate boundary conditions, the next four tests were to determine the region of fill displacement. The influence of fill density was studied in the following three tests. Parametric studies were carried out on both arches and the repeatability of the observed collapse loads was checked. Each arch geometry was tested to collapse with six different fill depths at the crown and applying the load at ten different positions. The investigators reached a conclusion that soil structure interaction contributed significantly to the capacity of the model arches, and the collapse load increased with the fill depth. The most critical load position was found between 1/3 span and 1/4 span. Although the model did not directly represent the situation in a real arch because of the inappropriate materials, their work provided useful insight into arch/fill interaction.
Some experimental studies of the backfill arch interactions have been carried by Gilbert et al. (2007) on small and large scale models. This included the testing on a series of model bridges with a span of 380 mm and a rise of 85 mm, while the full scale bridge had a span of 3000 mm and a span to rise ratio of four. The influence of different backfill materials was studied in the research. The Particle Image Velocimetry (PIV) technique was used to track the movement of backfill thereby assisting in the understanding of backfill-arch interaction. The tests results demonstrated that both load spreading and passive restraint enhance the load carrying capacity and that a limestone filled arch bridge proved capable of carrying significantly more load than its clay filled counterpart.

The possibility of using small scale models to predict behaviour of large scale prototypes is another area that has received significant research interest. A large body of research into this aspect has been carried out by the masonry research group based on Cardiff University. The different behaviour of the brickwork at prototype and model scales was reported by Mohammed (2006). Taunton (1997) used the centrifuge modelling technique in order to get similar stress conditions in typical arch bridges on small scale level, thereby increasing understanding of backfill/masonry structure interaction. Miri (2005) adopted the centrifuge modelling method and compared the outcomes of different repair techniques for masonry arch bridges. As a result of this research, it was determined that small scale models are suitable for qualitatively examining structural behavior and can be useful for calibrating numerical models.

From the above experimental work, three main failure modes have been identified for masonry arches and are summarised below (Hughes and Blackler, 1997):

- **Formation of a hinge mechanism**
  The most common failure type that has been observed for the majority tests on arch bridges. It is characterised by the progressive opening of cracks at the positions of the arch where the line of thrust is sufficiently eccentric. At least four “hinges” are needed to convert a single span arch into a mechanism.

- **Snap through failure of arch barrel**
  This mechanism involves rotations take place at a hinge so as to produce instability and local failure, prior to the formation of a global hinge failure
mechanism (Figure 2-15). It is most likely to occur in highly confined arches and precipitates the global collapse of the structure. The spread movement of arch abutments and the large non-linear deformation of the arch ring are the main causes of this type of failure

![Figure 2-15: A typical snap through buckling failure of arch](image)

- Material failure
  
  This failure type occurs when the compressive or tensile stress developed in the arch barrel exceeds the material strength and leads to a global failure of the whole structure.

The majority of failures of single span masonry arch bridges during experimental and field testing occurred in the above mentioned modes. However it is possible that more complex failure modes can develop as combinations of the above failure modes, as a result of local failure such as ring separation in multi-ring brickwork arch bridges, or from local spandrel wall failure.

Previous experimental tests in laboratory conditions mainly focus on the load carrying capacity of the arch barrel. There is generally a lack of testing on material samples from real masonry arch bridges. The model tests usually ignore the structural function of spandrel wall in masonry arch bridges and the deformations of the spandrel walls in either transverse or longitudinal direction are not well documented.
2.4 Research work on failure of spandrel wall

Previous experimental work has indicated the importance of spandrel walls on the overall behaviour of masonry arch bridges, but limited research has been undertaken in this area. In this section, some analytical and numerical modelling work related to the investigation of transverse behaviour of arch bridges, especially the spandrel walls, has been reviewed.

The failure of spandrel walls is mainly due to interactions with the arch barrel and the backfill which can lead to the instability of spandrel walls. The movement of spandrel walls may take the form of tilting, bulging or sliding over the extrados or longitudinal cracking immediately behind the spandrel wall which leads to spandrel wall separation from the arch barrel (Figure 2-16). The stability of spandrel walls is subjected to a variety of factors (McKibbins et al., 2006), including: a) inadequate design because of lack of knowledge on soil properties at construction; b) increased traffic load applied at the bridge surface; c) lateral forces exerted by the fill; d) deterioration of materials; e) thermal expansion or shrinkage due to temperature change; f) strengthening affecting overall behaviour. Considering the uncertainty of the traffic conditions and the difficulties in determining the mechanical properties of construction materials, it is difficult to determine the best overall approach of assessing the stability of spandrel walls by considering all these factors. However, this does not prevent carrying out parametric studies to investigate the sensitivity of spandrel wall to these influencing factors, thereby determining how critical these factors are to the failure and providing advices on how to manage the risk.

![Figure 2-16: Spandrel wall defect (McKibbins et al., 2006)](image)

Erdogmus and Boothby (2004) proposed an approach for the determination of the strength of spandrel wall in masonry arch bridges based on the conventional analysis with fill supported by retaining walls. The soil pressure in this method
is determined by Coulomb-Rankine analysis. An energy method named the fracture line method was used in their study. The fracture line method is similar to the yield line theory which is a commonly used plasticity method for reinforced concrete slabs (Johansen, 1962). This method can take several aspects into consideration when analysing the out of plane strength of an unreinforced masonry wall, such as the strength of masonry assembly, boundary conditions, and the geometry of the walls. The most important aspect when using this method is the determination of the potential fracture line (Figure 2-17). The principle for the selection of fracture line patterns was discussed by Nilson (1997). Once the fracture pattern is determined, the external virtual work can be calculated based on the Coulomb-Rankine theory and equated to the internal virtual work. The failure load which is an upper bound solution corresponding to the fracture pattern is derived. As there are a variety of potential fracture patterns, several trials are needed before the minimum load that causes a mechanism to form can be obtained. The proposed method was then applied for the analysis of two masonry arch bridges. They concluded that it is justifiable to consider the spandrel wall as a retaining wall for calculating the lateral soil pressures as good agreement was found between the analysis and observed bridge conditions. However, the proposed approach did not consider the interaction between the spandrel wall and the arch barrel, and it is not applicable to bridges with longitudinal cracks in the arch barrel.

![Fracture line pattern determination](image)

(a) Restrained on the bottom and by the arch (b) Restrained on the bottom by the arch barrel and by the wing wall

**Figure 2-17: Fracture line pattern determination (Erdogmus and Boothby, 2004)**

The influence of spandrel wall construction on arch bridge behaviour was studied by Harvey et al. (2007) using a two dimensional finite element model by taking the arch barrel and spandrel wall into consideration. Both the arch barrel and the spandrel wall were modelled as elastic continuum and the live load
was applied as a concentrated load at the quarter point. The load dispersion through the spandrel wall and the difference between a soft spandrel wall and a hard spandrel wall were investigated by using a different elastic modulus. The concentrated load for the soft spandrel wall case was found to be confined within a wedge with sides at $30^\circ$ to $45^\circ$ to the vertical. An equilibrium model using hybrid elements (Maunder et al., 1996; Almeida and Freitas, 1991) was produced to study the stress and traction distributions on the arch extrados, and it was concluded that the interaction between spandrel wall and arch ring was dominated by the shear flows. A simplified curved tapered beam model was adopted for the analysis of the interaction between the spandrel wall and arch extrados for the hard spandrel case. They concluded that the stiffness of the spandrel wall has a significant influence on the structural system for concentrated loads, but this could be limited by delamination of the arch ring from the wall. In this study, the spandrel wall was taken as a structural element rather than being ignored as in many previous analyses. The analytical studies were limited by the poor knowledge of interaction with the surrounding environment, but they are still encouraging. However, it is noted that the concentrated load which was applied directly on the spandrel wall, which is not consistent with real load cases. This is a problem that cannot be addressed in a two dimensional analysis.

Armstrong et al. (1995) studied the influence of spandrel wall separation on the dynamic response of masonry arch bridges using modal testing. Tests were carried out on two sets of segmental modal arches with different geometries. The arches in the first group had a span of $5\ m$ and span to rise ratio of $4$. The arch barrel and spandrels were not connected in order to simulate spandrel wall separation. The bridges in the second group had a span of $3\ m$ and rise of $730\ mm$. Loading was applied to the fill through concrete blocks at the third span. Static loads were applied to the arches to alter their mass and stiffness characteristics. Dynamic testing was then undertaken to investigate these changes. By comparing the mode shapes between the arches in the two different groups, the authors found that the deflections varied considerably across the bridge, and the deflections along the centre were significantly greater than those at the spandrels, indicating the restraint effects of the spandrel walls.

Cavicchi and Gambarotta (2009) developed a two dimensional finite element model for the study of load carrying capacity of masonry bridges while taking the action of spandrel wall into account. In the proposed model, the transverse
restraining effects of spandrel walls on the fill were considered by introducing a condition which limited the transverse compressive stress up to an admissible value. The out of plane stress component \( \sigma_3 \) was assumed to range from 0 to the limiting value \( \sigma_c \), and it was obtained as a function of the other principal stresses \( \sigma_1, \sigma_2 \). The limiting value can represent the maximum allowable pressure on the spandrel wall or the maximum allowable tensile strength in a tie rod which is used for strengthening of the spandrel wall. In their finite element model, the fill was modelled using three node triangular elements while the arches were approximated by straight beam elements as shown in Figure 2-18. Both the fill and masonry arches were assumed as perfectly plastic materials with no tensile strength. The proposed model was used for the analysis of Prestwood Bridge and compared with the experimental results obtained by Page (1987). The mainly improvement of this model is the introduction of out of plane stress to account the restrain effects of the spandrel wall. However, the determination of this stress value is difficult and different levels of stress may developed through the barrel up to parapet.

![Finite element model of the bridge (Cavicchi and Gambarotta, 2009)](image)

The transverse behaviour of masonry arch bridges was studied by Boothby et al. (Fanning et al., 2001; Boothby and Roberts, 2001; Fanning and Boothby, 2001) using three dimensional models (Figure 2-19) with a commercial finite element package (ANSYS). The complex behaviour of masonry was simplified to a homogeneous, isotropic material before cracking, it was modelled using solid
elements that can have their stiffness modified by the development of cracking and crushing. A failure criterion initially developed for concrete was used during the analysis. The fill was modelled as a Drucker-Prager material, and the interface between the masonry and fill was characterised by a frictional element with normal and sticking stiffness. The developed model was used to compare predictions with experimental testing under service load. Fanning et al. concluded that the lateral behaviour of a masonry arch bridge is as significant as the behaviour in the direction of the span. It was found that the model of the structure the three dimensional non-linear finite element analysis enabled accurate predictions to be made of the actual behaviour of a masonry arch bridge.

![Figure 2-19: 3D Finite element model of arch bridge (Fanning and Boothby, 2001)](image)

The relationship between a range of geometric and material parameters was examined with a series of finite element models under an ultimate load condition. Four different premature lateral failure mechanisms were identified from the numerical models: a) Overturning of the spandrel wall; b) Localised failure of the spandrel walls; c) Edge failure of the arch barrel; d) Punch through of the arch barrel (Boothby and Roberts, 2001). It was found that the thickness of the arch barrel and spandrel wall had a significant influence on the overall bridge strength and was directly related to the failure mode of the bridge. However, the depth of fill was less influential than expected. The authors claimed that the models have great accuracy for the prediction of vertical fill stresses, arch barrel displacements, lateral crack locations, collapse mechanisms, and ultimate strength of stone masonry structures in spite of neglecting the anisotropic behaviour of masonry. However, some of the results are highly dependent on
CHAPTER 2. LITERATURE REVIEW

the input material properties, and care should be taken when applied to brick masonry bridges. The spandrel wall in their model was considered as a continuum and modelled using the same concrete material model, the crack developments across spandrel wall found in many bridges were not discussed and documented in their studies.

2.5 Concluding remarks

Both theoretical and experimental research continue to be carried out on masonry arch bridges, although they have been subjected to scientific research for decades. The classical structural theory for the analysis of masonry has been refined and improved due to advances in scientific structural knowledge and computers. The development of computational and numerical techniques has enabled the analysis of more complex structures more easily, more quickly and more accurate than ever before. As the major thrust of arch bridge research, full scale testing has provided the essential information on behaviour of masonry arch bridge under service and ultimate load conditions. The structural function of backfill and spandrel wall has been proved, and the strengthening effect of passive soil pressures has been identified. These test results are of great value for the study of masonry arch bridges as it almost impossible to carry out tests on entire bridges due to financial and environmental reasons.

Most of the currently available arch bridge assessment methods are idealised representations of reality. Many unrealistic or subjective assumptions have been made in order to make the analysis easier or indeed even possible but these simplifications mean that some failure modes are neglected. Finite element modelling of such structures is sensitive to the variations of material properties. High quality data of material properties are required for a better reproduction of numerical models. There are still some gaps in understanding backfill/arch interaction and in the transverse behaviour of arch bridges. Further research efforts are required in this area. Finite/discrete element analysis, repeatability of large scale arch bridge tests, and modification to the mechanism method are still of interest to many researchers.
Chapter 3

Brick, mortar and brickwork
masonry material properties

3.1 Introduction

Masonry’s strength, durability and resilience to damage from water and fire have contributed to its widespread and continued use throughout history. Although some previous research work has investigated shear wall failure, there is a general scarcity of high quality published data on the mechanical properties of bricks, mortar and masonry under simple loading states that can be used to validate numerical modelling work. A series of laboratory tests were carried out to determine the material properties, and the objective of these tests was to obtain the required input data for the numerical model and have a better understanding of the composite behaviour of masonry assemblies under various loading conditions (compression, shear, flexure etc.). Owning to the nature of masonry, deformations are often localised to mortar joints, particularly when high strength engineering bricks are used with weak lime mortars. The measurement methodologies in standard tests can ignore this and measure deformations across a number of mortar joints leading to a lack of understanding of actual behaviour.

All the testing was carried out according to the relevant British Standard. Comparisons between the obtained experimental results and previous testing on similar materials are provided in this chapter. The tests are focussed on those required for a understanding of masonry arch bridge behaviour.
3.2 Materials

The materials used were Staffordshire Slate Blue Smooth Engineering Brick from 
Ibstock and Eminently Hydraulic Lime Mortar from LimeTechnology Ltd (Figure 3-1). These were selected as they are representative of the products used in some historic masonry arch bridges with a very high strength brick and a weak lime mortar. According to the factory specification, the fired clay bricks have a nominal size of 215 mm × 102 mm × 65 mm and an average density of 2220 kg/m³. The specified compressive strength is more than 75 N/mm². The mortar is a pre-mixed hydraulic lime mortar with a volume ratio of 1:2 (lime:sand). The specified compressive strength is 5 N/mm² at 91 days. The main advantage of this type of premixed mortar is its consistent mix proportions and quality. It can provide samples with consistent properties once the mortar/water ratio is determined. Although the bricks and mortar have specified strength for design purpose, for accurate modelling the actual properties must be measured.

(a) Brick unit
(b) Mortar

Figure 3-1: Materials used in this study

3.3 Brick tests

If the bricks are to be modelled in a computer package as a linear elastic material, the elastic modulus and Poisson’s ratio are required. These can be obtained via compression test and through accurately measuring the length change in principal directions. Axial compressive load was applied to the bricks until failure, which allowed calculation of a more accurate strength and determine the stress level for further tests. Temperature and water can influence the
CHAPTER 3. BRICK, MORTAR AND BRICKWORK MASONRY MATERIAL PROPERTIES

behaviour of masonry arch bridges, therefore it is of great value to obtain some information about the thermal properties and water absorption capacity of the bricks. The following material properties were tested in the lab: compressive and flexural strength, modulus of elasticity, thermal expansion and water absorption.

3.3.1 Compression test

One of the most commonly quoted mechanical properties of a clay brick is its compressive strength. Compression test were carried out in two directions on brick units for the determination of a more accurate compressive strength as shown in Figure 3-2. Three specimens were prepared for each condition and were capped using dental plaster to ensure a flat surface and a uniform bearing. The compressive strength of dental plaster was measured to be 70 N/mm$^2$ and therefore meets the requirement of BS EN 771-1. The bricks were oven (105 °C) dried and left in room temperature for a period of 24 hours to cool down. Given the maximum capacity of the machine, only half bricks were tested for the horizontal condition. Load was applied at a stress loading rate of 1.0 N/mm$^2$/s. The detailed test results are listed in Table 3.1.

![Brick compression test in two directions](image)

(a) Vertical  (b) Horizontal

**Figure 3-2:** Brick compression test in two directions

The experimental tests give an average compressive strength of 160 N/mm$^2$ and 100 N/mm$^2$ respectively for these two different conditions. It was noticed that the second specimen gave a relatively lower strength for the horizontal case. This may be explained by the inconsistency of brick material with pre-existing
cracks. The brick units showed brittle behaviour under the axial compressive load. According to the British Standard 771-1 (BSI, 2011), the measured values were then modified by a shape factor, and a normalized average strength of 145 $N/mm^2$ was obtained for the brick units. A report published by the British Railway Research at Derby (Temple and Kennedy, 1989) has shown that the bricks found in railway structures have a compressive strength ranging from 10 $N/mm^2$ to 100 $N/mm^2$. This indicates that the bricks used in this research have a higher compressive strength than those commonly found in railway structures.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Horizontal [$N/mm^2$]</th>
<th>Vertical [$N/mm^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>175.7</td>
<td>107.9</td>
</tr>
<tr>
<td>2</td>
<td>167.0</td>
<td>74.7</td>
</tr>
<tr>
<td>3</td>
<td>168.6</td>
<td>119.1</td>
</tr>
<tr>
<td>Shape factor</td>
<td>0.85</td>
<td>1.45</td>
</tr>
<tr>
<td>Normalised Average Strength</td>
<td>144.9</td>
<td>145.8</td>
</tr>
</tbody>
</table>

### 3.3.2 Thermal expansion test

As external masonry structures always experience environmental changes throughout life cycle, thermal effects are important factors when considering the behaviour of masonry arch bridges. Four bricks were used for the determination of the thermal expansion. A demountable mechanical strain gauge (DEMEC) was used to measure the change in length within a temperature range from 20$^\circ$C to 105$^\circ$C. The original length for measuring was set as 152 mm as shown in Figure 3-3. The whole test consisted of three cycles until stable results were obtained, each time three readings were taken for each specimen. An average value was recorded and is summarised in Table 3.2.

An average value of $8.22\times10^{-6}/^\circ$C was found for the thermal expansion coefficient of the bricks, and this is a little higher compared with the value reported by Ross (1941), where the thermal expansion coefficient of 90% tested bricks ranged between $5\times10^{-6}/^\circ$C and $7\times10^{-6}/^\circ$C.
3.3.3 Water absorption

Many arch bridges show defects that are caused by water, so it is useful to determine the water absorption of the bricks. Four bricks were fully immersed in water to determine the 24hrs water absorption according to BS EN 772-11 (BSI, 2000). They were stored at 20°C and 65% relative humidity in a curing room for 24 hours before immersion into water. The detailed results are listed in Table 3.3 and an average value of 1.5% was measured as the water absorption for the work of this study.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Temperature</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20°C</td>
<td>152.04</td>
<td>152.06</td>
<td>152.00</td>
<td>152.07</td>
</tr>
<tr>
<td>Measured Length</td>
<td>105°C</td>
<td>151.93</td>
<td>151.95</td>
<td>151.90</td>
<td>151.98</td>
</tr>
<tr>
<td></td>
<td>20°C</td>
<td>152.05</td>
<td>152.05</td>
<td>152.00</td>
<td>152.06</td>
</tr>
<tr>
<td></td>
<td>105°C</td>
<td>151.92</td>
<td>151.95</td>
<td>151.90</td>
<td>151.90</td>
</tr>
<tr>
<td>Average length change</td>
<td></td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Thermal expansion coefficient [10⁻⁶/°C]</td>
<td></td>
<td>9.86</td>
<td>8.22</td>
<td>8.22</td>
<td>6.57</td>
</tr>
</tbody>
</table>

Table 3.3: Water absorption test results

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass at curing room [g]</td>
<td>3162</td>
<td>3130</td>
<td>3127</td>
<td>3182</td>
</tr>
<tr>
<td>Mass after 24h immersion [g]</td>
<td>3208</td>
<td>3163</td>
<td>3188</td>
<td>3233</td>
</tr>
<tr>
<td>Water absorption [%]</td>
<td>1.5</td>
<td>1.1</td>
<td>1.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>
3.3.4 Flexural strength test

When masonry is subjected to out-of-plane flexural stress in the direction of the bed joints, failure can occur either at the mortar joints or through the units. The flexural resistance of the brick unit was determined using a three point bending test with a span of 175 mm (Figure 3-4). The flexural strength of brick units was then calculated by the following equation:

\[ F_f = \frac{M}{Z} \quad (3.1) \]

Where:

- \( M \) = the bending moment at failure, given as \( PL/4 \);
- \( Z \) = section modulus of test specimen, given as \( bd^2/6 \);
- \( P \) = maximum load at failure;
- \( L \) = distance between supports;
- \( b \) = width of brick unit;
- \( d \) = height of brick unit.

Ten specimens were prepared and tested. The flexural strengths ranged from 4.5 to 9.5 N/mm\(^2\), with an average value of 7.2 N/mm\(^2\). The standard deviation calculated for the flexural strength is 1.9 N/mm\(^2\).
3.3.5 Modulus of elasticity test

The modulus of elasticity of building material is of interest to engineers as it is directly related to deformation properties. Four bricks were tested under uniaxial compression loading to investigate the elastic properties of the brick. Linear Variable Differential Transducers (LVDT) were attached onto the face of units using aluminium holders and plastic padding to measure the displacement in both vertical and horizontal directions (Figure 3-5). Load was applied in three load cycles using a 100 kN compressive machine for two of the specimens with a 0.1 kN/s loading rate, while the other two specimens experienced only one load cycle. Test results indicated a linear relationship between the stress and strain in the range of stresses tested up to 15 N/mm$^2$ (Figure 3-6), an average elastic modulus of $25 \times 10^3$ N/mm$^2$ was calculated from the tests. Although LVDTs were attached on the side face of the brick unit in order to measure transverse strain and study the Poisson’s effect, the results showed similar value between the longitudinal strain and transverse strain, indicating that the determination of Poisson’s ratio using current method needs further review. Table 3.4 summarises the results obtained by other researchers from experimental test on bricks with similar compressive strength.
3.4 Mortar tests

The primary function of mortar is to bed and joint units. In the UK context, rail bridges were largely constructed during Victorian times (between 1840s and 1900s) and comprise either stone or brick masonry with a weak lime mortar. It is essential to undertake tests to determine the material performance characteristics in order to have a better understanding of the mortar contribution to the masonry assembly. The mortar used in this study was a premixed hydraulic lime mortar supplied by Lime Technology Ltd. Advantages of this mortar are its consistent quality, and its long working life. A water/dry material ratio of 0.19 was maintained throughout the production of the specimens, and it gave a flow value between 170 and 180 mm, the standard value for a mortar of this density, when measured in accordance with BS EN 1015-3 (BSI, 1999c). After 28 days storage in the laboratory conditions (20 °C, 65% relative humidity), the specimens were tested for compressive and flexural strength, modulus of elasticity and also
subjected to triaxial tests. The influence of moisture content on the strength of mortar at time of testing was also studied using 91 days old mortar specimens; 91 day strength has become the widely accepted ‘28 day’ equivalent used widely for Portland cement materials. The tests were carried out in line with relevant standards, while in some cases some changes were made to the standard procedure to better represent the real behaviour of mortar in this work.

### 3.4.1 Compressive and flexural strength

The strength of mortar as a jointing material is important for the overall strength of the composite masonry assembly. Three prisms measuring 40 mm × 40 mm × 160 mm were cast in steel moulds for the compressive and flexural strength test in accordance with BS EN 1015-11 (BSI, 1999b). They were tested after 28 days storage under 20°C and 65% relative humidity. The flexural strength was determined firstly using a three point bending test as shown in Figure 3-7(a). The broken half sample was then used for the determination of compressive strength (Figure 3-7(b)). These tests were under displacement control at a rate of 0.2 mm/min and 0.5 mm/min respectively. The average compressive and flexural strength obtained were 0.74 N/mm² and 0.44 N/mm² respectively. The standard deviations calculated are 0.11 N/mm² and 0.09 N/mm² respectively for the compressive and flexural strength. These values show good agreement with the work of Hendry (1990) which reported compressive strength between 0.5 N/mm² and 1.0 N/mm² for lime mortar on old masonry arch bridges.

![Mortar flexural and compressive tests](image-url)
3.4.2 Modulus of elasticity tests

Two prisms with dimensions of 75 mm × 75 mm × 135 mm (Figure 3-8(a)) were made for testing of elastic properties of the mortar, the same set up for the bricks was applied to the mortar samples. The test was carried out under load control conditions. As the mortar shows a low compressive strength, a slow loading rate of 0.02 kN/s was used during the test. The stress-strain relationship is shown in Figure 3-8(b). The stress increases linearly at the beginning and the gradient decreases gradually to zero as it undergoes plastic deformation. The maximum strain during the tests on the two specimens was 0.27% and 0.38%, with an average elastic modulus of 700 N/mm² based on the linear stage under 0.05% strain. As with the brick tests, although transducers were fitted in the horizontal directions, it was difficult to accurately determine the Poisson’s ratio.

![Mortar sample for elasticity test](image1)

![Stress strain relationship of mortar unit](image2)

Figure 3-8: Mortar elasticity tests

3.4.3 Triaxial test on mortar

In addition to the conventional unconfined tests used for mortars, the mortar was subjected to triaxial testing, commonly used to define the strength of soil materials, in order to determine its stiffness, cohesion and frictional properties.
CHAPTER 3. BRICK, MORTAR AND BRICKWORK MASONRY MATERIAL PROPERTIES

under increasing confining stress. Three cylinders with a diameter of 100 mm and height of 200 mm were prepared. In order to simulate very weak mortar in old masonry structures and compare with previous tested results, they were tested after only 28 days storage in the laboratory conditions (20 °C, 65% relative humidity). The specimens were tested under drained conditions and the confining pressures were 0, 0.2 and 0.4 N/mm² respectively. The setup for test was shown in Figure 3-9, two transducers were attached on the top of the cell to measure displacement, and for the specimen with no confining pressure, a radial transducer and two axial transducers were fixed on the specimen to allow more accurate determination of small-strain properties. Tests were carried out under displacement control conditions and the loading rate was 0.5 mm/min. The elastic modulus from the unconfined sample with on-sample strain measurement was similar to that for the mortar prisms. It was not possible to use on-sample instrumentation for the confined samples as they were within a latex membrane and the conventional fixing methods for on-sample transducers in soils did not work because of the cementation of the lime sample. The stiffness of the confined samples therefore could not be accurately determined using external displacement measurements, and only the stress-strain relationship for the specimen without confining pressure is presented.

The maximum principal stresses obtained were 0.43, 1.14 and 1.75 N/mm² for the samples with 0, 0.2 and 0.4 N/mm² confining stress respectively. The samples showed clear shear failure under axial compression with different confining pressures. The tested specimens were sprayed with Phenolphthalein, and the chemical reaction indicated that only the surface of the cylinders had

Figure 3-9: (a) Experimental set up and failure modes under different confining stress (b) 0 (c) 0.2 (d) 0.4 N/mm²

The maximum principal stresses obtained were 0.43, 1.14 and 1.75 N/mm² for the samples with 0, 0.2 and 0.4 N/mm² confining stress respectively. The samples showed clear shear failure under axial compression with different confining pressures. The tested specimens were sprayed with Phenolphthalein, and the chemical reaction indicated that only the surface of the cylinders had
carbonated as expected for a lime sample at early age. Additional carbonation would result in an increase in mortar strength (Lanas and Alvarez-Galindo, 2003), but this could take an unacceptably long time, particularly for the 100 mm diameter triaxial samples. Mortar carbonation rates are significantly influenced by interaction with the units within the masonry, with dewatering of the mortar by brick suction effects influence drying rates for the mortar and the resultant porosity.

![Stress and axial strain curve](a)

![Axial strain versus radial strain](b)

**Figure 3-10:** Test results for specimen with 0.2 N/mm² compressive stress (internal transducer)
The relationship between the stress and strain measured by the internal transducers fitted on the sample for zero confining stress is plotted in Figure 3-10(a), and comparison was made between the axial and radial strains (Figure 3-10(b)). As can be seen, the specimen shows similar behaviour to the elasticity tests on the unconfined prisms. The stress and strain curve indicated a linear stage when the strain is smaller than 0.05%, and it shows a plastic behaviour when the stress exceed 0.4 $N/mm^2$. The modulus of elasticity was then calculated based on the linear stage and gave a value of 750 $N/mm^2$. The Poisson’s ratio was then identified by comparing the strains in the axial and radial directions and gave a value of 0.075. According to the Mohr-Coulomb theory, the cohesion and friction angle were determined as 0.115 $N/mm^2$ and $34^\circ$ respectively (Figure 3-11). This indicates a constant strength for the mortar may not be a good representation of behaviour. The smallest diameter circle in Figure 3-11 represents the unconfined compressive strength and the value of 0.43 $N/mm^2$ is much lower than that of 0.74 $N/mm^2$ attained by the smaller prisms. This could be because of increased carbonation of the smaller sample, because of the different geometrical ratios (height: width was 1:1 for the smaller prisms but 2:1 for the triaxial sample), because increased moisture content of the larger triaxial sample which would not have fully dried or because of size effect. All of these factors would have resulted in an increased strength of the smaller prisms as measured, and this is an area requiring further investigation, but beyond the scope of this research.

![Figure 3-11: Mohr Coulomb circles obtained from triaxial test](image)

Table 3.5 provides a summary of the mortar properties that have been tested by other researchers on similar lime mortars (Baker, 1996; Sicilia, 2001; Mahmoud, 2005). The elastic modulus obtained from the tests is lower than
that in the other studies, and this may be attributed to the lower compressive strength of the specimen. As can be seen from the table, the modulus of elasticity shows an increased trend. Comparisons between the results from the elasticity test and triaxial test show that the size of the sample has little influence on the elastic modulus.

### Table 3.5: Mortar unit materials

<table>
<thead>
<tr>
<th></th>
<th>Compressive strength [N/mm²]</th>
<th>Flexural strength [N/mm²]</th>
<th>Modulus of elasticity [N/mm²]</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baker (1996)</td>
<td>3.5</td>
<td>0.25</td>
<td>11154</td>
<td>0.14</td>
</tr>
<tr>
<td>Miri (2005)</td>
<td>1.7</td>
<td>0.76</td>
<td>2900</td>
<td>0.09</td>
</tr>
<tr>
<td>Sicilia (2001)</td>
<td>1.4</td>
<td>0.62</td>
<td>1879</td>
<td>0.064</td>
</tr>
<tr>
<td>Current study</td>
<td>0.74</td>
<td>0.44</td>
<td>725</td>
<td>0.075</td>
</tr>
</tbody>
</table>

#### 3.4.4 Influence of water on mortar strength

For many exposed masonry structures, water is an important environmental factor which has influence on their overall behaviour. An effort was made to investigate the influence of moisture content at time of testing on the strength performance of the mortar. Eleven mortar prisms (40 mm × 40 mm × 160 mm) with 91 days air cured age under laboratory conditions were tested for compressive and flexural strength. These were divided into four groups. The first group was placed in the curing room for 91 days while the other three groups had the same curing conditions but were placed in a water filled tank 48hrs, 24hrs, 16hrs before testing. Moisture content was measured for each specimen after testing by putting them into a 105 °C oven for 48 hours.

### Table 3.6: Experimental results of mortar compressive and flexural strength

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Moisture content (%)</th>
<th>Immersion time (hour)</th>
<th>Average flexural strength [N/mm²]</th>
<th>Average compressive strength [N/mm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1</td>
<td>0</td>
<td>0.38</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>9.0</td>
<td>48</td>
<td>0.43</td>
<td>1.1</td>
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<td>3</td>
<td>11.0</td>
<td>24</td>
<td>0.35</td>
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<td>4</td>
<td>10.8</td>
<td>16</td>
<td>0.29</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Small differences (up to 24%) were found in terms of the flexural and compressive strength results among different groups. The flexural strength ranges from 0.27 to 0.45 N/mm$^2$, while the compressive strength ranges from 0.84 to 1.25 N/mm$^2$. The standard deviations calculated are 0.14 N/mm$^2$ and 0.06 N/mm$^2$ respectively for the compressive and flexural strength. The saturated specimens had an average moisture content of 10% and do not show great difference between groups. Compared to specimens with 28 days age, the compressive strength of the mortar has increased by 48% at 91 days. The flexural strength decreased by 14% over the same period, and this may be caused by the variety of the mixing as the specimens were not made at the same time or could be due to the higher variability of flexural results. There is no defined relationship between the strength and moisture content. An Analysis of Variance (ANOVA) was performed to identify the significance of moisture content on mortar strength, and it gives the conclusion that at the 95% confidence level, the moisture content has no significant influence on the compressive or flexural strength of the mortar. This may not hold for all mortars or if the mortar is saturated for a longer period where leaching may occur and curing may be affected. The results are summarised in Table 3.6.

3.5 Brickwork masonry tests

A number of brickwork masonry specimens were prepared for different tests in accordance with relevant standards. The details of the various masonry tests and their instrumentation are discussed in this section. Vertical and horizontal mortar joints in these specimens were maintained to a nominal 10 mm thickness by an experienced bricklayer. All brickwork specimens were covered with plastic after fabrication, and then after 14 days uncovered and stored in laboratory conditions until testing at between 91 and 100 days age.

3.5.1 Compressive strength test

Three small masonry walls with dimensions of 330 mm $\times$ 290 mm $\times$ 102 mm were produced for the study of stiffness characteristics. The size of the specimens was not exactly as recommended by BS EN 1052-1 (BSI, 1999d) and was modified.
by taking the maximum capacity of compression machine in the laboratory into account. Figure 3-12 shows the general arrangement of the test specimens. All the specimens were capped with dental plaster before testing in order to obtain a flat surface and ensure load was distributed uniformly. Two transducers were fitted on the surface to measure vertical movement. Given the relatively high compressive strength of the brick unit, load was applied at a 2 kN/s rate until failure of the walls.

![Figure 3-12: Small compressive wall test set up](image)

The experimental results showed that all the specimens exhibit initially linear behaviour between the stress and strain with brittle behaviour at failure (Figure 3-13). The average compressive strength was 33 $N/mm^2$ and an average elastic modulus of 2550 $N/mm^2$ was calculated by taking the whole range of data in consideration. From BS EN 1052-1, the characteristic compressive strength of masonry is given as:

$$f_k = f/1.2 \quad \text{or} \quad f_k = f_{i,min} \quad (3.2)$$

Where $f_{i,min}$ is the smallest compressive strength of an individual masonry specimen and $f$ is the mean compressive strength of the masonry. The characteristic compressive strength is then identified as 24.7 $N/mm^2$. The characteristic strength was also calculated using the equation provided by Eurocode 6 (BSI, 2005a) based on the compressive strength of brick and mortar units for comparison purpose.

$$f_k = K_f^{0.7} f_m^{0.3} \quad (3.3)$$
where:

\[ f_k \] is the characteristic compressive strength of the masonry, in \( N/mm^2 \)
\[ f_b \] is the normalised mean compressive strength of the units, in \( N/mm^2 \)
\[ f_m \] is the compressive strength of the mortar, \( N/mm^2 \)
\[ K \] is a constant which related to the type of mortar and masonry unit.

The predicted value from Equation 3.3 is 17.9 \( N/mm^2 \) which is 38% lower than the measured value. This could be because the Eurocode equation is an empirical equation which is calibrated for modern masonry with high strength mortars, or because the Eurocode equation is designed to return a conservative estimate of strength in the absence of masonry test data.

![Stress strain relationship of masonry wallette](image)

**Figure 3-13: Stress strain relationship of masonry wallette**

Particle Image Velocimetry (PIV) analysis was performed based on a series of digital images taken during testing. The general movement is illustrated by displacement vectors represented by the arrows showing 20 times magnification of the real displacement (Figure 3-12). The maximum horizontal and vertical displacements identified were 2.0 mm and 0.9 mm respectively by the analysis. It was noted that greatest strains were localised on the mortar before failure, but at failure one brick failed in tension induced by lateral expansion. This indicates
that a longer wall with increased lateral restraint may not fail in this manner and that the test method may therefore yield conservative strengths.

3.5.2 Triplet shear test

The friction and cohesion properties between the brick and mortar affect masonry behaviour under lateral loading. Fifteen triplet stacks were prepared and tested in accordance with BS EN 1052-3 (BSI, 1999c) to study the friction properties before and after initial failure, and the influence of the specimen moisture content on shear strength was also investigated. The failure load and load displacement relationships obtained from these tests are intended for calibration of computer modelling work.

Figure 3-14: Schematic arrangement for triplet shear test

Figure 3-14 shows the schematic arrangement of this test, two transducers were attached on the top surface, and one was fitted on the loading jack while another two were attached on the top and bottom brick. All the specimens were divided into five groups. The first four groups were tested under normal dry conditions with different compressive stresses which are 0 $N/mm^2$, 0.2 $N/mm^2$, 0.6 $N/mm^2$ and 1 $N/mm^2$ respectively. There was one specimen in each group to which load was applied through three loading cycles to about 33% of the expected failure load and then loaded to failure (except for the 0 $N/mm^2$ normal stress condition). The specimens in the final group were put into water to achieve a saturated condition, and they were then tested under 0 $N/mm^2$, 0.2 $N/mm^2$ and 0.6 $N/mm^2$ normal stress respectively.
The results show that there is no remarkable decrease of the shear strength for the saturated specimens compared with the corresponding dry ones. The relationship between shear strength and normal stress was linear for both the dry and saturated specimens, as was shown by the work carried out by Capozacca (2011). The characteristic angles of friction determined in accordance with BS EN 1052-3 (BSI, 2002) are 26.6° and 25° (coefficient between normal stress and shear strength of 0.50 N/mm² and 0.47 N/mm² for the dry and wet samples respectively). The average initial shear strengths were 0.084 N/mm² and 0.082 N/mm² respectively for the dry and saturated samples. This equates to a reduction in initial shear strength of approximately 26% from the cohesion measured during triaxial testing, and a reduction of masonry friction angle of approximately 22% (reduction of tangent of friction angle of 26%). Reductions in strength were expected and are attributed to the weaker brick/mortar interface compared with the mortar (Roca et al., 1998). As a result, the failure by delamination at the interface was the most likely failure mechanism. The main failure mode observed was delamination at the interface, accompanied with some cracking of mortar at the corner (Figure 3-15). The test results are listed in Table 3.7.

The characteristic initial shear strength gives a value of 0.07 N/mm², which is 50% of the figure reported by Zhou (2008) for similar mortars, while the designed value recommended by Eurocode 6 (BSI, 2005a) is 0.1 N/mm². Although blue bricks were used by Zhou, they had different geometry with the brick for this study having no perforations and those used by Zhou having three holes with a total of 18% perforations, and this is most likely the reason for the
variation between these figures and those by Zhou. The presence of perforations means some of the mortar has to shear as well as the brick/mortar interface, potentially increasing the shear capacity, but historic masonry arch bridges are unlikely to have bricks with perforations. The coefficient between normal stress and shear strength are significantly higher than the 0.40 used in Eurocode 6, indicating the equations Eurocode 6 will overpredict strength at low normal stresses and underpredict strength at higher normal stresses in this particular case. The partial factors for actions and resistances in the Eurocode will, however, ensure shear failure is not achieved for these materials.

Table 3.7: Shear strength under different normal stress levels

<table>
<thead>
<tr>
<th>Applied normal pre-compressive stress [N/mm²]</th>
<th>Mean shear strength initial shear strength [N/mm²]</th>
<th>Water content of mortar [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.09</td>
<td>Not tested</td>
</tr>
<tr>
<td>Dry specimen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.21</td>
<td>0.07</td>
</tr>
<tr>
<td>0.6</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>Saturated specimen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.06</td>
</tr>
<tr>
<td>0.6</td>
<td>0.43</td>
<td>9.3</td>
</tr>
</tbody>
</table>

Figure 3-16 shows the relationship between mean relative displacement of the central block to the adjacent ones and corresponding load obtained for the specimens under different stress levels. The stress increased almost linearly with the displacement until failure, and then behaviour was governed by the friction, which produced a relatively constant residual stress. Experimental work completed by Abdou et.al (2006) on mortar joint behaviour showed similar response between shear stress and deformation. As the work was carried out under displacement control conditions, and so the load decrease after initial failure was less pronounce with the displacement. The initial displacement of the specimen under 0.6 N/mm² at the beginning maybe attributed to the unloading and loading path for the last two load cycles. It therefore can be concluded that the linear behaviour has been stiffened after the first load cycle. For the 1 N/mm² stress level, as the load provided by friction is close to the maximum load, instead of abrupt load decrease after initial failure, the load decreased gradually accompanied by mortar crushing.
Figure 3-16: Shear stress displacement curves for triplet test under different normal stress levels

Figure 3-17: Vertical displacement under different normal stress levels
The average vertical movement throughout the tests are plotted versus the horizontal displacement in Figure 3-17. As the normal stress was applied, these specimens experienced an initial compression of 0.6 mm, 0.9 mm and 1.2 mm respectively. It can be seen from the chart that the specimens under 0.2 N/mm² and 0.6 N/mm² normal stresses dilated by 0.3 mm and 0.1 mm respectively after both joints opened, while the specimen under 1.0 N/mm² stress level behaved in the opposite way; it was compressed with a final displacement of 0.3 mm accompanied with the mortar cracking near the brick/mortar interface. This behaviour of increasing dilation under lower normal stress is common in shear box tests on sands which have similar stress conditions to the triplet shear test (Burland and Yu, 2007).

3.5.3 Bond wrench strength test

Two stacks with five mortar joints were made for the determination of the bond strength of masonry using the bond wrench method and tested in accordance with the specifications of the BS EN 1052-5 (BSI, 2005b). The stack was put on a rigid platform (Figure 3-18); the top two contiguous bricks were clamped and load was applied on the top-most brick through a lever arrangement. The main failure mode observed for the specimens was tensile fracture along the interface between the brick bed face and the opposing mortar joint. The bond strength for each individual specimen was then calculated using the following formula (Equation 3.4)

\[
f_{wi} = \frac{F_1e_1 + F_2e_2 - \frac{2}{3}d(F_1 + F_2 + \frac{W}{4})}{Z}
\]

\[
Z = bd^2/6
\]

Where:

- \(b\) mean width of the bed joint tested in \(mm\);
- \(d\) mean depth of the specimen in \(mm\);
- \(e_1\) distance from the applied load to the tension face the specimen in \(mm\);
- \(e_2\) distance from the centre of gravity of the upper clamp from
the tension face of the specimen in \( mm \);

\( F_1 \) applied load in \( N \);
\( F_2 \) weight of the bond wrench in \( N \);
\( W \) weight of the masonry unit pulled off the specimen and any adherent mortar.

![Figure 3-18: Bond wrench test set up](image)

Seven groups of meaningful results were obtained, and the details of the test results have been listed in Table 3.8. As can be seen from the table, the individual bond strength ranges from 0.1 \( N/mm^2 \) to 0.28 \( N/mm^2 \) with an average value of 0.18 \( N/mm^2 \). These figures are close to the work done by Zhou (2008) on similar materials. The presence of perforations in the bricks used by Zhou should not affect bond strength as the failure is in flexurally induced tension. The characteristic bond strength of the samples used for this research was then calculated according to BS EN 1052-3 and gave a value of 0.08 \( N/mm^2 \).

**Table 3.8: Bond wrench test results**

<table>
<thead>
<tr>
<th>Joint</th>
<th>Applied load ( F_1 ) (( N ))</th>
<th>Pull out unit weight ( W ) (( N ))</th>
<th>Individual bond strength (( N/mm^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.5</td>
<td>30.9</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>93.2</td>
<td>33.4</td>
<td>0.28</td>
</tr>
<tr>
<td>3</td>
<td>67.7</td>
<td>30.9</td>
<td>0.21</td>
</tr>
<tr>
<td>4</td>
<td>44.1</td>
<td>34.0</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>52.0</td>
<td>33.8</td>
<td>0.17</td>
</tr>
<tr>
<td>6</td>
<td>76.5</td>
<td>34.2</td>
<td>0.24</td>
</tr>
<tr>
<td>7</td>
<td>37.3</td>
<td>34.2</td>
<td>0.13</td>
</tr>
</tbody>
</table>
3.5.4 Shear wall tests

Three brickwork walls with dimensions of 665 mm × 740 mm × 102 mm were constructed for the evaluation of the in-plane shear failure under static compression. Construction of these walls was performed manually by the same mason to ensure consistent workmanship, and they were built on timber stands for easy transportation and to avoid local damage. They were tested under three loading conditions as shown in Figures 3-19(a), 3-20(a), and 3-21(a). The specified axial pre-compression stress was 0.2 N/mm² for specimen 1 and 3, and the load was applied using two jacks through low friction Teflon bridge bearings. Steel beams and a timber frame were laid on the top and bottom to help distribute the vertical load uniformly. The applied load was adjusted manually during the test to maintain a constant stress level. Applying a constant stress at the top of the wall would allow horizontal movement and rotation of the top. Four transducers were attached on the top two at jacks, loading jack, and the reaction frame respectively to monitor the movement.

![Figure 3-19: Shear wall 1 test](image)

For shear wall 1, the first course of the wall was horizontally supported by steel blocks, while the load was applied at the top course through the steel beam. For shear wall 2, the designed compressive stress was 1 N/mm² and the load
was applied on the top right corner over an area of 150 mm × 102 mm. As the same as shear wall 1, the bottom two courses were horizontally supported by steel blocks, while the load was applied at the top course in order to simulate a diagonal shear condition for the specimen. For shear wall 3, both the bottom and top courses were horizontally supported against the reaction frame, while the
load was applied at the fifth course through a steel plate. Care was taken when pushed the jacks to avoid out of plane movement during load application.

The maximum shear failure loads obtained were 20.5 kN, 14.1 kN and 28.7 kN respectively for these three specimens, and the corresponding shear stresses were calculated as 0.30 N/mm², 0.21 N/mm² and 0.21 N/mm². The applied load versus the corresponding displacement of the loading cell is plotted in Figure 3-22. PIV analysis was performed based on a series of pictures before and after failure, giving the general movement of each component represented by the arrows. The design value of shear resistance was then calculated by equation 3.6 given by Eurocode 6 (BSI, 2005a):

\[ V_{Rd} = f_{vd}c_t \]  \hspace{1cm} (3.6)

\[ f_{vk} = f_{veko} + 0.4\sigma_d \]  \hspace{1cm} (3.7)

Where:

- \( f_{vd} \) design value of the shear strength of masonry at that normal stress (can be obtained from characteristic shear strength);
- \( t \) the wall thickness;
- \( l_c \) that portion of the wall which carries compressive stress;
- \( f_{veko} \) the characteristic initial shear strength, under zero compressive stress;
- \( \sigma_d \) the applied normal stress.

It is difficult to determine where the failure occurs, since the crack patterns are related to the loading which the wall is subjected. The shear resistance was firstly calculated based on the initial condition by assuming the walls were under compression across the entire width. As the normal stress caused by self-weight is rather small compared with applied normal stress, it was not included in the calculation. The shear resistance was first calculated with the suggested values in the Eurocode. The characteristic initial shear strength was taken as 0.1 N/mm² and it gives a maximum shear resistance of 12.2 kN for shear wall 1 and 2, with 24.4 kN for shear wall 3. Equation 3.7 was then modified with experimental results, which gives:

\[ f_{vk} = f_{veko} + 0.5\sigma_d \]  \hspace{1cm} (3.8)
The shear resistance was calculated with the measured value and gives a maximum load of 11.5 kN for shear wall 1 and 2, and 23.0 kN for shear wall 3. As can be seen, both of the value are rather small compared with experimental results, indicating the standard is conservative in design.

For shear wall 1, the observed failure mode is stepped cracking through the mortar joint at the bottom four courses as shown in Figure 3-19(b). The stepped failure across different courses is consistent with the theory proposed by Mann et al. (1982) and the corresponding failure shear stress was calculated by the equation provided and gives a value of 0.15 N/mm². The detailed failure mode is greatly affected by the pre-compression load and material strength according to Senthivel and Lourenco (2009). The maximum horizontal and vertical displacement obtained from the PIV analysis was 18.2 mm and 10.0 mm respectively. The overturning of the shear wall has been observed during the test, and lower and upper bound approaches have been developed by Milani (2006) for the limit analysis of shear walls with overturning effects. The load displacement curve is characterised by an approximately linear increase with several drops before reaching the maximum load, and then the load remains almost constant. The reason for the fluctuation is due to manual adjustment of normal load, which is inevitable given the set-up as the load was applied by hand and could not be controlled automatically. The initial load displacement response before failure is similar to the results reported by Vermeltfoort (1993) in his work on a shear wall under 0.3 N/mm² compressive stress. The only exception was the large corresponding horizontal displacement when maximum load was reached, and this may be caused by the lower compressive stress and different loading system adopted here.

For shear wall 2, this masonry wall behaved almost linearly up to peak load (Figure 3-22) and then failed suddenly at small deformations with a diagonal cracking failure mode as shown in Figure 3-20(b). The general movement of the specimen after failure was identified by the PIV analysis, and it gave a value of 10 mm and 3.5 mm for the maximum horizontal and vertical displacement respectively. A series of tests was carried out by Najif et al. (2011) for the study of diagonal shear behaviour of unreinforced masonry walls. Two different load configurations were adopted in their study, one group was tested following the standard guidelines in ASTM E-519-02 (ASTM, 2002), while the other group used a similar set up to that described in this study. Similar behaviour in terms of load
displacement relationship was observed between these two studies. Comparisons between the experimental results in the two groups performed by Najif et al. indicated a lower shear strength for the current configuration compared with the standard test procedure (ASTM, 2002).

For shear wall 3, a similar failure mode was observed in the lower part of the wall while the wall failed in sliding along the loading course (Figure 3-21(b)). The PIV analysis gave 7.0 mm and 1.0 mm for the maximum horizontal and vertical movement. The maximum shear stress and the trend of the load-displacement curve are consistent with the triplet shear test, where the load increased linearly until the maximum was achieved, then dropped abruptly and remained constant thereafter. This has provided evidence of the potential using data from simple masonry specimens for the analysis of complex masonry structures, especially for the prediction of failure load. The load and displacement characteristics can be used for validation of future computer modelling.

**Figure 3-22: Load displacement curves for shear walls**
3.5.5 Flexural strength tests

The flexural strength with planes of failure parallel and perpendicular to the bed joint were determined in accordance with BS EN 1052-2 (BSI, 1999a). Two specimen formats were tested; one 2 brick units wide by 10 courses high rectangular specimen for the determination of the masonry flexural strength across the bed joints and a second 4 units wide by 4 courses high wallette for the determination of the flexural strength parallel to the bed joints. Five specimens were constructed for each condition and the general set up of these tests is shown in Figure 3-23. Linear Variable Differential Transducers (LVDT) were located at three points on each wall, one near the mid-span and one near each support. Magnetic bases attached the LVDT to unloaded elements of the

Figure 3-23: Experimental set up for flexural strength tests
test rig. Loading was applied at a slow and repeatable rate using a hydraulic jack until failure of the specimen. For the tests with plane of failure parallel to the bed joints (Figure 3-23(a)), fracture planes occurred along the interface between the brick and mortar joint. Most of the failure planes were located within the inner bearings, except for one specimen which failed with the cracks between the inner and outer bearings on the top, indicating a shear rather than flexural failure. A combined failure along the joint interface and within the depth of mortar was observed for the tests with plane of failure perpendicular to the bed joint (Figure 3-23(b)), and all the fractures occurred within the inner bearings. All specimen showed similar behaviour in terms of the load displacement relationship. Typical load displacement curves measured by the transducers at different locations which include the post failure behaviour are plotted in Figure 3-24 for the test with plane of failure perpendicular to the bed joint. The detailed results are summarised in Table 3.9.

![Graph showing load deflection relationships for wall](image)

**Figure 3-24:** *Load deflection relationships for wall*

The tests in both directions show similar behaviour in terms of the load displacement relationships. All graphs reveal an almost linear relationship from the first application of load up to 25% of the failure load. The graphs become non-linear as the curvature increased until the ultimate load. All the specimens failed in a brittle manner with an immediate and complete loss of strength. Figure 3-25 shows a typical load displacement response for conventionally constructed wallets in previous research by Willis et al. (2004) and Lourenco et al. (1999).
The curve was divided into four regions with the change of slope, while the points of inflection represent the first cracking point, the ultimate load point and residual strength respectively. As shown in Figure 3-24, it seems that wallette experiences a rigid body movement at the beginning since transducers at different locations give close values. The load increased slowly at the initial stage, and this may due to the small gap between the specimen and test rig. The load has a sudden decrease when the wallette failed and remained almost constant thereafter. Similar behaviour has been observed by Kanyeto (2011) for the experimental work carried on concrete blockwork. It was argued by the author that the different behaviour may because the bond condition at the joint might be different due to the potential micro-cracking in the material, some regions with weaker bond start to fail while the stronger regions are beginning to carry load, and this leads to an increase in the stiffness and gives non-linear behaviour in the graph. As the weaker areas fail, the remaining bond is stronger and stiffer than the failed parts which results in the unusual behaviour.

The characteristic flexural strength calculated from the standard (BSI, 1999a) for the specimens failed parallel and perpendicular to bed joints were 0.08 \(N/mm^2\) and 0.37 \(N/mm^2\) respectively, and these values are about 50% and 30% smaller than the values reported by Zhou (2008). Although both tests share the same failure mechanism, the failure strength parallel to bed joints is lower
<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Failure stress ( \sigma ) [N/mm²]</th>
<th>Calculated moment of resistance ( M_{c} ) [kNm]</th>
<th>Predicted failure stress ( \sigma_{p} ) [N/mm²]</th>
<th>Calculated moment of resistance ( M_{c} ) [kNm]</th>
<th>Characteristic value ( M ) [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.56</td>
<td>0.29</td>
<td>0.07</td>
<td>0.1</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.49</td>
<td>0.24</td>
<td>0.11</td>
<td>0.04</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>0.61</td>
<td>0.3</td>
<td>0.09</td>
<td>0.04</td>
<td>0.37</td>
</tr>
<tr>
<td>4</td>
<td>0.57</td>
<td>0.29</td>
<td>0.13</td>
<td>0.04</td>
<td>0.37</td>
</tr>
<tr>
<td>5</td>
<td>0.56</td>
<td>0.29</td>
<td>0.1</td>
<td>0.04</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>Characteristic value ( M ) [kNm]</td>
<td>Perpendicular to bed joint</td>
<td>69</td>
<td>0.36</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Parallel to bed joint</td>
<td>69</td>
<td>0.36</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.3</td>
<td>69</td>
<td>0.36</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9</td>
<td>69</td>
<td>0.36</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.9: Flexural strength test results
than achieved in the bond wrench test (average 0.18 N/mm$^2$) as An increase to a similar value to that of Zhou (2008) would therefore have been expected, with an anticipated decrease in strength for failure perpendicular to bed joints as a result of Zhou’s tests being undertaken on bricks with 18% perforations, resulting in both interface and mortar failure. The corresponding ultimate bending moment $M_{ult}$ was studied and compared with the value calculated from the mathematical model proposed by Willis et al. (2004) for tests with plane of failure perpendicular to the bed joint. The ultimate horizontal bending moment is mainly governed by torsional capacity of the bed joints, which could be calculated using the following equation:

$$M_{ult} = n_b t_b k_b 0.5(l_u + t_m)t^2$$  \hspace{1cm} (3.9)

Where:

- $n_b$ number of bed joint in failure;
- $t_b$ ultimate shear stress on a bed joint;
- $k_b$ numerical factor used in calculation of $t_b$ (for $(l_u + t_m)=225$ mm, $t=102$ mm, $k_b = 0.21$ (Timoshenko and Goodier, 1970));
- $l_u$ length of brick unit;
- $t_m$ thickness of mortar joint;
- $t$ thickness of masonry section.

As shown in Table 3.9, the value predicted by Equation 3.9 is 0.07 kNm which is only 25% of the measured value from the tests. The mathematical model has greatly underestimated the failure load; it gives a much conservative value for design and might not be applicable for the masonry structures with weak mortar and strong masonry unit. The bending moment is also compared with the value given by Eurocode 6 by assuming NHL lime mortar is a ‘general mortar’, the experimental value is about three times higher than the code value. This demonstrates the necessity of using measured data for the prediction of performance.

### 3.6 Concluding remarks

A series of tests was undertaken to determine the properties of engineering blue brick units and premixed hydraulic lime mortar, relevant British Standards
procedures were followed in most cases. The shear behaviour between the bricks and mortar under different stress levels and different loading conditions was also investigated. Comparison of the test results with test results reported elsewhere have shown good agreement, but it was noted that designed values from Eurocode 6 are conservative. The following conclusions could be drawn from the experimental study.

- Compressive test on brickwork specimens shows linear behaviour between stress and strain under compressive load, and the failure was caused by crushing of the brick.

- Linear relationship between the shear strength and normal stresses has been found for both the mortar and the brick/mortar interface, and a reduction in initial shear strength has been found for the brick/mortar interface.

- Shear failure of brickwork walls is characterised by stepped cracking and sliding through the mortar joint. The magnitude of the decrease is higher than the initial shear strength, indicating there is a breaking of the bond as well as a decrease in friction once failure commences. Comparison between the experimental results and design values given by relevant standard shows that the standard is conservative in design, particularly at higher normal stress levels.

- Failure planes for flexural testing occurred along the interface between the brick and mortar joint. For the flexural test, the proposed mathematical model (Willis et al., 2004) and Eurocode 6 both give conservative values for design. The load displacement relationship of NHL mortar brickwork reveals different behaviour compared with conventional masonry wall panels under lateral load. More research work is needed to compare the theoretical model by Willis et al. (2004) with the results obtained in this study.

- Based on results of this testing and on previous tests by Zhou (2008) with similar mortars and bricks but containing perforations, the presence of perforations appears to increase shear strength by forcing failure to be both along the brick/mortar interface and through the mortar in the perforation.

The material properties and small-scale masonry wall behaviour presented in this chapter are used for validation of future computer modelling work in the following chapters.
Chapter 4

Modelling of small masonry structures

4.1 Introduction

While the finite element method (FEM) is widely and successfully used when analysing both steel and reinforced concrete structures, there is still some dispute about its application for the analysis of masonry structures. The main reason is due to the discontinuous characteristic of masonry, which consists of brick/stone units connected by mortar joints. Numerical representation can be achieved either by focusing on the individual components, brick and mortar units, or at a macro level by considering the masonry as a continuum composite.

In this chapter, a brief description of modelling techniques that have been used for masonry structures is given. This is focused on micro modelling and macro modelling, and different constitutive laws developed by various researchers. The modelling work that have been done by the author is based on the commercial finite element analysis package, ANSYS. The work focused on the application of models to the problem of masonry arch bridges rather than the development of constitutive materials. The modelling work was initially carried out on the triplet shear tests, and several approaches have been proposed for a better simulation in terms of load displacement relationships. The work was then extended to the masonry walls as described in Chapter 3. A discussion on previous approaches with large structures is provided and the influence of boundary conditions is
discussed. The main purpose of the work in this chapter is to investigate the sensitivity of the parameters used in the current model and provide information for its further application to masonry arch bridges.

4.2 Review of masonry modelling

The numerical modelling of masonry has experienced significant improvement due to the development of computer hardware, however, the knowledge of masonry mechanics is still not as well developed as with concrete and steel structures. There are still no accurate constitutive laws to represent the basic material properties. The material and geometry non-linearity can cause convergence difficulties during the analysis largely because a discontinuous material is often modelled as a continuum with such materials. Care should be therefore taken when using the results from a finite element analysis.

Figure 4-1: Modelling of masonry structures

Generally there are two modelling approaches which are widely used. These are micro modelling and macro modelling (shown in Figure 4-1). The first approach aims to simulate the actual texture of masonry structures, as the units and mortar joints are considered separately and characterized with different properties. The second attempts to define an equivalent continuum to represent the global behaviour of masonry. The two approaches are complementary and may be used in different situations and both could be used in a single situation where areas of the domain are considered separately. Micro modelling is often used for the detail analysis of small structures where the stress and displacement states are of great interests. The micro modelling approach requires more data from small-scale laboratory tests and a relatively fine mesh is frequently used in the finite element model, especially if non-linear properties are involved in the
problem. On the other hand, macro-modelling is suitable for the global analysis of a structure with sufficient size, where the interaction between brick and mortar joints is not important. This approach often requires a coarser mesh, and the necessary material properties can be obtained from tests on masonry assemblages with sufficient size under homogeneous applied stress conditions.

4.2.1 Micro modelling

For masonry structures, the main feature for modelling is their discrete composition of individual brick units with mortar. The micro modelling technique treats each component of masonry separately with its own specific constitutive law and failure criteria. This strategy can be further divided into two levels depending on the accuracy and simplicity desired: detailed micro modelling and simplified micro modelling.

![Diagram of micro modelling strategies](image)

Figure 4-2: Micro modelling strategies for masonry

For detailed modelling (shown in Figure 4-2(a)), the brick/stone and mortar unit are modelled using continuum elements, while the brick/mortar interfaces for both the bed and head joints were taken into consideration. In this approach, the brick and mortar units can be treated as either elastic or inelastic materials, and even the interface could be assigned as having inelastic behaviour. In principle, all possible failure mechanisms can be reproduced using this approach. The disadvantages of this level of modelling include a high computational resource requirement. This is because a highly refined mesh is required for the model in order to capture the behaviour of different components, especially when inelastic materials are involved. In addition, detailed material properties and constitutive laws for the brick (stone) units and mortar joints are required. Another difficulty for the implementation of this approach is the junction area between the bed
joints and head joints (as indicated by the red circle in Figure 4-2(a)). As discussed in Chapter 3, one of the dominant failure patterns of masonry structures is delamination at the brick/mortar joint interface. For the detailed modelling approach, there are two possible ways to achieve this: one is to create a contact between the bed and head joint using interface elements, where there will be three types of interfaces in the masonry system. These are the interface between a brick unit, bed joint, and the interface between a brick unit and head joint, the interface between the bed and head joint. The identification the contacts at different interfaces is of great difficulty and modelling process can be very time consuming. The other way is to consider all the mortar joint as a continuum, and fracture mechanics needs to be introduced at the intersection between mortar joint to allow cracking and fracture to occur. This means specific material properties and constitutive laws need to be developed. Given the complexity and high demand for resources, this method is mostly likely to be used for small structural elements where the states of stress and displacement are of great interest.

At the level of simplified micro modelling, the masonry sample is considered to consist of only the unit and joint, as shown in Figure 4-2(b). In this case, the unit is represented by continuum elements while the behaviour of the mortar joints and unit-mortar interface are represented by a modified ‘joint’. This joint includes the mortar and the two unit-mortar interfaces, while the units are expanded in order to keep the geometry unchanged. Masonry is thus considered as a set of elastic blocks bonded by potential fracture / slip lines at the joints. Compared with the detailed modelling approach, only one interface is required for the mortar joint, and it greatly increases the computation efficiency as considerably fewer elements were involved in the analysis. A significant disadvantage of this approach is the loss of accuracy under some conditions as the Poisson’s effect of mortar is not included (Lourenco et al., 1995).

Chiostrini and Vignoli (1989) proposed a simple micro-model for the analysis of masonry panels under seismic loads. The bricks were assumed as isotropic elastic materials while the mortar joints were modelled by introducing a ‘gap’ contact element characterized by a stiffness to address the gap opening and a friction coefficient for interface sliding. As the load increases, the cracking evolution of the panel is represented by the progressive opening of these gap elements. The authors argue that the load displacement relationships as well as the collapse mechanism for a shear panel could be successfully reproduced.
by the proposed model, as shown in Figure 4-3. However, it is noted that the physical constants were not derived from experimental tests but chosen in order to reproduce the global experimental results. The model is only appropriate when the bricks are not involved in the failure.

Figure 4-3: Analytical and experimental load-displacement curves and predicted collapse mechanism (Chiostrini and Vignoli, 1989)

Ali and Page (1998) further developed a finite element model based on the detailed micro modelling approach for the analysis of masonry subjected to concentrated load. In their model, both the bond failure between the brick and mortar joint and the material failure (cracking/crushing of brick or mortar units) were taken into account. To predict bond failure at the brick-mortar interface, a three-dimensional failure surface in terms of the normal, parallel, and shear stresses at the interface ($\sigma_x$, $\sigma_p$, $\tau_{xy}$) was used (see Figure 4-4). The brick and mortar were assumed as elastic brittle materials with similar properties to concrete, and conventional concrete failure criteria were adopted to model failure of the masonry materials. The Von Mises failure criterion with a tension cutoff has been adopted in the model to predict the cracking and crushing failure modes (see Figure 4-5). The brick and mortar materials were incorporated into a plane stress finite element program using simple four-node quadrilateral elements. Concentrated load tests were carried out on masonry panels to check the validity of the finite element model. With a fine mesh near the loading point, the model was used successfully to simulate the progressive failure that takes place beneath a concentrated load. The above mentioned model is claimed by the authors to have a wide application to any type of brick-mortar combination owning to the detailed modelling technique. It does not include tension softening effects in the
CHAPTER 4. MODELLING OF SMALL MASONRY STRUCTURES

Figure 4-4: Bond failure surface for mortar joints (Ali and Page, 1998)

Figure 4-5: Cracking and crushing failure surfaces of brick or mortar (Ali and Page, 1998)
post cracking regions of masonry assembly.

Based on the simplified modelling strategy, in which interface elements are used as potential crack, slip or crushing planes, Lourenco (1996) developed a new interface model. It is defined by a convex composite yield criterion which consists of a tension cut-off for tensile failure. The Coulomb friction model for shear failure and an elliptical cap mode for compressive failure (shown in Figure 4-6) are included. The brick units were discretized with continuum elements and the joints were discretized with interface elements. The interface elements were also placed in the middle of each brick unit in order to simulate the possible cracking occurred in the units. The proposed model was validated with experimental tests on shear walls and a deep beam. The model was shown to reproduce the complete path of the structures up to and beyond peak until total degradation of strength. Parametric studies identified the crucial roles of the dilatation angle, compressive capacity and cracking of the units, and the mesh density of the interface model.

![Diagram](image)

**Figure 4-6: Proposed cap model for interfaces (Lourenco, 1996)**

Macorini and Izzuddin (2011) have written a paper to describe their work on the development of a non-linear interface element for three dimensional mesoscale analysis of brick-masonry structures. A simplified micro modelling approach was adopted in their work. The blocks were modelled using three dimensional continuum solid elements, whereas the mortar and brick–mortar interfaces were modelled by means of a two dimensional non-linear interface element. The interface element consists 16 nodes where nodes 1 to 8 lie on the top face, while the remained nodes lie on the bottom face as shown in Figure 4-7. This element
represent both the geometric and material non-linearity of the interface. These two faces coincide with each other at the initial undeformed condition. A co-rotational approach was employed in order to account for the large displacement of the interface at failure, and a local reference system that moved with the interface mid-plane was defined. The internal contact forces through the interface were simulated by means of a multi-surface plasticity criterion. An elasto-plastic contact law that follows a Coulomb slip criterion was used to model failure in tension and shear, whereas a cap model was employed to account for crushing in compression. In addition, the energy dissipation, decohesion and residual frictional behaviour as well as the dilatancy due to shear failure were also taken into account in the model. The proposed model was then used for the analysis of masonry walls and compared with experimental results (Vermeltfoort and Rajmakers, 1993) and the numerical modelling results obtained by Lourenco and Rots (1997). Both the current model and the model used by Lourenco and Rots (1997) showed good agreement with the experimental results in terms of the load displacement relationships at the initial stage before the peak load was reached. Macorini and Izzuddin’s (2011) model exhibited great improvement for prediction of post failure behaviour compared with the model used by Lourenco and Rots. This may be due to the consideration of tension softening behaviour.
in the Macorini and Izzuddin’s model. The proposed model was applied to a brickwork masonry panel subjected to lateral pressure load. The large vertical crack that ran along the head mortar joints and bricks as well as the continuous diagonal cracks observed in the tests by Chee Liang (1996) were well represented using the proposed model with non-linear interface elements. This indicates that it is possible to use simplified micro modelling to represent deformations in individual joints in small-scale masonry structures. However, the required parameters for this model, especially the non-linear properties for the interface are difficult to determine. The consideration of both geometry and material non-linearity during analysis require high computational resources, which has limited the application of the model.

4.2.2 Macro modelling

For large scale masonry structures, it is the global structural behaviour rather than stress and strain status of a specific area that is of great interest, so it is possible to ignore the interaction between units and mortar in this case. The macro-modelling approach does not make a distinction between individual brick units and joints but only treats masonry as a homogeneous anisotropic composite. The material properties of such a composite can only be determined from masonry tests of sufficient size under homogeneous states of stress. There are not many macro models currently available due to the intrinsic complexity of introducing orthotropic behaviour. A complete macro-model must reproduce an orthotropic material with different tensile and compressive strengths along the material axes as well as different inelastic behaviour for each material axis (Lourenço et al., 1995).

For the development of yield surfaces for orthotropic materials, it is noted that the surface cannot be fully represented in terms of only principal stresses. For plain stress situations, there are two possible methods for the establishment of a yield surface. If the material axes are assumed to be defined by the bed joint direction (x-direction) and the head joints direction (y-direction), the yield surface can be defined by the full stress vectors ($\sigma_x$, $\sigma_y$, and $\tau_{xy}$) as shown in Figure 4-8a. The other possible way is to set up the relationship in terms of principal stresses and an angle $\theta$ (see Figure 4-8b). The angle measures the rotation between the principal stress axes and the material axes, and the stress
diagrams will change according to the value of $\theta$.

![Figure 4-8: Representation of masonry strength in plane stress: (a) Full stress vector component; (b) Principal stresses and angle between principal and material axes (Lourenço et al., 1998)](image)

A series of tests were carried by Page (1981; 1983) and Samarasinghe and Hendry (1980) to study the biaxial behaviour of masonry walls under compression and tension. It was found that the biaxial failure of brickwork masonry is greatly related to the orientation of mortar joints. An idealized failure surface under the condition of biaxial tension-compression derived from the test results was proposed by Samarasinghe et al. (1982) in terms of principal stresses and the joint orientation relative to the principal stress (shown in Figure 4-9). In the model by Samarasinge et al, the brickwork was supposed to be linear isotropic elastic as long as the failure surface was not violated, and it was modelled using rectangular plane stress elements with average properties. Once the failure surface was reached, the residual stiffness and strength of the corresponding element was taken as zero. The proposed model was then used for a comparison with experimental tests on shear walls subjected to vertical precompression and monotonic racking load applied at varying height. Good agreements were found between the experimental and predicted results. However, biaxial compression failure was not included in the model (Samarasinghe et al., 1982).

Contro and Sacchi (1985) proposed an incremental elastic-plastic model with zero strength in tension and work-hardening in biaxial compression for layered materials. They argued that post failure behaviour and creep effects
were negligible under a short term loading, so attention was more focused on the elastic range rather than the post failure. The model was based on a simplified version of the anisotropic failure criterion derived by Nova and Sacchi (1979) as an orthotropic generalization of the Mohr-Coulomb condition. A failure domain could be defined based on the above hypothesis with zero tensile strength. The shape of the domain changes with layer orientation $\theta$, which is the angle between one of the principal stress directions and the bed joint orientation. For a given $\theta$, the failure domain was bounded by six straight lines as shown in Figure 4-10. Although the limit elastic domain was similar in shape to the failure domain, the elastic domain was assumed rectangular in order to simplify the model. The basic hypothesis assumes as current yield surface the intersection of the failure domain, henceforth referred to as ultimate, and a domain (dash-dot lines in Figure 4-10) evolving from the initial elastic one as a consequence of plastic strains. The development of the ultimate surface was governed by a second order matrix, hardening parameters which were dependent on angle $\theta$. This proposed constitutive model attempts to describe the inelastic response of rocklike or masonry materials subjected to short-term loads. However there are several problems that were not well addressed in this model, including the assumptions of linear elasticity for the material, the flow rule adopted in the
model and including incremental procedures during numerical calculation. The proposed model did not take the post failure behaviour into consideration and there is a lack of implementation of the model and verification from experimental tests.

Figure 4-10: For a given angle $\theta$: initial elastic (dashed line), current yield (solid line) and ultimate (dotted line) surfaces (Contro and Sacchi, 1985).

The models outlined above were originally developed for masonry structures based on the experimental testing, taking into account the anisotropic effects due to the mortar joints. Due to similar behaviour under compression and tension between masonry and concrete structures, macro models that were initially developed for concrete have also been used by many researchers with appropriate modification for the analysis of masonry structures. The development of numerical models for concrete under biaxial stresses is still an ongoing research. Chen (2007) gave a comprehensive review of constitutive laws for macro concrete model. According to Chen, there are two main types of model, which are the elastic-hardening-fracture models and non-linear elastic-fracture models. The main difference between these two types of models is the description of non-linear response under biaxial compressive stresses. Plasticity theory is used in the first approach to describe the behaviour, while the non-linear elasticity theory was adopted for the other model. For the macro model, as there are not
any interfaces involved, cracking and crushing behaviour are modelled using a smeared crack approach. In this approach that the concrete is still considered as a continuum after cracking, and the stiffness matrix is then modified by several factors to represent the strength loss after cracking. These models are sometimes used without any fundamental changes to analyse masonry. The only difference is finding appropriate material properties (elastic modulus, strength values) for masonry.

Shing et al. (1998) adopted the concept of the former concrete model and used a element with elastic-hardening-brittle in biaxial compression and elastic-brittle behaviour for the other conditions for the analysis of reinforced masonry panels under monotonic loading. In their model, cracking and crushing were determined according to the ultimate compressive or tensile strain. In the post cracking-failure range, the smeared crack approach was used and the tension-stiffening effect was simulated by allowing a gradual drop in tensile stress. The analytical results were then compared with experimental tests, and the load displacement curves are plotted in Figure [4-11]. As can be seen that the load displacement curves showed good agreement until crushing occurred at the toe. However, diagonal crack opening could not be well modelled using the smeared crack approach, and the author suggested a finite element analysis using discrete cracking modelling. The model also gave a lower value for the shear cracking load, while it overestimated the ultimate shear strength.

Figure 4-11: Comparison of experimental and analytical results (Shing et al., 1998)
Loo and Yang (1991) carried out a study on cracking and crushing of masonry arch bridges. In their work, the brick and mortar are considered similar materials in terms of the strain and stress curves. Averaged masonry properties were assumed based on the experimental results reported by Ali (1987). An elastic-brittle material was assumed for masonry, where the elastic material matrix was kept constant up to the state of stress that violates the appropriate failure criteria. The failure criterion developed by Chen and Saleeb (1982) for tension fracture in plain concrete was adopted in their study. Before cracking, the masonry is assumed to have isotropic behavior. The same approach that has been used by Ali and Page (1998) for the prediction of cracking and crushing patterns was used here.

The post failure behaviour of masonry was identified by one of the following forms (Loo and Yang, 1991):

- Pure crushing failure:
  The masonry is assumed to lose its resistance completely against further deformation by taking the normal stresses and shear stress down to zero.

- Pure cracking failure
  This condition indicates only partial tension damage of the material across the plane of cracking. The smeared crack approach then assumed an infinite number of parallel fissures to exist in the direction normal to the principal tensile stress. The tensile stress at the cracking area experienced a sudden drop to zero, and the overall strength of the material was then reduced to zero in direction normal to the crack plane.

- Mixed-Fracture Behaviour
  For this type of behaviour, the normal tensile stress and the shear stress across the crack plus a proportion of the normal stress parallel to the crack direction are released.

The proposed model has considered the non-linearity of the materials and includes the post failure behaviour. However, the averaged material properties lead to stiffer structure, and it did not take the anisotropy of masonry into account. The contribution to the strength and failure criterion of the mortar joints as planes of weakness were ignored.
Whilst significant research efforts have been made for the development of appropriate material model for masonry structures, there is still no model could accurately represent the anisotropic behaviour under different conditions. Most of the models were developed at two dimensional level and there is a lack of the consideration of three dimensional behaviour. The applications of some of the models in practice are limited by the difficulties in determination of required parameters. Different modelling approaches should be adopted when investigating specific problem such as material failure, and structure instability issues.

4.3 Triplet shear modelling

In this section, finite element modelling of the triplet shear test is described and comparisons with experimental results are presented. The work is mainly focused on the prediction of failure load and the reproduction of load displacement relationships. Based on the micro modelling concept, two different approaches were adopted. The first approach uses a continuum model to represent the masonry assembly without considering the bond slip between the brick unit and mortar joint. The brick and mortar units were modelled separately with different materials. A second model was produced for comparison, and the previously mentioned simplified micro modelling approach was followed. A series of contacts following the Mohr-Coulomb failure criterion were introduced, and they were assumed to represent the mortar joint as well as two brick/mortar interfaces. The brick units were expanded to keep the geometry unchanged and an averaged material property was used for the new ‘brick’ unit.

4.3.1 Micro modelling without interface

A continuum model was produced in ANSYS Version 13 for the triplet shear tests and aimed to predict maximum failure load. As indicated from the experimental tests described in section 3.5.3 of Chapter 3, shear failure always occurred at the brick-mortar interfaces or across the mortar joints. Given pre-compression normal stresses (0.2N/mm², 0.6N/mm² and 1.0N/mm²) are far less than the failure stress of brick units, and the brick unit was assumed as an
elastic isotropic material. For comparison, the 10 \( mm \) thick mortar joint was modelled using two separate material models: the Drucker-Prager (DP) model and a concrete model.

An eight node three dimensional solid element was used to generate the finite element model. The detailed geometry of this element is illustrated in Figure 4-12. The element was defined by eight nodes, and each node has three degrees of freedom which are in the nodal \( X, Y \) and \( Z \) directions. This element was chosen because of its compatibility with non-linear material properties. The element is capable of modelling of cracking, crushing, plastic deformation and creep when used with an appropriate material model. It can also take the reinforcement into account by introducing rebars within the element in concrete applications. The SOLID 65 element has a linear behaviour until a yield surface is reached in compression. Beyond the yield surface, the material can exhibit perfectly plastic behaviour or present other material nonlinearities defined by an isotropic or kinematic hardening rule or the DP criterion.

![Figure 4-12: Geometry of SOLID 65 element (ANSYS, 2009b)](image)

The mortar joint was initially assumed to follow the DP material behaviour. The DP failure criterion is a three-dimensional pressure-dependent model to represent the stress state at which the material reaches its ultimate strength. The model was originally developed for modelling rock and soils, but can also be used for other frictional materials. The criterion is established on linear relationship between the octahedral shear stress and the octahedral normal stress through material constants. The Drucker–Prager failure criterion (Drucker and...
Prager, 1952) was established as a generalization of the Mohr–Coulomb criterion for soils and can be expressed as:

$$\sqrt{J_2} = \lambda I'_1 + k$$  \hspace{1cm} (4.1)

where $\lambda$ and $k$ are material constants, $J_2$ is the second invariant of the stress deviator tensor and $I'_1$ is the first invariant of the stress tensor, and are defined by the following equations:

$$I'_1 = \sigma'_1 + \sigma'_2 + \sigma'_3$$

$$J_2 = \frac{1}{6}[(\sigma'_1 - \sigma'_2)^2 + (\sigma'_1 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_2)^2]$$  \hspace{1cm} (4.2)

where $\sigma'_1, \sigma'_2, \sigma'_3$ are the principal effective stresses. The parameters $\lambda$ and $k$ can be determined from triaxial tests by plotting the results in the $I'_1$ and $\sqrt{J_2}$ space. Alternatively, the parameters can be obtained from standard compression triaxial tests and can be expressed in terms of internal friction angle and cohesion (Colmenares and Zoback, 2002; Yi et al., 2005; Yi et al., 2006):

$$\lambda = \frac{2 \sin \phi}{\sqrt{3(3 - \sin \phi)}}$$  \hspace{1cm} (4.3)

$$k = \frac{6c \cos \phi}{\sqrt{3(3 - \sin \phi)}}$$  \hspace{1cm} (4.4)

where $c$ and $\phi$ are the cohesion intercept and internal friction angle of the material, respectively. The DP yield surface has a circular cone and can correspond to the outer boundary of the hexagonal Mohr-Coulomb yield surface with appropriate material constant (see Figure 4-13). The DP yield surface is often considered as a smooth version of the Mohr-Coulomb yield surface. The yield surface does not change with progressive yielding, so there is no hardening rule and the material is elastic-perfectly plastic (Figure 4-14). Detailed information on the numerical implementation of the DP criterion in finite element model is available in the ANSYS Theory Reference for Mechanical Applications (2009c).

The finite element mesh of the triplet shear model is illustrated in Figure 4-15. The brick solids were divided into 20, 6 and 10 segments in the $X$, $Y$ and $Z$ direction. The solid represented mortar joints which have the same divisions in
the \( X \) and \( Z \) directions as the brick units, while two segments were made in the \( Z \) directions. The whole model consisted of 4400 elements and 5313 nodes. The material properties of brick units and mortar joints during the analysis were listed in Table 4.1. The elastic modulus used in the analysis were obtained from the compression test on the brick and mortar units as discussed in Chapter 3. The Poisson’s ratio of the brick and mortar joints were determined according to the work has been done by Mahmoud (2005) and Sicilia (2001) on similar materials. As the intention was for modal behaviour based on individual material properties rather than back-calculate from triplet shear tests, the inelastic properties of the mortar (cohesion and friction angle) come directly from the triaxial tests on mortar specimens.

Figure 4-13: Drucker-Prager and Mohr-Coulomb yield surface

Figure 4-14: Stress and Strain relation

Figure 4-15: Finite element mesh for shipment of Drucker-Prager material

Figure 4-15: Finite element mesh for triplet shear model
Table 4.1: Material properties for finite element model

<table>
<thead>
<tr>
<th>Property</th>
<th>Brick unit</th>
<th>Mortar joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $kg/m^3$</td>
<td>2200</td>
<td>1850</td>
</tr>
<tr>
<td>Elastic modulus $N/mm^2$</td>
<td>$2.5 \times 10^4$</td>
<td>700</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>Cohesion $N/mm^2$</td>
<td>-</td>
<td>0.12</td>
</tr>
<tr>
<td>Internal friction angle</td>
<td>N/A</td>
<td>34°</td>
</tr>
<tr>
<td>Dilatation angle</td>
<td>-</td>
<td>0°</td>
</tr>
</tbody>
</table>

The whole analysis was completed in four steps. Firstly, the lower surface of bottom brick was fixed in all the three directions, and the influence of self weight was considered by introducing gravity. The normal compressive stress was applied on the top surface as a pressure load in the second step. Then the right side surface of the top and bottom bricks were fixed horizontally, and the movement in the Z direction was also constrained. A small horizontal displacement load was divided into small substeps and gradually applied on the surface of the central brick (see Figure 4-16). Initial modelling results showed that a large displacement causes convergence problems. This may be due to the non-linear material properties, or the mesh density of the finite element model which is also considered to affect this aspect (Mei et al., 2009). As determined from the experimental results, all the specimens failed with a horizontal displacement under $2\ mm$. The modelling work initially started with a $2\ mm$ displacement, and it was divided into 100 increments.

Figure 4-16: Boundary conditions for triplet shear model
Results Discussion

The maximum failure load was determined when the specimen experienced plastic deformation, which can be seen from the load displacement curves. The specimen under different normal stresses showed similar behaviour. The horizontal displacement versus the corresponding reaction force is plotted in Figure 4-17. As can be seen from the graph, the specimen goes into the plastic deformation stage when the horizontal movement is more than 0.2 mm. The model experienced convergence problems when the displacement more than 0.9 mm. As previously discussed, this may because of involvement of a non-linear material and a low mesh density used in the model. A parametric study was performed on the material properties and mesh density levels, and these results are discussed later in this section.

![Load Displacement relationship for specimen under 0.6 N/mm² normal stress](image)

**Figure 4-17:** Load Displacement relationship for specimen under 0.6 N/mm² normal stress

The contour plot for the plastic shear strain in the XY plane (for the maximum horizontal deformation) is shown in Figure 4-18. The largest plastic shear occurred at the middle of both mortar joints. The whole mortar experienced relatively large shear deformation, indicating a interface delamination was likely to occur, and this is consistent with experimental results. The experimental and modelling results under different normal compressive stresses in terms of the failure load are summarised in Table 4.2 along with the ratio of modelled peak strength to experiment peak strength, $F_m/F_e$. 

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As shown in the table, the proposed model accurately predicted the failure load. The accuracy increased accordingly with the increase of normal compressive stress. A parametric study was performed to investigate the effects of the mesh density on the specimen with a 0.6 \text{ N/mm}^2 normal stress level. The brick units are considered to have minimum contribution in terms of the shear strength, so only the mortar joints were modified with either a finer or coarser mesh. As shown in Figure 4-19, the number of elements for the mortar joints has reduced from the standard case of 400 to 200 and increased to 800 for the coarse mesh model and fine mesh model respectively. The corresponding load displacements are plotted in Figure 4-20 for these models. It is concluded from the study that the mesh density has an impact on the convergence during the analysis, but shows little influence on the prediction of maximum failure load. The coarse mesh model shows improved convergence in the analysis, indicating a coarse mesh is preferable when a DP material is used in the finite element model.
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(a) Coarse mesh level
(b) Fine mesh level

Figure 4-19: Modified triplet shear model with different mesh levels

Figure 4-20: Load displacement curves for different mesh models

Modelling with concrete material properties

In the section, the generation of a finite element model using a concrete material model instead of the DP material for the mortar joints is described. The concrete material model in ANSYS was designed for use in conjunction with the SOLID 65 element. It was initially developed for the modelling of concrete material by taking the material non-linearity into consideration, including cracking and crushing. A smeared crack model was used for this concrete material, the material is initially considered as an isotropic homogeneous continuum and orthotropic after cracking. The cracking is modelled by the
modification of the stiffness matrix and material properties when the maximum tensile stress was reached. The cracking occurred in the model is considered as a ‘smeared band’, rather than discrete cracks.

When used in line with the SOLID 65 element, the concrete material model has a linear behaviour in compression until a yield surface was reached, followed by perfectly plastic behaviour. However, it can present other material non-linearity following an isotropic or kinematic hardening rule or Drucker-Prager criterion. In terms of the behaviour of this element in tension, it shows linear behaviour up to a limit tensile stress, and the material starts to crack when this stress level is achieved.

Prior to the application of the concrete material model, it is necessary to review its theoretical foundations that have been adopted by ANSYS for developing this model.

**Failure Criterion**

The failure criterion adopted in ANSYS concrete model is mainly developed from the Willam-Warnke five parameter model (Willam and Warnke, 1975). The concrete material fails due to a multiaxial stress states with both cracking and crushing failure modes accounted for. It can be expressed in the form:

\[ \frac{F}{f_c} - S \geq 0 \]  

(4.5)

Where:

- \( F \) a function of the principal stress state \((\sigma_{xp}, \sigma_{yp}, \sigma_{zp})\);
- \( S \) failure surface defined by principal stresses and five input parameters \( f_t, f_c, f_{cb}, f_1 \) and \( f_2 \)
- \( \sigma_{xp}, \sigma_{yp}, \sigma_{zp} \) principal stresses in principal directions
- \( f_t \) ultimate uniaxial tensile strength
- \( f_c \) ultimate uniaxial compressive strength
- \( f_{cb} \) ultimate biaxial compressive strength
- \( f_1 \) ultimate compressive strength for a state of biaxial compression superimposed on hydrostatic stress state \( \sigma_h^p \)
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\( f_2 \) ultimate compressive strength for a state of uniaxial compression superimposed on hydrostatic stress state \( \sigma_h^a \).

This failure surface can be further simplified using a minimum of two constants, which are the \( f_c \) and \( f_t \). The other parameters can be redefined as:

\[
\begin{align*}
\sigma_{cb} &= 1.2 f_c & \text{(4.6)} \\
\sigma_1 &= 1.45 f_c & \text{(4.7)} \\
\sigma_2 &= 1.725 f_c & \text{(4.8)}
\end{align*}
\]

However, it is noted that the modified values are valid only where the condition described in Equation \( 4.9 \) is satisfied.

\[
|\sigma_h| \leq \sqrt{3} f_c
\]

\[
\sigma_h = \text{hydrostatic stress state} = \frac{1}{3}(\sigma_{xp} + \sigma_{yp} + \sigma_{zp})
\]  

As noticed, Equation \( 4.9 \) specifies a stress condition with a relatively low hydrostatic stress. All five failure parameters should be specified when a large hydrostatic stress component is expected.

For the concrete material, ANSYS provides an option to suppress the crushing ability during analysis by assigning \( f_c = -1.0 \), and the material cracks whenever a principal stress component exceed \( f_t \). The function \( F \) and the failure surface \( S \) can also be expressed in terms of principal stresses, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \).

The failure of concrete can be categorized into four domains depends on the state of the principal stresses:

1. \( 0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3 \) (compression - compression - compression)
2. \( \sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3 \) (tensile - compression - compression)
3. \( \sigma_1 \geq \sigma_2 \geq 0 \geq \sigma_3 \) (tensile - tensile - compression)
4. \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0 \) (tensile - tensile - tensile)

In each domain, independent functions are taken to describe \( F \) and the failure surface \( S \).
CHAPTER 4. MODELLING OF SMALL MAISONRY STRUCTURES

The compression - compression - compression domain

In this region, the material follows the failure criterion defined by Willam and Warnke, and the function \( F \) and failure surface \( S \) were defined in Equation 4.10 and Equation 4.11 respectively.

\[
F = F_1 = \frac{1}{\sqrt{15}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] 
\] (4.10)

\[
S = S_1 = \frac{2r_2(r_2^2 - r_1^2)\cos \eta + r_2(2r_1 - r_2)[4(r_2^2 - r_1^2)\cos^2 \eta + 5r_1^2 - 4r_1r_2]^{1/2}}{4(r_2^2 - r_1^2)\cos^2 \eta + (r_2 - 2r_1)^2} 
\] (4.11)

The terms involved in \( S \) are defined as follows:

\[
\cos \eta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{\sqrt{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} 
\] (4.12)

\[
r_1 = a_0 + a_1 \xi + a_2 \xi^2 
\] (4.13)

\[
r_2 = b_0 + b_1 \xi + b_2 \xi^2 
\] (4.14)

\[
\xi = \frac{\sigma_h}{f_c} 
\] (4.15)

where \( \sigma_h \) is hydrostatic stress, \( \eta \) is the angle of similarity which describes the relative magnitudes of the principal stresses, \( r_1 \) and \( r_2 \) are the stress components perpendicular to the hydrostatic axis at \( \eta = 0 \) and \( \eta = 60^\circ \) respectively, and \( a_0 \), \( a_1 \), \( a_2 \), \( b_0 \), \( b_1 \) and \( b_2 \) are material constants.

Figure 4-21 shows the failure surface defined by the above functions. According to Equation 4.12, the uniaxial compression and biaxial tension conditions are considered by taking \( \eta = 0^\circ \), which indicates any stress state such that \( \sigma_3 = \sigma_2 < \sigma_1 \). If \( \eta = 0^\circ \), it represents a stress state where \( \sigma_3 < \sigma_2 = \sigma_1 \), such as uniaxial tension, biaxial compression. All other multiaxial stress states can be represented when \( 0^\circ \leq \eta \leq 60^\circ \).
The tension - compression - compression domain

In this regime, \( F \) and \( S \) have a form described in Equations 4.16 and 4.17:

\[
F = F_2 = \frac{1}{\sqrt{15}} [(\sigma_2 - \sigma_3)^2 + \sigma_2^2 + \sigma_3^2]^{1/2} \quad (4.16)
\]

\[
S = S_2 = (1 - \frac{\sigma_1}{f_t}) \frac{2p_2(p_2^2 - p_1^2) \cos \eta + p_2(2p_1 - p_2)[4(p_2^2 - p_1^2) \cos^2 \eta + 5p_2^2 - 4p_1p_2]^{1/2}}{4(p_2^2 - p_1^2) \cos^2 \eta + (p_2 - 2p_1)^2} \quad (4.17)
\]

While the other terms were defined as follows:

\[
p_1 = a_0 + a_1 \chi + a_2 \chi^2
\]

\[
p_2 = a_0 + b_1 \chi + b_2 \chi^2
\]

\[
\chi = \frac{\sigma_2 + \sigma_3}{3f_c}
\]

The coefficients \( a_0, a_1, a_2, b_0, b_1 \) and \( b_2 \) are the same material constants that used in the previous section. If the failure criterion is satisfied, cracking occurs in the plane perpendicular to principal stress \( \sigma_1 \).

The tension - tension - compression domain

In this regime, \( F \) take the form:

\[
F = F_3 = \sigma_i; \quad i = 1, 2
\]

\[
\text{Figure 4-21: Failure surface in principal stress space}
\]
and S is defined as:

\[ S = S_3 = \frac{f_t}{f_c} (1 + \frac{\sigma_3}{f_c}) : \quad i = 1, 2 \]  

(4.20)

In this domain, two types of crack patterns are involved. Cracking occurs in the planes perpendicular to both principal stresses \( \sigma_1 \) and \( \sigma_2 \) if the criterion for \( i=1, 2 \) was satisfied. If the failure criterion is satisfied only for \( i=1 \), cracking occurs only in the plane perpendicular to principal stress \( \sigma_1 \).

The tension - tension - tension domain

In this regime, \( F \) and \( S \) have a form of:

\[ F = F_4 = \sigma_i; \quad i = 1, 2, 3 \quad \text{ (4.21)} \]

\[ S = S_4 = \frac{f_t}{f_c} \]  

(4.22)

This failure criterion involves three different patterns, cracking occurs in the plane perpendicular to principal stress \( \sigma_1, \sigma_1 \) and \( \sigma_2 \) or in all principal directions once the criterion is satisfied in direction 1, 1 and 2 or all three directions respectively.

The 3D failure surface for the stress state which is biaxial or nearly biaxial is plotted in Figure 4-22. The graph shows three different surfaces that stand for where the principal stress in \( Z \) direction (\( \sigma_{zp} \)) are slightly greater than zero, equal to zero and slightly smaller than zero. The principal stresses in the \( X \) and \( Y \) directions are considered much higher than that in the \( Z \) directions and are of non-zero values. The failure of the material is defined as a function of \( \sigma_{zp} \). As shown in the figure, if the principal stresses in \( X \) and \( Y \) directions are both negative, while \( \sigma_{zp} \) is positive, cracking will occur in the direction perpendicular to the \( \sigma_{zp} \). However, if \( \sigma_{zp} \) equals zero or smaller than zero, crushing failure will be predicted for the material.

As mentioned above, the concrete model considers the material as an isotropic linear material before a yield surface is reached. A smeared crack approach was introduced to model the cracks presented at integration points. The material properties were modified which effectively treats the cracking as a 'smeared band' of cracks, rather than discrete cracks. The initial linear behaviour
of the material is defined by the following stress-strain matrix in the model:

\[
[D^c] = \frac{E}{(1 + v)(1 - 2v)} \begin{bmatrix}
(1 - v) & v & v & 0 & 0 & 0 \\
v & (1 - v) & v & 0 & 0 & 0 \\
v & v & (1 - v) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{(1 - 2v)}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{(1 - 2v)}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{(1 - 2v)}{2} \\
\end{bmatrix}
\]

A tensile failure occurs once a principal stress exceed the tensile failure stress. Cracks are assumed to locate at an integration point, and this is achieved by modifying the stress-strain relations with a plane of weakness in the direction normal to the crack face. Also, a shear transfer coefficient \( \beta \) is introduced to account for the shear strength reduction for those subsequent loads. The normal stress is reduced by modifying the Young’s modulus as shown in Figure 4-23.

The stress-strain relationships for a material that has cracked in one direction only becomes:
The stress-strain relations for concrete that has cracked in two directions are:

\[
[D^c_k] = \frac{E}{(1 + \nu)} \begin{bmatrix}
\frac{R^t(1 + \nu)}{E} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \nu & 0 & 0 & 0 \\
0 & \frac{1}{(1 - \nu)} & \frac{1}{(1 - \nu)} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\beta_t}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\beta_t}{2}
\end{bmatrix}
\]

The stress-strain relations for concrete that has cracked in all three directions are:

\[
[D^c_k] = E \begin{bmatrix}
\frac{R^t}{E} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{R^t}{E} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\beta_t}{2(1 + \nu)} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\beta_t}{2(1 + \nu)} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\beta_t}{2(1 + \nu)}
\end{bmatrix}
\]

The stress-strain relations for concrete that has cracked in all three directions are:

\[
[D^c_k] = E \begin{bmatrix}
\frac{R^t}{E} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{R^t}{E} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{R^t}{E} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\beta_t}{2(1 + \nu)} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\beta_t}{2(1 + \nu)} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\beta_t}{2(1 + \nu)}
\end{bmatrix}
\]
As illustrated in Figure 4-23, $f_t$ is defined as the uniaxial tensile stress, $R^t$ is the modified Young’s Modulus normal to the tensile failure plane, and it decreases to zero as the solution converges. $T_c$ is a multiplier adopted for the consideration of stress relaxation (defaults to 0.6).

The material model also takes the condition when an opened crack closes into consideration. The cracks status at integration point cracking is checked and determined by a strain value called the crack strain. For a closed crack, the compressive stresses can still transmitted across the crack in the normal direction of the cracks, however a shear transfer coefficient $\beta_c$ is introduced to account for the stress reduction the tangential direction. The stress strain matrix is adjusted accordingly by this coefficient.

In the concrete material model, crushing is defined by the failure at an integration point either in uniaxial, biaxial, or triaxial compression status. Once the crushing occurred, it was assumed that the stiffness matrix of that element has no contribution to the whole structure and can therefore be ignored.

The above described concrete model was then used for the modelling of mortar joints in the triplet shear test. Similar to the previous model, the brick unit is still modelled as a linear elastic material. The same elastic material properties (elastic modulus, Poisson’s ratio) as listed in Table 4.1 were used in the current model for the brick and mortar units. The parameters used for the concrete material are listed in Table 4.3.
Table 4.3: Material properties for concrete model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear transfer coefficient for open crack $\beta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Shear transfer coefficient for close crack $\beta_c$</td>
<td>0.9</td>
</tr>
<tr>
<td>Uniaxial tensile strength $f_t$ $N/mm^2$</td>
<td>0.4</td>
</tr>
<tr>
<td>Uniaxial compressive strength $f_c$ $N/mm^2$</td>
<td>1.1</td>
</tr>
<tr>
<td>Tensile stress relaxation factor $T_c$</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The shear transfer coefficient represents the shear strength reduction for an open or closed crack. It normally has a value ranges 0.3 to 0.5 and 0.9 to 1.0 for open and closed cracks respectively (Li et al., 2006), while Morbiducci (2003) suggested smaller values of 0.2 and 0.8 respectively. An average experimental value from the compressive and flexural strength on mortar specimens (see Chapter 3) was used for the compressive and tensile strength here. The tensile stress relaxation factor was assigned the default value. Initial modelling studies with the above parameters experienced convergence difficulty during the analysis. Some sensitivity tests carried by the author have identified it is the relatively low compressive strength that lead to the convergence problem, especially when the compressive and tensile stress do not affect this. This may because the crushing element provides no contribution to the stiffness and leads to the adjustment of the overall stiffness matrix. As a results, a singular matrix might form during the process and cause convergence difficulty. The problem is identified by several researchers (Li et al., 2006; Lu and Jiang, 2003), and it is suggested to suppress the crushing ability by assigning $f_c = -1.0$. The failure of the specimen is then defined as when the majority of cracks (represented by the small red circles) were found in the mortar joints as shown in Figure 4-24. The prediction of failure loads for the specimens under different normal stress levels are summarised in Table 4.4 and compared with the previous model as well as the experimental results.

Table 4.4: Predicted load of concrete model and comparison with experimental results

<table>
<thead>
<tr>
<th>Normal stress $N/mm^2$</th>
<th>Experimental $F_e [kN]$</th>
<th>DP model $[kN]$</th>
<th>Concrete model $[kN]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>8.6</td>
<td>12.4</td>
<td>13.8</td>
</tr>
<tr>
<td>0.6</td>
<td>20.5</td>
<td>25.5</td>
<td>23.7</td>
</tr>
<tr>
<td>1.0</td>
<td>33.2</td>
<td>36.5</td>
<td>35.4</td>
</tr>
</tbody>
</table>

As can be seen, the predictions of the concrete model show good agreement with experimental tests under high normal stress level. It gives an overestimated
strength when the normal compressive stress is lower. Predictions are more accurate than the DP material model under high stress levels ($0.6 \text{N/mm}^2$, $1.0 \text{N/mm}^2$), but not for the lower stress level.

![Finite model shows mortar cracking failure](image)

**Figure 4-24:** Finite model shows mortar cracking failure

It was noticed from the analysis that the failure of the specimen was with small horizontal displacement (0.2 mm). This may be due to the assumption of an linear elastic material of mortar before cracking, as a small deformation of the continuum model may cause significant change of stress conditions. Several researchers have investigated to the use of a Multi-linear Kinematic Hardening material to redefine the stress-strain relationship of masonry and introduce plastic behaviour (Wang et al., 2010; Jiang et al., 2004). It aims to have a better representation of the deformation properties, however, it is out of the scope of the research and so is not discussed here.

### 4.3.2 Micro modelling with interface

In this section, the modelling work with introduced interfaces between brick and mortar joints is described. Unlike the previous models, it is mainly focused on the load displacement relationship rather than the failure load. The simplified micro modelling approach was employed as the interface demonstrated little benefit of more complex modelling. The mortar joints as well as the brick/mortar
interfaces were represented by a series of contact elements. The brick units were expanded to keep the original geometry and were assigned average properties of the masonry assembly. As evidenced in Chapter 3 no cracking was found in the brick, the new ‘brick’ unit is still assumed as an isotropic elastic material, and the failure is caused by the separations at the interface. Firstly, a brief description of the contact elements used in the model is given.

**Contact elements**

In ANSYS, when studying the contact between two bodies, it is considered that there is always a contact surface and a target surface involved at an interface. For a rigid-flexible contact, the deformable body is always taken as contact surface while target surface must be the rigid surface. In a flexible-flexible contact situation, the deformable bodies can be either assigned with a contact element or a target element. The concept of ‘contact pair’ which includes a contact and target surface is widely used in finite element analysis.

![Contact element and associated target surface](image)

**Figure 4-25:** (a) Contact element and associated target surface (b) Contact element type

A contact pair is defined by a real constant set number which shared by the contact and target element. The contact element is located on the surface of a 3D solid or shell elements (called underlying elements) and has the same geometry as the underlying body (Figure 4-25a). There are several types of contact elements available with different geometries for the modelling of 2D and 3D problems (Figure 4-25b).
Contact elements follow the behaviour defined by Coulomb friction model with a slight modification. The contacting surfaces remain sticky until the shear stress developed at the interface reaches a certain value. The equivalent shear stress $\tau$, at which sliding on the surface begins in the Coulomb’s friction model is defined as follows:

$$\tau = \mu p + c$$  \hspace{1cm} (4.23)

where $\mu$ is the friction coefficient and $c$ specifies the cohesion sliding resistance. A maximum frictional stress is introduced in the model which is the modification from the original Coulomb model. The maximum frictional stress defines a point where sliding between the two surfaces will occur once this value was reached, while the shear stress developed from normal contact pressure is suppressed. The maximum frictional stress is used when the contact pressure becomes very large and has a default value of $1.0 \times 10^{20} N/mm^2$ (if the units for length and force are $N$ and $mm$). The sliding resistance of the contact surface in terms of normal pressure is plotted in Figure 4-26 showing the different components used in the model.

![Figure 4-26: Sliding contact resistance](image)

The behaviour of these elements is controlled by many parameters with different features. Among the most important ones are the normal contact stiffness $FKN$, friction coefficient $\mu$, penetration between contact and target surface $Pe$ and tangential contact stiffness $FKT$. The stress state along the contact area is primarily determined by these factors which have the following relationships:

$$Cs = Cp \times \mu$$  \hspace{1cm} (4.24)
where $Cs$ is the contact friction stress and $Cp$ is the contact normal stress. The contact normal and tangential stiffness can be input as a factor or a constant. The contact normal stiffness is initially determined by the programme based on the material properties, element size, and the total number of degrees of freedom in the model. The tangential contact stiffness is defined as a function of $\mu$ and the normal stiffness $FKN$. The amount of penetration between contact and target surfaces is mainly determined by the normal stiffness. The $FKN$ and $Pe$ are important parameters for finite element solution of sliding condition. In a numerical analysis, a higher stiffness values could reduce the amount of penetration. However, this may also lead to a ill-conditioning of the global stiffness matrix and cause convergence difficulties. On the other hand, lower stiffness values may contribute to a larger penetration or slip, producing a less accurate solution. Care should be taken for choosing these values especially when there is a lack of the material properties. For the detailed numerical implementation in the programme can be referred to the ANSYS contact technology guide (ANSYS, 2009a).

**Assessment of the contact element**

The introduction of contact techniques enables the use of traditional FEM in analysing problems with a discontinuity, and several built in contact elements are available in ANSYS for the analysis of this type of problem. ANSYS presently supports five contact models: node to node, node to surface, surface to surface, line to line, and line to surface. Each type of model uses a different set of ANSYS contact elements and is appropriate for specific types of problems. Since information about the application of these contact elements is limited, the main purpose of this section is to evaluate the performance of the contact elements on true contact problems. Comparisons have been made between known analytical solutions and numerical results for the Hertzian contact problem (Hertz et al., 1896). The specifics of this plane strain contact problem are as follows: a rigid cylinder of radius $R$ is pressed into an elastic half-space by a force per unit length of $P$ (Figure 4-27). The contact surface is assumed to be frictionless and the stress conditions at the contact area is the area under investigation.

The exact solutions of vertical stress at the contact area are given by Hertz
Figure 4-27: Hertzian contact problem

as shown below:

\[ E = \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1} \]  
(4.26)

\[ a = \left( \frac{4PR}{\pi E} \right)^{1/2} \]  
(4.27)

\[ \sigma_y = \frac{2P}{\pi a^2} (a^2 - x^2)^{1/2} \]  
(4.28)

Where:

- \( E_1 \) Young’s modulus of the cylinder
- \( E_2 \) Young’s modulus of the elastic space
- \( \nu_1 \) Poisson’s ratio of the cylinder
- \( \nu_2 \) Poisson’s ratio of the elastic half space
- \( a \) Contact length
- \( E \) Equivalent modulus
- \( P \) Applied load
- \( \sigma_y \) Vertical stress
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For the finite element analysis of this problem, the half space is represented by a rectangular block and both the base and the indenter were meshed. For this discretization, a built-in eight-node quadrilateral element (ANSYS PLANE 82 plain strain) was used. The contact conditions between the geometries were simulated by surface-to-surface (CONTACT 172 and TARGET 169) elements. These contact elements overlie the spatial elements on contact surfaces at \( y = 0 \). Before the analysis, a certain contact length of 0.1 \( m \) (2a) was assumed, and then the applied load was calculated using Equation 4.27, thereby obtaining the theoretical solution from Equation 4.28. All the materials were assumed to be elastic, and the parameters used during the analysis are summarised in Table 4.5.

Table 4.5: Material properties for the Hertzian contact problem

<table>
<thead>
<tr>
<th>Material</th>
<th>Rigid Cylinder</th>
<th>Elastic base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus E ( E )</td>
<td>( 1 \times 10^{15} ) ( N/m^2 )</td>
<td>( 1 \times 10^6 ) ( N/m^2 )</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Element edge length</td>
<td>0.01 ( m )</td>
<td>0.01 ( m )</td>
</tr>
<tr>
<td>R</td>
<td>5 ( m )</td>
<td></td>
</tr>
<tr>
<td>Dimensions of the base</td>
<td>Width 4 ( m ) height 2 ( m )</td>
<td></td>
</tr>
<tr>
<td>Proposed contact area</td>
<td>0.1 ( m )</td>
<td></td>
</tr>
<tr>
<td>Applied load</td>
<td>1635 ( N )</td>
<td></td>
</tr>
</tbody>
</table>

The geometry and applied boundary conditions of this problem as well as the detailed meshing of the contact area are illustrated in Figures 4-28(a) and 4-28(b). In order to assist convergence during the calculation, a small friction coefficient of 0.001 was assigned to the contact elements. The analytical results calculated from Equation 4.28 and the numerical results are plotted in Figure 4-29.

As can be seen in Figure 4-29, both the analytical and numerical results show stress decrease as the increase of contact length. The greatest variance (up to 20\%) between the analytical and numerical models are beneath the contact area at 0.1 \( m \). The numerical model reproduces the shape of the stress profile. Considering the difference between the proposed model and reality (boundary conditions, mesh density etc.), it is fairly reasonable to say that the built-in contact element in ANSYS can accurately represent the real situation.
(a) Finite element model and boundary conditions

(b) Detailed mesh for contact area

Figure 4-28: Numerical model

Figure 4-29: Results comparison between analytical and numerical solutions
Modelling of triplet test with contact element

The contact elements were then used for the generation of the finite element model using the simplified micro modelling approach. The model had a mesh density with 20, 10 and 6 divisions in the X, Z and Y directions respectively for the 'unit'. The main material properties used during the analysis for both the unit and contact elements are listed in Table 4.6.

<table>
<thead>
<tr>
<th>Material properties for simplified micro model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit</strong></td>
</tr>
<tr>
<td>Density [kg/m³]</td>
</tr>
<tr>
<td>Elastic Modulus E [N/mm²]</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Cohesion c [N/mm²]</td>
</tr>
<tr>
<td>Friction coefficient µ</td>
</tr>
<tr>
<td>Normal stiffness FKN [N/mm³]</td>
</tr>
<tr>
<td>Tangential stiffness FKT [N/mm³]</td>
</tr>
</tbody>
</table>

The elastic modulus used for the unit was determined from the compressive strength tests on the masonry assemblies (see Chapter 3). The value of Poisson’s ratio comes from the experimental tests by Mahmoud (2005) on similar materials. The cohesion, friction coefficient properties and tangential stiffness for the contact element were determined according to the initial shear strength under different normal stress levels. There is generally a lack of information on the contact normal stiffness, therefore a reference value used by Claxton et al. (2005) on their numerical analysis of retaining walls was adopted.

The same boundary conditions as described in section 4.3.1 for the modelling without interfaces were used. A 10 mm displacement load was applied by dividing into 100 increments, and the reaction force was recorded and plotted in Figure 4-30. As seen from Table 4.7, the numerical model with contact elements enables good prediction for all the specimens under different compressive stress levels. Similar to the DP model, it gives a slightly higher failure load for the specimen under 0.2 N/mm² normal stress. However, for the specimens with 0.6 N/mm² and 1.0 N/mm² normal stresses, lower values were obtained for the maximum load, and they are 4% and 9% smaller than the experimental results. It is also noticed from the load displacement curves that the corresponding
displacement when the maximum load was reached shows good agreement with the experimental tests. However, the model does not capture the slight decrease in strength immediately after the peak, and this is discussed in more details later. The maximum loads obtained from the finite element analysis are presented and compared with the experimental results and previous modelling results with the DP material model.

Figure 4-30: Load displacement relationships with contact elements

Table 4.7: Predicted load of contact model and comparison with experimental results

<table>
<thead>
<tr>
<th>Normal compression stress</th>
<th>Experimental $[kN]$</th>
<th>DP model $[kN]$</th>
<th>Contact model $[kN]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 $N/mm^2$</td>
<td>8.6</td>
<td>12.4</td>
<td>9.7</td>
</tr>
<tr>
<td>0.6 $N/mm^2$</td>
<td>20.5</td>
<td>25.5</td>
<td>19.7</td>
</tr>
<tr>
<td>1.0 $N/mm^2$</td>
<td>33.2</td>
<td>36.5</td>
<td>30.2</td>
</tr>
</tbody>
</table>

For finite element analysis, the mesh density influences the accuracy of the modelling results (Mei et al., 2009). A parametric study was then carried out for the specimen under normal stress 0.6 $N/mm^2$ to identify the influence of element size. Two additional models were generated with a fine and coarse mesh respectively. The fine mesh model has 15300 elements compared with 4400 for the original model, while the coarse mesh model consists of only 650 elements.

Figure 4-31 shows the load displacement relationships for the models with different meshes. As shown all the curves show similar behaviour and almost
coincide with each other, indicating that the mesh size has little influence on the failure load of this model. It can be concluded that a coarse mesh is preferable for a contact model in this case since it can greatly decrease the solution time and maintain considerable accuracy. This is most likely because in this case the masonry units are stiff and have flat surfaces, limiting geometrical relations which would be affected by mesh density.

Figure 4-31: Load displacement relationships for model with different meshes

An assessment of the cohesive zone material

Masonry can be considered as a composite material consisting of individual brick (stone) units bonded by mortar joints. The interface between layers of a composite structure is of special interest when it is subjected to certain types of external loading. The cohesive zone model has been used to simulate interface delamination by several researchers working on masonry structures. This approach introduces a failure mechanism by using the hardening-softening relationships between the separations and incorporating the corresponding tensile stress across the interface, and may assist in modelling the initial drop in strength not captured by the frictional model only with contact elements.

Silva et al. (2013) used an interface layer with a cohesive zone constitutive model to simulate the adhesion loss between mortar and substrate. His research demonstrated that the proposed model can reproduce in-service factors that influence adherence. A cohesive zone model was proposed by Alfano and Sacco (2006) for combining interface damage and friction. The interface in the model
was divided into an undamaged part and a fully damaged part. It is assumed that friction occurs only on the fully damaged part. The proposed model was then used for the simulation of a fibre push-out test and a masonry wall loaded in compression and shear. The comparisons with available experimental results show the effectiveness of the proposed model to predict the failure mechanisms and the overall structural response for the problem.

ANSYS allows for the simulation of interface separation using a cohesive zone material model. This model can be used with the contact elements, and the interface separation is defined in terms of contact gap or penetration and tangential slip distance. The following section gives a brief description of the cohesive zone model and evaluates its performance for the simulation of shear failure during the triplet shear test.

The bilinear cohesive zone material model used in the programme is based on the model proposed by Alfano and Crisfield (2001). There are three different types of debonding (interface delamination) behaviour defined in terms of the main direction where the separation occurs. For the first debonding mode, separation of the interface surfaces where the separation normal to the interface is considered as the main failure pattern and more important than the slip tangential to the interface, for example between a head joint as one brick slides horizontally away from another. The second debonding mode is defined where tangential slip at the interface is the predominant behaviour rather than the separation normal to the interface. If the separation behaviour is determined by both normal and tangential components, it is called mixed mode debonding. Using the first debonding mode as an example, the relationship between the contact stress and contact gap is discussed.

The relationship between the contact stress and contact gap is illustrated in Figure 4-32. The behaviour is featured with a linear elastic stage (OA) followed by linear softening (AC). The debonding at interface starts once the maximum normal stress is reached (at point A). It is considered to be completed at point C with the normal stress decreases to zero. The area surrounded by the curve OAC represents the energy dissipated during the debonding behaviour and is called the critical fracture energy. Once the debonding has begin at the interface, any unloading and subsequent reloading follows a similar linear elastic behaviour as defined by line OB. Compared to the initial elastic loading, it has a more gradual
The equation for curve OAC can be expressed as:

\[ P = K_n U_n (1 - d_n) \]  \hspace{1cm} (4.29)

Where:

- \( P \) = normal contact stress (tension);
- \( K_n \) = normal contact stiffness;
- \( U_n \) = contact gap;
- \( U_{n_{\text{max}}} \) = contact gap at the maximum normal contact stress;
- \( U_{c_n} \) = contact gap at the completion of debonding;
- \( d_n \) = debonding parameter.

The debonding parameter is defined as:

\[ d_n = \left( \frac{U_n - U_{n_{\text{max}}}}{U_n} \right) \left( \frac{U_{c_n}}{U_{c_n} - U_{n_{\text{max}}}} \right) \]  \hspace{1cm} (4.30)
The normal critical fracture energy is computed as:

\[ G_{cn} = \frac{1}{2} \sigma_{\text{max}} U^c_n \]  

(4.31)

Where:

\[ \sigma_{\text{max}} = \text{maximum normal contact stress}. \]

The tangential contact stress and tangential slip have the same behaviour as the normal contact stress and contact gap, it can be defined as:

\[ \tau_t = K_t U_t (1 - d_t) \]  

(4.32)

Where:

\[ \tau_t = \text{tangential contact stress}; \]
\[ K_t = \text{tangential contact stiffness}; \]
\[ U_t = \text{tangential slip distance} \]

For the second type of debonding, it follows the same behaviour defined by the above equations. The only difference is instead of the separation in the normal direction at the interface, the tangential slip is considered as the main failure criterion and dominates the contact behaviour.

For the mixed mode debonding mode, the interface separation depends on both normal and tangential components. The behaviour could be defined by the following equations:

\[ P = K_n U_n (1 - d_m) \]  

(4.33)

\[ \tau_t = K_t U_t (1 - d_m) \]  

(4.34)

The debonding parameters are defined as:

\[ d_m = (\frac{\Delta_m - 1}{\Delta_m}) \chi \]

\[ \Delta_m = \sqrt{\Delta_n^2 + \Delta_t^2} \]
\[ \Delta_n = \frac{U_n}{U^c_n} \]
\[ \Delta_t = \frac{U_t}{U^c_t} \]  

(4.35)

\[ \chi = (\frac{U^c_n}{U_n - U^c_n}) = (\frac{U^c_t}{U_t - U^c_t}) \]
In this condition, both the normal and tangential contact stresses contribute to the total fracture energy and debonding is completed before the critical fracture energy values are reached. A power law based energy criterion is used to define the completion of debonding:

\[
\left( \frac{G_n}{G_{cn}} \right) + \left( \frac{G_t}{G_{ct}} \right) = 1 \tag{4.36}
\]

Where: \( G_n = \int PdU_n \) and \( G_t = \int \tau_dU_t \) are the normal and tangential fracture energies respectively.

The cohesive zone material model was used together with contact elements in order to reproduce the interface failure. According to the experimental tests, it is reasonable to consider the separation in the tangential direction is more important for the triplet shear test in this case than that in the normal direction. The evaluation of the material model was carried out by assuming the shear test related to the second debonding type. The analysis was performed for the specimen under 0.6 N/mm\(^2\) normal stress based on a coarse mesh model. The parameters used during the analysis are listed in Table 4.8.

### Table 4.8: Material properties for cohesive zone model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion c ([N/mm^2])</td>
<td>0.09</td>
</tr>
<tr>
<td>Friction coefficient (\mu)</td>
<td>0.6</td>
</tr>
<tr>
<td>Normal stiffness FKN ([N/mm^3])</td>
<td>1.0 (Claxton et al., 2005)</td>
</tr>
<tr>
<td>Tangential stiffness FKT ([N/mm^3])</td>
<td>0.5</td>
</tr>
<tr>
<td>Maximum tangential stress (\tau_{max}) ([N/mm^2])</td>
<td>0.47</td>
</tr>
<tr>
<td>Tangential slip at debonding completion ([mm])</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Apart from the parameters for the contact elements, there are two additional parameters required for the cohesive zone material, which are the maximum tangential stress and the tangential slip distance at the completion of debonding. They were both determined using the triplet shear test as described in Chapter 3. The maximum tangential stress was taken as the shear strength. The slip distance was determined as the distance when the shear strength remained constant with increased displacement (see Figure 3-16). The load displacement relationships are plotted in Figure 4-33 and compared with the experimental results.

As can be seen from Figure 4-33, the load displacement curves of the cohesive
zone material model have similar behaviour to the model with contact element before the maximum load is achieved. After reaching the peak load, the load start to decrease until the pre-defined slip distance is reached. It then increases again with the same rate as the initial stage and remained almost constant when the value provided by friction was reached. The initial behaviour was dominated by the cohesive zone material, and the frictional properties were not involved in this stage. The frictional behaviour was activated after the cohesive separation was completed. This material model is not suitable for the modelling of load displacement response for shear test in such a small scale level. However, it uses a similar assumption to Alfano and Sacco (2006), and its application on relatively large scale structures therefore needs further review and is discussed in later sections.

Consideration of load decrease after initial failure

As described in previous section, the contact elements with a Mohr-Coulomb failure surface show good agreement for the maximum failure load and corresponding displacement. However, it is also noticed from the experimental tests that there is a load drop after the initial failure especially for the specimens under 0.2 N/mm² normal compression stress. The frictional model did not take
this condition into account and gave a higher residual strength after failure.

The effects of load decrease have been considered using the following two approaches. Cohesion specifies resistance to sliding when the contact pressure is zero, but once there is relative movement at the interface, this ability is lost. The first approach was to remove the cohesive strength at the interface by reassigning a zero value once the maximum load was reached. The other approach was to introduce a reduced friction coefficient (0.4 used in the analysis) after failure. The load displacement relationships for the revised models are presented in Figure 4-34 and compared with the experimental results and original contact element model.

![Figure 4-34: Load displacement relationship with reduced friction and cohesion (0.2 N/mm²)](image_url)

For the above mentioned two methods, the key issue is identification of the point when the maximum strength has been reached. To achieve this, the frictional stress of a specific node located at the interface was monitored. Comparisons were made for the frictional stress between adjacent load increments, once the difference between these two values was within 5%, it was assumed that the maximum strength was reached and the analysis continued with a modified cohesion or friction property. Reductions in post peak load of 19% and 42% decrease have been found for the models with reduced friction and cohesion respectively, these reductions can be varied through the material.
properties. The proposed approaches can be used to simulate the peak and residual strength for small scale structures. It is noted that the drop in strength occurs abruptly, and this can be improved by defining a function between the friction (or cohesion) and displacement, but, this would require the development of a new constitutive model and is therefore beyond the scope of this research.

Non-linear spring element

A unidirectional element in ANSYS can be used in any analysis with the feature of user defined load-deflection relationships. The element is capable of use in either one, two or three dimensional analysis with pure longitudinal or torsional behaviour. If a longitudinal behaviour is specified, the element only can take uniaxial compressive/tensile load at each node. Translations in all the three (nodal X, Y, and Z) directions can be taken into account. No bending or torsion is considered. The element can be used as a purely rotational element. In this case, the axial loads are not considered. The rotational behaviour about the nodal x, y, and z axes can be included. The geometry, node locations, and the coordinate system for this element are shown in Figure 4-35. Ideally, the element is defined by two coincident points with a user defined load-deflection curve. The element has large displacement capability for which there can be two or three degrees of freedom at each node.

For the Mohr-Coulomb friction model, once the cohesive strength was specified, this component of sliding resistance exists regardless of deflection, and the load decrease after interface separation cannot be represented. A cohesive
zone model has been proposed in this research using the non-linear spring element to reproduce the load displacement relationships. The cohesion was replaced by a bilinear model described in the following section.

The cohesive model is assumed to behave within a specific range, and experimental results show it affects in the 1 mm displacement range (as indicated by the red rectangle in Figure 4-36). A bilinear model (as shown in Figure 4-37) was defined between the horizontal displacement and corresponding stress for individual spring elements. The model is assumed to act when the maximum stress provided by friction is reached. The corresponding displacement was identified as $X$ by monitoring the shear stress at the interface as described in previous section. The model provides a resistance load with an increasing value, and the slope of the increasing stage has the same value of tangential contact stiffness. It starts to decrease when load reaches the specified cohesion (0.09 N/mm$^2$ in this case) and drops to zero after 1 mm displacement.

The load displacement relationships of the proposed model with non-linear spring element for the specimen under 0.2N/mm$^2$ compression stress are plotted in Figure 4-38. It is also compared to the experimental results and the modelling results with the contact element only. The proposed model shows good prediction of initial linear behaviour as well as the maximum shear strength. But the residual strength is lower than the experimental value. This indicates the assumption that the total loss of cohesion after interface separation might be conservative for specimens under shear failure.
In this section, several approaches were proposed and discussed for the reproduction of the triplet shear test under different compressive stresses. These approaches give similar response, however, this needs to be extended to large structures with the aim to model with full-scale bridges. This is discussed in the following section.

4.4 Shear wall modelling

Previous work on modelling masonry walls under shear and compression has been reported by several researchers (Chaimoon and Attard, 2004; Mohebkah and Tasnimi, 2008). Though their work has shown it is possible to predict peak load and deformations, some of the parameters used in the model are difficult to
obtain directly from experimental tests. From previous triplet shear modelling, the contact elements are able to simulate shear behaviour especially at high normal stresses. Some of the other methods presented earlier are difficult to scale up because the maximum frictional stress, which gives the peak strength, is difficult to determine for complex structures. The contact element model was therefore extended for the analysis of shear wall tests. The finite element model and mesh are presented in Figure 4-39.

![Shear wall model and finite element mesh](image)

**Figure 4-39: Shear wall model and finite element mesh**

A simplified micro modelling approach was adopted here, the mortar joint was lumped into as average interface which consists of mortar and two brick-mortar interfaces, while the brick units were expanded in order to keep the geometry unchanged. As no cracks were noted in the experimental test, potential cracks in the units were not considered in the modelling work. Masonry is thus considered as a set of elastic blocks bonded by potential fracture lines at the joints. The work was initially focused the shear failure of masonry walls under uniform load, and the two brickwork walls (see Chapter 3) with uniform compression stress were studied by the finite element analysis. The same material properties (see Table 4.6) for the triplet shear models were used during the analysis.

The boundary conditions used for the modelling work were illustrated in Figure 4-40. Similar as the triplet shear specimen, the analysis was completed in four steps. Firstly, the bottom course bricks were fixed in all three directions, and the gravity was applied to take the self-weight into account in the first step.
The 0.2 $N/mm^2$ normal compression stress was applied as a pressure load on the surface of top course bricks in the second step. The brick in the bottom left was constrained in the $X$ and $Z$ directions for shear wall 1, while both the bottom and top left bricks were constrained for shear wall 3. A 30 $mm$ and 10 $mm$ displacement were applied to shear wall 1 and 3 at the top and middle courses respectively.

![Figure 4-40: Boundary conditions for (a) shear wall 1 (b) shear wall 3](image)

The experimental crack patterns and scaled deformed meshes obtained from the simulations for the two specimens are illustrated in Figure 4-41 and 4-42 respectively. For both cases, the main failure characteristics are predicted well, which includes the stepped diagonal shear cracks. The rotational trend of shear wall 1 under horizontal load was also observed from the modelling results. A 5 $mm$ maximum vertical displacement was experienced by the brick at top right corner, while the experimental test gave a measurement of 7 $mm$.

The load displacement response from the numerical analysis are plotted in Figure 4-43 and compared with experimental results. The load curves for both specimens show steady increase until the maximum is reached and keep almost constant thereafter. The maximum loads predicted by the model were 6.8 $kN$ and 24.2 $kN$ respectively for the two specimens, and they are approximately 65% and 15% lower than experimental values.

The proposed model shows great accuracy for the prediction of crack patterns for both specimens under current load conditions. It also gives a close prediction
CHAPTER 4. MODELLING OF SMALL MASONRY STRUCTURES

(a) Experimental failure mode

(b) Scaled deformed mesh obtained from analysis (scale=2)

Figure 4-41: Comparisons of experimental and numerical crack patterns for shear wall 1

(a) Experimental failure mode

(b) Scaled deformed mesh obtained from analysis (scale=4)

Figure 4-42: Comparisons of experimental and numerical crack patterns for shear wall 3

in terms of the maximum load for shear wall 3. However, the load obtained for shear wall 1 is much lower than the experimental value. It is believed the problem lies on the boundary conditions used during analysis does not reproduce the real
conditions properly. As it was difficult to apply a constant stress across the top of specimen, and the tested wall experienced rigid body rotation which was not allowed by the model boundary conditions.

According to the work by Chaimoon and Attard (2004) and Attard et al. (2004), the loading conditions and confinement control through the upper loading beam were identified as very important and proved to be the most difficult aspect of the simulation in order to simulate the experimental results. It is noted from the experimental tests that the upper beam was not kept at a horizontal position throughout the test. This is also found by Van Zijl et al. (2001) from the testing on masonry walls under shear and compression. Different modifications for the loading conditions were adopted by different researchers. Lourenco (1996) introduced rollers for modelling the top boundary and made an assumption that both the bottom and top boundary are always horizontal and preclude any vertical movements. Giambanco et al. (2001) introduced spring elements between loading beam and top brick course, while Attard et al. (2004) applied a layer of soft material placed between the rollers and the loading beam. In the current study, since the compression load was applied directly on the surface of top course brick, and there was some restraint by the loading beam and flat jacks, it would be expected that a smaller load was obtained for modelling shear wall 1.

Figure 4-43: Load displacement relations for shear walls
For the finite element model, the previously described contact elements were used for both the head and bed joints. The computational approach used to ensure contact conditions is based on the penalty method (David, 1980), where a contact spring with a contact stiffness is placed between the vertex of one block and the corresponding edge of the other, producing a contact force. In the present study, the spring was placed between the contact surface and target surface with a normal and tangential stiffness. According to Thavalingam et al. (2001), the accuracy of solution when using such an approach depends on the contact stiffness. As the proposed model gave an accurate prediction of the crack patterns for both specimens, it is reasonable to conclude that the selected normal contact and tangential stiffness are appropriate for the simulation of shear tests.

**Cohesive zone material in shear wall**

In this section, the previously described cohesive zone material was introduced for the modelling of masonry walls. Its performance was evaluated through the simulation of shear wall 3, it was selected because it has similar load conditions to the triplet shear tests and experiences additional failure modes.

For the shear wall, the bed joint was modelled by the cohesive zone material, in which the slip in the tangential direction dominates the separation in the normal direction. According to the experimental results, the maximum shear stress and slip distance were set as 0.21 N/mm$^2$ and 1 mm. For the head joints in the wall, it is assumed the behaviour is governed by separation in the normal direction. Considering the construction process of the masonry walls, where there was no load applied laterally, the bond strength at the vertical joint is rather small compared with the horizontal joints, where the self weight assist adhesion. A small value of 0.001 N/mm$^2$ maximum contact stress was therefore assumed for vertical joints and the separation was assumed to be completed within 0.5 mm. The initial bond status for both the bed and head joints was modelled by the cohesive zone material and cohesion properties was set as zero in the Mohr-Coulomb friction model. The friction was assumed to occur once the interface was fully damaged (Alfano and Sacco, 2006). The material properties are summarised in Table 4.9.

For finite element analysis, in an algorithmic sense, the accuracy of the solution depends on the penalty stiffness if the penalty method is selected as the computational approach (Thavalingam et al., 2001). There is a general lack of
Table 4.9: Material properties for bed and head joints

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum shear stress [bed joint]</td>
<td>$N/mm^2$</td>
</tr>
<tr>
<td>Tangential slip $mm$</td>
<td>1.0</td>
</tr>
<tr>
<td>Maximum Normal stress [head joint]</td>
<td>$N/mm^2$</td>
</tr>
<tr>
<td>Contact gap $mm$</td>
<td>0.5</td>
</tr>
<tr>
<td>Cohesion $c [N/mm^2]$</td>
<td>0</td>
</tr>
<tr>
<td>Friction coefficient $\mu$</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Information about the values of the contact stiffness of head joints. A sensitivity analysis was therefore carried out in this study which showed that the stiffness value used in the previous model did not work well when the cohesive zone material was involved. The augmented Lagrangian method was then introduced for the analysis, which is an iterative series of penalty methods. Compared to the penalty method, it is less sensitive to the magnitude of the contact stiffness (Zienkiewicz and Taylor, 1989).

In the current model, instead of specifying a constant value for the contact stiffness, the normal contact stiffness was determined automatically by the programme depending on the material properties of unit and the element size, and it was also updated by the programme at each sub-step. The tangential contact stiffness had a value that was proportional to the friction coefficient and the normal stiffness. An stiffness factor was introduced to control the magnitude of the contact stiffness. This factor was initially assigned as 0.001, and this gives a normal stiffness ($2 \ N/mm^3$) close to that used in the previous analysis. The same load conditions as described in the previous section were used in the current analysis. The deformed meshes for the models with different stiffness factors are illustrated in Figure 4-44, 4-45 and 4-46 and the load displacement relationships are plotted in Figures 4-47.

As can been seen, various crack patterns were obtained for the models with different stiffness factors. For the model with the lowest stiffness, no cracks were found across the specimen and the load displacement curve features with a straight line with increasing load. This indicates that the stress level at the interface is still within the limit of the maximum stress for the cohesive material zone. Cracks were mainly found in the upper part of the specimen for the model with a stiffness factor of 0.005. The load increased with displacement until the maximum load was reached, and it did not change significantly in the following
2 mm movement. The maximum load obtained from the model was 22.2 kN, which is 20% lower than the experimental value. A load drop was also found in the experimental results and the load kept almost constant thereafter. For the model with the stiffness factor of 0.01, cracks were found across the whole wall. The load displacement curve shows a similar trend to the experimental one. The maximum load obtained for this condition is 25.6 kN, which is about 90% accuracy of the experimental results. The load experienced steady decrease once the maximum load was reached, and it gave a residual strength of 16.7 kN.
The cohesive zone material shows the ability for the prediction of maximum failure load under shear loading. It can be also used to reproduce the load displacement response including the post failure behaviour. However, the accuracy of the results are highly dependent on the selection of parameters during the analysis, and the contact stiffness has been proved to have great influence on the final results. The assumption that friction only occurs at a fully damaged interface is more applicable to masonry walls than triplet specimens. Care should
be taken when selecting values for the definition of complete separation of the interface.

4.5 Flexural wall modelling

In this section, the flexural behaviour is studied using finite element analysis and it focuses on the maximum failure load of the specimen. The modelling results are then compared with experimental tests reported in Chapter 3. The intention was to determine if properties obtained using one test are applicable to another. The same simplified model used for the shear walls was adopted here and the same material properties were used (see Table 4.6). The finite element mesh and the boundary conditions were illustrated in Figure 4-48.

![Figure 4-48: Finite element mesh and boundary conditions for flexural wall test](image)

The whole analysis was completed in three steps. In the first step, the bottom of the specimen was fixed in the Y direction and the gravity was applied. The nodes (on the back face) which have a distance of 50 mm and 75 mm to the edges were constrained in the Z direction to simulate the support beams. In the last step, points loads were applied to the node (in the front face) which located at 300 mm and 600 mm in the X direction. The maximum load was identified by the unconverged solution. Two analyses were performed, one with the contact element only while the other model include the cohesive material model. The
same material properties (see Table 4.9) used in the shear wall modelling for the cohesive model were used here. The maximum shear stress was changed to 0.09 \( N/mm^2 \) according to the triplet shear test without normal compressive stress as described in Chapter 3.

![Deformed mesh for flexural wall (scale = 5)](image1)

![Experimental failure](image2)

(a) Deformed mesh for flexural wall (scale = 5)

(b) Experimental failure

Figure 4-49: Deformed mesh for model I and experimental failure of flexural wall

Both models showed similar crack patterns at failure, the deformed mesh for the model without the cohesive material and experimental failure are presented in Figure 4-49. Symmetric cracks were found in the finite element model across the specimen, while stepped cracks were observed in the experimental tests. This is expected as the real walls can have irregularity not present in the numerical
model. The maximum load obtained from the analyses and the displacement measured in the middle point are summarised in Table 4.10.

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Model I</th>
<th>Model II (with cohesive material)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (kN)</td>
<td>2.2</td>
<td>1.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Displacement (mm)</td>
<td>4.0</td>
<td>3.7</td>
<td>4.8</td>
</tr>
</tbody>
</table>

The predicted maximum loads are 1.1 kN and 3.8 kN for the models without and with cohesive zone materials, while the experimental average value is 2.2 kN. The model without the cohesive material gave a much lower prediction, this may be attributed to the load configurations used in the analysis. As there was no normal stress applied on the specimen, the load resistance mainly comes from the cohesion at the interface. Once the contact status at the interface changes from closed to open, it indicates separation happens at joints. In this condition, the cohesion no longer has any contribution to the resistance, and a much lower load was then obtained. The cohesive material model (Model II) gave a much higher value, and it might be because the different crack pattern obtained from the numerical analysis. The bed joints were assumed to provide the most resistance to deformation. Once the maximum shear stress was assigned for the cohesive zone material, the resistance mainly depends on the crack area across the bed joints. The cracks that were predicted by the numerical model are more than that measured in the experiments, and a higher failure load was obtained.

### 4.6 Conclusion

In this chapter, the modelling techniques used for masonry structures and constitutive laws that developed by various researcher were briefly reviewed. Modelling work on the triplet shear tests, shear wall and flexural wall tests were described and compared with experimental results. A discussion about the applications and restrictions about the proposed models and approaches has been provided.

The shear strength of brickwork masonry specimens under different normal compressive stresses was studied using the finite element analysis with two different types of models. Both the continuum and interface model described
CHAPTER 4. MODELLING OF SMALL MASONRY STRUCTURES

The interface model has better performance in terms of the load displacement response and the prediction of failure load at a low stress level. The contact elements with a Mohr-Coulomb failure surface are suitable for the modelling of shear failure at the interface, especially for specimens with relatively high normal compressive stress level.

The load decrease after the initial failure of the triplet shear tests could be simulated by the reduction of friction or cohesion at interface. The key issue for this method lies in the identification of the point when the maximum load was achieved. The assumption of the loss of cohesion is conservative for the current situation, and it gives a smaller residual strength compared with experimental tests.

The contact elements show great accuracy for the prediction of crack patterns in the shear walls with appropriate contact stiffness. The predicted load depends highly on the load conditions and confinement of the upper loading beam. The cohesive zone material is applicable for the analysis of shear walls under the assumption that the friction only occurs at the fully damaged interface. The model with the cohesive material reproduced the load drop after the maximum load was reached in the load displacement curve and gave a almost constant residual strength. However, the accuracy of the results depends the selection of the parameters defining the cohesive zone material and magnitude of the contact stiffness.

The numerical model generated for the flexural wall tests with the contact elements gave a lower prediction of the failure load. The application of cohesive material model with the contact elements overestimated the strength of the specimen. The load configuration used during the analysis might need further review in order to get more accurate results. It has, however, been shown that it is possible to use the FEM with interface elements to successfully predict the failure pattern in masonry walls, along with reasonable predictions of load/deformation behaviour. These methods show potential for modelling masonry arch bridges.
Chapter 5

Modelling of masonry arch bridges

5.1 Introduction

In this chapter, the overall behaviour of masonry arch bridges is described and studied using finite elements models at both small and large scale. Numerical results are compared with those obtained from tests reported in the literature. The finite element models were constructed using an available commercial finite element package (ANSYS). In the first part, the centrifuge tests carried out by Mahmoud (2005) on an arch-backfill model are discussed. The arch behaviour under two different loading conditions was modelled. One of the load configurations consisted of a moving load across the top of the backfill from one abutment to the other. The other was a static concentrated load applied at the quarter span position. The load and pressure properties obtained from numerical models as part of this research were compared with the experimental results of Mahmoud (2005).

In the second part of this chapter, a three dimensional model was produced including an arch barrel, backfill and spandrel wall. The model has the same geometry of a full scale bridge tested by Melbourne and Walker (1990). The behaviour of the bridge under concentrated load was studied and compared with experimental observations. It mainly focused on the tensile stress developed in the arch barrel and the development of the cracks across the spandrel wall.
The material properties used for the modelling were primarily from experimental tests reported by the authors of the experimental work. Reasonable estimation was made based on available literature where if an experimental value was not provided. Parametric studies were also carried out to study the sensitivity of the results to some of these parameters. The simplified modelling approach described in the previous chapter was adopted for the small scale arch-backfill model and the 3D model.

5.2 Small scale bridge modelling

In this section, an arch-backfill finite element model was constructed using ANSYS, and its modeller program was used for the pre and post processing of the data. The pressures on the extrados of the arch barrel and the arch deformation at different locations were predicted using the finite element model under rolling loads and compared with experimental results. A brief description of the experimental set up is provided.

5.2.1 Description of the experimental tests

The arches under test (Mahmoud, 2005) were 1/12th scale models of a notional 6 m span brickwork arch bridge tested at Cardiff university using a centrifuge. The major geometric parameters of the bridge are listed in Table 5.1, whilst the general view of the arch model and the arrangement of measurements are illustrated in Figure 5-1. For this model, there were no spandrel walls involved, so it was essentially a two dimensional test carried out with a three dimensional model.

<table>
<thead>
<tr>
<th>Table 5.1: Dimensions of arch model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span (mm)</td>
</tr>
<tr>
<td>Span/rise ratio</td>
</tr>
<tr>
<td>Ring number</td>
</tr>
<tr>
<td>Ring thickness (mm)</td>
</tr>
<tr>
<td>Arch width (mm)</td>
</tr>
<tr>
<td>Depth of the fill at crown (mm)</td>
</tr>
</tbody>
</table>
CHAPTER 5. MODELLING OF MASONRY ARCH BRIDGES

After the construction of the scale model, it was placed in a centrifuge gondola with suitable supports. The required centrifuge speed was then calculated with $12g$ as the required acceleration. The rotation acceleration was increased slowly in $3g$ increments. At each $3g$ model acceleration rate, the speed was kept constant until a stabilized condition was achieved. The centrifuge tests were carried out at a stable $12g$ acceleration condition with a rolling load initially being applied to the model. The load was moved slowly enough, so dynamic loading effects were negligible.

![Figure 5-1: Geometry and experimental set up of arch backfill model (in mm)](image)

The arch deflection and backfill/masonry interaction were measured during the test. Two rows of displacement transducers were used and they were placed at the centreline and the edge of the arch barrel respectively. Several pressure sensors were placed at the extrados of the arch barrel to record the pressures and they were calibrated before use to ensure accuracy. The model first experienced a rolling load moving from one abutment to the other. After completion of the roller load test, an increasing load was applied at the quarter span position across the whole width of the arch. The load was applied to the arch through a load spread beam which was 17.5 mm wide. The detailed description of the rolling and increasing load system is presented in the studies carried by Baralos (2002) and Burroughs (2002).

5.2.2 Finite element model

The previously described eight-node SOLID 65 element was used to model the arch barrel. A twenty node solid element, which is a higher order version of the
SOLID 65 element, was used for the modelling of backfill because deformations were likely to be larger and more concentrated. A preliminary parametric study carried out by the author showed that the higher order element has better performance in terms of convergence during calculation, especially when non-linear material properties were used. The finite element mesh of the arch backfill model is illustrated in Figure 5-2.

In order to improve the solution time and save computer resource, only half the model was generated with a symmetric boundary condition. The model simplifies the masonry arch bridges into two main elements: the arch barrel and the backfill as there was no spandrel wall presents. The complex behaviour of masonry is simplified to a homogeneous, isotropic material. The arch barrel was built with forty individual blocks which were modelled using an elastic material connected by the frictional contact elements. Each block was divided into three segments in the radial direction and twenty segments in the transverse direction across the barrel. The fill element was modelled with a Drucker-Prager material. This material requires three parameters: the cohesion $c$, the angle of internal friction angle $\phi$ and the angle of dilation $\varphi$. The Drucker-Prager yield criteria gives a yield surface around Mohr-Coulomb surface with a smooth surface defined by the above parameters as shown in Figure 4-13. The yielding of the fill is
assumed as elastic perfectly plastic. According to the construction procedure and experimental test, the fill elements were divided into three layers as shown in Figure 5.2, with an increasing elastic modulus from the top to the bottom. The same material model was used by Boothby and Roberts (2001) for the backfill in their analysis work on the transverse behaviour of masonry arch bridges. A layer of friction contact elements were placed between the arch and backfill.

The main material properties of the arch barrel and backfill used during the analysis are summarized in Table 5.2. Most of the material properties were initially selected from the material tests carried out by Mahmoud (2005). In the cases where no experimental values were available, they were assumed according to values found in previous literature. For the arch barrel, the density and Poisson’s ratio properties are experimental values. A smaller value for the elastic modulus than the experimental value was used by Mahmoud (2005) in order to fit his numerical model to experimental results and the same value was used in this study. As mentioned above, the backfill elements consist of three layers of material with an increasing elastic modulus to represent the increase in stiffness with increased confinement. The density, friction angle and elastic modulus properties of the fill material are measured values from the experimental tests. There were no measured values for the Poisson’s ratio and dilation angle for the fills. Previous studies showed that sand (clay) materials have a Poisson’s ratio ranging from 0.15 to 0.5 (Das, 2010). A value of 0.4 was selected in this study for the backfill, and the same value was recommended by Mahmoud (2005) in his work.

| Table 5.2: Material properties for arch backfill model |
|-----------------|-------------|----------|-----------|-------------|-----------------|
|                | Modulus of Elasticity [N/mm²] | Poisson’s Ratio | Density [kg/m³] | Friction angle [N/mm²] | Cohesion | Dilation angle |
| Arch barrel     | 4000        | 0.3      | 2100       | -           | -            | -               |
| Backfill        | 18/42/66    | 0.4      | 2100       | 53          | 0.02         | 0°              |

There is generally a lack of information on the dilation angle of soils. The research work performed by Vermeer and de Borst (1984) shows that dense sand has a dilation angle of about $15^\circ$, while the dilation angle for loose sand is smaller than $10^\circ$. They also suggested a value of $0^\circ$ for normally consolidated clay. Deeyvid (2010) carried out a series of tests on clean sand for determination of
dilation angle in both loose and dense states. The experimental results gave an average value of 3.7° and 9.8° for loose and dense sand respectively. It also showed that the compaction state and moisture content have great influence on the dilation angle. As little information was known about the fill materials and in order to keep the simplicity of the model, a zero value was initially selected for dilation angle in the current study.

For the contact elements at the arch/backfill interface, the Mohr-Coulomb friction model was used. It was assumed to have the same cohesion property as the backfill material at the interface. The friction coefficient for the interfaces in the arch barrel was assigned a value of 0.5 based on the experimental results obtained by the author on triplet shear test (see Chapter 3). A series of parametric studies with various material properties was carried out to investigate the sensitivity of the parameters in this study, and this is presented in a later section of this chapter.

5.2.3 Boundary conditions

The general support conditions for the finite element model are shown in Figure 5-3 in a plane view. The base of the arch barrel and backfill were fixed in all three directions to simulate the rigid arch abutments. The backfill was fixed horizontally at both vertical ends. As mentioned before, the model used in the analysis is only a half model and has a symmetry plane in the transverse direction (Y direction as indicated in the figure). A symmetry displacement condition is applied on the plane of symmetric. This means that for the nodes located on the plane, the displacement vector component perpendicular to the plane is zero and the rotational vector components parallel to the plane are zero. The movement of the backfill in the transverse direction (Y direction) was also restrained at the edge.

The loading on a masonry arch bridge is a combination of permanent self weight loading and variable traffic loading. As a small scale simulation, the tests were carried out in a centrifuge which increases stress levels to that in the full-scale bridge. The stresses developed across the structure due to the self weight effects are considered as important for the strength of bridges. For the modelling of this type of bridge test, an initial gravity loading step is required to produce the in-situ stresses to which the bridge is subjected. In the current model, the self weight
effects were introduced by assigning all the elements with appropriate density
property, and a $12g$ gravity was applied to simulate the centrifuge acceleration
conditions. The moving roller load and the concentrated load are both idealised
as pressure loads and applied directly on the backfill. The exact value of the
load was calculated according to the arch width and mass of the roller weight.
The rolling load was applied using several load steps with its position changed
at each step to simulate the moving load, while the increasing concentrated load
was applied with several increments.

Figure 5-3: Boundary conditions of arch backfill model

5.2.4 Analysis procedure

As noted in previous experimental research work presented in Chapter 2,
the mechanism failure with formation of hinges in the arch barrel is the main
failure mode for masonry arch bridges. The definition of the failure is a key
issue for a numerical analysis, as it is difficult to reach the point of collapse
with conventional finite element procedures. There are some other ways that
can be used to help the identification of the failure status. One possible way
is by visual inspection of the modelling result to identify the point when the
fourth hinges is about to form. There are several constitutive laws with failure
criterion that have been developed for the materials in the finite element models,
and the failure of arch bridges could be related to the failure of material which
can be either a compression or tensile failure (Boothby and Roberts, 2001). The
method that has been used for the shear strength of masonry can also be used
for the determination of the strength of arch bridges. The method plots the load
displacement curve, the corresponding load for the point where the load does not
increase with displacement is taken as the failure load (Ng, 1999).
An alternative way of defining failure is to link it to non-convergence of the solution. In a finite element analysis, when an iterative solution algorithm is used, a convergence criteria needs to be introduced to define when equilibrium has been achieved. There are generally two ways of monitoring convergence in ANSYS by either performing a load check or displacement check. The load check is the default option in the programme and more frequently used during analysis. The convergence criterion can be expressed in the following equation:

\[
\kappa = \frac{\gamma}{R}
\]  

(5.1)

Where \(\gamma\) is the square root sum of the squares (SRSS) of the force imbalances, while \(R\) is the SRSS of all applied loads. The default tolerance in the programme is 0.005, which means if \(\kappa\) is smaller than the tolerance value, the solution is considered as converged and an equilibrium state is achieved (ANSYS, 2009c). In a finite element analysis, a tighter convergence criteria will improve the accuracy of the results, but at the cost of more equilibrium iterations. When analysing an engineering problem such as for masonry arch bridges, a relatively loose tolerance (\(\kappa=0.05\)) value can used to improve convergence.

### 5.2.5 Results under moving roller load

The numerical model has the same scale and units as the centrifuge models, and the comparison between the numerical and experimental results is therefore straightforward. The self weight was considered before the application of the moving load. To simulate the centrifuge acceleration in the finite element model, a 12\(g\) gravity was initially applied in small increments to all the elements. The moving roller load was then applied as pressure load on the backfill across the whole width of the arch from one abutment to another. To simulate the roller movement after application of the self weight, the moving load was applied as different load cases. These load cases are of the same magnitude and were applied on the top surface of the backfill at different arch span positions. These load cases were applied after the application of gravity, therefore the roller load was added to the self weight effect. Once the self weight was applied and stable condition was achieved, the pressure load was applied at specific positions, while in the next load step, the previous load case was deleted and new load was added at a
different location.

Previous analysis on the shear walls show the contact stiffness properties have great influence on the modelling results. As there is a general lack of knowledge about the friction properties and contact stiffness between the backfill and masonry units, a series of parametric studies were carried out to determine the sensitivity of the model to these parameters. The deflection and pressure at arch extrados for mid span (at the edge) point were recorded and compared.

Published data on stiffness properties for rock joints are limited, and a summary of these data can be found in the work carried out by Barton (1976), Kulhawy (1975) and Rosso (1976). Values for normal and shear stiffness for rock joints typically can range from roughly 0.1 $N/mm^3$ (for joints with soft clay), to over 100 $N/mm^3$ (for tight joints in granite and basalt). Previous studies completed by various researchers on numerical modelling of masonry arch bridges using similar method shows the contact stiffness of arch barrel range between 0.5-150 $N/mm^3$ (Giordano et al., 2002; Idris et al., 2008; Toth et al., 2009; Schlegel and Will, 2007).

In terms of the friction properties, according to previous research on masonry retaining walls (Day, 2001; Kerry and Skinner, 2001; Fan and Fang, 2010; Lambe and Whitman, 2008), there are generally three ways to deal with the friction between the soil and the walls in a numerical analysis:

- The friction is neglected;
- The friction is taken as 0.67-1.0 of the internal friction angle of the soil;
- A typical friction coefficient of 0.57 may be used.

A series of numerical analyses was performed to identify the importance of the properties. Investigated parameters include the normal contact stiffness and friction between arch barrel and backfill as well as the contact stiffness and friction between the elastic blocks comprising the arch barrel. The normal contact stiffness was assumed to range from 20 to 200 $N/mm^3$ in this study, and the contact stiffness between the arch barrel and backfill was assumed to have a smaller value which ranges from 1 to 50 $N/mm^3$. The friction between the arch barrel and backfill was initially taken as 0.67 of the internal friction angle of soil. A value of 0.5 was selected as the coefficient of friction in the arch barrel.
based on the triplet shear test in Chapter 3, since similar materials were used in the experimental programme of the current study and the tests carried out by Mahmoud (2005).

The normal contact stiffness was initially selected as 100 and 10 N/mm$^3$, while the friction coefficient had a value of 0.5 and 0.7 for the interfaces in the arch barrel and between arch barrel and backfill. The numerical analysis using above values was considered as the benchmark during the parametric analysis. The maximum deflection and contact pressure at the middle span point from the numerical analyses is summarised in Tables 5.3 and 5.4, and they were compared with experimental values. These results are obtained from the node located at the edge of the barrel, and comparisons between the results obtained from the edge and middle points show little difference. The maximum loads were obtained from a non-convergence solution. The non-convergence is believed due to the deformation of arch barrel, which make the structure into a unequilibrium state indicating a mechanism failure.

Table 5.3: Parametric analysis of contact at arch barrel (Results with contact stiffness between backfill/arch barrel 10 N/mm$^3$, friction 0.7)

<table>
<thead>
<tr>
<th>Normal contact stiffness [N/mm$^3$]</th>
<th>Deflection [mm]</th>
<th>Pressure [kPa]</th>
<th>Failure load [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.5</td>
<td>76.5</td>
<td>3.2</td>
</tr>
<tr>
<td>100</td>
<td>0.16</td>
<td>82</td>
<td>6.9</td>
</tr>
<tr>
<td>200</td>
<td>0.10</td>
<td>83.6</td>
<td>7.6</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.16</td>
<td>82</td>
<td>6.9</td>
</tr>
<tr>
<td>0.8</td>
<td>0.15</td>
<td>81.7</td>
<td>7.0</td>
</tr>
<tr>
<td>Experimental results</td>
<td>0.08</td>
<td>38</td>
<td>5.3</td>
</tr>
</tbody>
</table>

As can be seen from the tables, the proposed model generally over predicts both the deflection and pressure compared with the experimental results. The normal contact stiffness in the arch barrel has a large effect on the predicted results. The deflection at the mid span revealed a general decreasing trend with the increase in the contact stiffness. The maximum deflection obtained at middle span is 0.5 mm with a contact stiffness of 20 N/mm$^3$, while this figure has decreased by 80% when the normal contact stiffness increased to 200 N/mm$^3$. The pressure at the extrados shows the opposite trend, it experienced a slight
Table 5.4: Parametric analysis of contact between arch barrel/backfill (Results with contact stiffness at arch barrel 100 N/mm³, friction 0.5)

<table>
<thead>
<tr>
<th>Normal contact stiffness [N/mm³]</th>
<th>Deflection [mm]</th>
<th>Pressure [kPa]</th>
<th>Failure load [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17</td>
<td>74.7</td>
<td>7.2</td>
</tr>
<tr>
<td>10</td>
<td>0.16</td>
<td>82</td>
<td>6.9</td>
</tr>
<tr>
<td>50</td>
<td>0.15</td>
<td>82.7</td>
<td>5.3</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.16</td>
<td>82.6</td>
<td>6.5</td>
</tr>
<tr>
<td>0.7</td>
<td>0.16</td>
<td>82</td>
<td>6.9</td>
</tr>
<tr>
<td>Experimental results</td>
<td>0.08</td>
<td>38</td>
<td>5.3</td>
</tr>
</tbody>
</table>

increase with the increase in contact stiffness. The predicted value is 76.5 kPa under the condition of 20 N/mm³ contact stiffness, and it is about twice the experimental value. The pressure shows a 10% rise as the normal contact stiffness increased to 200 N/mm³.

The friction property in the arch barrel was shown to be less important for the predicted results compared with the contact stiffness. There are no significant changes found in both the arch deflection and pressure at the extrados with changes in arch barrel friction. Both the predicted deflection and pressure show a slightly decreasing trend with a larger friction value.

The parametric studies on the normal contact stiffness and friction coefficient for the interface between the arch barrel and backfill reveal similar effects as the interfaces in the arch barrel. The deflection at middle span shows only 12% decrease even when the normal contact stiffness has increased 50 times. The pressure experienced 10% rise when the contact stiffness increased from 1 to 10 N/mm³, while there is only about 1% increase as the contact stiffness increased 5 times more. Similar to the friction property in the arch barrel, the change of friction coefficient between the arch barrel and backfill shows little influence on the predicted results. Both the deflection and pressure indicated almost no changes with the increase in friction.

Modelling results show both the normal contact stiffness within the arch barrel and between the arch barrel and backfill have significant impacts on the failure load, while the corresponding friction at the interfaces reveal little effect.
The experimental tests showed the arch bridge failed when the load reached 5.3 kN. The numerical analysis for the benchmark model gave a prediction of 6.9 kN of the failure load, which is 30% higher than the test value. The failure load indicated a trend of steady rise with an increase of the contact stiffness within the arch barrel. The failure load is only 3.2 kN for the model with a contact stiffness of 20 N/mm$^3$ for the interfaces within the arch barrel. A 10% rise has been found when the contact stiffness increased to 200 N/mm$^3$ compared with the benchmark analysis. The increasing trend may be explained that for a certain amount of deformation of the arch barrel, a higher load is required for interfaces with a larger contact stiffness. The influence of the contact stiffness between the arch barrel and backfill indicated the opposite trend. The predicted load has increased by 4% and decreased by 23% for the model with a contact stiffness of 1 and 50 N/mm$^3$ respectively when compared with the benchmark model. The decreasing trend may be because the load is more easily transferred from the backfill to the arch barrel with a higher contact stiffness at the interface.

The predictions of arch deflections at 50% and 75% of arch span with various roller locations are plotted in Figures 5-4 and 5-5 respectively. The experimental results and numerical modelling results obtained by Mahmoud (2005) were also included in the figures for comparison. The predictions for both deflection and pressure show general consistency with the experimental results as the roller load moved across the span. The maximum deflections were experienced when the roller load moved to the corresponding measured position as expected for both conditions. The experimental tests revealed a maximum deformation of 0.08 mm at the middle section. The proposed model in the current study gives a prediction of 0.16 mm which is twice that of the experimental value, however, the modelling result obtained by Mahmoud is about 30% smaller than the average test results. This may due to the different techniques when constructing the finite element models. The finite element model in Mahmoud’s work treated the arch barrel as a continuum without any interface, while in the current model the arch barrel consists several individual blocks connected by contact elements. The model by Mahmoud is therefore limited in predicting large deformation and failure as noted in Chapter 4. It can be concluded that the existence of the interface in the arch barrel will lead to larger deflections when loaded.

As can be seen in Figure 5-4, there are small differences between the test and both numerical model when the roller load was located at both ends of
the arch (0%-20% and 75%-100% span). The difference is not significant and indicates upward movements. This effect is more obvious for the modelling results calculated at 75% span. An upward movement was observed when the roller load moved from 0% to 50% of the span (Figure 5-5) and a maximum of 0.02 mm was recorded. Mahmoud argued that this may be due to the movement direction of the roller in the experiments and was not properly considered in the numerical model. Another possible reason may be attributed to the difference in the load dispersion between the real test condition and the numerical model because of the uncertainty in the backfill properties.

![Figure 5-4: Numerical and experimental arch deflection at 50% span](image)

The other parameter that was measured in the experimental work is the pressure on the extrados of the arch barrel at various locations. The predicted pressures are presented in Figures 5-6 and 5-7. The experimental and modelling results obtained by Mahmoud (2005) were also plotted in the figures as a comparison. The numerical analysis shows good agreement in terms of the general behaviour of predicted pressure when compared with the experimental results. However, the maximum pressure of the finite element model is much higher than that of the experimental value. A maximum pressure of about 40 kPa and 30 kPa were measured for the roller load located at 50% and 75% of arch span respectively in the experimental tests. The pressure obtained from the finite element model is about 80 kPa in the current study. It is about twice the experimental value and 10% smaller than that of the modelling results obtained by Mahmoud. The
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Figure 5-5: Numerical and experimental arch deflection at 75% span

predicted pressure at 75% of arch span according to Mahmoud is almost the same as at 50% of span. However, the calculated pressure from the current finite element model gives a more accurate prediction with a value of about 45 kPa, which is 50% higher than the measured figure in the experiments.

Figure 5-6: Numerical and experimental pressure at 50% span
5.2.6 Discussion of concentrated load results

The behaviour of the small scale model under concentrated load was also simulated by the numerical model. Although the increasing load was applied after the completion of moving roller load in the experimental tests, the effects of the rolling load were not considered in the numerical analysis. This is because the magnitude of the moving load is rather small and the corresponding deformation is negligible when compared with the maximum increasing load. Similar to the modelling of moving load tests, the increasing load was applied directly on the surface of backfill after the application of 12\(g\) of gravity load. The ultimate load was identified by using the failure criteria described in section 5.2.4, and a mechanism failure of the arch barrel was assumed.

The numerical analysis was carried out on a model with the parameters used in the benchmark analysis. The failure load identified by the numerical model is 6.9 \(kN\), and it is about 38\% higher than the experimental average. The deformed mesh of the finite element model is presented in Figure 5-8 in which the deformation has a scale factor of ten for clarity. The stress distributions in the horizontal direction across the arch barrel are illustrated in Figure 5-9. The positive and negative values indicate a tensile and compressive stress respectively. Four hinges were found across the arch barrel as represented by the black dots.
Two of them were located near the springings, while one was found under the load position and the other one occurred at about 3/4 quarter span in the arch intrados in this case. The numerical results in the current study show great consistency with the previous research on the mechanism analysis and experimental tests on arch bridges (Melbourne and Gilbert, 1995; Harvey, 1988; Pippard and Baker, 1957).

![Figure 5-8: Deformed mesh of arch backfill model (scale factor=10)](image)

![Figure 5-9: Stress distribution in X direction across arch barrel (scale factor=10)](image)

The predicted load displacement relationships were studied and compared with the experimental and modelling results obtained by Mahmoud (2005). As shown in Figure 5-10, the numerical and experimental are generally in good agreement. However, the finite element results are stiffer than the experimental one as with a relatively larger load and a smaller corresponding deflection. The maximum deflection obtained in the current study is 0.9 mm, while a maximum value of 1.5 mm was found in the experimental test. The corresponding load of the experimental test when the finite element model failed is much smaller and is about only 57% of the predicted value. The modelling work carried out by Mahmoud (2005) showed a similar trend with even stiffer results. Similar results of stiffer load displacement relationships have been found and reported by Ng
(1999) on his numerical modelling work of a single span arch model. In addition, a study presented in the design manual for road and bridges from the Department of Transport (2001) has also confirmed the stiffer behaviour when comparing the experimental and numerical results. It is believed that a smaller value of the elastic modulus of the arch barrel, the decrease of normal contact stiffness of the interfaces within arch barrel will lead to a larger deflection. However, it might be unrealistic if a very small value is used for the elastic modulus of arch barrel. Further review is needed for the selection the normal contact stiffness in terms of the deflection.

![Figure 5-10: Load displacement relationships](image)

Apart from the load displacement response, the load distribution across the backfill to the arch barrel is of interest. Figure 5-11 shows the stress conditions in the vertical direction within the whole structures. It seems the concentrated load only has limited influence on stress conditions within a certain area, and a dispersion angle of 31° was identified as indicated in the figure. There is approximately ten times difference between the stress at the loading area and at the far end of backfill. While in the arch barrel, an increasing trend is found for the stresses from loading position to the left springing near the arch intrados. A positive value of stress is found at the springing in the right side and this is due to the upward deformation caused at the counter part of loading position within arch barrel.
5.3 Large scale bridge modelling

In this section, the modelling work is extended to large scale masonry arch bridges. A simulation was carried out for a six metre span brick arch bridge. The bridge was tested in a laboratory and reported by Melbourne and Walker (1990). The analysis aimed to investigate the general behaviour of arch bridges in the transverse direction and therefore as a basis for studying failures related to the spandrel wall in particular. The diagonal cracks developed in the spandrel wall and the stress conditions underneath the spandrel wall at the loading area were of interest.

The bridge was simplified into three main parts in the finite element model: the arch barrel, backfill and spandrel wall. Some different techniques were employed here to improve efficiency and save solution time. The backfill was modelled with the same DP material used in the small scale bridges. The spandrel wall consisted of a series of elastic blocks in contact with each other, as an accurate prediction of the crack pattern within the spandrel wall was required. The contact analysis involved in the finite element model in highly non-linear behaviour and requires much computer resource. The analysis is mainly concerned with the interactions between the arch barrel and spandrel wall and between the spandrel wall and backfill. In order to save solution time and computer resource, the interfaces between the arch barrel and backfill and the interfaces within arch barrel (used in the small scale bridge) were ignored. The arch barrel in this model is considered as a continuum and simulated by the smeared crack continuum...
concrete model described in Chapter 4. The following section gives a brief justification for the use of the concrete model for masonry structures.

5.3.1 The use of concrete model

A concrete material model has been used by several researchers for the modelling of masonry bridges (Fanning et al., 2001; Boothby and Roberts, 2001; Wang, 2001). There are many similarities between brickwork masonry and concrete. Both concrete and masonry fail in forms of cracking and crushing under various loadings, and they both show an elastic behaviour under low stress levels and exhibit non-linear properties for higher stress levels. Previous experimental work revealed that both concrete and masonry show non-linear stress-strain behaviour with an initial linear stage under uniaxial compressive load (Nilson, 1997; Hossain et al., 1997; Powell and Hogkinson, 1976). In terms of material behaviour under direct tension, the experimental work carried out by Hughes and Chapman (1966) and Van der Pluijm (1997) for concrete and masonry separately show similarity in the stress-strain curves form (see Figure 5-12). Both materials have obvious brittle behaviour in tension and strain softening effects have been identified.

(a) Concrete (Hughes and Chapman, 1966)  (b) Brickwork masonry (Van der Pluijm, 1997)

Figure 5-12: Stress-strain curves in tension

The experimental behaviour of concrete and masonry under biaxial loading conditions have been investigated and reported by several researchers (Kupfer et al., 1969; Page, 1981; Hendry, 1998). Figure 5-13 illustrates the biaxial strength...
CHAPTER 5. MODELLING OF MASONRY ARCH BRIDGES

Envelope obtained by Kupfer et al. (1969) for concrete. It can be seen that under biaxial compression condition, the strength of concrete has increased by about 25% when compared with the uniaxial compressive strength. With a combination of compression and tension loads, a decreased strength has been identified. Under biaxial tension loading, little difference has been found compared with the strength obtained under uniaxial loading.

Figure 5-13: Biaxial failure surface for concrete (Kupfer et al, 1969)

For brickwork masonry, the biaxial compression and tension-compression behaviour were studied and reported by Page (1981) and Hendry (1998). The failure surfaces for both situations are plotted in Figure 5-14 and the influence of the joint angle was included. Under biaxial compression, the failure occurred abruptly by splitting in a plane parallel to the surface of the specimen for most stress conditions, and the bed joint angle (as shown in Figure 5-14(a)) showed little influence on the strength. However, for the condition when one principal stress is overwhelmed by the other, the specimen failed through cracking and sliding at the joints by involving both brick and joints. The bed joint angle was shown to have great influence on strength. An increase in the strength is also found compared with the uniaxial compressive condition. For the tension-compression condition, both principal stress ratio and bed joint angle exhibit remarkable effects on the strength, and a similar strength decrease as found in concrete materials was experienced for masonry. There is little information
available for the specimen under biaxial tension conditions. An idealized failure surface of brickwork masonry proposed by Dhanasekar et al. (1985) has a similar shape to that for the concrete material. However, the strength of the specimen is greatly affected by the bed joint angle.

(a) Biaxial compression failure (Page, 1981)

(b) Biaxial compression-tension failure (Hendry, 1998)

Figure 5-14: Failure surface for concrete and masonry under biaxial loading
The stress-strain behaviour of concrete when subjected to triaxial loading can be found in Balmer (1949) and Richart et al. (1928). The experimental results revealed that concrete can act as a quasi-brittle, plastic-softening or plastic hardening material depending on the confining stress. It was also found that the failure in three dimensional space is fairly consistent and can be expressed in the three principal stresses (Chen, 2007) as shown in Figure 5-15. For masonry materials, the determination of triaxial behaviour is very difficult and little information is available in the literature. However, it is believed that there should be similarity between masonry and rock behaviour. This is an area that needs further review, but it is interesting to know how these failure criteria work with masonry.

![Schematic failure surface in three dimensional stress space (Chen, 2007)](image)

**Figure 5-15:** Schematic failure surface in three dimensional stress space (Chen, 2007)

On the numerical level, the concrete model has been used by many researchers for masonry structures. The failure criteria used by Ali and Page (1998) in their finite element model for brick and mortar materials has the same shape as the concrete criteria. Fanning et al. (2001) reported their modelling work on the transverse loading effects on assessment of masonry arches using the concrete model described in Chapter 4. Their model gave an accurate prediction for the arch barrel displacement below the centre-line under a service load. In addition, the predicted cracks in the longitudinal direction at the inside of the arch barrel shows agreement with visual observations of a real
structure (Boothby and Domalik, 1994). The same material model was used by Wang (2001) for the investigation of three dimensional behaviour of masonry arches. The proposed model closely predicted the failure load as well as the corresponding displacements. The cracks developed in the arch barrel are found mainly underneath the loading position across the arch barrel in the transverse direction. As discussed in the above section, the concrete model could be used for the modelling of masonry structures and has been shown to accurately predict the general behaviour of arch bridges in terms of load displacement curves. However the prediction of crack patterns cannot be easily represented with a continuum model.

5.3.2 FE model and boundary conditions

Figure 5-16 shows the general view of the 3D model and corresponding finite element meshes. The geometry of the investigated bridge is summarised in Table 5.5. As mentioned before, the whole model was simplified into three main parts: the arch barrel, backfill and spandrel wall. The arch barrel in this model is considered as a continuum with the ability of cracking and crushing. The SOLID 65 element and a failure criterion initially designed for concrete as described in Chapter 4 was used. This was used instead of interfaces to improve computational efficiency and because the intention is to focus on crack development in the spandrel rather than failure of the arch barrel. Because of the geometry, cracking will occur in the spandrel before the arch collapses. The concrete failure criterion was defined by a uniaxial tensile cracking stress $f_t$, a uniaxial crushing stress $f_c$ and shear transfer coefficients for open/closed cracks. The material cracked whenever a principal stress component exceeded $f_t$. For the backfill, a eight node solid element (SOLID 45) was used in the current model. This element is a lower order version of the SOLID 95 element which has been used in the analysis of the small scale bridge modelling. The element has plasticity, creep, swelling, stress stiffening, and large deflection capabilities. It aims to improve the efficiency and reduce solution time with fewer nodes and because collapse was not investigated, large strain concentrations were unlikely to occur. The same DP material model was employed for the backfill elements.
CHAPTER 5. MODELLING OF MASONRY ARCH BRIDGES

(a) General view of 3D arch bridge model

(b) Finite element mesh of 3D model

Figure 5-16: 3D model of the masonry arch bridge

Table 5.5: Geometry information of Bolton bridge (Melbourne and Walker, 1990)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Span</td>
<td>6000 mm</td>
</tr>
<tr>
<td>Rise</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Spandrel wall thickness</td>
<td>436/216 mm</td>
</tr>
<tr>
<td>Arch thickness</td>
<td>220 mm</td>
</tr>
<tr>
<td>Fill at depth</td>
<td>300 mm</td>
</tr>
<tr>
<td>Width</td>
<td>6000 mm</td>
</tr>
</tbody>
</table>
In order to simulate the real structure as accurately as possible, the spandrel wall was built with a series of elastic blocks connected by frictional contact elements which have been used in the small scale bridge modelling. The interaction behaviour between the spandrel wall and barrel as well as the backfill are of main concern. Interfaces were introduced only between the spandrel wall/arch and the backfill/spandrel wall. Special care should be taken when generating the part of the spandrel wall which is connected to the arch barrel. The blocks at these locations should not have sharp corners, as this may cause convergence difficulties as noted in some initial analyses. The finite element model as shown in Figure 5-16 consists of 37236 nodes, and a number of 37827 elements is included.

The main material properties used in the current model for the arch barrel and backfill are listed in Table 5.6. The strength properties and the density of the arch barrel used in the analysis come from the experimental test by Melbourne and Walker (1990) on masonry samples, while the cohesion and friction properties for the backfill are from triaxial tests on soil samples. The other material properties used in the analysis are based on previous research on numerical modelling of masonry arch bridges (Ng, 1999; Mahmoud, 2005). The values of elastic modulus for the arch barrel and backfill as well as the dilation angle are based on the modelling work carried out by Boothby and Roberts (2001) on transverse behaviour of masonry arch bridges. A value of 0.2 and 0.4 were adopted as the Poisson’s ratio for masonry and soil respectively, the same values were used by Ng (1999) and Mahmoud (2005) in their work on the analysis of masonry arch bridges.

<table>
<thead>
<tr>
<th>Table 5.6: Material properties for full scale arch model</th>
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<tbody>
<tr>
<td><img src="image_url" alt="Table" /></td>
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</tbody>
</table>

For the interfaces involved in the model, as previously described, the three main parameters that govern the behaviour are the normal contact stiffness, friction coefficient and cohesion. There is generally a lack of information on
these parameters, and assumed values based on the previous literature and the experimental and modelling carried out by the author were used. Instead of using a constant value of the normal contact stiffness as used by the author for the modelling of small masonry structures (see Chapter 4), the contact stiffness was initially determined by the programme based on material properties and element size, and a scale factor was then introduced for the modification of this parameter. A value of 0.1 was used as the scale factor for the interface between arch barrel and spandrel wall, this gave a normal contact stiffness of $120 \text{ N/mm}^3$, which is close to the value used in the small scale bridge modelling. For the interface between the backfill and spandrel wall, the same scale factor was used and the corresponding normal contact stiffness was $1.2 \text{ N/mm}^3$. The contacts within the spandrel wall have a scale factor of 0.001 and this gave a contact stiffness of about $1.0 \text{ N/mm}^3$, and this is much similar to the value used in the shear wall modelling, which gave satisfactory results in terms of deformed shape. The cohesion properties between arch barrel and spandrel as well as the contacts across spandrel wall were set as $0.09 \text{ N/mm}^2$ as learned from the experimental results by the author (see Chapter 3). The cohesion between spandrel wall and backfill was ignored. A uniform value of 0.5 is used as the friction coefficient for all the contacts for simplicity. The effect of this parameter is discussed in Chapter 6.

Figure 5-17 shows the boundary conditions used during the analysis in a 3D and front plane view. As with the small scale bridge, only half the model with symmetry to the $XZ$ plane was produced to improve efficiency. A symmetry displacement condition was applied on the plane of symmetry. The bottom of the backfill and spandrel wall as well as both the springings were fixed in all three directions to simulate the rigid abutments. The movement in the $X$ direction of the backfill at both far ends was constrained. The whole analysis was carried out in two steps. After the boundary conditions were applied to the model, the self weight of the bridge was taken into account by introducing gravity to all the elements. The concentrated load was idealised as point load and applied directly on the surface of backfill. The load was applied at the quarter span across the whole width of the bridge. It had a width of 850 mm in the span direction to simulate the load through a concrete beam in the experimental condition.
5.3.3 Results discussion

In this section, the modelling results are presented and compared with the experimental findings by Melbourne and Walker (1990), including the crack patterns within spandrel wall, failure load, maximum surface deflection and arch barrel deformation underneath the load. It also provides a discussion about the accuracy of the concrete model in terms of the crack development in the arch barrel. The modelling and experimental results are summarised in Table 5.7.

The failure load was determined by the tensile failure criteria of the concrete
model when the pre-defined tensile strength was reached in the arch barrel and the formation of hinges. The convergence difficulty and cracks found during the calculation were also used to identify the failure load. As shown in the table, the proposed model has great accuracy in terms of load capacity and road surface deformation, indicating that the failure criteria and material model of the backfill are appropriate for the determination of failure. Figure 5-18 shows the outward movement of the numerical model. The maximum spandrel wall movement occurred at the loaded quarter point which is consistent with experimental observations. The outward deformation increased higher up the spandrel wall, indicating the possibility of the overturning of spandrel wall. The maximum outward movement was found at the top of the spandrel wall with a value of 3.8 mm, which is about 85% higher than that of the experimental observation.

<table>
<thead>
<tr>
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<th>Experimental</th>
<th>FE model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure load [kN]</td>
<td>1173</td>
<td>1154</td>
</tr>
<tr>
<td>Road surface deflection [mm]</td>
<td>10</td>
<td>8.4</td>
</tr>
<tr>
<td>Spandrel wall outward movement [mm]</td>
<td>2</td>
<td>3.7</td>
</tr>
<tr>
<td>Vertical displacement at 3/4 span [mm]</td>
<td>4.5</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Figure 5-18: Outward movement of the model (mm)
The crack patterns at failure load from experimental observation and finite element predictions are illustrated in Figure 5-19(a) and 5-19(b) respectively. As shown in the figure, the diagonal cracking which started from about the middle of arch extrados as observed in the experiment test was successfully predicted by the finite element model. The cracks developed in an opposite direction at the connection area near arch barrel underneath the loading point (as indicated by the red circle in Figure 5-19(b)). This is also consistent with the experimental results. It is believed that the crack pattern predicted by the model is related to the size of the block unit. A smaller size of the unit may lead to better results as it gives more accurate representation of the spandrel wall, however it will require much more effort for the model generation and calculation. As model predicts the general failure pattern, the output is considered acceptable.

Figure 5-19: Crack patterns in spandrel wall

The load displacement curves obtained at 3/4 quarter span in the arch intrados are plotted in Figure 5-20 and the measured average results from the test
was included for comparison purpose. The experimental results exhibits an initial linear behaviour before the load has reached 700 kN, then the load continues to rise with the increase of displacement at lower rate. The curves obtained from the finite element model show almost linear behaviour through the whole range. This may due to the assumption that the concrete material model has linear elastic behaviour before cracking. The constraining effects of the spandrel wall have been demonstrated as the modelled deflection in the middle of arch barrel is 14% higher than the deformation at the edge of the barrel. This effect is also confirmed by the experimental test. The deflections measured at the edge and in the middle have a difference of 0.7 mm with an average deflection of 5.2 mm at 3/4 quarter span. It is noted that the stiffness of the bridge predicted by the finite element model is much smaller than of the experimental test, this may be attributed to the modification of elastic stiffness made for the concrete material, as a much lower value was used once cracking occurs.

The stress conditions in the X direction across arch barrel and the stress distribution in the vertical direction in the backfill are illustrated in Figures 5-21 and 5-22 respectively. Similar to the small scale bridge, the possibility of formation of hinges were found in the arch barrel. The black dots shown in Figure 5-21 stand for the potential hinges as identified by the relatively highest stress in that area. Two of them were identified at the springings, one was found

Figure 5-20: Load displacement of relationship under loading point
underneath the loading point, and the other is located near the middle span point in the intrados. The load distribution condition in the vertical direction is comparable similar to that found in small scale model. A higher load dispersion angle with an average value of 41° was found for the current model.

Figure 5-21: Stress distribution in X direction across arch barrel

Figure 5-22: Stress distribution in vertical direction across backfill

As mentioned before, the concrete model used for the arch barrel in conjunction with the SOLID 65 element has the ability of cracking and crushing. The element cracks when the principal stress exceeds the defined compressive/tensile strength. It is noted from the finite element model that a tensile failure with cracks across the arch barrel occurred. The maximum compressive principal stress is still within the range of defined strength. The cracking conditions (indicated by the red dots) within the arch barrel under
different levels of load are illustrated in Figure 5-23 and 5-24. As shown in Figure 5-23, the cracks are initially found in the arch intrados underneath the line load and in the extrados at the springing with a load of 310 kN. The experimental results revealed the formation of the first hinge in the barrel underneath the line load when the load has increased to 400 kN (Melbourne and Walker, 1990). The cracks experienced at the springing may be explained by the boundary conditions used in the analysis. Since the springing was totally fixed in all three directions, the tensile stress in the adjacent area may lead to the cracking in this area.

As the load increases, there are more cracks found in the barrel. Cracks in the transverse direction across the whole width of bridge were found when the load reached 714 kN. About 2/3 of the arch thickness was in a tensile status. In addition, longitudinal cracks in the barrel were found underneath the spandrel wall (as indicated by the black oval in Figure 5-24a). This may due to the strengthening effects of the spandrel wall against the deflection of arch barrel. The bending moment experienced in that area could lead to a cracking in a transverse direction, which will further lead to the separation of the spandrel wall from the barrel. The maximum tensile principal stress was also found in that area. This also provides evidence that potential cracks may develop in this area and cause the separation of the spandrel wall, which has been observed in many historic masonry arch bridges.

The cracks are found to spread across the whole barrel under a load of 1154 kN, when the bridge lost strength in this condition (Figure 5-24b). It is featured with two groups of cracks in the transverse direction located under the load line and near the 1/3 span position. Several longitudinal cracks were also noted near the spandrel wall and across the barrel. The longitudinal cracks developed at the plane of symmetry may be because of the bending moment caused by the symmetry displacement boundary conditions applied. There are no experimental results of the crack patterns in the barrel to compare the model with. However, it is worth knowing how the proposed material model works in masonry structures, and the modelling results provided some information which could be used for the explanation of some phenomena found in historical structures.
Figure 5-23: Cracks developed in arch barrel Load = 310 kN
Figure 5-24: Crack patterns in spandrel wall

(a) Cracks developed in arch barrel Load = 714 kN

(b) Cracks developed in arch barrel Load = 1154 kN
5.4 Conclusions

A three dimensional finite element model was constructed to study the general behaviour of a small scale bridge under rolling and increasing load, as well as a full scale bridge under concentrated load until failure. The modelling results were compared with corresponding experimental data found in the literature. The following conclusions can be drawn from modelling work and the comparison for the small scale bridge which did not include spandrel walls:

- The proposed model and modelling techniques can simulate the general behaviour with good consistency in terms of the deflection and pressure in the arch extrados under rolling load.
- The finite element models predicted failure due to the formation of four hinges, and the finite element model give a good prediction of the locations of the hinges. However, the model used could not take the ring separation into consideration, and further research may be required to account for this.
- The proposed model provided a good prediction in terms of the magnitude of failure load, however the corresponding deflections were much lower than the measured value from experimental tests, which may be because the real model may not have fully fixed abutments.
- The contact stiffness of the interfaces that were located either between the barrel and backfill or within the barrel shows great influence on the failure load.
- The increase of contact stiffness within the arch barrel will lead to an increase in bridge strength and a decrease in arch deflection. The contact stiffness between the arch barrel and backfill exhibited a similar effect on the arch deflection but the opposite impact on the failure load.
- The coefficient of friction is less important in the numerical model and shows little influence on the results.

For the full scale bridge with spandrel walls, the transverse behaviour was considered by the implementation of a three dimensional model. The following findings are worth noting from the modelling work:

- The proposed model shows good agreement for the predictions of failure
load and arch deflection as well as the crack pattern in the spandrel wall under increasing load.

- The strengthening effects of spandrel wall have been proved by both experiments and the finite element model from the load displacement curves. Small differences were found for the load displacement curves with a bilinear and linear behaviour for the experimental and numerical results respectively.

- The outward movement of the spandrel wall has been captured by the finite element model.

- The concrete material model can be used for the identification of failure of structure, and it can also provide information about the crack development in the barrel. However care should be taken and further reviews may need when using these results.

- The crack pattern in the spandrel wall depends on the contact stiffness of the interfaces, and more work may be needed in this area to identify appropriate interface model parameters to accurately capture this behaviour.
Chapter 6

Parametric study

6.1 Introduction

As discussed in previous chapters, the finite element results show good correlations when compared with corresponding model tests for both small and large scale bridge in terms of modes of failure, stress conditions at the arch extrados and arch deflections. In this chapter, the numerical results for the full scale model bridge were used as the basis for a parametric study. A series of computer models were constructed to investigate the relationship between a range of geometric and material parameters and the lateral behaviour of masonry arch bridges. The following aspects were considered: the depth of backfill at the crown, the stiffness of the arch barrel, the thickness of the spandrel wall and the fill properties. This study is necessary also because during the modelling of each bridge, various estimations were made for some of the parameters required by the finite element model in the absence of measured experimental values. It is important to understand the sensitivity of these parameters to the results.

The constructed finite element models discussed in the following sections have the same boundary conditions as used in Chapter 5 for the large scale bridge modelling. For each model, only one parameter (either geometric or material) was changed for comparison purposes. The ultimate load carrying capacity of the bridge during analysis was identified when the principle tensile stress was exceeded and the majority of cracks were found in the arch barrel. This criterion was adopted because the concrete material for the arch barrel
uses a smeared crack approach, and the formation discrete cracks and large
deformation are not allowed. The cracks in the arch barrel are considered to
represent the formations of hinges, which turn the structure into a mechanism
failure. A separate finite element model was constructed to simulate the existing
longitudinal cracks found in the arch barrel of some old bridges. The general
behaviour under a concentrated load was studied and discussed. A strengthened
finite element model was also built with spandrel wall tie bars passing through
the full width of the bridge. It aims to provide quantitative information about
the effectiveness of the tie bar and the influence on the general behaviour of
the bridge. Numerical analyses were performed to investigate the influence of
the stiffness ratio between the spandrel wall and arch barrel on the behaviour of
model bridge, and it focus on the initial crack load in the spandrel wall.

6.2 Influence of geometric and material properties

In this section, the results of a parametric study based on the numerical
analysis described in previous chapters. The studied bridge is a single span
brickwork masonry bridge and is featured with a straight arch. As mentioned
in Chapter 1, there are currently more than 40000 masonry arch bridges still in
service in the UK. Among them, there are not only single span bridges but
also multi-span arch bridges built with different material or a skewed arch.
The findings from the numerical analysis may be limited to single span arch
bridges and the application to multi-span or skewed arch bridges need further
investigations. The numerical models used in this chapter were generated using
the same technique as described in Chapter 5. It is noted that the model which
involves the change of material properties is relatively easy to produce. For the
model with a different geometry profile, it requires a complete re-build. This is
because the interfaces on the spandrel wall were manually produced by identifying
the numbers of each contact surface. The change of geometry causes the disorder
of the contact surfaces, which leads to the re-generation of the interfaces. The
analysis takes an average time of two hours to run, and time varies slightly for
different models materials used.
CHAPTER 6. PARAMETRIC STUDY

6.2.1 Backfill depth

The depth of fill has been shown to have great influence on the failure load from previous research (Mahmoud, 2005; Boothby and Roberts, 2001), and it is believed the lateral pressure from the fill is of great importance to the overall behaviour. In the current analysis, a finite element model was constructed with the backfill depth at the crown increased from 300 mm to 1250 mm (as shown in Figure 6-1). The model experienced the same concentrated load at quarter span position and the spandrel geometry was unchanged.

Figure 6-1: High backfill model

As determined from the analysis, the increase in depth of the backfill shows substantial strengthening effects of the arch bridge with a value of 2633 kN for failure load obtained from the analysis for this condition. It is more than twice the failure load of the benchmark analysis. This could be explained by the thicker backfill helping to distribute the load more uniformly on the arch extrados, while keeping the arch under compression. The vertical stress distribution in the backfill and the stress condition across the arch barrel are illustrated in Figures 6-2 and 6-3. The applied load does not disperse much in the upper part of the backfill (with a spandrel wall thickness of 216 mm) and the load was gradually transferred to the arch extrados between the springing and load area. In terms of the stress in the X direction, the potential hinges could be formed at four areas across the arch barrel. Compared with the benchmark analysis, the majority area in
the arch barrel was found in a compression condition rather than in tension (see Figure 5-21 for comparison).

![Stress distribution in vertical direction through the backfill (front view in N/mm$^2$)](image)

**Figure 6-2:** Stress distribution in vertical direction through the backfill (front view in N/mm$^2$)

![Stress distribution in horizontal direction across arch barrel (front view in N/mm$^2$)](image)

**Figure 6-3:** Stress distribution in horizontal direction across arch barrel (front view in N/mm$^2$)

Apart from the strengthening effects, the lateral behaviour of the backfill is clear with symmetric diagonal cracks across the spandrel wall near the loaded area. The outward movement of the spandrel wall has increased from 1.6 mm near the arch barrel to 14.5 mm at the top of parapet (Figure 6-4), which shows the potential for overturning of the spandrel wall near the loaded area. This failure mechanism cannot be predicted in a 2D analysis.

It is worth noting that in the current analysis, the maximum deflection at the arch barrel was found at the front edge (underneath the spandrel wall) at the loading point rather than in the middle across bridge width as found in the benchmark analysis (see Figure 6-5). This may be explained by the different
thickness of the spandrel wall used in the numerical model. As listed in Table 5.3, the finite element model used in the analyses have spandrel wall thickness of 436 mm at the lower part and which decreased to 216 mm for the upper part (as shown in Figure 6-1). For the benchmark analysis, the backfill was always adjacent to the thicker portion of the spandrel. In the current analysis, the depth of backfill has increased and fill material was also placed above the step in the spandrel wall. When load was applied on the surface of backfill, it was transferred to the stepped portion of the spandrel wall directly in the vertical direction. The relatively larger deflection of the arch barrel as well as the diagonal cracks in the spandrel wall may be attributed to the transferred load on the spandrel wall from the backfill.

**Figure 6-4:** Outward movement of the whole structure (mm)

**Figure 6-5:** Arch deflection in front view (16 times magnification of real displacement)
6.2.2 Fill properties

As mentioned before, the fill material used for construction of masonry arch bridges was often local material, which ranges from dense, cohesionless sands to soft clays. The proposed model in the current analysis considered the backfill as a homogeneous material. The Drucker-Prager material model defines a shear failure, which is controlled by the cohesion, friction angle and dilation angle, and many other effects such as soil creep, water pressure, and frost effect have not been considered in the analysis. While backfill stiffness can affect overall arch bridge behaviour, it will not directly affect the failure strength of the material. In this section, a series of numerical studies was carried out to examine the effects of cohesion, friction and dilation angle of backfill to the overall behaviour of masonry arch bridges.

Firstly it is important to identify an appropriate range of values for these parameters. The typical values of the friction angle for different soils obtained by Das (2010) can be found in Table A.1. Research work carried out by Lekarp et al. (1996), Garg and Thompson (1997), Theyse (2000) on shear strength of unbound aggregates shows that the cohesion can range from 4 kPa to 145 kPa and friction angle can range from 31° to 67°. The details of these properties for unbound aggregates can be found in Table 6.1.

<table>
<thead>
<tr>
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<th>φ (deg)</th>
<th>c (kPa)</th>
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<tbody>
<tr>
<td>Lekarp et al. (1996)</td>
<td>58-67</td>
<td>49-145</td>
</tr>
<tr>
<td>Garg and Thompson (1997)</td>
<td>31-51</td>
<td>48-124</td>
</tr>
<tr>
<td>van Niekerk et al. (2000)</td>
<td>37-44</td>
<td>4-142</td>
</tr>
<tr>
<td>Theyse (2000)</td>
<td>48-55</td>
<td>26-121</td>
</tr>
</tbody>
</table>

Numerical analysis was carried out on finite element models with the same geometry as the benchmark model, and each time only one parameter was changed. The modelling results are summarised in Table 6.2. Analysis results show that for this particular bridge there is about 40% increase in the bridge strength as the cohesion of the backfill increases to 50 kPa. The friction angle seems to have more significant impact on bridge strength than the cohesion, with a smaller friction angle leading to a decrease bridge load capacity. An approximately 63% lower load capacity was obtained when the friction angle
decreased from 54° to 30°. However these two parameters were not found to have significant impact on spandrel wall failures.

Table 6.2: Effects of fill properties

<table>
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<th>Bridge</th>
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<th>ϕ</th>
<th>Load [kN]</th>
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<td>36°</td>
<td>1154</td>
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<td>Model 3</td>
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<td>54°</td>
<td>0°</td>
<td>190</td>
<td>overturning of Spandrel wall</td>
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</tbody>
</table>

Figure 6-6: Outward movement of structure with zero dilation angle (mm)

There is a general scarcity of published data for the dilatation angle of different backfill, particularly densely compacted crushed soils, and its importance has not previously been recognized. A dilatation angle of 30° was used in the work done by Fanning et al. (2001). The modelling results in this study shows that the dilatation angle of the soil in the Drucker-Prager model has the greatest impact.
on the lateral behaviour. A zero dilatation angle caused the bridge to fail through overturning of the spandrel wall (Figure 6-6) at a very low load of approximately 15% of the peak load for the same model with a dilation angle of 36°. Numerical analysis carried out by De Borst and Vermeer (1984) on bearing capacity of a footing showed that the dilatation angle has great impact on the deformation patterns of soils surrounding the footing. As shown in Figure 6-7, the soil with a larger dilation angle shows a much larger surface depression. It is noted that the maximum movement was found at the both ends in the parapet rather than the loading position. It may be explained that a low dilatation angle alters the load dispersion through the backfill and is more likely to cause the backfill to deformed in the transverse direction which may lead to the over-turning of spandrel wall.

![Figure 6-7: Deformation patterns below circular footing (De Borst and Vermeer, 1984)](image)

6.2.3 Spandrel wall thickness

The thickness of the spandrel wall is believed to have an definite impact on the lateral behaviour of arch bridges. The numerical analysis performed by Boothby and Roberts (2001) shows that the spandrel wall thickness affects not only the load carrying capacity of bridge, but also the failure mechanism. In the current study, a finite element model was built with a uniform spandrel wall thickness of 216 mm thickness for analysis rather than the stepped 436 mm to 216 mm wall in the standard case. The failure load and deformation in the transverse direction are presented and discussed here.
Similar to the benchmark model, the model bridge with a thin spandrel wall fails in a four hinge mechanism with cracks developed in the spandrel wall. The failure load obtained from the analysis was $771 \text{ kN}$ and this is about 34% lower than that of the benchmark result. This lower failure load could be explained by the decrease of the strengthening contribution of the thinner spandrel wall which leads the barrel to a tensile condition more easily. The push out effects exhibited by the backfill are more critical for a bridge with thinner spandrel walls. The spandrel wall is more likely to experience a local failure as the maximum outward movement of the spandrel wall was found at the same level of backfill surface near the loaded area (as indicated by the red circle in Figure 6-8). The spandrel wall thickness also has impacts on the load distribution in the backfill. As shown in Figure 6-9, the applied load was transferred to the arch barrel with a larger dispersion angle compared with the benchmark model, and this could be one of the reasons for the lower bridge strength obtained in the current analysis.

Figure 6-8: Outward movement for the model with thin spandrel wall (mm)
6.2.4 Elastic modulus of masonry

As it is very difficult to determine the material properties accurately, and reasonable estimates have been employed during the analysis when model parameters were not available from test data. It is likely that the development of cracks in the spandrel wall is related to the stiffness of arch barrel, so it is necessary to examine the influence of elastic modulus of masonry on the overall behaviour of the model. Previous research on masonry testing and modelling showed that the stiffness of masonry ranges from 2000 $N/mm^2$ to 7000 $N/mm^2$ (Mahmoud, 2005; Baker, 1996). In the current analysis the standard bridge was modelled with another two different stiffness values for the arch barrel, which are 2500 $N/mm^2$ and 6000 $N/mm^2$ respectively. The predicted results for the failure load and the crack pattern developed across the spandrel wall are discussed and compared with the benchmark analysis.

<table>
<thead>
<tr>
<th>Bridge</th>
<th>E [GPa]</th>
<th>Load [kN]</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>4.0</td>
<td>1154</td>
<td>Four hinge mechanism and Spandrel wall cracking</td>
</tr>
<tr>
<td>Model 1</td>
<td>2.5</td>
<td>1244</td>
<td>Four hinge mechanism and Spandrel wall cracking</td>
</tr>
<tr>
<td>Model 2</td>
<td>6.0</td>
<td>912</td>
<td>Four hinge mechanism and Spandrel wall cracking</td>
</tr>
</tbody>
</table>

The predicted load and failure mode are listed in Table 6.3. The predicted failure load are 912 $kN$ and 1244 $kN$ for the models with a stiffness of 6000 $N/mm^2$. The predicted load and failure mode are listed in Table 6.3.
CHAPTER 6. PARAMETRIC STUDY

$N/mm^2$ and 2500 $N/mm^2$ respectively. The bridge strength exhibits a decreasing trend with an increase in the arch stiffness. This may be explained by a more flexible arch usually having larger deformation which forces the load to distribute more widely in the soil. Slightly different crack patterns were found in the spandrel wall for different models. For the model with low stiffness, a larger deformation in the vertical direction was obtained (as shown in Figure 6-10), and several cracks were found in above arch barrel beneath the load area. The development of diagonal cracks is more obvious for the model bridge with a higher stiffness even though it has a smaller deformation, indicating a higher arch stiffness contributes to the development of the cracks across different courses.

Figure 6-10: Vertical displacement and crack patterns in spandrel wall with different masonry stiffness (mm)
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6.3 Simulation of spandrel wall separation

Longitudinal cracks across the arch barrel are found in many old masonry arch bridges (Page et al., 1991). This kind of defect is likely to cause a further separation between spandrel wall and arch barrel. In this section, a finite element model was constructed to simulate the existing longitudinal cracks in the arch barrel underneath the spandrel wall. The general behaviour of the model bridge under a concentrated load at quarter span position was studied and the potential failure modes are discussed.

6.3.1 FE model and boundary conditions

The finite element model used in the current analysis has the same geometry as the benchmark model described in Chapter 5. Similarly, the model simplified the bridge into three main elements: arch barrel, backfill and spandrel wall. In order to simulate the existing cracks in old bridges, the arch barrel was divided into two parts. One is connected to the spandrel wall, while the other part was attached to the backfill (see Figure 6-11). The same material models were used for the three elements and the material properties can be found in Table 5.6.

Since the main purpose of this study is to simulate separation between the spandrel wall and arch barrel, the interaction between two parts of the arch barrel is the main concern. Interfaces were only introduced between the backfill and spandrel wall and between the arch barrels using frictional contact elements. For the construction of the spandrel wall in the finite element model, instead of a series of elastic blocks, the spandrel was modelled using the same concrete material as the arch barrel. Although the use of a homogeneous continuum for the spandrel wall will limit crack development with interface separation, it can still provide some information related to the stress conditions across the wall. In order to ensure only one parameter changes in each analysis, initial analysis were carried out for a model without longitudinal cracks for comparison purpose. The material properties used for the contact elements during the analysis are listed in Table 6.4. The finite element meshes of the numerical model together with boundary conditions are illustrated in Figure 6-12. The same boundary conditions adopted for the benchmark analysis were used in the current analysis. A detailed description can be found in Chapter 5.
Figure 6-11: FE model bridge without(top)/with(bottom) longitudinal cracks in arch barrel underneath spandrel wall.

Table 6.4: Contact properties

<table>
<thead>
<tr>
<th></th>
<th>Friction coefficient</th>
<th>Cohesion ([N/mm^2])</th>
<th>Stiffness factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between arch barrels</td>
<td>0.5</td>
<td>0.09</td>
<td>0.1</td>
</tr>
<tr>
<td>Spandrel/backfill</td>
<td>0.67</td>
<td>0.001</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 6-12: Finite element mesh and boundary conditions.
6.3.2 Results and discussions

The finite element model experienced a concentrated load at quarter span position and the failure load was identified when the principal tensile stress exceed the tensile strength of masonry and cracks were found in arch barrel. The maximum failure load obtained for the model without cracks in the arch barrel was 1190 kN which is close to the experimental value. Figure 6-13 shows the deformation in the transverse direction. The maximum displacement was found at the top centre of the parapet instead of near the load area in the benchmark analysis. The maximum value for the outward movement and vertical displacement under the loading point are 2.7 mm and 5.3 mm, while the experimental measurements gave a value of 2 mm and 5.8 mm respectively.

![Figure 6-13: Transverse deformation for model without longitudinal cracks in the arch barrel (mm)]

The predictions of the cracks across the spandrel using the concrete material model are illustrated in Figure 6-14. As can been seen that, different crack patterns were found in the current model compared with benchmark analysis. Cracks were found in the vertical direction near the crown point, with development of cracks through the parapet. The areas near the springings experienced several cracks, and this may due to the boundary conditions used.
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during the analysis. Taking the cracks in the left side as an example (as indicated in the circle in Figure 6-14), once the load was applied at the quarter point, the barrel near the springing is more likely to move upward and lift up the spandrel wall with it. As the bottom of the wall was fixed in all directions, cracks occurred in this area. The cracks found in the middle of the wall can be explained by the same reason, and it is worth noting that vertical cracks were found in the experimental test (see Figure 6-15) carried out by Melbourne and Walker (1990). However, the diagonal cracks developed in spandrel wall were not captured using the concrete material for the spandrel wall.

![Figure 6-14: Predicted cracks in the spandrel wall using the concrete material](image)

Table 6.5: Results for different models

<table>
<thead>
<tr>
<th>Bridge</th>
<th>$\varphi$</th>
<th>Load [kN]</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard model (no arch cracks)</td>
<td>36°</td>
<td>1190</td>
<td>Four hinge mechanism and Spandrel wall cracking</td>
</tr>
<tr>
<td>Crack model 1</td>
<td>36°</td>
<td>904</td>
<td>Four hinge mechanism and fill failure</td>
</tr>
<tr>
<td>Crack model 2</td>
<td>0°</td>
<td>63</td>
<td>Spandrel wall separation</td>
</tr>
</tbody>
</table>

Numerical analysis was then carried out using the model with longitudinal cracks in the arch barrel. A maximum failure load obtained from the current analysis was 904 kN was recorded, and it was approximately 24% smaller.
compared with the standard bridge. This has proved that the spandrel wall has a strengthening effect on the bridge. Figure 6-16 illustrates the principal stress conditions of the whole structure with details of the arch intrados and crack conditions in the arch barrel. The maximum tensile stress was found at the arch intrados under the loading point as expected, while the maximum compression stress was experienced at the springing position near the load point.

Figure 6-15: Development of cracking in spandrel wall west face (Melbourne and Walker, 1990)

Figure 6-16: Principal stress conditions in the structure and crack development in arch barrel (N/mm$^2$)
The cracks were found across the arch barrel under the loading point. Compared to the results for the standard bridge, there are no cracks found in the spandrel wall for this condition, which indicates the interaction between the arch barrel and spandrel in more critical for the development of cracks in the spandrel wall. It was also noticed that the surface backfill elements exhibited relatively large deformations under the point load, which indicates a shear failure occurred at this area. This was not observed for the standard bridge analysis. It can be concluded that the change of interface also has significant on the load transfer across the backfill.

However, this modified model did not show significant changes in terms of the transverse behaviour. The maximum movement was again found at the top parapet near the load point. Slight separation between the spandrel wall and backfill was observed at the load area. It can be concluded that the backfill pressure in the transverse direction is not a critical factor for spandrel wall separation problem in the current situation.

As determined from the study in section 6.2 that the dilation angle of the fill materials has a large impact on the general behaviour in terms of bridge strength and failure mode. In this section, a parametric study was carried out with a zero dilation angle for the backfill for comparison purpose. The modelling results are listed in Table 6.5 and the deformation in the transverse direction is presented in Figure 6-17(b).

Similar to the analysis performed for the bridges without spandrel wall defects, the dilation angle of soil shows great influence on the failure load. The model bridge failed with a load of 63 kN, and the load is about three times smaller when compared with the benchmark bridge without spandrel wall defects and the same dilation angle. In terms of the outward movement, as the failure load is rather small, a relatively small deformation was experienced. The maximum movement was found at the top of the parapet as shown in the figure. Unlike the model with a 36° dilation angle, there was no backfill failure for the model with zero dilation angle and the maximum principal tensile stress found in the arch barrel was still within predefined material strength. The failure of the structure may be explained by the instability caused by the separation between the spandrel wall and backfill, as larger movements were identified in the spandrel than the backfill at the same position.
Figure 6-17: Deformation in transverse direction (mm)
6.4 Spandrel wall strengthening with pattress plates and ties

As mentioned in section 6.3, the longitudinal cracks in the arch barrel have been found in many historic masonry arch bridges, and these cracks have contributed to the outward movement and separation of the spandrel wall. The horizontal forces experienced by the spandrel wall are mainly from the spread of fill, increased ballast depth and increased live load. For the outward movement of spandrel walls, one of the most commonly used approaches is the use of tie bars with pattress plates as shown in Figure 6-18. The method consists of tie bars placed into bored holes that are perpendicular to the spandrel walls and across the whole barrel (backfill). The ends of the tie bars are fixed with the pattress. The aim of this method is to stabilize the arch spandrel walls pushed by the lateral forces due to the traffic load. This method is more practical as it requires minimal disruption to the traffic on existing structures. It is also cost effective compared with other available methods and could avoid rebuilding the walls (Apreutesei, 2005). However, their effectiveness has not been proven and there is no appropriate guidance on the suitable locations for tie bars. The numbers and positioning of the tie bars are largely determined due to the consideration of appearance. An important criterion for the repair work is to maintain the symmetry of the structures (Ryall et al., 2000).

The current assessment does not give any guidance for the strength calculation of masonry arch bridge with strengthening tie bars. In this section, a finite element model was developed based on previous studies with spandrel wall tie bars passing through the full width of the bridge. It aims to provide quantitative information about the effectiveness of the tie bar and the influence on the general behaviour of the bridge.

6.4.1 FE model

A schematic view of the numerical model used in the analysis is plotted in Figure 6-19. There are two tie bars with a diameter of 60 mm placed symmetrically across the bridge. The pattress plates fixed at the ends have a diameter of 400 mm and thickness of 50 mm. For the convenience of generating
the meshes of the finite element model and determination of the locations of the tie bars, the whole spandrel wall was divided into several parts. The tie bars were placed at approximately the centroid of the area indicated by the rectangles in Figure 6-19. The detailed finite element mesh of the numerical model is illustrated in Figure 6-20. The same modelling techniques were adopted as described in section 6.3. The three main components: arch barrel, backfill and spandrel wall were considered as continuum. The arch barrel and spandrel wall followed a strength criterion defined by the concrete material model, while the backfill was modelled using the Drucker-Prager material model. Interfaces were introduced between the arch and spandrel wall as well as between the spandrel wall and backfill.
The spandrel tie bars were made of steel. A typical stress strain relationship for commonly used low carbon steel which is described by Williams and Todd.
(2000) have been plotted in Figure 6-21(a). The curve shows a linear elastic stage until it reaches the yield stress, \( f_y \). After that plastic behaviour starts, where the strain increases quickly with no change in stress. The specimen experiences a slight increase in stress and achieves ultimate strength before failure. In the current model, this relationship was simplified into a bilinear behaviour as shown in Figure 6-21(b). The behaviour is initially linear elastic before the yield stress is reached and then undergoes a plastic stage defined by a tangent modulus \( E_t \).

There are also interfaces placed between the tie bars and spandrel wall as well as the backfill. However, these interfaces were assigned a bonded behaviour, where the slip between the tie bars and structures was not taken into consideration. This was aimed to represent the good cohered conditions between the spandrel wall and tie bars.

![Typical stress strain curve of steel](a) Typical stress strain curve of steel bar ![Bilinear stress strain behaviour](b) Bilinear stress strain behaviour

**Figure 6-21:** Stress strain behaviour of steel bar

The material properties used for the steel tie bars during the analysis are listed in Table 6.6. They mainly come from the experimental tests carried out by Wang et al. (2013) on high strength steel. The test results give a average elastic modulus of 20900 \( N/mm^2 \) and a yield stress of 500 \( N/mm^2 \). The tangent modulus is taken as 1/15 of the elastic modulus and has a value of 1400 \( N/mm^2 \). The density and Poisson’s ratio of steel can be referred to the work performed by Spittel (2009). The boundary conditions used in the current analysis is the same as described in section 6.3 for the standard bridge. The analysis took about eight hours to run, which is much longer compared with previous model. This is
because much finer mesh was used in this model, especially for the area adjacent to the tie bars. The contacts between the spandrel wall and tie bars (plates) and the non-linear material properties also contribute to the longer solution time.

<table>
<thead>
<tr>
<th>Table 6.6: Material properties for steel tie bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($kg/m^3$)</td>
</tr>
<tr>
<td>Elastic modulus $E$ ($N/mm^2$)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Yield stress $f_y$ ($N/mm^2$)</td>
</tr>
<tr>
<td>Tangent modulus ($N/mm^2$)</td>
</tr>
</tbody>
</table>

6.4.2 Results and discussions

In this section, the obtained results for the strengthened model are discussed and compared with the standard bridge described in section 6.3. The main focus is on the influence of tie bars on the outward movements of spandrel wall and the cracks condition across the wall. The failure load obtained for the current model was 1155 $kN$ with a maximum vertical displacement of 5.2 mm, occurred at the quarter span position under the loading point. They are very close with the values obtained for the standard bridge, indicating the spandrel tie bars have little impact on the bridge strength.

The deformation in the transverse direction for these two models is presented in Figure 6-22. In order to make a direct comparison, the deformations shown for the two models are under a load of 1000 $kN$. The outward movement of the point where the tie bar (right side) is placed is only 0.25 mm for the strengthened model, and this is 60% smaller than the corresponding value found for the standard model. It is also noticed that the maximum deformation occurred near the arch extrados for the model with tie bars rather than at the top of parapet for the standard bridge. The tie bars are therefore effective for the prevention of the spandrel wall movement, however they do not increase maximum load and may contribute to the failure of the wall in adjacent area.

Slightly different crack patterns were found in the spandrel walls for these two models (see Figure 6-23). The connection area near the loading point of the strengthened model is more critical in terms of the occurrence of cracks.
This can be explained by the difference of stiffness between two materials. The stress concentration due to the deformation resistance at the interface can cause cracking of masonry in the adjacent area. There are also cracks found above the tie bar in the wall which developed through the parapet. The existence of the tie bar and plates shows potential influence on the stress distribution in the wall and more research efforts are needed in this area.

**Figure 6-22: Deformations in the transverse direction (mm)**
Figure 6-23: Crack conditions in spandrel walls
The principal stress condition in the tie bars is illustrated in Figure 6-24. As can be seen, the maximum tensile stress, which is $43.3 \text{ N/mm}^2$, was experienced at the contact area with the spandrel wall. It is far smaller than the yield stress of the steel bar.

Figure 6-24: Principal stress conditions in the tie bar near load position ($\text{N/mm}^2$)

6.5 Investigation of stiffness ratio between arch barrel and spandrel wall

There have been several structural failures of spandrel walls over in the past decade. Most of the failures have showed the unpredictability and the potential to cause a serious accident. Longitudinal cracks underneath the spandrel wall across the arch barrel are common defects found in old masonry arch bridges (Figure 6-25(a)). This type of defect can be relevant to all masonry arch bridges, but in particular for those bridges built with different materials (e.g. between the voussoir and the arch barrel or arch strengthening using different material). Structures with brick barrels and a stone voussoir (e.g. multi span viaducts with tall / slender piers) with spandrel wall defects, typically exhibit longitudinal cracks in the arch intrados between the brick and stone components. The reason for this is that there is a significant change of elasticity and stiffness between the two components / materials coupled with change in bond and difference in weathering. This results in a discontinuity that will concentrate cracking at that
point. These cracks can further lead to separation between the spandrel wall and arch barrel as shown in Figure 6-25(b). It is therefore essential and beneficial to have an assessment tool which could provide information on the bridges that is more likely to experience such failure for monitoring purposes. The possible parameters that could be used in such a tool may include: backfill material, arch stiffness and geometry, pier geometry, and foundation conditions. Among them, the different stiffness between the arch barrel and spandrel wall are considered as the most critical factor for the longitudinal cracks as the cracks occurred when the deformation of arch barrel was constrained by the relatively stiffer spandrel wall (Harvey et al., 2007). Numerical analyses were performed to investigate the influence of the stiffness ratio between the spandrel wall and arch barrel on the behaviour of model bridge, and it focus on the initial crack load in the spandrel wall. It aims to provide information for the identification of bridges which is most likely to experience spandrel wall separation defects.

The finite element model described in section 6.3 was used in the current analysis. Similarly, the model simplified the bridge into three main elements: arch barrel, backfill and spandrel wall. Both the spandrel wall and arch barrel were modelled using the concrete material model, while the backfill was simulated with Drucker-Prager material model. Interfaces were introduced between arch barrel and spandrel wall and between spandrel wall and backfill as in section 6.3. The material properties and boundary conditions adopted in the analysis can be found in Table 5.6 and Figure 5-17.
The stiffness of the arch barrel ($E_1$) has an initial value of 4000 $N/mm^2$ as used in previous analyses, while the stiffness of spandrel wall ($E_2$) ranges from 2000 $N/mm^2$ and 20000 $N/mm^2$. A further study was also performed with an increased stiffness of 6000 $N/mm^2$ for the arch barrel. As identified from section 6.2, the backfill properties have significant influence on the strength of the bridge, the influence of cohesion property of backfill was investigated. Numerical analysis was completed with the cohesion of backfill increased from 6.5 $kPa$ to 50 $kPa$, while the stiffness of arch barrel was set as 6000 $N/mm^2$. The crack load when the first crack were found in the spandrel wall (Figure 6-26) was recorded and plotted in Figure 6-27 versus the stiffness ratio between arch barrel and spandrel wall.

![Figure 6-26: Initial cracks in spandrel wall](image)

![Figure 6-27: Relationship between the initial crack load and stiffness ratio between arch barrel and spandrel wall](image)
As can be seen, the cracks were initially found near the crown and developed across the parapet. The initial crack load shows an increasing trend with the increase of stiffness ratio between arch barrel and spandrel wall in all conditions. The maximum initial crack load recorded is 690 kN when the stiffness ratio is 2 with 6000 N/mm$^2$ for the stiffness of arch barrel, and the load has decreased to 309 kN as the ratio dropped to 0.3. By comparing the two sets of results with different stiffness values for the arch barrel, the spandrel wall shows more vulnerable to cracking with an increased stiffness of arch barrel. Under the condition when the stiffness ratio is 1, the crack load is 19% smaller for the model with a stiffness of 6000 N/mm$^2$ for the arch barrel compared with the model whose stiffness is 4000 N/mm$^2$ for the arch barrel. The increase of cohesion property for backfill material has delayed the occurrence of cracking in the spandrel wall, the crack load shows 11% increase under the stiffness ratio of 1. It is noted that the initial crack load exhibits close value when the stiffness ratio decreases to 0.3, indicating the absolute high stiffness value of the spanrel wall becomes the predominant factor in terms of cracking development across the wall.

6.6 Conclusions

A series of finite element models were constructed and the relationship between a range of geometric and material parameters and the lateral behaviour of arch bridges was studied. Because of time limitations not all parameters could be investigated and it is proposed these form a future research programme. A separate model was developed to simulate longitudinal cracks underneath the spandrel wall in the arch barrel, and strengthening of spandrel wall with tie bars. The following conclusions could be drawn from the above analysis:

- The bridge with deeper fill is stronger compared with the standard bridge, and it results in more critical lateral earth pressure which is likely to cause the overturning of the spandrel wall.
- A local failure near the loaded area of a thin spandrel wall is more likely to occur rather than a global overturning failure. The thickness of the spandrel wall also influences the load distribution across the backfill in the vertical
direction because of the wall geometry used in this analysis.

- For the soil model used in this research, though the friction and cohesion shows a strong relationship with bridge strength, they were not found to have significant impact on lateral behaviour. Care should be taken when choosing a dilatation angle for the soil model, as it shows a strong negative relationship with failure load, and more research effort is required in this area.

- The load capacity of bridges is not sensitive to the arch stiffness and there is a decreasing trend as for load capacity with an increase of stiffness. A stiff arch barrel helps the development of diagonal cracks in the spandrel wall.

- The spandrel wall has been shown to influence the load capacity by comparing the results between the standard bridge and model with longitudinal cracks in arch barrel. This indicates bridges with a crack in the arch below the spandrel have much lower capacity than those which do not.

- The earth pressure developed in the transverse direction is rather small, and it is not a critical factor for spandrel wall separation problem in the current situation. The interaction between arch barrel and spandrel wall is more important for the spandrel wall separation problem.

- Strengthening with spandrel tie bars did not have great influence on bridge strength.

- The tie bars perform well for the prevention of spandrel wall movement, however they might contribute to the failure of the wall in an adjacent area. The existence of tie bars influences the stress distribution across the spandrel wall, and makes the connection area more critical for crack development.

- The initial crack load shows an increasing trend with the increase of stiffness ratio between arch barrel and spandrel wall.
Chapter 7

Conclusions and recommendations

Numerical analyses of masonry arch bridges have been carried out, with the aim to investigate the potential reasons of spandrel wall failure. This was done by taking the interaction of the arch barrel, backfill and spandrel wall into consideration. The research work initially composed material testing on brickwork masonry specimens, followed by numerical analyses on those small masonry structures. The modelling work was then extended to masonry arch bridges together with a series of parametric studies. A summary of the conclusions and limitations of the study are presented in this chapter, and some suggestions for future work in this field are also presented.

7.1 Summary

An experimental programme for an engineering blue brick and a hydraulic premixed mortar was performed to obtain the mechanical properties which can be used to validate numerical models. The shear behaviour of brickwork mortar joints under normal compression was studied and the influence of specimen moisture content at the time of testing on strength was investigated. Tests were performed on brickwork walls to study the shear failure under flexure and different shear loading configurations. The experimental results were compared to Eurocode 6 predictions and theoretical calculations.
Numerical analyses based on the simplified micro modelling technique were performed for those small masonry structures. The modelling results were compared with the experimental measurements. Different approaches were proposed in order to obtain an improved simulation of the load displacement relationships.

Three dimensional finite element models have been proposed for small/large scale masonry arch bridges, and comparisons and validation of numerical analysis against the experimental results from the literature were performed.

The influence of the geometric parameters and material properties on the lateral behaviour of masonry arch bridge has also been investigated.

7.2 Conclusions

As noted from experimental tests on materials and finite element analyses on different masonry structures, this section gives a brief summary of the conclusions from the main individual research areas described within this thesis.

From experimental programme, the following conclusions have been drawn:

- Linear behaviour has been found between stress and strain relationship for brickwork specimen under compressive load, and the failure is caused by crushing of brick. This is consistent with the assumption of concrete material used in numerical analysis.

- A linear relationship between the shear strength and normal stresses has been found for both the mortar and the brick/mortar interface.

- Shear failure of brickwork walls was characterised by stepped cracking and sliding through the mortar joint. Comparison between the experimental results and design values given by the relevant standard shows that the standard is conservative in design, particularly at higher normal stress levels.

- The mathematical model proposed by Willis et al. (2004) and Eurocode 6 both give conservative design values for flexural strength. Different behaviour has been obtained in this study in terms of the load displacement relationships and more research work may be needed in this area.
CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

- The presence of perforations in the bricks appears to increase shear strength by forcing failure to be both along the brick/mortar interface and through the mortar in the perforation.

From the numerical analyses on small masonry structures:

- The proposed models show good ability for the reproduction of the load displacement relationships and predictions of failure load with appropriate material parameters.
- There is an increasing trend in accuracy with an increase in the normal compressive stress for the triplet shear tests. The contact elements with the Mohr-Coulomb failure surface are suitable for the modelling of shear failure at the interface, especially for specimen with relatively high normal compressive stress level.
- The load decrease after the initial failure for the triplet shear tests can be simulated by the reduction of friction or cohesion at the interface. The assumption of the loss of cohesion properties is conservative for the current situation, as gives a lower residual strength than experimental tests.
- The contact elements shows great accuracy for the prediction of crack patterns of the shear walls with appropriate contact stiffness. However, the predicted load is highly dependent on the load conditions and confinement controls of the upper loading beam during the tests.
- The cohesive zone material is applicable for the analysis of shear walls under the assumption that friction only occurs at a fully damaged interface. It gives a better representation of the load displacement behaviour, however, the accuracy of the results depend on the selection of the parameters defining the cohesive zone material and the magnitude of the contact stiffness.

From the numerical analyses on small scale masonry arch bridges:

- The proposed model and modelling techniques can simulate the general behaviour with good consistency in terms of the deflection, pressure and failure load.
- The finite element models failed due to the formation of four hinges, and the finite element model provides a good prediction of the locations of the
hinges. However the corresponding deflections indicate the model is much more stiffer than the measured value from experimental tests which may be because the experimental tests did not have fully fixed abutments.

- The increase in contact stiffness within the arch barrel will lead to an increase in the bridge strength and a decrease arch deflection. The contact stiffness between the arch barrel and backfill exhibited a similar effect on the arch deflection while the opposite impact on the failure load which decreased with increasing stiffness.
- The friction property was shown to be less important than the interface stiffness in the numerical model and shows little influence on the results.

From the numerical analyses on large scale masonry arch bridges:

- The proposed model shows good agreement for the predictions of failure load and arch deflection as well as the crack pattern in the spandrel wall under increasing load.
- The strengthening effects of the spandrel wall have been shown by both experiments and comparisons between the benchmark model and a model with longitudinal cracks in the arch barrel.
- The concrete material model can be used for the identification of failure of structure, and it can also provide information about crack development in the barrel. However care should be taken and further reviews may be needed when using these results.
- The crack pattern in the spandrel wall depends on the contact stiffness of the interfaces.
- The bridge with deeper fill depth is much stronger, and it results in more lateral earth pressure which is likely to increase the possibility of overturning of the spandrel wall.
- A local failure of a thin spandrel wall is more likely to occur near the loaded area rather than a global overturning failure. The thickness of the spandrel wall also shows influence on the load distribution across the backfill in the vertical direction.
- For the soil model used in this research, though the friction and cohesion shows a strong relationship with bridge strength, they were not found to have significant impact of lateral behaviour which is largely governed by
dilation angle.

- The load capacity the full scale of bridges is not sensitive to the masonry stiffness and there is a decreasing trend of load with the increase of stiffness. A stiff arch barrel does help, however, the development of diagonal cracks in the spandrel wall.
- The earth pressure developed from the backfill in the transverse direction is rather small, and it is not a critical factor for the spandrel wall separation problem. The interaction between arch barrel and spandrel wall is more important for spandrel wall separation problem.
- Strengthening with spandrel tie bars perform well for the prevention of spandrel wall movement, however it did not show great influence on bridge strength.
- The existence of tie bars influences the stress distribution across the spandrel wall, and they might contribute to the failure of the wall in an adjacent area.
- The initial crack load shows an increasing trend with the increase of stiffness ratio between arch barrel and spandrel wall.

7.3 Limitations

There are a number of limitations of the model and there are largely due to time constraint, computational efficiency and lack of available of data.

- A compromise was made for modelling the large scale bridge by using concrete material to simulate brickwork masonry for the arch barrel.
- For the modelling of shear walls and spandrel wall in the large scale bridge model, cracks are assumed to occur at the interface of the brick/mortar joints. The cracking of brick unit was not considered.
- In order to improve computational efficiency, only half models were constructed for both small and large scale bridges, although slightly different behaviour (deflection, crack patterns in spandrel wall etc) were obtained from experimental tests on different sides.
- The springings were fixed in all three dimensions to simulate rigid abut-
ments, and the influences of abutment movement was not studied.

- For the large scale bridge modelling, as the main focus was on the spandrel wall, the interaction behaviour between the backfill and arch barrel was ignored.

### 7.4 Future recommendations

The work presented in this thesis is a major improvement in understanding spandrel wall defects in masonry arch bridges and identification of potential important parameters for this problem. The research is still in a preliminary stage due to the time limits of the project, and the work could be taken further in certain areas. It is suggested that the following areas may worth to put research effort:

**Experimental work level:**

- Further experimental tests on masonry arch bridges is necessary to record the behaviour in the transverse direction, such as the outward movement of the spandrel wall, crack development across spandrel wall and the separation between the arch barrel and spandrel wall.
- Mechanical tests according to relevant standards should be performed on samples from real structures to obtain a better representation of the material properties.
- The behaviour of masonry specimens under biaxial stress state and three dimensional stress states is necessary for the development of appropriate constitutive laws which could be used for numerical analysis.
- Experimental tests for the investigation of the interface stiffness and soil dilation angle are required.
- Apart from the general behaviour under concentrated load for arch bridges, cyclic loading tests should be undertaken to investigate long-term effects.

**Numerical work level:**

- Numerical models can be constructed for arch bridges with different span/rise ratio as well as for multi-span masonry arch bridges.
• The general behaviour of large scale bridge under moving/cyclic load could be studied.

• The interaction between the backfill and arch barrel may be included to provide better representation of the real structure.

• The abutments may be included in the finite element models as another main element, so that spandrel wall defects caused by abutments movement can be considered.
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## Appendix A

### Typical soil material properties from previous literature

**Table A.1**: Typical values of drained angle of friction for sands and silts (Das, 2010)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>φ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand: Rounded grains</td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>27-30</td>
</tr>
<tr>
<td>Medium</td>
<td>30-35</td>
</tr>
<tr>
<td>Dense</td>
<td>35-38</td>
</tr>
<tr>
<td>Sand: Angular grains</td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>30-35</td>
</tr>
<tr>
<td>Medium</td>
<td>35-40</td>
</tr>
<tr>
<td>Dense</td>
<td>40-45</td>
</tr>
<tr>
<td>Gravel with some sand</td>
<td>34-48</td>
</tr>
<tr>
<td>Silts</td>
<td>26-35</td>
</tr>
</tbody>
</table>
### Table A.2: Typical values of elastic modulus and Poisson’s ratio (Das, 2010)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Elastic modulus, $E$ [$N/mm^2$]</th>
<th>Poisson’s ratio, $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose sand</td>
<td>10.4-27.6</td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>Medium sand</td>
<td></td>
<td>0.25-0.4</td>
</tr>
<tr>
<td>Dense sand</td>
<td>34.5-69</td>
<td>0.30-0.45</td>
</tr>
<tr>
<td>Silty sand</td>
<td></td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>Soft clay</td>
<td>1.4-3.5</td>
<td>0.15-0.25</td>
</tr>
<tr>
<td>Medium clay</td>
<td></td>
<td>0.2-0.5</td>
</tr>
<tr>
<td>Hard clay</td>
<td>5.9-13.8</td>
<td></td>
</tr>
</tbody>
</table>

### Table A.3: Typical values for dilation angle (Vermeer and de Borst, 1984)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$\varphi$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense sand</td>
<td>15</td>
</tr>
<tr>
<td>Loose sand</td>
<td>&lt;10</td>
</tr>
<tr>
<td>Normally consolidated clay</td>
<td>0</td>
</tr>
<tr>
<td>Granulated and intact marble</td>
<td>12-20</td>
</tr>
<tr>
<td>Concrete</td>
<td>12</td>
</tr>
</tbody>
</table>

### Table A.4: Test values for dilation angle of clean sand for the loose and dense state (Deeyvid, 2010)

<table>
<thead>
<tr>
<th>Test number</th>
<th>State condition</th>
<th>Dilation angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Loose</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>Loose</td>
<td>3.5</td>
</tr>
<tr>
<td>3</td>
<td>Loose</td>
<td>4.3</td>
</tr>
<tr>
<td>5</td>
<td>Dense</td>
<td>9.8</td>
</tr>
<tr>
<td>6</td>
<td>Compacted</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>moisture content=2%</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Compacted</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>moisture content=4%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Compacted</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>moisture content=6%</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

Published papers

