Method to measure the Stokes parameters of GPS signals

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[1] As Global Positioning System (GPS) signals travel to the ground, they are affected by the medium through which they propagate. Consequently, measurements of signal amplitude and phase are used in a variety of remote sensing applications. A limitation of current receiver systems is that they do not measure polarization. While GPS signals are transmitted with right-hand circular polarization, the same cannot necessarily be said for the signal reaching the antenna, and this difference will contain further information about the propagation path. This paper describes a method for combining two GPS receivers into a device that is capable of measuring the Stokes parameters of the signal. A mathematical procedure for converting the receiver data into the Stokes parameters is derived, and the results of proof-of-concept experiments are presented.


1. Introduction

[2] As Global Positioning System (GPS) signals travel to the ground, they are affected by the medium through which they propagate. This has led to the use of GPS signals in applications beyond positioning. For example, by observing the frequency dependence of the phase shift caused by passage through the ionosphere, it is possible to construct maps of large-scale electron density [Mitchell and Spencer, 2003]. By observing rapid fluctuations in phase and amplitude (known as scintillation), it is possible to obtain information about ionospheric structure at sub-kilometer scales [Kintner et al., 2007]. GPS data have been used to measure the density of volcanic ash clouds [Larson, 2013], and atmospheric properties, such as water vapor density [e.g., Duan et al., 1996]. Reflected GPS signals can be used to infer information about the reflection geometry and the material properties of the reflecting object. This includes measurement of sea height [e.g., Martin-Neira et al., 2001], snow depth [e.g., Larson et al., 2009], the position of unknown reflecting objects [e.g., Benton and Mitchell, 2011], and soil moisture content [e.g., Larson et al., 2008].

[3] All current systems measure the incoming signal as if it was a scalar quantity and thus ignore polarization. While GPS signals are transmitted with right-hand circular (RHC) polarization [Kaplan and Hegarty, 2006], the same cannot necessarily be said for the signal reaching the antenna, and this difference will contain information about the propagation path. Furthermore, if the GPS antenna is itself circularly polarized to match the signal, any distortion in polarization will cause a false impression of amplitude fading due to polarization mismatch, and so the ability to detect polarization would provide more reliable amplitude measurements.

[4] A potential source of polarization distortion is the ionosphere. If the irregularities underlying scintillation are strongly anisotropic, they may alter the polarization state. Although theory suggests that this effect will be very weak at GPS wavelengths [Lee et al., 1982; Wheelon, 2003], experiments can be justified on the grounds on scientific rigor. Homogeneous ionospheric plasma can also in principle cause changes to polarization, as electromagnetic waves in a magnetized plasma have two characteristic polarization states with different refractive indices. Any other polarization will be a superposition of these states and will change as these component states propagate at different speeds. GPS signals, however, are usually very close to one of the characteristic states and so are unlikely to be affected in this manner. [Perrie et al., 2011].

[5] Reflection also causes changes to polarization. A right-hand circularly polarized electromagnetic wave, when reflected, will usually form a left-hand circularly polarized wave. If, however, the reflector is anisotropic, another polarization state may be created, thus imparting information the reflector’s physical properties. Similarly, an anisotropy in the medium through which a GPS signal is propagating may change the polarization. In the experiments described below, wire grids are used to create strongly anisotropic media.

[6] GPS signals are useful for remote sensing, as a relatively cheap receiver can operate continuously for years at a time, and can observe signals in 10 or more directions simultaneously. Their analysis also presents several challenges, as the signals are far weaker than ambient noise, are broadcast by multiple satellites onto the same frequency, and have data modulated onto them by phase shifting.

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The GPS system copes with noise by using a signal with massive redundancy. The L1 coarse acquisition code has a chip rate (i.e., phase shift rate) of 1.023 MHz, but each data bit is represented with 20 repetitions of a 1023 chip Gold code, giving a data rate of only 50 bits/s. A Gold code is a quasi-random binary sequence and is unique to each satellite, thus allowing a receiver to tell them apart [Kaplan and Hegarty, 2006]. Due to this signal structure, the obvious approach of directly combining antenna signals into a reconstruction of the changing electric field, and hence the polarization state, will not work. The field will be lost in noise, will be a superposition of multiple satellite signals, and will be interrupted by rapid phase shifts. Instead, the reconstruction must rely upon GPS receivers and make use of the data provided.

This paper demonstrates how a pair of GPS receivers can be combined into a device capable of resolving the Stokes parameters of the incoming L1 signal. These are a set of four numbers which completely describe the polarization state, even if the signal is only partially polarized. This paper is not intended as a practical guide to the construction of such a device but as a description of the underlying theory and as a demonstration of the method’s feasibility. In section 2 the layout of the device and its principles of operation are described. In section 3, the mathematical procedure for converting the receiver data into the Stokes parameters is derived. In section 4, the results of proof-of-concept experiments are presented.

### 2. Receiver Design

The apparatus uses two dipole antennas. These consist of two colinear lengths of wire, connected to a coaxial feed line at the point where they meet. The antennas are oriented horizontally but are perpendicular to one another, as is shown in Figure 1. Only the component of the wave’s electric field in the direction of the wire is capable of driving a current, and so the strength of the received signal is highly dependent on the wave’s linear polarization. It will be shown that by appropriately combining amplitude and phase measurements from the two antennas, the polarization state can be fully determined. The technique of inferring the Stokes parameters by combining signals from orthogonal dipole antennas is well established [e.g., Hatanaka et al., 1955]. However, this is the first time that anything like it has been attempted with GPS. We also believe that the calibration procedure described below is novel.

In theory, the gain of a dipole antenna is maximized when its total length is half the wavelength, due to the current forming a standing wave, with an antinode at the coaxial feed point [Chen, 2005]. In practice, the optimal length is slightly less than this, due to factors such as the metal having finite conductivity. The antennas were tuned to give optimal gain at the GPS L1 wavelength of $\lambda = 19.029$ cm. This was done by starting with a half-wave antenna and using a network analyzer to determine the frequency at which the highest proportion of a driving signal was radiated. The ends of the antenna were steadily trimmed away, until this frequency coincided with the L1 frequency. This occurred at a total length of $0.473\lambda = 90$ mm.

A conductive ground plane, 300 mm square, was placed beneath the dipoles, at a distance of $\lambda/4$. This both suppresses unwanted reflections from below and increases the gain in the skyward direction. The distance was chosen so that the path difference between the direct and the reflected signals was half a wavelength, at least when the satellite is vertical. Due to the half-cycle phase shift that occurs when an electromagnetic wave is reflected, this ensures that the direct and reflected signal constructively interfere, thus maximizing gain.

Two GPS receivers were used, allowing the signals from the two antennas to be measured independently. These were scintillation receivers, a form of modified GPS receiver, which take phase and amplitude measurements at rapid intervals, allowing variations to be detected and quantified [Van-Dierendonck et al., 1993]. Novatel GPS Station-2 receivers, modified into GSV4004 scintillation receivers, were used. These are capable of taking measurements at a rate of 50 Hz.

To accurately measure phase, it is necessary to have a highly stable frequency reference. GPS scintillation receivers contain an oven-controlled crystal oscillator (OCXO) for this purpose. When determining polarization, the relative phase difference is of particular importance. To increase the accuracy of such measurements, the receivers were modified to use the same oscillator, so that errors would affect both phase measurements equally, and so cancel out when the difference was taken.

The receivers possess external ports which are directly connected to the internal OCXO module. This allows one of the receivers to act as a frequency source for the other. To modify the other receiver to accept this signal, it was opened up and the OCXO circuit board removed. The internal cable from the frequency output port was then attached directly to the main GPS circuit board, thus creating a frequency input port. We have confirmed with the design engineers that this is a viable way of modifying the device [Van Dierendonck and Bobyn, 2012].

### 3. Procedure for Deriving the Stokes Parameters From the Receiver Data

The signal from the satellite will be effectively a plane wave when it reaches the receiver. Therefore, the
between vertical and horizontal polarization, 
vertical basis vectors for the electric field are given by 
east, and down, respectively. The horizontal and quasi-
polarization, and 
represented by the real part of 

The amplitudes 
are defined with respect to a reference field strength 
the complex arguments. The amplitudes are dimensionless,
and position 

The direction of the incoming signal is given by the wave-
direction of 
being defined with respect to a reference field strength 

where 
represents total intensity, 
represents the difference between right-handed and left-handed circular polarization. Despite the use of complex amplitudes, it follows from these definitions that the parameters are always real numbers. The parameters are often supplied as mean values, in which case they are capable of describing partially polarized waves. In the case of completely unpolarized light, the means of the \( Q \), \( U \), and \( V \) parameters will all be zero.

There is an important limitation, in that if \( E_v \) and \( E_h \) vary within a length of time shorter than the sampling interval, then averaged values rather than true values will be recorded. This averaging happens at the wrong stage of the calculation, and so the Stokes parameters will be distorted. The implications of this limitation are discussed in the conclusions.

There is no universal convention as to what the terms left handed and right handed mean in the context of circular polarization. This paper uses the convention of the International Astronomical Union and the Institute of Electrical and Electronics Engineers: A right-hand circularly polarized wave is defined so that if one looks away from the source, in the direction of propagation, the electric field vector will appear to rotate clockwise. If the \( \vec{E} \) field definition in equation (1) is adhered to, then the \( V \) parameter as defined by equation (5) will be positive for right-handed polarization [Hamaker and Bregman, 1996].

### 3.1. Accounting for the Antenna Gain Pattern

A dipole antenna is only sensitive to the component of the electric field along its axis. For simplicity, the axes of antennas labeled \( x \) and \( y \) were aligned to point northward and eastward, respectively. The amplitude and phases of the received signals are given by the complex amplitudes \( A_v \) and \( A_h \). These will be linear superpositions of \( E_v \) and \( E_h \) for the incoming wave. In the general case, this can be written as

\[
\begin{bmatrix}
A_v \\
A_h 
\end{bmatrix} = \hat{G}(\alpha, \psi) \begin{bmatrix}
E_v \\
E_h 
\end{bmatrix}
\]

The gain pattern matrix \( \hat{G} \) is specific to the precise antenna geometry and is a function of the satellite azimuth \( \alpha \) and elevation \( \psi \). A diagram of these coordinates is given in Figure 2. A procedure for calculating \( \hat{G}(\alpha, \psi) \), for the receiver system used, is given in Appendix A. The matrix is normalized so that for a vertical raypath, where (for a suitable choice of azimuth) \( \vec{x} \) and \( \vec{y} \) correspond to \( \vec{v} \) and \( \vec{h} \), it becomes the identity matrix.

The resulting transformation can be thought of as defining a virtual antenna, positioned perfectly for each satellite. By applying the inverse of \( \hat{G} \) to the measured values of \( A_v \) and \( A_h \), the values that would have been measured by antennas orientated along the \( \vec{v} \) and \( \vec{h} \) directions can be calculated.

### 3.2. Derivation of Conversion Equations

The definitions of the Stokes parameters, as given by equations (2)–(5), must be reformulated to use the information available from a GPS scintillation receiver and to accommodate the antenna gain pattern calculations. The equations can be rewritten in a common matrix form as

\[
S = \begin{bmatrix}
E_v \\
E_h 
\end{bmatrix} \hat{M}_S \begin{bmatrix}
E_v \\
E_h 
\end{bmatrix}
\]
where the dagger symbol represents the Hermitian conjugate, \( S \) is an arbitrary element of the set \( S = \{ I, Q, U, V \} \), and \( M_S \) is the corresponding element of the set of matrices defined as

\[
\begin{align*}
\hat{M}_I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
\hat{M}_Q &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
\hat{M}_U &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
\hat{M}_V &= \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}
\end{align*}
\]

The Stokes parameters can be calculated in terms of the field components along the physical antennas, by substituting equation (6) into equation (7), giving

\[
S = \left[ A_x A_y \right] \setminus (G^{-1}) \setminus M_S \setminus G \setminus \left[ A_x A_y \right]
\]

From a GPS receiver, the complex amplitudes can be reconstructed as

\[
\begin{align*}
A_x &= g_x F_x e^{i(\theta_x - b_x)} \\
A_y &= g_y F_y e^{i(\theta_y - b_y)}
\end{align*}
\]

where \( F_x \) and \( F_y \) are the square roots of the recorded powers and \( \theta_x \) and \( \theta_y \) are recorded phases. This reconstruction depends on four unknown coefficients. The coefficients \( b_x \) and \( b_y \) are phase biases, which reflect the fact that the measurement of phase starts from whenever the receiver happens to gain lock. This time is effectively arbitrary, and so phase is being added to an arbitrary value.

Equations (13) and (14) can be reformulated as

\[
\begin{bmatrix} A_x \\ A_y \end{bmatrix} = e^{i(\theta_x - b_x)} \hat{F} \hat{C}
\]

where the matrix \( \hat{F} \) holds the measured parameters, and the column vector \( \hat{C} \) holds the (to be determined) calibration parameters. These are defined as

\[
\hat{F} = \begin{bmatrix} F_x & 0 \\ 0 & F_y e^{i(\theta_y - \theta_x)} \end{bmatrix}
\]

\[
\hat{C} = \begin{bmatrix} g_x \\ g_y e^{i\beta} \end{bmatrix}
\]

where \( \beta = b_y - b_x \). Substituting equation (15) into equation (12) gives

\[
S = \hat{C} \setminus \hat{F} \setminus (G^{-1}) \setminus M_S \setminus G \setminus \hat{F} \hat{C}
\]

which is the master equation for converting the receiver measurements into the Stokes parameters. This formulation reduces the number of unknown coefficients from the original four, to the three coefficients encoded in \( \hat{C} \). This can be done because the Stokes parameters are not dependent on absolute phase.

### 3.3. Determining the Calibration Vector

To find the calibration vector \( \hat{C} \), the fact that the signal is usually in a RHC, polarized state can be used. This has complex amplitudes given in terms of the \( \tilde{v} \) and \( \tilde{h} \) basis vectors by

\[
\hat{E}_R = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}
\]

A given set of measurements can then be related to an assumption of RHC polarization, by assuming the calibration vector to be

\[
\hat{C}_R = e^{i\zeta} \hat{F}^{-1} G \hat{E}_R
\]

where \( \zeta \) is a constant chosen so that the first element of \( \hat{C}_R \) is a positive real number, thus conforming to the definition given by equation (17). This sets the \( I, Q, U, \) and \( V \) Stokes parameters to

\[
\hat{S}_R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}
\]

which correspond to RHC polarization as required. The definition of \( \hat{C}_R \) has an implicit scaling factor of 1, which has the effect of scaling all of the reconstructed Stokes parameters, such that the value of \( I \) at the calibration point is 1. This relates to the fact that the electric field defined by equation (1) contains an unknown scaling coefficient \( E_{ref} \). By using this choice of scaling coefficient, the Stokes parameters are being measured in intrinsic units, where \( E_{ref} \) is by definition 1.

In practice, the calculated values of \( \hat{C}_R \) will differ from both the true value of \( \hat{C} \), and from each other. This is due to both noise in the receiver system, and the fact that the signal is unlikely to be perfectly RHC polarized at any instant. Therefore, many values of \( \hat{C}_R \) must be calculated using the general form of equation (20)

\[
\hat{C}_{R_n} = e^{i\zeta_n} \hat{F}_n^{-1} G \hat{E}_R
\]

and then be combined into an estimate of \( \hat{C} \). The distribution of the values of \( \hat{C}_R \) around the true value of \( \hat{C} \) can be analyzed by defining

\[
\hat{C}_{R_n} = \hat{C} + \hat{e}_n
\]

where \( \hat{C}_{R_n} \) is the calculated value of \( \hat{C} \) for the \( n \)th calibration point. The corresponding error term is given by

\[
\hat{e}_n = \begin{bmatrix} \mu_n \\ v_n + \xi_n i \\ \xi_n \end{bmatrix}
\]

where the error terms \( \mu_n, v_n, \) and \( \xi_n \) are real numbers. The upper element of \( \hat{e}_n \) is a purely real number, whereas the lower element is a complex number, thus matching the structure of \( \hat{C} \) and \( \hat{C}_{R_n} \).

It is shown in Appendix B that if the true Stokes parameters for the \( n \)th calibration point differ only slightly from RHC polarization, then for short time intervals, the contributions to the error terms are given by the linear transformation

\[
\begin{bmatrix} \mu_n \\ v_n \\ \xi_n \end{bmatrix}_\text{Stokes} = \hat{A}^{-1} \begin{bmatrix} Q_n \\ U_n \\ V_n - i \\ \end{bmatrix}
\]

where \( \hat{A} \) is a 3 \( \times \) 3 matrix, which can be considered constant over short time intervals. This means that if the Stokes
parameters are symmetrically distributed around those corresponding to RHC polarization, then the values of $\mu_n$, $\nu_n$, and $\xi_n$ will be symmetrically distributed about zero. Similarly, it is shown in Appendix C that if relative noise is small, then for short time intervals, the contributions to the error terms are given by the linear transformation

$$\begin{bmatrix} \mu_n \\ \nu_n \\ \xi_n \end{bmatrix} = \mathbf{F}_n \begin{bmatrix} \mathbf{P}_{n,1}^{1,1} \\ 3(\mathbf{P}_{n,2}^{2,2}) \end{bmatrix}$$

(26)

where $\mathbf{P}_{n,1}^{1,1}$ and $\mathbf{P}_{n,2}^{2,2}$ are the top-left and bottom-right elements of $\mathbf{P}_n$, and where $\mathbf{F}_n$ is a $3 \times 3$ matrix, which can be considered constant over short time intervals.

The first set of terms asserts the need for similar values of $\mathbf{C}_{\theta_0}$. A suitable set of calibration data can be found by starting with a time range in which the data seems free from scintillation and other changes. The time interval used for calibration can be found by systematically searching through this range and choosing a subinterval that maximizes some figure of merit. This was chosen to be

$$\Xi = -\left[ \sigma^2 (\mu) + \sigma^2 (\nu) + \sigma^2 (\xi) \right] - a_1 \left[ \gamma_1^2 (\mu) + \gamma_1^2 (\nu) + \gamma_1^2 (\xi) \right] - a_2 \left[ \gamma_2^2 (\mu) + \gamma_2^2 (\nu) + \gamma_2^2 (\xi) \right]$$

(27)

The first set of terms asserts the need for similar values of $\mathbf{C}_{\theta_0}$, by minimizing the standard deviation $\sigma$. This is important, as if the values of $\mathbf{C}_{\theta_0}$ differ substantially from one another, then the polarization may be changing, and so is less likely to be centered around RHC polarization.

The other terms assert the need for a normal distribution, by minimizing the magnitudes of the skewness $\gamma_1$ and the excess kurtosis $\gamma_2$. The relative importance of these statistical measures are specified by the constants $a_1$ and $a_2$. A normal distribution is desirable, as by the central limit theorem, such distributions arise from the aggregate of multiple effects, as is expected for background effects during the calibration period. In terms of ionospheric effects, phase scintillation is indeed observed to follow a normal distribution [Fremouw et al., 1980], and amplitude scintillation is observed to follow a Nakagami distribution [Fremouw et al., 1980], which approaches a normal distribution for small scintillations. Conversely, a strongly skewed or otherwise nonnormal distribution is likely to be due to a systematic bias, which cannot be removed by taking the mean.

The calibration vector should be calculated for each individual satellite pass. This is essential for the complex phases, as at the start of each pass, phase measurement will once again start from an arbitrary value. The magnitudes can in theory be retained, but problems may arise from the fact that power values are not true measurements but proxy measurements taken after multiple processing steps. Using this proxy is acceptable for a satellite pass lasting a few hours, but over longer timescales it could drift due to temperature changes, or over even longer time scales, hardware aging. The performance of the different physical channels within the receiver is also likely to differ, and so it would be necessary to log the channel that each satellite was being received on.

4. Experimental Results

An antenna was placed in a location with a clear sky view and data recorded for multiple satellites. After some time, a wire-grid polarizer was placed in close proximity to the device. This was constructed from a 60 cm square of polystyrene foam, with a total of 13 wires, spaced at $\lambda/4$ intervals, running from one end to the other. The wires interact with the component of the electric field along their length but have relatively little effect on the perpendicular component. The polarizer was oriented with its wires running vertically, therefore reflecting vertically polarized signals, and transmitting horizontally polarized signals. It was
positioned about 26 cm to the west of the antenna (relative to the coaxial feed points), such that the axis of the $y$ dipole intersected it perpendicularly, about 18 cm from the bottom, along its vertical centerline. This arrangement allowed it to intersect the raypaths from certain satellites but to leave the signals from other satellites alone. After 5 min, the polarizer was removed. The reconstructed Stokes parameters are shown in Figures 3–6. A chart of the positions of these satellites in the sky, in relation to the polarizer, is given in Figure 7.

[30] These figures also show the parameters normalized by dividing by $I$, thus removing variations in signal strength, and hence giving a clearer view of the polarization state. In all of these figures, the $V$ parameter tracks the $I$ parameters for the time when the polarizer was absent, thus indicating that the signal is remaining RHC polarized.

[31] In Figure 3, the satellite (PRN 17) is to the west, and so the polarizer is directly between it and the antenna. As expected, the $Q$ parameter gains a strongly negative value, indicating horizontal polarization, as does (to a lesser extent) the $U$ parameter. The $I$ parameter falls, due to much of the signal being reflected away, as does the $V$ parameter. In Figures 4 and 5, the satellites (PRN 20 and PRN 23) are directly overhead and to the south, respectively. The signals neither pass through the polarizer, nor see the reflected signal. As expected, the Stokes parameters are largely unaffected, although spikes are visible at the points where the polarizer was added and removed. This is probably due to a
different geometry being present during the moments when the polarizer was being lifted away, thus briefly creating a different polarization state.

[32] In Figure 6, the satellite (PRN 32) is to the east, and so the antenna has both a direct view of the satellite, and its reflection in the polarizer. The results are as expected for constructive interference between the two signals. The $I$ parameter is boosted, because the antenna is now seeing signals from two directions. The unnormalized $V$ parameter is largely unaffected, because a direct view to the satellite remains. The $Q$ parameter is boosted, due to a vertically polarized signal reaching the antenna from the reflector.

The normalized $V$ parameter does fall however, due to the combined signal no longer being RHC polarized.

[33] The results were calibrated using data from the initial part of the experiment, before the polarizers were introduced. The optimum minute-long (3000 data point) sets of data were determined using the metric defined by equation (27). The constants $a_1 = 1$ and $a_2 = 1$ were chosen, relative to power values that had been normalized to a mean value of 1. Statistics for the chosen calibration data sets are displayed in Figures 8–11. These include scatterplots of parameters against each other, to assess their range and codependence. They also include quantile-quantile (Q-Q) plots of the individual parameters, to assess how they are distributed. These plots the quantiles of the cumulative distribution function of the parameters against those of a standardized normal distribution, so that normally distributed values form straight lines [Wilk and Gnanadesikan, 1968]. In all cases, the statistics reveal the distributions to be compact, approximately normal, and roughly independent.

[34] The results demonstrate that the calibration method works. By asserting a small section of the data to be RHC polarized, the method correctly reports RHC polarization in all cases besides those when the signal is deliberately distorted. Therefore, the calibration persists, despite fluctuations in the signal strength, changes in the antenna gain due to satellite motion, and intervening periods of artificial distortion.

[35] A possible confounding factor in this experiment is that the supposedly unmodified signals were not RHC polarized in the first place. Ionospheric effects are unlikely, as in addition to the expectation that polarization distortions due to scintillation are improbable, the experiments were conducted in England, where scintillation is rare. In the week before the experiment, no solar flare above C class had been detected [Space Weather Prediction Center (SWPC), 2012]. Such flares are weak and can only produce slight ionospheric disturbance [Afraimovich et al., 2002]. Similarly,
Figure 8. Statistics for the calibration used to create Figure 3. (top) Scatter graphs of the calibration parameters against one another. (bottom) Q-Q plots for the calibration parameters.

Figure 9. Statistics for the calibration used to create Figure 4. (top) Scatter graphs of the calibration parameters against one another. (bottom) Q-Q plots for the calibration parameters.
the $K_p$ index of geomagnetic disturbance was at its lowest possible value of zero [SWPC, 2012], which renders scintillation highly improbable [Aquino et al., 2005]. Effects due to objects in close proximity to the antenna are also unlikely, as only high elevation satellites with clear sky views were chosen.

5. Conclusions

[36] It has been demonstrated that two GPS scintillation receivers can be combined into a device capable of measuring the Stokes parameters of the incoming signal. Preliminary experiments have shown that the device both reports unmodified polarization under normal conditions and
can detect deviations in polarization due to artificial obstructions. When the signals from different satellites have different polarization states, the device correctly distinguishes between them.

[37] For a practical device, a better antenna and better characterization of its gain pattern is required. For this initial demonstration, dipole antennas were chosen because they are easy to fabricate, and they have extremely simple gain patterns. However, in comparison to other GPS antennas, their performance was poor. They are also fragile and can easily be distorted by mechanical contact. A better solution may be a patch antenna, with two independent driving points, thus providing two independent polarization measurements. The Stokes parameters could be calculated using the above procedure, but the functions within the gain matrix \( \hat{G} \) would be much more complicated. Determining them

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![image] 

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[38] In section 3 an important limitation is mentioned, whereby if the signals change over time scales shorter than the sampling interval, then the implicit use of average values will distort the calculated Stokes parameters. Ionospheric scintillations are usually observed to occur below 10 Hz [Forte, 2012], and multipath effects almost always occur below 1 Hz [e.g., Benton and Mitchell, 2011]. These values are substantially less than the 25 Hz bandwidth provided by a scintillation receiver sampling at 50 Hz. If higher sampling rates are required, there are two ways to proceed. First, scintillation receivers with higher sampling frequencies are coming onto the market. Second, a software-defined radio system, configured as a GPS receiver, would allow higher sampling rates still.

[39] In the case of reflected signals, another problem arises, in that the direction of arrival becomes ambiguous, causing a similar ambiguity in the values of \( \hat{G} \). This probably rules out the measurement of Stokes parameters in cases where the reflection geometry is completely unknown, but the procedure remains possible in cases where the geometry is either known beforehand or sufficiently constrained for it to be determined using a method such as that described in [Forte and Mitchell, 2011]. Once the direction of the reflected signal is known, it can be disentangled from the direct signal by making use of the interference pattern. As the signals alternate between intervals of constructive and destructive interference, the measured complex amplitudes will correspondingly alternate between the sum and the difference of the direct and reflected components, allowing them to be separated.

### Appendix A: Calculating the Antenna Gain Matrix

[40] In section 3.1, a matrix \( \hat{G} \) describing the gain pattern of the antenna system is introduced but is not defined. The first stage in calculating it is to calculate the sensitivity for a dipole without a reflector. By the electromagnetic reciprocity theorem, this is an equivalent task to calculating its emission pattern when actively driven. For a half-wave dipole antenna, a driving current of the form \( I_0 e^{i \omega t} \) will drive an electric field of the form \( [\text{Chen}, 2005] \)

\[
\vec{E} = \frac{I_0 \eta_0 \cos \left( \frac{\pi}{2} \cos (\theta) \right)}{2\pi r} \phi^{\omega t-\psi r} \hat{\theta} \tag{A1}
\]

where \( r \) is the distance from the center of the dipole, \( \eta_0 \) is the impedance of free space, \( \theta \) is the angle between the direction of propagation and the axis of the dipole, and \( \hat{\theta} \) is a local unit vector in the direction of increasing \( \theta \). This formula is in fact the far-field formula, applying only at a distance of many wavelengths, where nonradiating near-field components have died away.

[41] The matrix \( \hat{G} \) is to be normalized, and so any factor common to all values of \( \theta \) can be divided out. This includes the time-dependent oscillating term, which can be removed because \( \hat{G} \) is defined in terms of amplitude coefficients, rather than instantaneous values of the electric field. In equation (A1), the angle \( \theta \) and direction \( \hat{\theta} \) are specific to the orientation of the dipole. For a pair of dipoles, separate variables are required. The normalized fields for the \( x \) and \( y \) dipoles are therefore given by

\[
\vec{E}_x' = \frac{\cos \left( \frac{\pi}{2} \cos (\theta_x) \right)}{\sin(\theta_x)} \hat{\theta}_x \tag{A2}
\]

\[
\vec{E}_y' = \frac{\cos \left( \frac{\pi}{2} \cos (\theta_y) \right)}{\sin(\theta_y)} \hat{\theta}_y \tag{A3}
\]

Resolving the directions \( \hat{\theta}_x \) and \( \hat{\theta}_y \) into Cartesian coordinates described by Figure 2 gives

\[
\vec{E}_x' = \frac{\cos \left( \frac{\pi}{2} \cos(\theta) \right)}{\sin(\theta)} \left[ -\sin(\theta) \hat{x} + \cos(\theta) \cos(\phi) \hat{y} + \cos(\theta) \sin(\phi) \hat{z} \right] \tag{A4}
\]

\[
\vec{E}_y' = \frac{\cos \left( \frac{\pi}{2} \cos(\theta) \right)}{\sin(\theta)} \left[ \cos(\theta) \sin(\phi) \hat{x} - \sin(\theta) \hat{y} + \cos(\theta) \cos(\phi) \hat{z} \right] \tag{A5}
\]

where \( \phi \) and \( \phi \) are the azimuthal angles about the dipoles. The dipole intrinsic coordinates can be converted into ray-path dependent coordinates by evaluating the trigonometric functions as

\[
\cos (\theta_x) = -\cos (\alpha) \cos (\psi) \tag{A6}
\]

\[
\sin (\theta_x) = \sqrt{1 - \cos^2 (\alpha) \cos^2 (\psi)} \tag{A7}
\]

\[
\cos (\phi_x) = \frac{-\sin (\alpha) \cos (\psi)}{\sqrt{\sin^2 (\alpha) \cos^2 (\psi) + \sin^2 (\psi)}} \tag{A8}
\]

\[
\sin (\phi_x) = \frac{\sin (\psi)}{\sqrt{\sin^2 (\alpha) \cos^2 (\psi) + \sin^2 (\psi)}} \tag{A9}
\]

\[
\cos (\theta_y) = -\sin (\alpha) \cos (\psi) \tag{A10}
\]

\[
\sin (\theta_y) = \sqrt{1 - \sin^2 (\alpha) \cos^2 (\psi)} \tag{A11}
\]

\[
\cos (\phi_y) = \frac{\sin (\psi)}{\sqrt{\cos^2 (\alpha) \cos^2 (\psi) + \sin^2 (\psi)}} \tag{A12}
\]

\[
\sin (\phi_y) = \frac{-\cos (\alpha) \cos (\psi)}{\sqrt{\cos^2 (\alpha) \cos^2 (\psi) + \sin^2 (\psi)}} \tag{A13}
\]

where the positive value of the square root should be taken. This formulation ensures the trigonometric functions will always have the correct sign. The sensitivity for the quasi-vertical and horizontal polarization states can be calculated
from the projection of this normalized electric field onto the \( \tilde{v} \) and \( \tilde{h} \) directions. These are given by

\[
\tilde{v} = -\cos(\alpha) \sin(\psi) \tilde{x} - \sin(\alpha) \sin(\psi) \tilde{y} - \cos(\psi) \tilde{z} \tag{A14}
\]
\[
\tilde{h} = \sin(\alpha) \tilde{x} - \cos(\alpha) \tilde{y} \tag{A15}
\]

Making these projections amount to taking the scalar product. Doing so for all the possible matrix elements gives a gain pattern matrix without the reflector of

\[
\hat{G}' = \begin{bmatrix} \tilde{v} \cdot \tilde{E}_r & \tilde{h} \cdot \tilde{E}_r & \tilde{d} \cdot \tilde{E}_r \\ \tilde{v} \cdot \tilde{E}_r & \tilde{h} \cdot \tilde{E}_r & \tilde{d} \cdot \tilde{E}_r \end{bmatrix} \tag{A16}
\]

The ground plane was accounted for with the method of images. Each dipole was assumed to be accompanied by another dipole beneath the plane, in a mirror-imaged position, but with the phase reversed. At the level of the ground plane, this sets the tangential part of the electric field to zero, as is expected for an electrical conductor. The sum of the fields from the real and virtual dipoles gives, at least above the level of the plane, the true electric field as influenced by the conductive ground plane. The phase difference between the direct and reflected signals is

\[
\Phi = \pi + \pi \sin(\phi) \tag{A17}
\]

where the first term is due to the phase difference between the real and virtual dipoles (or equivalently the phase reversal which happens when an electromagnetic wave is reflected by a conductor) and the second term is due to the path difference. Superposing the real and virtual gain matrices, with a phase shift \( \Phi \) gives the gain pattern matrix as

\[
\hat{G} = \frac{1}{2} \left( 1 - e^{i\sin(\Phi)} \right) \hat{G}' \tag{A18}
\]

where the factor of \( \frac{1}{2} \) is to maintain the normalization described in section 3.1. Substituting the above equations into equation (A18) is possible but would produce an impractically large expression. Therefore, the best method of evaluating it is to follow the piecemeal approach shown here.

**Appendix B: Linear Transform Formulation of Calibration Errors From Perturbations in Signal**

It is stated in section 3.3 that when a short section of calibration data contain small deviations from RHC polarization, the resulting contributions to the calibration error terms \( \mu_n, v_n \), and \( \xi_n \) are given by a linear transformation of the complex amplitudes of the noise. The proof starts by considering equation (22) in the special case of RHC polarization being perfectly measured. For an ideal measurement \( \hat{F}_{0n} \), the calculated calibration vector \( \hat{C}_{Rn} \) is the true calibration vector \( \hat{C} \), giving

\[
\hat{C} = e^{i\xi_n} \hat{F}_{0n}^{-1} \hat{G}_n \hat{E}_R \tag{C1}
\]

Substituting this and the unmodified equation (22) into equation (23) gives

\[
\hat{e}_n = e^{i\xi_n} (\hat{F}_{n} - \hat{F}_{0n}) (\hat{F}_{0n}^{-1}) \hat{G}_n \hat{E}_R \tag{C2}
\]

where the higher powers of the error term \( \hat{e}_n \) have been removed. By defining

\[
\hat{J}_{0n} = -e^{i\xi_n} \hat{F}_{0n}^3 (\hat{G}_n^{-1}) \hat{M}_S \hat{E}_R \tag{B2}
\]

the changes to the complex amplitudes are defined by the column vector \( \Delta \hat{F}_n \). This is related to the error \( \hat{e}_n \) by the
diagonal matrix $\hat{K}$. This is slowly varying, and so over short time periods can be treated as a constant.

[44] Splitting the bottom-right element of $\hat{F}_n$ into its real and imaginary part allows equation (C4) to be rewritten as

$$\begin{bmatrix} \mu_n \\ \nu_n \\ \xi_n \end{bmatrix} = \hat{\Gamma} \begin{bmatrix} F_n^{1,1} \\ \Re(F_n^{2,2}) \\ \Im(F_n^{2,2}) \end{bmatrix}$$

(C7)

where $\hat{\Gamma}$ is a constant matrix derived from $\hat{K}$. This is the linear transformation stated in section 3.3.

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References


